Computer Algebra Independent Integration Tests

Summer 2023 edition

1-Algebraic-functions/1.1-Binomial-products/1.1.2-Quadratic/24-1.1.2.8-P-x-c-x-^m-a+b-x^2-^p

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CHAPTER 1

INTRODUCTION

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [174]. This is test number [24].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (174)	0.00 (0)
Mathematica	100.00 (174)	0.00 (0)
Maple	97.70 (170)	2.30 (4)
Fricas	97.70 (170)	2.30 (4)
Maxima	97.70 (170)	2.30 (4)
Giac	95.40 (166)	4.60 (8)
Sympy	88.51 (154)	11.49 (20)
Mupad	74.14 (129)	25.86 (45)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

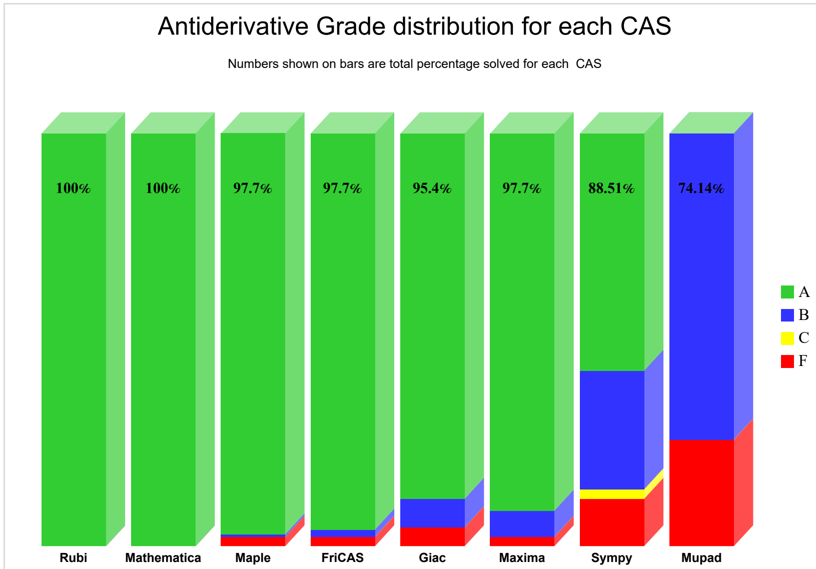
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

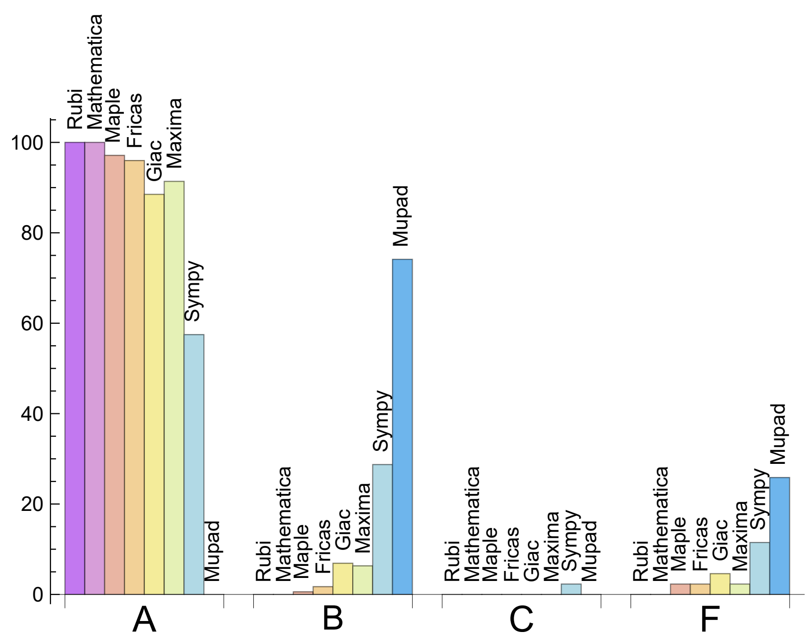
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	100.000	0.000	0.000	0.000
Maple	97.126	0.575	0.000	2.299
Fricas	95.977	1.724	0.000	2.299
Maxima	91.379	6.322	0.000	2.299
Giac	88.506	6.897	0.000	4.598
Sympy	57.471	28.736	2.299	11.494
Mupad	0.000	74.138	0.000	25.862

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	4	100.00	0.00	0.00
Maple	4	100.00	0.00	0.00
Maxima	4	100.00	0.00	0.00
Giac	8	50.00	0.00	50.00
Sympy	20	0.00	100.00	0.00
Mupad	45	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.11
Mathematica	0.22
Maxima	0.23
Giac	0.30
Fricas	0.30
Maple	3.32
Mupad	5.45
Sympy	11.87

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	113.44	0.91	103.00	0.91
Maple	115.09	0.90	102.00	0.90
Rubi	129.82	1.00	121.00	1.00
Mupad	129.83	1.02	108.00	0.98
Giac	151.84	1.09	125.00	0.98
Maxima	164.95	1.14	122.00	0.99
Fricas	279.44	2.09	223.50	2.10
Sympy	647.16	4.11	199.00	1.68

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

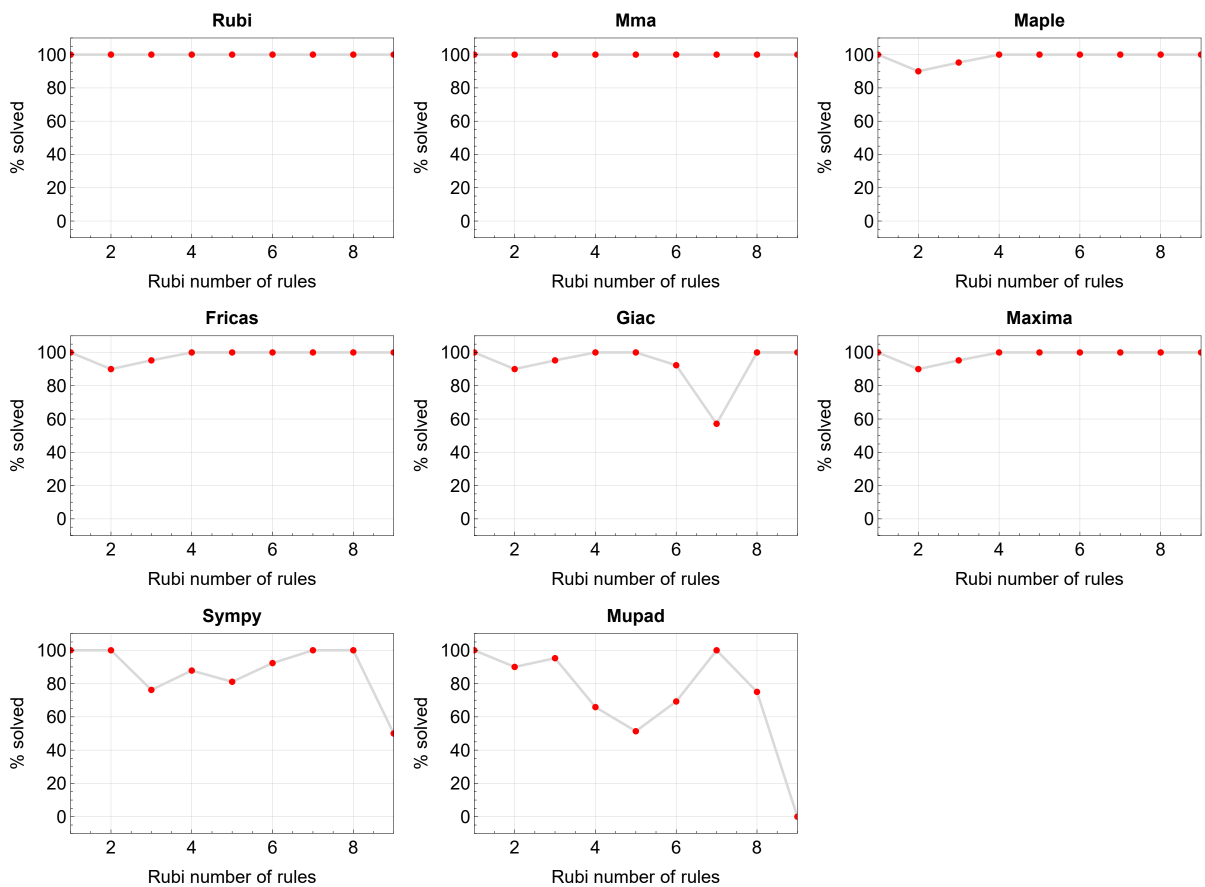


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

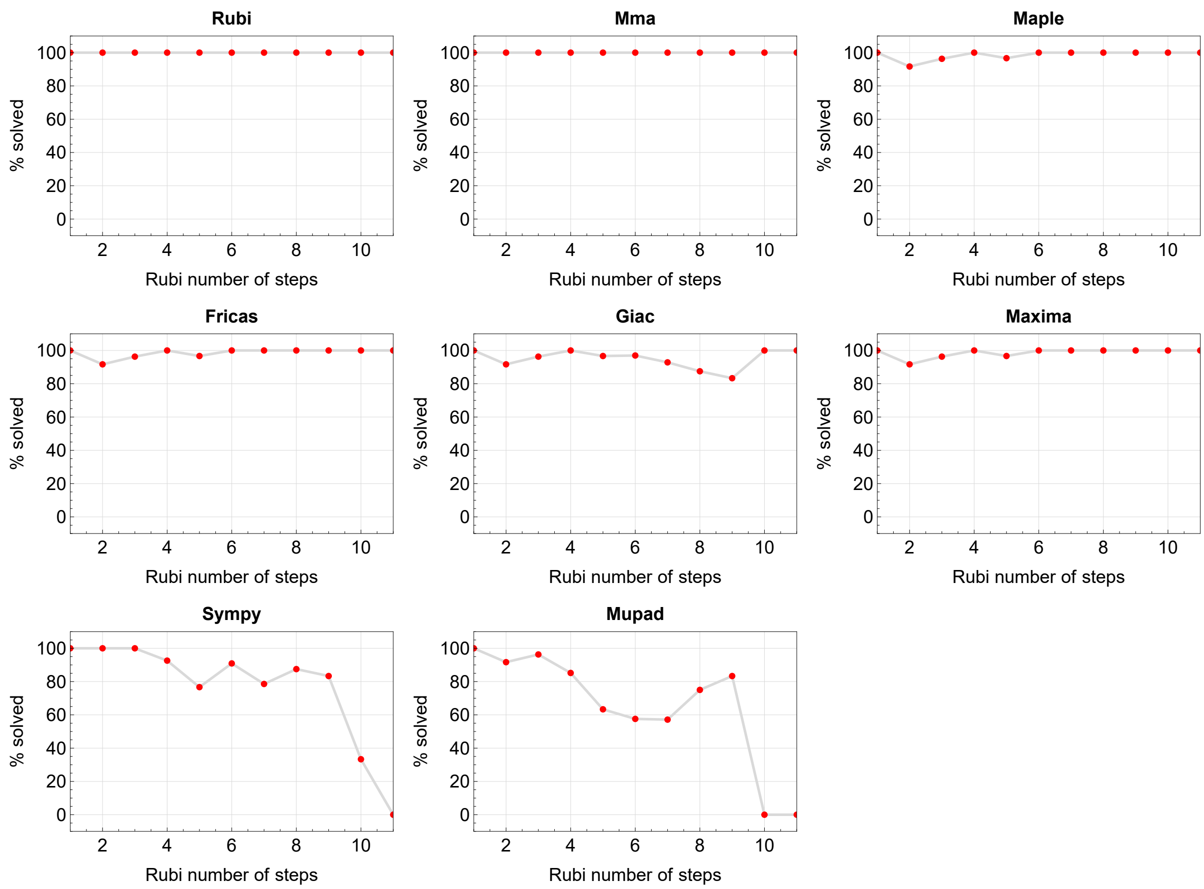


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

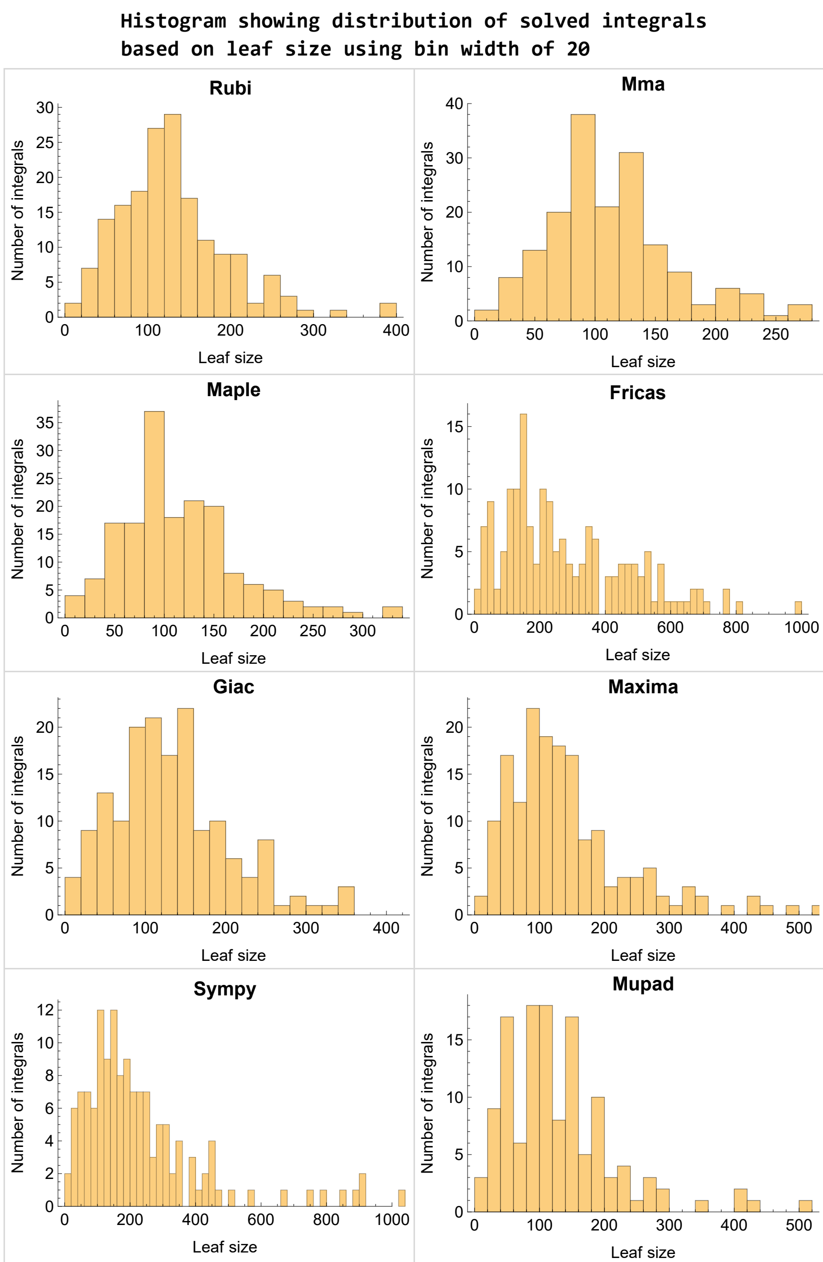


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

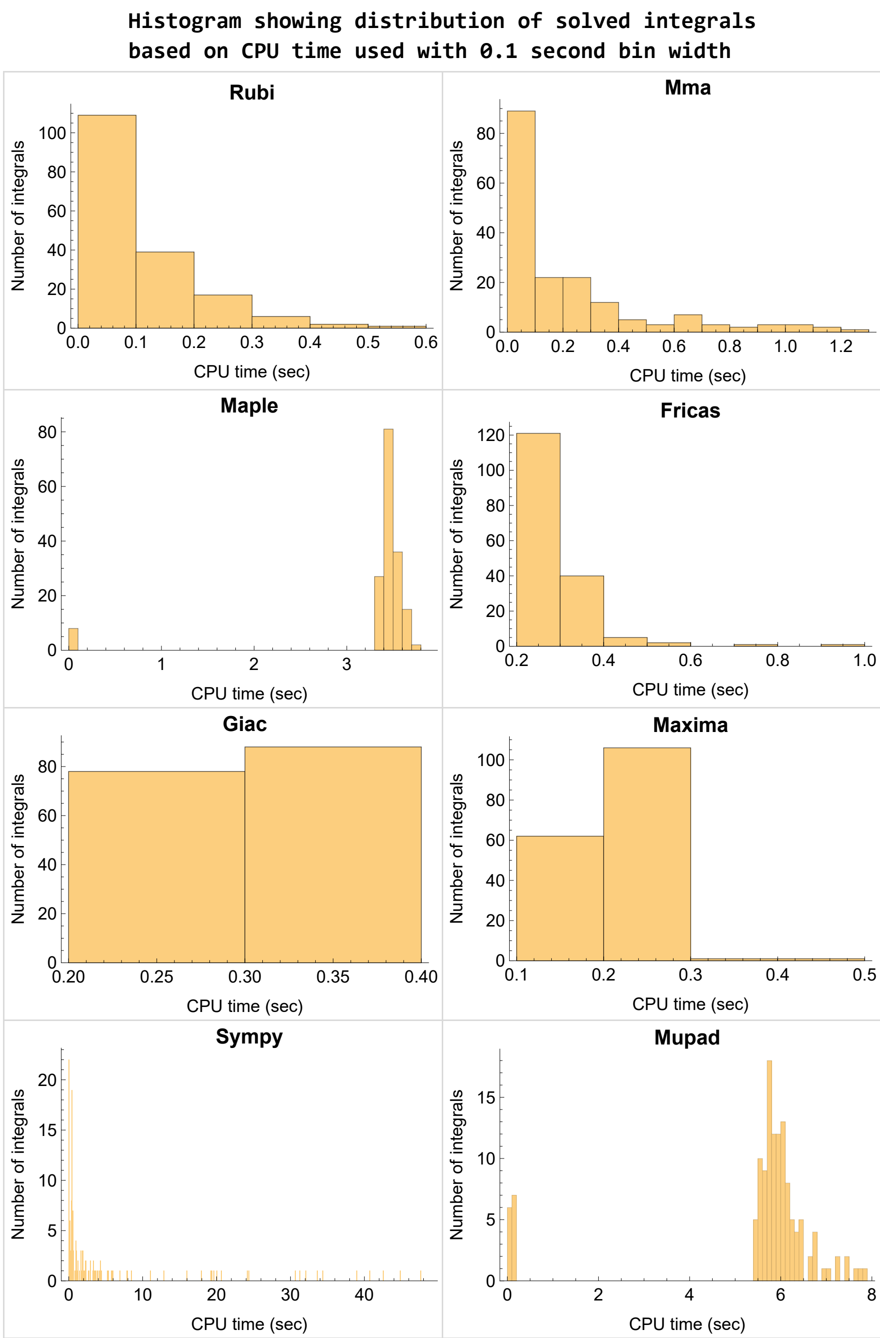


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

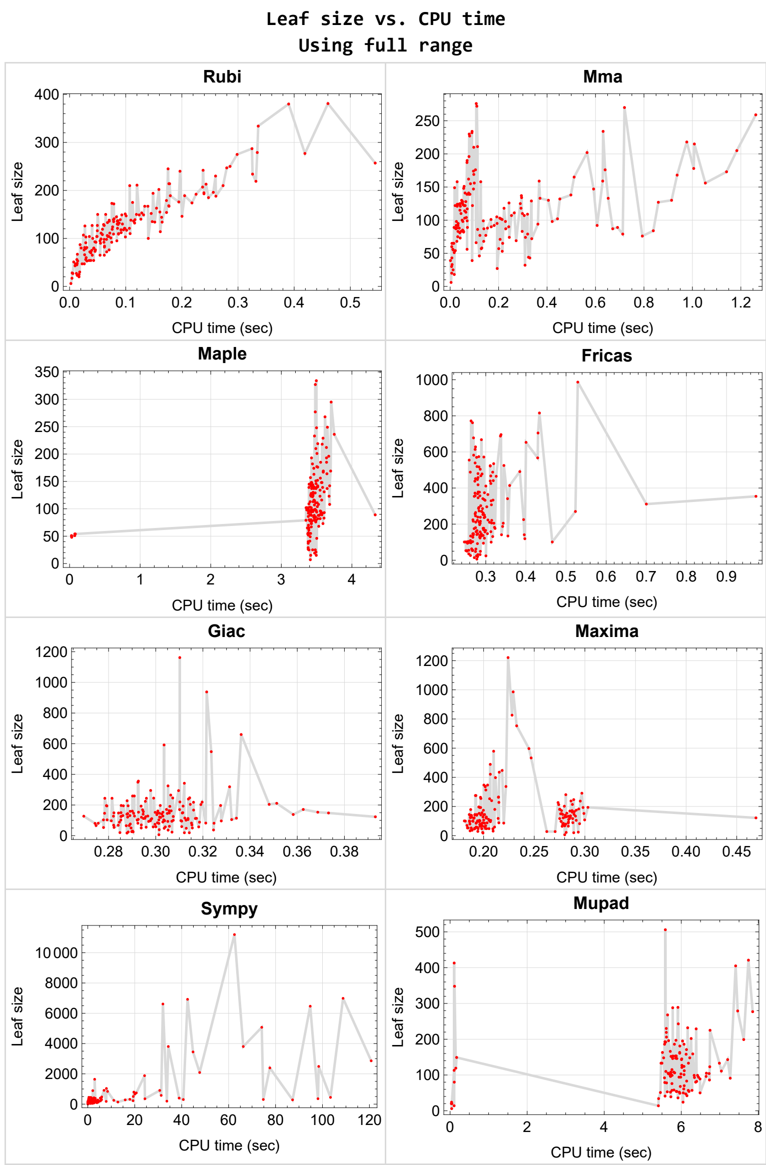


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

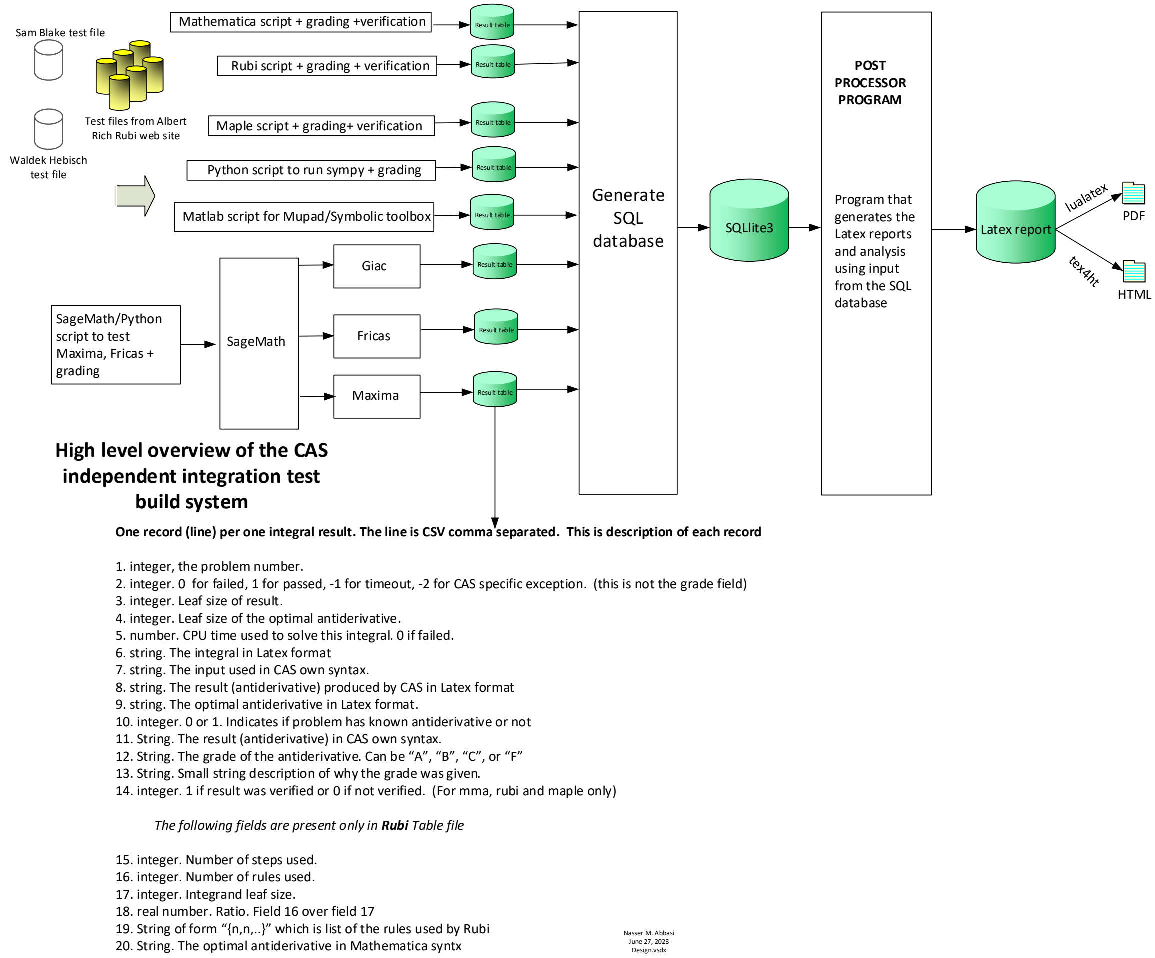
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS 22
2.2 Detailed conclusion table per each integral for all CAS systems 26
2.3 Detailed conclusion table specific for Rubi results 62

2.1 List of integrals sorted by grade for each CAS

Rubi 22
Mma 22
Maple 23
Fricas 23
Maxima 24
Giac 24
Mupad 24
Sympy 25

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade { 48 }

C grade { }

F normal fail { 58, 59, 60, 61 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade { 107, 108, 109 }

C grade { }

F normal fail { 58, 59, 60, 61 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 52, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 164, 165, 166, 167, 168, 169, 170, 171 }

B grade { 47, 48, 49, 159, 160, 161, 162, 163, 172, 173, 174 }

C grade { }

F normal fail { 58, 59, 60, 61 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 6, 8, 9, 10, 11, 13, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 27, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 169, 170, 171, 172, 173, 174 }

B grade { 7, 14, 28, 35, 45, 150, 156, 157, 158, 166, 167, 168 }

C grade { }

F normal fail { 58, 59, 60, 61 }

F(-1) timedout fail { }

F(-2) exception fail { 5, 12, 19, 26 }

Mupad

A grade { }

B grade { 4, 5, 6, 7, 11, 12, 13, 14, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 90, 92, 93, 96, 98, 100, 101, 102, 104, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 155, 156, 157, 158, 166, 167, 169, 170, 171 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 8, 9, 10, 15, 16, 17, 22, 29, 36, 47, 48, 58, 59, 60, 61, 86, 87, 88, 89, 91, 94, 95, 97, 99, 103, 105, 107, 151, 152, 153, 154, 159, 160, 161, 162, 163, 164, 165, 168, 172, 173, 174 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35, 36, 38, 43, 44, 45, 46, 47, 50, 51, 53, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 105, 106, 110, 111, 112, 113, 114, 116, 117, 120, 123, 124, 125, 126, 127, 128, 129, 130, 133, 134, 135, 136, 137, 138, 146, 147, 148, 151, 152, 153, 154, 155, 156 }

B grade { 33, 34, 37, 39, 40, 41, 42, 48, 49, 52, 54, 55, 56, 57, 86, 87, 88, 89, 90, 94, 95, 96, 97, 98, 102, 103, 104, 115, 118, 119, 121, 122, 143, 144, 145, 149, 150, 157, 158, 161, 162, 163, 164, 165, 169, 170, 171, 172, 173, 174 }

C grade { 58, 59, 60, 61 }

F normal fail { }

F(-1) timedout fail { 91, 92, 93, 99, 100, 101, 107, 108, 109, 131, 132, 139, 140, 141, 142, 159, 160, 166, 167, 168 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	101	89	107	206	138	93	0
N.S.	1	1.00	0.80	0.70	0.84	1.62	1.09	0.73	0.00
time (sec)	N/A	0.062	0.169	4.330	0.200	0.314	0.444	0.305	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	87	80	86	175	122	81	0
N.S.	1	1.00	0.84	0.77	0.83	1.68	1.17	0.78	0.00
time (sec)	N/A	0.035	0.139	3.394	0.201	0.271	0.421	0.292	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	77	65	67	157	107	68	0
N.S.	1	1.00	0.96	0.81	0.84	1.96	1.34	0.85	0.00
time (sec)	N/A	0.021	0.123	3.376	0.197	0.286	0.428	0.284	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	68	54	45	128	90	55	52
N.S.	1	1.00	1.01	0.81	0.67	1.91	1.34	0.82	0.78
time (sec)	N/A	0.014	0.139	3.375	0.188	0.285	0.346	0.296	5.720

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	90	79	59	341	129	0	68
N.S.	1	1.00	1.14	1.00	0.75	4.32	1.63	0.00	0.86
time (sec)	N/A	0.043	0.170	3.349	0.193	0.293	1.966	0.000	5.976

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	88	78	59	333	124	102	89
N.S.	1	1.00	1.17	1.04	0.79	4.44	1.65	1.36	1.19
time (sec)	N/A	0.039	0.154	3.405	0.211	0.318	1.947	0.285	6.487

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	90	73	83	377	107	163	94
N.S.	1	1.00	1.12	0.91	1.04	4.71	1.34	2.04	1.18
time (sec)	N/A	0.043	0.242	3.427	0.198	0.284	1.992	0.306	6.447

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	118	113	126	254	168	115	0
N.S.	1	1.00	0.79	0.75	0.84	1.69	1.12	0.77	0.00
time (sec)	N/A	0.064	0.222	3.450	0.188	0.294	0.518	0.312	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	107	104	105	223	150	103	0
N.S.	1	1.00	0.84	0.82	0.83	1.76	1.18	0.81	0.00
time (sec)	N/A	0.040	0.253	3.510	0.207	0.305	0.473	0.303	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	101	89	86	205	134	89	0
N.S.	1	1.00	0.98	0.86	0.83	1.99	1.30	0.86	0.00
time (sec)	N/A	0.029	0.215	3.409	0.220	0.344	0.449	0.303	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	70	61	176	119	76	54
N.S.	1	1.00	1.00	0.80	0.70	2.02	1.37	0.87	0.62
time (sec)	N/A	0.019	0.229	3.384	0.201	0.323	0.400	0.289	6.202

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	110	109	88	439	274	0	83
N.S.	1	1.00	1.04	1.03	0.83	4.14	2.58	0.00	0.78
time (sec)	N/A	0.067	0.298	3.391	0.203	0.311	4.260	0.000	6.140

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	111	113	88	411	243	124	86
N.S.	1	1.00	1.03	1.05	0.81	3.81	2.25	1.15	0.80
time (sec)	N/A	0.059	0.265	3.408	0.199	0.300	2.276	0.296	6.732

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	109	102	112	425	224	191	91
N.S.	1	1.00	0.98	0.92	1.01	3.83	2.02	1.72	0.82
time (sec)	N/A	0.056	0.319	3.424	0.205	0.313	2.910	0.306	7.272

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	137	137	145	302	199	140	0
N.S.	1	1.00	0.79	0.79	0.84	1.75	1.15	0.81	0.00
time (sec)	N/A	0.075	0.294	3.430	0.186	0.287	0.626	0.295	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	125	128	124	271	184	128	0
N.S.	1	1.00	0.83	0.85	0.83	1.81	1.23	0.85	0.00
time (sec)	N/A	0.050	0.290	3.422	0.202	0.290	0.593	0.303	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	118	108	105	253	168	114	0
N.S.	1	1.00	0.94	0.86	0.83	2.01	1.33	0.90	0.00
time (sec)	N/A	0.027	0.292	3.527	0.196	0.293	0.552	0.334	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	107	86	77	224	150	101	54
N.S.	1	1.00	1.00	0.80	0.72	2.09	1.40	0.94	0.50
time (sec)	N/A	0.025	0.306	3.382	0.196	0.268	0.460	0.313	5.873

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	130	139	119	539	474	0	101
N.S.	1	1.00	0.98	1.05	0.90	4.08	3.59	0.00	0.77
time (sec)	N/A	0.088	0.406	3.391	0.190	0.273	5.944	0.000	6.073

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	133	157	120	519	420	150	104
N.S.	1	1.00	0.98	1.15	0.88	3.82	3.09	1.10	0.76
time (sec)	N/A	0.085	0.371	3.413	0.203	0.278	2.623	0.311	6.724

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	132	145	143	535	381	219	111
N.S.	1	1.00	0.94	1.03	1.01	3.79	2.70	1.55	0.79
time (sec)	N/A	0.079	0.453	3.394	0.200	0.320	3.210	0.320	7.041

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	77	65	88	158	121	74	0
N.S.	1	1.00	0.74	0.62	0.85	1.52	1.16	0.71	0.00
time (sec)	N/A	0.048	0.147	3.391	0.215	0.276	0.399	0.300	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	74	56	67	127	102	61	93
N.S.	1	1.00	0.91	0.69	0.83	1.57	1.26	0.75	1.15
time (sec)	N/A	0.027	0.311	3.384	0.199	0.268	0.393	0.306	6.343

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	58	46	47	109	87	50	82
N.S.	1	1.00	1.04	0.82	0.84	1.95	1.55	0.89	1.46
time (sec)	N/A	0.013	0.131	3.502	0.209	0.262	0.411	0.294	6.400

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	46	37	29	92	71	39	36
N.S.	1	1.00	1.07	0.86	0.67	2.14	1.65	0.91	0.84
time (sec)	N/A	0.010	0.121	3.381	0.194	0.279	0.317	0.301	5.915

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	66	52	33	273	70	0	42
N.S.	1	1.00	1.25	0.98	0.62	5.15	1.32	0.00	0.79
time (sec)	N/A	0.031	0.107	3.383	0.187	0.283	1.204	0.000	6.148

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	57	49	37	101	41	65	39
N.S.	1	1.00	1.21	1.04	0.79	2.15	0.87	1.38	0.83
time (sec)	N/A	0.022	0.128	3.409	0.189	0.466	1.079	0.287	5.730

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	65	55	56	123	66	146	58
N.S.	1	1.00	0.90	0.76	0.78	1.71	0.92	2.03	0.81
time (sec)	N/A	0.036	0.217	3.415	0.194	0.286	1.652	0.302	5.810

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	74	77	85	197	117	70	0
N.S.	1	1.00	0.91	0.95	1.05	2.43	1.44	0.86	0.00
time (sec)	N/A	0.031	0.244	3.428	0.193	0.285	3.633	0.294	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	69	68	64	164	83	58	61
N.S.	1	1.00	1.05	1.03	0.97	2.48	1.26	0.88	0.92
time (sec)	N/A	0.027	0.272	3.530	0.196	0.278	2.714	0.300	6.065

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	53	55	46	147	66	48	53
N.S.	1	1.00	1.10	1.15	0.96	3.06	1.38	1.00	1.10
time (sec)	N/A	0.014	0.215	3.378	0.191	0.304	2.345	0.313	5.638

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	26	31	35	46	23	24
N.S.	1	1.00	0.96	0.93	1.11	1.25	1.64	0.82	0.86
time (sec)	N/A	0.005	0.194	3.381	0.209	0.258	1.874	0.307	6.046

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	57	61	48	146	206	59	50
N.S.	1	1.00	1.21	1.30	1.02	3.11	4.38	1.26	1.06
time (sec)	N/A	0.026	0.201	3.390	0.203	0.289	3.302	0.311	6.489

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	71	80	68	169	235	96	70
N.S.	1	1.00	1.01	1.14	0.97	2.41	3.36	1.37	1.00
time (sec)	N/A	0.038	0.206	3.421	0.202	0.294	4.145	0.328	6.077

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	83	88	89	211	124	171	94
N.S.	1	1.00	0.87	0.93	0.94	2.22	1.31	1.80	0.99
time (sec)	N/A	0.054	0.297	3.523	0.193	0.275	3.961	0.305	5.942

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	72	97	102	239	400	70	0
N.S.	1	1.00	0.91	1.23	1.29	3.03	5.06	0.89	0.00
time (sec)	N/A	0.029	0.337	3.412	0.194	0.280	5.333	0.315	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	44	41	70	63	141	36	51
N.S.	1	1.00	0.83	0.77	1.32	1.19	2.66	0.68	0.96
time (sec)	N/A	0.014	0.323	3.404	0.184	0.265	4.252	0.305	5.459

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	47	32	29	51	49	95	26	34
N.S.	1	0.94	0.64	0.58	1.02	0.98	1.90	0.52	0.68
time (sec)	N/A	0.010	0.309	3.380	0.192	0.287	3.876	0.290	5.412

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	43	40	48	62	146	37	41
N.S.	1	1.00	0.84	0.78	0.94	1.22	2.86	0.73	0.80
time (sec)	N/A	0.007	0.329	3.381	0.198	0.275	3.593	0.325	5.580

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	79	98	80	239	840	82	80
N.S.	1	1.00	1.04	1.29	1.05	3.14	11.05	1.08	1.05
time (sec)	N/A	0.046	0.320	3.460	0.186	0.298	8.428	0.315	6.374

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	94	122	100	264	910	119	96
N.S.	1	1.00	0.90	1.17	0.96	2.54	8.75	1.14	0.92
time (sec)	N/A	0.068	0.362	3.442	0.182	0.275	6.905	0.302	6.645

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	102	146	122	307	1034	197	123
N.S.	1	1.00	0.79	1.13	0.95	2.38	8.02	1.53	0.95
time (sec)	N/A	0.086	0.443	3.492	0.191	0.314	7.939	0.302	6.740

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	39	25	28	31	24	19	20
N.S.	1	1.00	1.44	0.93	1.04	1.15	0.89	0.70	0.74
time (sec)	N/A	0.006	0.091	3.481	0.271	0.286	0.083	0.314	0.043

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	39	25	28	31	24	19	20
N.S.	1	1.00	1.44	0.93	1.04	1.15	0.89	0.70	0.74
time (sec)	N/A	0.012	0.001	3.392	0.262	0.280	0.082	0.312	0.033

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	33	15	22	26	27	30	14
N.S.	1	1.00	1.94	0.88	1.29	1.53	1.59	1.76	0.82
time (sec)	N/A	0.004	0.008	3.416	0.282	0.262	0.036	0.290	5.407

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	5	6	6
N.S.	1	1.00	1.00	1.17	1.00	1.00	0.83	1.00	1.00
time (sec)	N/A	0.002	0.004	3.411	0.280	0.278	0.036	0.301	0.041

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	156	295	435	522	3806	204	0
N.S.	1	1.00	0.73	1.38	2.04	2.45	17.87	0.96	0.00
time (sec)	N/A	0.243	1.053	3.706	0.215	0.311	34.303	0.348	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	127	334	447	467	3448	138	0
N.S.	1	1.00	0.85	2.23	2.98	3.11	22.99	0.92	0.00
time (sec)	N/A	0.119	0.859	3.496	0.218	0.312	44.840	0.358	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	89	95	240	137	740	112	196
N.S.	1	1.00	0.67	0.72	1.82	1.04	5.61	0.85	1.48
time (sec)	N/A	0.116	0.691	3.557	0.208	0.293	20.663	0.314	5.672

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	79	76	253	122	575	81	186
N.S.	1	1.00	0.53	0.51	1.70	0.82	3.86	0.54	1.25
time (sec)	N/A	0.132	0.712	3.488	0.206	0.297	31.270	0.325	5.567

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	84	85	179	131	660	95	133
N.S.	1	1.00	0.60	0.61	1.29	0.94	4.75	0.68	0.96
time (sec)	N/A	0.117	0.839	3.485	0.187	0.317	20.049	0.311	5.499

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	87	88	197	134	904	94	133
N.S.	1	1.00	0.63	0.63	1.42	0.96	6.50	0.68	0.96
time (sec)	N/A	0.090	0.671	3.426	0.195	0.355	30.654	0.307	5.475

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	76	73	123	119	796	82	99
N.S.	1	1.00	0.64	0.61	1.03	1.00	6.69	0.69	0.83
time (sec)	N/A	0.059	0.793	3.590	0.187	0.397	19.626	0.321	5.763

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	92	96	153	137	1880	112	115
N.S.	1	1.00	0.72	0.76	1.20	1.08	14.80	0.88	0.91
time (sec)	N/A	0.049	0.607	3.494	0.196	0.285	24.192	0.317	5.759

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	130	193	157	465	6613	152	159
N.S.	1	1.00	0.94	1.40	1.14	3.37	47.92	1.10	1.15
time (sec)	N/A	0.109	0.914	3.428	0.198	0.326	32.006	0.300	6.299

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	168	277	228	525	6922	239	225
N.S.	1	1.00	0.89	1.47	1.21	2.79	36.82	1.27	1.20
time (sec)	N/A	0.261	0.937	3.481	0.215	0.345	42.510	0.314	6.747

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	173	327	265	688	11198	325	279
N.S.	1	1.00	0.79	1.49	1.21	3.14	51.13	1.48	1.27
time (sec)	N/A	0.332	1.141	3.482	0.215	0.337	62.494	0.305	7.465

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	45	45	43	0	0	0	97	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	2.16	0.00	0.00
time (sec)	N/A	0.013	0.007	0.000	0.000	0.000	0.614	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	82	0	0	0	189	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	2.08	0.00	0.00
time (sec)	N/A	0.029	0.055	0.000	0.000	0.000	1.674	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	56	0	0	0	201	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	2.64	0.00	0.00
time (sec)	N/A	0.026	0.070	0.000	0.000	0.000	1.817	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	99	0	0	0	291	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	2.40	0.00	0.00
time (sec)	N/A	0.085	0.137	0.000	0.000	0.000	2.309	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	53	53	60	57	57
N.S.	1	1.00	1.00	0.83	0.82	0.82	0.92	0.88	0.88
time (sec)	N/A	0.054	0.015	0.081	0.188	0.250	0.017	0.304	5.760

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	53	53	60	57	57
N.S.	1	1.00	1.00	0.83	0.82	0.82	0.92	0.88	0.88
time (sec)	N/A	0.062	0.012	0.078	0.185	0.253	0.017	0.318	6.280

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	53	53	60	57	57
N.S.	1	1.00	1.00	0.83	0.82	0.82	0.92	0.88	0.88
time (sec)	N/A	0.040	0.007	0.074	0.184	0.267	0.019	0.299	6.182

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	50	50	56	54	54
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.93	0.90	0.90
time (sec)	N/A	0.030	0.006	0.073	0.191	0.253	0.017	0.282	5.885

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	51	48	48	54	53	52
N.S.	1	1.00	1.00	0.91	0.86	0.86	0.96	0.95	0.93
time (sec)	N/A	0.029	0.011	0.027	0.198	0.274	0.061	0.278	5.950

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	50	48	55	49	50	49
N.S.	1	1.00	1.00	0.93	0.89	1.02	0.91	0.93	0.91
time (sec)	N/A	0.038	0.021	0.031	0.195	0.258	0.069	0.290	5.964

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	51	48	48	55	51	48	47
N.S.	1	1.00	0.94	0.89	0.89	1.02	0.94	0.89	0.87
time (sec)	N/A	0.043	0.017	0.030	0.205	0.267	0.146	0.304	6.118

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	55	49	49	55	54	50	50
N.S.	1	1.00	1.02	0.91	0.91	1.02	1.00	0.93	0.93
time (sec)	N/A	0.035	0.013	0.030	0.202	0.268	0.370	0.316	6.052

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	98	102	101	101	110	105	108
N.S.	1	1.00	0.90	0.94	0.93	0.93	1.01	0.96	0.99
time (sec)	N/A	0.087	0.026	3.360	0.189	0.253	0.023	0.332	5.804

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	92	102	101	101	110	105	108
N.S.	1	1.00	0.84	0.94	0.93	0.93	1.01	0.96	0.99
time (sec)	N/A	0.078	0.032	3.382	0.181	0.247	0.023	0.314	5.729

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	92	102	101	101	110	105	107
N.S.	1	1.00	0.88	0.98	0.97	0.97	1.06	1.01	1.03
time (sec)	N/A	0.049	0.025	3.351	0.186	0.250	0.025	0.298	5.729

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	88	99	98	98	107	102	105
N.S.	1	1.00	0.89	1.00	0.99	0.99	1.08	1.03	1.06
time (sec)	N/A	0.056	0.021	3.381	0.190	0.258	0.026	0.292	5.658

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	88	96	96	96	104	100	103
N.S.	1	1.00	0.96	1.04	1.04	1.04	1.13	1.09	1.12
time (sec)	N/A	0.047	0.028	3.383	0.186	0.263	0.089	0.307	5.728

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	88	98	96	103	99	98	92
N.S.	1	1.00	0.98	1.09	1.07	1.14	1.10	1.09	1.02
time (sec)	N/A	0.055	0.038	3.360	0.196	0.267	0.101	0.311	5.565

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	87	95	96	103	100	97	103
N.S.	1	1.00	0.89	0.97	0.98	1.05	1.02	0.99	1.05
time (sec)	N/A	0.062	0.024	3.448	0.207	0.256	0.182	0.304	5.671

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	83	92	97	103	100	97	106
N.S.	1	1.00	0.85	0.94	0.99	1.05	1.02	0.99	1.08
time (sec)	N/A	0.066	0.032	3.384	0.186	0.256	0.447	0.298	5.830

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	149	148	145	145	163	153	153
N.S.	1	1.00	1.00	0.99	0.97	0.97	1.09	1.03	1.03
time (sec)	N/A	0.124	0.018	3.576	0.191	0.269	0.027	0.293	5.871

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	125	148	145	145	165	153	153
N.S.	1	1.00	0.84	0.99	0.97	0.97	1.11	1.03	1.03
time (sec)	N/A	0.093	0.041	3.431	0.201	0.268	0.027	0.303	5.845

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	124	148	145	145	163	153	153
N.S.	1	1.00	0.90	1.07	1.05	1.05	1.18	1.11	1.11
time (sec)	N/A	0.071	0.036	3.424	0.193	0.268	0.027	0.369	5.785

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	121	144	142	142	158	149	149
N.S.	1	1.00	0.91	1.08	1.07	1.07	1.19	1.12	1.12
time (sec)	N/A	0.068	0.029	3.446	0.197	0.278	0.028	0.282	5.770

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	121	142	140	140	158	148	147
N.S.	1	1.00	0.94	1.10	1.09	1.09	1.22	1.15	1.14
time (sec)	N/A	0.061	0.042	3.421	0.193	0.269	0.122	0.301	5.853

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	123	145	139	147	150	145	121
N.S.	1	1.00	0.99	1.17	1.12	1.19	1.21	1.17	0.98
time (sec)	N/A	0.077	0.054	3.428	0.187	0.268	0.135	0.305	6.016

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	124	142	139	147	151	144	143
N.S.	1	1.00	0.92	1.05	1.03	1.09	1.12	1.07	1.06
time (sec)	N/A	0.079	0.040	3.430	0.188	0.280	0.221	0.288	6.006

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	124	141	142	147	155	146	148
N.S.	1	1.00	0.89	1.01	1.02	1.06	1.12	1.05	1.06
time (sec)	N/A	0.078	0.029	3.430	0.191	0.269	0.481	0.291	6.221

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	130	141	145	332	316	161	0
N.S.	1	1.00	0.86	0.93	0.96	2.20	2.09	1.07	0.00
time (sec)	N/A	0.099	0.047	3.441	0.278	0.298	0.545	0.301	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	114	128	127	270	274	137	0
N.S.	1	1.00	0.88	0.98	0.98	2.08	2.11	1.05	0.00
time (sec)	N/A	0.085	0.060	3.438	0.280	0.290	0.526	0.300	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	95	95	98	238	245	112	0
N.S.	1	1.00	0.86	0.86	0.88	2.14	2.21	1.01	0.00
time (sec)	N/A	0.072	0.033	3.412	0.276	0.287	0.484	0.295	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	81	85	82	180	211	88	0
N.S.	1	1.00	0.88	0.92	0.89	1.96	2.29	0.96	0.00
time (sec)	N/A	0.061	0.040	3.459	0.295	0.275	0.453	0.319	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	68	65	64	157	219	66	79
N.S.	1	1.00	0.93	0.89	0.88	2.15	3.00	0.90	1.08
time (sec)	N/A	0.045	0.028	3.369	0.278	0.298	0.420	0.289	5.572

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	73	73	65	158	0	66	0
N.S.	1	1.00	1.01	1.01	0.90	2.19	0.00	0.92	0.00
time (sec)	N/A	0.066	0.041	3.423	0.284	0.313	0.000	0.275	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	75	67	67	165	0	68	78
N.S.	1	1.00	0.99	0.88	0.88	2.17	0.00	0.89	1.03
time (sec)	N/A	0.070	0.031	3.425	0.280	0.300	0.000	0.282	5.618

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	84	89	76	205	0	80	97
N.S.	1	1.00	0.91	0.97	0.83	2.23	0.00	0.87	1.05
time (sec)	N/A	0.075	0.050	3.417	0.279	0.320	0.000	0.301	5.715

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	139	134	150	468	335	159	0
N.S.	1	1.00	0.79	0.76	0.85	2.66	1.90	0.90	0.00
time (sec)	N/A	0.194	0.082	3.549	0.276	0.288	1.866	0.289	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	128	124	127	372	289	131	0
N.S.	1	1.00	0.83	0.81	0.82	2.42	1.88	0.85	0.00
time (sec)	N/A	0.173	0.052	3.449	0.286	0.273	1.712	0.286	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	100	103	108	357	284	111	152
N.S.	1	1.00	0.75	0.77	0.81	2.66	2.12	0.83	1.13
time (sec)	N/A	0.152	0.050	3.406	0.300	0.274	1.602	0.298	5.735

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	92	78	84	287	212	81	0
N.S.	1	1.00	0.91	0.77	0.83	2.84	2.10	0.80	0.00
time (sec)	N/A	0.073	0.032	3.435	0.279	0.282	1.250	0.276	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	83	88	89	257	233	88	110
N.S.	1	1.00	0.89	0.95	0.96	2.76	2.51	0.95	1.18
time (sec)	N/A	0.045	0.053	3.466	0.288	0.273	0.988	0.285	5.600

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	85	99	87	296	0	93	0
N.S.	1	1.00	0.89	1.04	0.92	3.12	0.00	0.98	0.00
time (sec)	N/A	0.084	0.051	3.479	0.284	0.303	0.000	0.297	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	110	96	105	336	0	103	133
N.S.	1	1.00	1.00	0.87	0.95	3.05	0.00	0.94	1.21
time (sec)	N/A	0.108	0.045	3.552	0.291	0.296	0.000	0.279	6.186

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	112	127	117	441	0	126	158
N.S.	1	1.00	0.83	0.94	0.87	3.27	0.00	0.93	1.17
time (sec)	N/A	0.147	0.065	3.450	0.285	0.319	0.000	0.285	6.127

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	139	140	165	574	357	157	232
N.S.	1	1.00	0.75	0.76	0.89	3.10	1.93	0.85	1.25
time (sec)	N/A	0.247	0.070	3.463	0.290	0.283	98.075	0.295	6.166

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	126	115	136	480	282	122	0
N.S.	1	1.00	0.81	0.74	0.88	3.10	1.82	0.79	0.00
time (sec)	N/A	0.161	0.049	3.471	0.286	0.281	87.259	0.302	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	122	123	146	447	304	128	195
N.S.	1	1.00	0.90	0.90	1.07	3.29	2.24	0.94	1.43
time (sec)	N/A	0.114	0.062	3.421	0.289	0.279	74.789	0.291	6.034

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	99	97	111	357	178	97	0
N.S.	1	1.00	0.83	0.82	0.93	3.00	1.50	0.82	0.00
time (sec)	N/A	0.076	0.070	3.493	0.284	0.285	7.835	0.281	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	104	98	122	346	184	106	163
N.S.	1	1.00	0.90	0.84	1.05	2.98	1.59	0.91	1.41
time (sec)	N/A	0.049	0.067	3.433	0.279	0.281	3.412	0.293	5.960

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	117	130	133	488	0	128	0
N.S.	1	1.00	0.90	1.00	1.02	3.75	0.00	0.98	0.00
time (sec)	N/A	0.094	0.070	3.450	0.284	0.317	0.000	0.297	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	141	125	152	524	0	141	202
N.S.	1	1.00	0.98	0.87	1.06	3.64	0.00	0.98	1.40
time (sec)	N/A	0.164	0.062	3.416	0.299	0.311	0.000	0.303	6.257

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	147	169	172	696	0	162	229
N.S.	1	1.00	0.84	0.97	0.99	4.00	0.00	0.93	1.32
time (sec)	N/A	0.218	0.096	3.595	0.287	0.338	0.000	0.307	6.387

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	18	24	15	25	20
N.S.	1	1.00	1.00	0.95	0.90	1.20	0.75	1.25	1.00
time (sec)	N/A	0.017	0.006	3.472	0.199	0.300	0.035	0.290	0.038

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	15	22	14	22	17	14
N.S.	1	1.00	0.78	0.65	0.96	0.61	0.96	0.74	0.61
time (sec)	N/A	0.014	0.017	3.495	0.199	0.271	0.110	0.288	0.105

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	20	20	20	20	20
N.S.	1	1.00	1.00	0.84	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.017	0.012	3.491	0.289	0.280	0.037	0.285	0.041

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	22	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.73	0.80	0.80
time (sec)	N/A	0.018	0.006	3.444	0.293	0.277	0.041	0.288	0.038

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	210	233	213	452	384	244	289
N.S.	1	1.00	1.00	1.11	1.01	2.15	1.83	1.16	1.38
time (sec)	N/A	0.107	0.097	3.470	0.274	0.274	0.463	0.281	5.914

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	162	185	172	368	337	195	243
N.S.	1	1.00	0.94	1.08	1.00	2.14	1.96	1.13	1.41
time (sec)	N/A	0.079	0.076	3.557	0.277	0.280	0.446	0.288	5.924

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	128	137	133	286	185	148	193
N.S.	1	1.00	0.94	1.01	0.98	2.10	1.36	1.09	1.42
time (sec)	N/A	0.071	0.065	3.491	0.285	0.292	0.405	0.284	5.576

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	98	94	94	236	160	103	96
N.S.	1	1.00	0.98	0.94	0.94	2.36	1.60	1.03	0.96
time (sec)	N/A	0.041	0.052	3.501	0.284	0.296	0.365	0.301	5.724

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	83	79	80	211	150	84	76
N.S.	1	1.00	0.99	0.94	0.95	2.51	1.79	1.00	0.90
time (sec)	N/A	0.062	0.045	3.468	0.278	0.284	0.515	0.301	5.758

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	83	79	79	216	151	80	80
N.S.	1	1.00	1.01	0.96	0.96	2.63	1.84	0.98	0.98
time (sec)	N/A	0.062	0.055	3.493	0.281	0.287	0.989	0.274	0.109

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	103	94	97	246	167	103	94
N.S.	1	1.00	0.99	0.90	0.93	2.37	1.61	0.99	0.90
time (sec)	N/A	0.070	0.060	3.425	0.277	0.274	2.246	0.278	5.986

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	139	129	134	292	301	148	127
N.S.	1	1.00	1.01	0.94	0.98	2.13	2.20	1.08	0.93
time (sec)	N/A	0.086	0.078	3.480	0.291	0.283	5.860	0.311	6.017

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	174	163	175	374	354	197	161
N.S.	1	1.00	0.99	0.93	1.00	2.14	2.02	1.13	0.92
time (sec)	N/A	0.112	0.096	3.462	0.292	0.297	24.351	0.327	5.959

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	211	201	214	458	398	244	197
N.S.	1	1.00	1.00	0.95	1.01	2.17	1.89	1.16	0.93
time (sec)	N/A	0.120	0.113	3.505	0.288	0.281	38.915	0.296	5.863

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	227	229	227	572	444	246	413
N.S.	1	1.00	0.95	0.95	0.95	2.38	1.85	1.02	1.72
time (sec)	N/A	0.197	0.080	3.591	0.274	0.295	1.155	0.307	0.106

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	187	182	183	478	257	196	288
N.S.	1	1.00	0.93	0.90	0.91	2.37	1.27	0.97	1.43
time (sec)	N/A	0.159	0.069	3.441	0.294	0.291	1.078	0.279	5.778

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	148	139	140	418	221	148	153
N.S.	1	1.00	0.91	0.85	0.86	2.56	1.36	0.91	0.94
time (sec)	N/A	0.155	0.056	3.458	0.281	0.281	0.977	0.373	5.866

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	122	114	117	364	201	123	113
N.S.	1	1.00	1.03	0.97	0.99	3.08	1.70	1.04	0.96
time (sec)	N/A	0.092	0.058	3.465	0.275	0.305	0.804	0.393	0.104

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	115	107	117	354	197	120	112
N.S.	1	1.00	1.03	0.96	1.04	3.16	1.76	1.07	1.00
time (sec)	N/A	0.092	0.040	3.469	0.288	0.271	2.034	0.294	6.079

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	125	116	130	378	212	121	119
N.S.	1	1.00	1.03	0.96	1.07	3.12	1.75	1.00	0.98
time (sec)	N/A	0.107	0.051	3.462	0.282	0.281	5.298	0.288	0.149

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	151	137	151	438	226	148	145
N.S.	1	1.00	0.99	0.90	0.99	2.88	1.49	0.97	0.95
time (sec)	N/A	0.145	0.058	3.461	0.277	0.273	19.290	0.306	6.013

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	190	174	194	488	0	197	181
N.S.	1	1.00	1.01	0.92	1.03	2.58	0.00	1.04	0.96
time (sec)	N/A	0.205	0.070	3.460	0.303	0.261	0.000	0.285	5.964

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	230	210	238	582	0	247	219
N.S.	1	1.00	1.00	0.91	1.03	2.53	0.00	1.07	0.95
time (sec)	N/A	0.260	0.079	3.489	0.286	0.270	0.000	0.314	5.613

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	272	268	281	762	503	294	506
N.S.	1	1.00	0.95	0.93	0.98	2.66	1.75	1.02	1.76
time (sec)	N/A	0.325	0.110	3.619	0.281	0.267	19.388	0.310	5.587

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	232	219	237	668	316	244	348
N.S.	1	1.00	0.94	0.89	0.96	2.70	1.28	0.99	1.41
time (sec)	N/A	0.280	0.090	3.516	0.295	0.289	17.925	0.278	0.116

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	176	177	193	614	280	195	206
N.S.	1	1.00	0.85	0.86	0.93	2.97	1.35	0.94	1.00
time (sec)	N/A	0.237	0.105	3.503	0.298	0.273	15.945	0.286	5.616

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	156	151	169	555	260	169	163
N.S.	1	1.00	0.93	0.90	1.01	3.32	1.56	1.01	0.98
time (sec)	N/A	0.177	0.100	3.489	0.285	0.259	5.710	0.294	5.512

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	141	139	154	504	243	146	148
N.S.	1	1.00	0.96	0.95	1.05	3.43	1.65	0.99	1.01
time (sec)	N/A	0.101	0.076	3.499	0.282	0.286	3.400	0.278	5.543

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	155	140	161	517	250	150	149
N.S.	1	1.00	1.01	0.92	1.05	3.38	1.63	0.98	0.97
time (sec)	N/A	0.129	0.084	3.516	0.288	0.283	11.076	0.285	0.170

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	169	152	181	570	0	167	166
N.S.	1	1.00	1.01	0.90	1.08	3.39	0.00	0.99	0.99
time (sec)	N/A	0.177	0.098	3.513	0.275	0.277	0.000	0.316	5.715

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	196	176	202	628	0	194	192
N.S.	1	1.00	1.00	0.90	1.03	3.20	0.00	0.99	0.98
time (sec)	N/A	0.256	0.078	3.516	0.280	0.274	0.000	0.279	5.595

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	234	212	247	678	0	245	230
N.S.	1	1.00	1.00	0.91	1.06	2.90	0.00	1.05	0.98
time (sec)	N/A	0.326	0.090	3.639	0.289	0.270	0.000	0.299	5.612

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	276	248	291	772	0	295	268
N.S.	1	1.00	1.00	0.90	1.05	2.79	0.00	1.06	0.97
time (sec)	N/A	0.419	0.108	3.499	0.297	0.263	0.000	0.298	5.645

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	158	142	347	177	442	259	186
N.S.	1	1.00	0.74	0.66	1.62	0.83	2.07	1.21	0.87
time (sec)	N/A	0.177	0.126	3.515	0.204	0.295	0.461	0.290	5.823

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	122	111	263	134	340	193	146
N.S.	1	1.00	0.73	0.66	1.57	0.80	2.04	1.16	0.87
time (sec)	N/A	0.140	0.092	3.487	0.197	0.262	0.378	0.312	5.778

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	89	82	180	94	238	127	103
N.S.	1	1.00	0.74	0.68	1.49	0.78	1.97	1.05	0.85
time (sec)	N/A	0.098	0.071	3.470	0.275	0.254	0.294	0.269	5.721

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	86	82	122	205	134	125	99
N.S.	1	1.00	0.83	0.80	1.18	1.99	1.30	1.21	0.96
time (sec)	N/A	0.099	0.113	3.516	0.469	0.286	4.479	0.285	6.335

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	92	88	104	210	138	113	99
N.S.	1	1.00	0.92	0.88	1.04	2.10	1.38	1.13	0.99
time (sec)	N/A	0.140	0.182	3.537	0.206	0.309	12.825	0.295	6.423

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	102	95	128	221	194	140	133
N.S.	1	1.00	0.89	0.83	1.12	1.94	1.70	1.23	1.17
time (sec)	N/A	0.162	0.210	3.561	0.203	0.315	33.693	0.288	6.990

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	126	116	193	261	303	228	199
N.S.	1	1.00	0.86	0.79	1.32	1.79	2.08	1.56	1.36
time (sec)	N/A	0.200	0.243	3.572	0.196	0.294	40.705	0.288	7.622

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	159	148	275	341	444	356	277
N.S.	1	1.00	0.82	0.76	1.41	1.75	2.28	1.83	1.42
time (sec)	N/A	0.239	0.367	3.568	0.192	0.354	103.418	0.293	7.854

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	218	169	339	414	243	218	0
N.S.	1	1.00	0.89	0.69	1.38	1.69	0.99	0.89	0.00
time (sec)	N/A	0.175	0.977	3.699	0.201	0.360	0.613	0.316	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	159	137	255	329	199	170	0
N.S.	1	1.00	0.82	0.71	1.31	1.70	1.03	0.88	0.00
time (sec)	N/A	0.149	0.630	3.649	0.197	0.313	0.571	0.310	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	129	108	174	250	155	125	0
N.S.	1	1.00	0.89	0.74	1.20	1.72	1.07	0.86	0.00
time (sec)	N/A	0.084	0.335	3.685	0.200	0.297	0.407	0.307	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	105	93	118	216	246	113	0
N.S.	1	1.00	0.90	0.79	1.01	1.85	2.10	0.97	0.00
time (sec)	N/A	0.102	0.193	3.608	0.214	0.269	0.918	0.317	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	95	83	102	210	196	171	143
N.S.	1	1.00	0.86	0.75	0.93	1.91	1.78	1.55	1.30
time (sec)	N/A	0.087	0.194	3.605	0.196	0.284	1.088	0.363	7.205

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	98	90	128	221	427	319	105
N.S.	1	1.00	0.83	0.76	1.08	1.87	3.62	2.70	0.89
time (sec)	N/A	0.096	0.228	3.555	0.199	0.279	1.490	0.331	6.653

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	103	92	193	100	891	548	124
N.S.	1	1.00	0.74	0.66	1.38	0.71	6.36	3.91	0.89
time (sec)	N/A	0.127	0.205	3.583	0.197	0.313	2.123	0.324	6.072

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	134	123	275	141	1642	660	171
N.S.	1	1.00	0.71	0.65	1.46	0.75	8.69	3.49	0.90
time (sec)	N/A	0.179	0.295	3.677	0.192	0.396	2.922	0.336	6.136

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	259	236	1221	987	0	342	0
N.S.	1	1.00	0.68	0.62	3.20	2.59	0.00	0.90	0.00
time (sec)	N/A	0.460	1.262	3.750	0.224	0.529	0.000	0.312	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	215	196	986	816	0	265	0
N.S.	1	1.00	0.77	0.70	3.53	2.92	0.00	0.95	0.00
time (sec)	N/A	0.334	1.008	3.672	0.229	0.434	0.000	0.306	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	178	164	753	653	6467	203	0
N.S.	1	1.00	0.85	0.78	3.59	3.11	30.80	0.97	0.00
time (sec)	N/A	0.273	1.005	3.646	0.233	0.400	94.777	0.319	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	192	147	139	533	491	3803	160	0
N.S.	1	1.07	0.82	0.78	2.98	2.74	21.25	0.89	0.00
time (sec)	N/A	0.225	0.592	3.585	0.247	0.385	66.249	0.290	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	98	89	335	141	2088	131	0
N.S.	1	1.00	0.73	0.66	2.50	1.05	15.58	0.98	0.00
time (sec)	N/A	0.165	0.421	3.575	0.202	0.339	47.665	0.289	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	179	133	123	313	182	2392	211	0
N.S.	1	0.97	0.72	0.66	1.69	0.98	12.93	1.14	0.00
time (sec)	N/A	0.173	0.652	3.548	0.202	0.322	77.574	0.351	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	165	156	337	225	2861	349	0
N.S.	1	1.00	0.68	0.64	1.39	0.93	11.82	1.44	0.00
time (sec)	N/A	0.238	0.512	3.553	0.222	0.394	120.749	0.292	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	275	202	185	398	270	0	592	405
N.S.	1	0.98	0.72	0.66	1.42	0.96	0.00	2.11	1.44
time (sec)	N/A	0.298	0.565	3.641	0.212	0.523	0.000	0.304	7.412

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	234	218	489	311	0	938	421
N.S.	1	1.00	0.70	0.65	1.46	0.93	0.00	2.81	1.26
time (sec)	N/A	0.336	0.631	3.579	0.207	0.700	0.000	0.322	7.744

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	392	380	270	249	579	354	0	1162	0
N.S.	1	0.97	0.69	0.64	1.48	0.90	0.00	2.96	0.00
time (sec)	N/A	0.390	0.720	3.653	0.210	0.973	0.000	0.310	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	158	142	347	177	442	259	186
N.S.	1	1.00	0.74	0.66	1.62	0.83	2.07	1.21	0.87
time (sec)	N/A	0.178	0.029	3.685	0.200	0.293	0.458	0.291	6.072

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	122	111	263	134	340	193	146
N.S.	1	1.00	0.73	0.66	1.57	0.80	2.04	1.16	0.87
time (sec)	N/A	0.134	0.020	3.508	0.215	0.289	0.365	0.294	5.886

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	89	82	180	94	238	127	103
N.S.	1	1.00	0.74	0.68	1.49	0.78	1.97	1.05	0.85
time (sec)	N/A	0.098	0.016	3.550	0.211	0.294	0.291	0.282	5.992

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	257	205	193	826	705	6987	224	0
N.S.	1	0.98	0.79	0.74	3.16	2.70	26.77	0.86	0.00
time (sec)	N/A	0.544	1.183	3.640	0.228	0.430	108.770	0.296	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	250	176	167	597	567	5071	204	0
N.S.	1	1.17	0.82	0.78	2.79	2.65	23.70	0.95	0.00
time (sec)	N/A	0.286	0.640	3.594	0.245	0.429	74.123	0.315	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	138	131	421	187	2490	220	0
N.S.	1	1.00	0.72	0.68	2.18	0.97	12.90	1.14	0.00
time (sec)	N/A	0.239	0.498	3.607	0.207	0.342	98.387	0.310	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [13] had the largest ratio of [.4000000000000000022]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	1.00	20	0.250
2	A	5	5	1.00	20	0.250
3	A	4	4	1.00	18	0.222
4	A	4	4	1.00	17	0.235
5	A	7	7	1.00	20	0.350
6	A	7	7	1.00	20	0.350
7	A	7	7	1.00	20	0.350
8	A	7	5	1.00	20	0.250
9	A	6	5	1.00	20	0.250
10	A	5	4	1.00	18	0.222
11	A	5	4	1.00	17	0.235
12	A	8	7	1.00	20	0.350
13	A	8	8	1.00	20	0.400
14	A	8	7	1.00	20	0.350
15	A	8	5	1.00	20	0.250
16	A	7	5	1.00	20	0.250
17	A	6	4	1.00	18	0.222
18	A	6	4	1.00	17	0.235
19	A	9	7	1.00	20	0.350
20	A	9	8	1.00	20	0.400
21	A	9	8	1.00	20	0.400
22	A	5	4	1.00	20	0.200
23	A	4	4	1.00	20	0.200
24	A	3	3	1.00	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	3	3	1.00	17	0.176
26	A	6	6	1.00	20	0.300
27	A	4	4	1.00	20	0.200
28	A	5	5	1.00	20	0.250
29	A	4	4	1.00	20	0.200
30	A	4	4	1.00	20	0.200
31	A	3	3	1.00	18	0.167
32	A	1	1	1.00	17	0.059
33	A	5	5	1.00	20	0.250
34	A	5	5	1.00	20	0.250
35	A	6	6	1.00	20	0.300
36	A	4	4	1.00	20	0.200
37	A	2	2	1.00	20	0.100
38	A	2	2	0.94	18	0.111
39	A	2	2	1.00	17	0.118
40	A	6	5	1.00	20	0.250
41	A	6	5	1.00	20	0.250
42	A	7	6	1.00	20	0.300
43	A	2	2	1.00	18	0.111
44	A	3	3	1.00	19	0.158
45	A	2	2	1.00	13	0.154
46	A	2	2	1.00	13	0.154
47	A	7	5	1.00	25	0.200
48	A	6	5	1.00	25	0.200
49	A	5	4	1.00	25	0.160
50	A	4	4	1.00	25	0.160
51	A	4	4	1.00	25	0.160
52	A	4	4	1.00	25	0.160
53	A	4	4	1.00	23	0.174
54	A	5	4	1.00	22	0.182
55	A	8	6	1.00	25	0.240
56	A	8	5	1.00	25	0.200
57	A	9	6	1.00	25	0.240
58	A	2	2	1.00	16	0.125
59	A	3	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	2	2	1.00	22	0.091
61	A	5	3	1.00	25	0.120
62	A	2	1	1.00	26	0.038
63	A	2	1	1.00	26	0.038
64	A	2	1	1.00	24	0.042
65	A	2	1	1.00	23	0.043
66	A	2	1	1.00	26	0.038
67	A	2	1	1.00	26	0.038
68	A	2	1	1.00	26	0.038
69	A	2	1	1.00	26	0.038
70	A	2	1	1.00	28	0.036
71	A	2	1	1.00	28	0.036
72	A	3	2	1.00	26	0.077
73	A	3	2	1.00	25	0.080
74	A	3	2	1.00	28	0.071
75	A	3	2	1.00	28	0.071
76	A	2	1	1.00	28	0.036
77	A	2	1	1.00	28	0.036
78	A	2	1	1.00	28	0.036
79	A	2	1	1.00	28	0.036
80	A	3	2	1.00	26	0.077
81	A	3	2	1.00	25	0.080
82	A	3	2	1.00	28	0.071
83	A	3	2	1.00	28	0.071
84	A	2	1	1.00	28	0.036
85	A	2	1	1.00	28	0.036
86	A	5	4	1.00	28	0.143
87	A	5	4	1.00	28	0.143
88	A	5	4	1.00	28	0.143
89	A	5	4	1.00	26	0.154
90	A	5	4	1.00	25	0.160
91	A	5	4	1.00	28	0.143
92	A	5	4	1.00	28	0.143
93	A	5	4	1.00	28	0.143
94	A	6	5	1.00	28	0.179

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	6	5	1.00	28	0.179
96	A	6	5	1.00	28	0.179
97	A	6	5	1.00	26	0.192
98	A	4	4	1.00	25	0.160
99	A	6	5	1.00	28	0.179
100	A	6	5	1.00	28	0.179
101	A	6	5	1.00	28	0.179
102	A	7	5	1.00	28	0.179
103	A	6	5	1.00	28	0.179
104	A	5	4	1.00	28	0.143
105	A	4	4	1.00	26	0.154
106	A	3	3	1.00	25	0.120
107	A	7	6	1.00	28	0.214
108	A	7	5	1.00	28	0.179
109	A	7	5	1.00	28	0.179
110	A	4	3	1.00	17	0.176
111	A	4	3	1.00	17	0.176
112	A	4	4	1.00	21	0.190
113	A	6	5	1.00	15	0.333
114	A	3	2	1.00	30	0.067
115	A	3	2	1.00	30	0.067
116	A	3	2	1.00	30	0.067
117	A	3	2	1.00	27	0.074
118	A	3	2	1.00	30	0.067
119	A	3	2	1.00	30	0.067
120	A	3	2	1.00	30	0.067
121	A	3	2	1.00	30	0.067
122	A	3	2	1.00	30	0.067
123	A	3	2	1.00	30	0.067
124	A	5	4	1.00	30	0.133
125	A	5	4	1.00	30	0.133
126	A	5	4	1.00	30	0.133
127	A	4	3	1.00	27	0.111
128	A	4	3	1.00	30	0.100
129	A	4	3	1.00	30	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	4	3	1.00	30	0.100
131	A	4	3	1.00	30	0.100
132	A	4	3	1.00	30	0.100
133	A	6	5	1.00	30	0.167
134	A	6	5	1.00	30	0.167
135	A	6	5	1.00	30	0.167
136	A	6	5	1.00	30	0.167
137	A	4	4	1.00	27	0.148
138	A	4	4	1.00	30	0.133
139	A	5	4	1.00	30	0.133
140	A	5	3	1.00	30	0.100
141	A	5	3	1.00	30	0.100
142	A	5	3	1.00	30	0.100
143	A	3	2	1.00	32	0.062
144	A	3	2	1.00	32	0.062
145	A	3	2	1.00	30	0.067
146	A	5	4	1.00	32	0.125
147	A	6	5	1.00	32	0.156
148	A	6	6	1.00	32	0.188
149	A	6	6	1.00	32	0.188
150	A	7	7	1.00	32	0.219
151	A	7	6	1.00	32	0.188
152	A	6	6	1.00	32	0.188
153	A	5	5	1.00	29	0.172
154	A	6	6	1.00	32	0.188
155	A	6	6	1.00	32	0.188
156	A	6	6	1.00	32	0.188
157	A	5	3	1.00	32	0.094
158	A	6	4	1.00	32	0.125
159	A	11	9	1.00	32	0.281
160	A	10	9	1.00	32	0.281
161	A	9	9	1.00	32	0.281
162	A	8	8	1.07	32	0.250
163	A	5	4	1.00	29	0.138
164	A	6	5	0.97	32	0.156

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	7	5	1.00	32	0.156
166	A	8	4	0.98	32	0.125
167	A	9	5	1.00	32	0.156
168	A	10	5	0.97	32	0.156
169	A	4	3	1.00	33	0.091
170	A	4	3	1.00	33	0.091
171	A	4	3	1.00	31	0.097
172	A	10	9	0.98	37	0.243
173	A	6	5	1.17	34	0.147
174	A	6	4	1.00	37	0.108

CHAPTER 3

LISTING OF INTEGRALS

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3.84 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^3}\,dx$ 533
3.85 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$ 538
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3.87 $\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$ 549
3.88 $\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$ 555
3.89 $\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$ 560
3.90 $\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx$ 565
3.91 $\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)}\,dx$ 570

3.92 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)}\,dx$ . . . 574
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3.112 $\int \frac{-x^2+2x^4}{1+2x^2}\,dx$ . . . 682
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3.132 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)^2}\,dx$ . . . 790

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3.136 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$ . . . 820

3.137 $\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^3}\,dx$ . . . 826

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3.139 $\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)^3}\,dx$ . . . 838

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3.141 $\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)^3}\,dx$ . . . 850

3.142 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)^3}\,dx$ . . . 856

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3.144 $\int \frac{x^3(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$ . . . 870

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3.149 $\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx$ . . . 898

3.150 $\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx$ . . . 905

3.151 $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$ . . . 913

3.152 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$ . . . 921

3.153 $\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}}\,dx$ . . . 929

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3.155 $\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx$ . . . 942

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3.157 $\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx$ 955
3.158 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx$ 962
3.159 $\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$ 969
3.160 $\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$ 980
3.161 $\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$ 990
3.162 $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$ 1003
3.163 $\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx$ 1014
3.164 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(a+bx^2)^{9/2}}\,dx$ 1022
3.165 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(a+bx^2)^{9/2}}\,dx$ 1030
3.166 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6(a+bx^2)^{9/2}}\,dx$ 1039
3.167 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(a+bx^2)^{9/2}}\,dx$ 1049
3.168 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}(a+bx^2)^{9/2}}\,dx$ 1060
3.169 $\int \frac{cx^5+dx^7+ex^9+fx^{11}}{\sqrt{a+bx^2}}\,dx$ 1071
3.170 $\int \frac{cx^3+dx^5+ex^7+fx^9}{\sqrt{a+bx^2}}\,dx$ 1078
3.171 $\int \frac{cx+dx^3+ex^5+fx^7}{\sqrt{a+bx^2}}\,dx$ 1084
3.172 $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx$ 1090
3.173 $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}}\,dx$ 1104
3.174 $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2(a+bx^2)^{9/2}}\,dx$ 1115

3.1 $\int x^3(A+Bx)\sqrt{a+bx^2}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 127

$$\int x^3(A+Bx)\sqrt{a+bx^2}\,dx=\frac{a^2Bx\sqrt{a+bx^2}}{16b^2}+\frac{Ax^2\left(a+bx^2\right)^{3/2}}{5b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}-\frac{a(16A+15Bx)\left(a+bx^2\right)^{3/2}}{120b^2}+\frac{a^3B\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}$$

```
[Out] 1/5*A*x^2*(b*x^2+a)^(3/2)/b+1/6*B*x^3*(b*x^2+a)^(3/2)/b-1/120*a*(15*B*x+16*
A)*(b*x^2+a)^(3/2)/b^2+1/16*a^3*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2
)+1/16*a^2*B*x*(b*x^2+a)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {847, 794, 201, 223, 212}

$$\int x^3(A+Bx)\sqrt{a+bx^2}\,dx=\frac{a^3B\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}+\frac{a^2Bx\sqrt{a+bx^2}}{16b^2}-\frac{a\left(a+bx^2\right)^{3/2}(16A+15Bx)}{120b^2}+\frac{Ax^2\left(a+bx^2\right)^{3/2}}{5b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}$$

```
[In] Int[x^3*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*B*x*Sqrt[a + b*x^2])/(16*b^2) + (A*x^2*(a + b*x^2)^(3/2))/(5*b) + (B*x
^3*(a + b*x^2)^(3/2))/(6*b) - (a*(16*A + 15*B*x)*(a + b*x^2)^(3/2))/(120*b^
2) + (a^3*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{Bx^3(a+bx^2)^{3/2}}{6b} + \frac{\int x^2(-3aB+6Abx)\sqrt{a+bx^2}\,dx}{6b} \\
&= \frac{Ax^2(a+bx^2)^{3/2}}{5b} + \frac{Bx^3(a+bx^2)^{3/2}}{6b} + \frac{\int x(-12aAb-15abBx)\sqrt{a+bx^2}\,dx}{30b^2} \\
&= \frac{Ax^2(a+bx^2)^{3/2}}{5b} + \frac{Bx^3(a+bx^2)^{3/2}}{6b} - \frac{a(16A+15Bx)\,(a+bx^2)^{3/2}}{120b^2} + \frac{(a^2B)\int\sqrt{a+bx^2}\,dx}{8b^2}
\end{aligned}$$

$$= \frac{a^2Bx\sqrt{a+bx^2}}{16b^2} + \frac{Ax^2(a+bx^2)^{3/2}}{5b} + \frac{Bx^3(a+bx^2)^{3/2}}{6b} - \frac{a(16A+15Bx)\left(a+bx^2\right)^{3/2}}{120b^2} + \frac{(a^3B)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{16b^2}$$

$$= \frac{a^2Bx\sqrt{a+bx^2}}{16b^2} + \frac{Ax^2(a+bx^2)^{3/2}}{5b} + \frac{Bx^3(a+bx^2)^{3/2}}{6b} - \frac{a(16A+15Bx)\left(a+bx^2\right)^{3/2}}{120b^2} + \frac{(a^3B)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{16b^2}$$

$$= \frac{a^2Bx\sqrt{a+bx^2}}{16b^2} + \frac{Ax^2(a+bx^2)^{3/2}}{5b} + \frac{Bx^3(a+bx^2)^{3/2}}{6b} - \frac{a(16A+15Bx)\left(a+bx^2\right)^{3/2}}{120b^2} + \frac{a^3B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80

$$\int x^3(A+Bx)\sqrt{a+bx^2}\,dx = \frac{\sqrt{a+bx^2}(-32a^2A-15a^2Bx+16aAbx^2+10abBx^3+48Ab^2x^4+40b^2Bx^5)}{240b^2} - \frac{a^3B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{16b^{5/2}}$$

```
[In] Integrate[x^3*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-32*a^2*A - 15*a^2*B*x + 16*a*A*b*x^2 + 10*a*b*B*x^3 + 48
*A*b^2*x^4 + 40*b^2*B*x^5))/(240*b^2) - (a^3*B*Log[-(Sqrt[b]*x) + Sqrt[a +
b*x^2]])/(16*b^(5/2))
```

Maple [A] (verified)

Time = 4.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.70

method	result	size
risch	$-\frac{\left(-40b^{2}B\,x^{5}-48A\,b^{2}x^{4}-10Bab\,x^{3}-16aAb\,x^{2}+15a^{2}Bx+32a^{2}A\right)\sqrt{b\,x^{2}+a}}{240b^{2}}+\frac{a^{3}B\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{16b^{\frac{5}{2}}}$	89
default	$B\left(\frac{x^{3}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4b}\right)}{2b}\right)+A\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{5b}-\frac{2a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{15b^{2}}\right)$	120

```
[In] int(x^3*(B*x+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/240*(-40*B*b^2*x^5-48*A*b^2*x^4-10*B*a*b*x^3-16*A*a*b*x^2+15*B*a^2*x+32*
A*a^2)/b^2*(b*x^2+a)^(1/2)+1/16*a^3*B/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.62

$$\int x^3(A+Bx)\sqrt{a+bx^2}\,dx$$

$$=\left[\frac{15\,Ba^3\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(40\,Bb^3x^5+48\,Ab^3x^4+10\,Bab^2x^3+16\,Aab^2x^2-15\,B}{480\,b^3}\right.$$

$$-\frac{15\,Ba^3\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(40\,Bb^3x^5+48\,Ab^3x^4+10\,Bab^2x^3+16\,Aab^2x^2-15\,Ba^2bx-32\,Aa^2b)\sqrt{}}{240\,b^3}$$

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(15*B*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(40*B*b^3*x^5 + 48*A*b^3*x^4 + 10*B*a*b^2*x^3 + 16*A*a*b^2*x^2 - 15*B*a^2
*b*x - 32*A*a^2*b)*sqrt(b*x^2 + a))/b^3, -1/240*(15*B*a^3*sqrt(-b)*arctan(s
qrt(-b)*x/sqrt(b*x^2 + a)) - (40*B*b^3*x^5 + 48*A*b^3*x^4 + 10*B*a*b^2*x^3
+ 16*A*a*b^2*x^2 - 15*B*a^2*b*x - 32*A*a^2*b)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.09

$$\int x^3(A+Bx)\sqrt{a+bx^2}\,dx = \begin{cases} \frac{Ba^3\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{16b^2} + \sqrt{a+bx^2}\left(-\frac{2Aa^2}{15b^2}+\frac{Aax^2}{15b}+\frac{Ax^4}{5}-\frac{Ba^2x}{16b^2}+\frac{Bax^3}{24b}+\frac{Bx^5}{6}\right) & \text{for } b \neq \\ \sqrt{a}\left(\frac{Ax^4}{4}+\frac{Bx^5}{5}\right) & \text{other} \end{cases}$$

```
[In] integrate(x**3*(B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((B*a**3*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b
), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(16*b**2) + sqrt(a + b*x**2)*(
-2*A*a**2/(15*b**2) + A*a*x**2/(15*b) + A*x**4/5 - B*a**2*x/(16*b**2) + B*a
*x**3/(24*b) + B*x**5/6), Ne(b, 0)), (sqrt(a)*(A*x**4/4 + B*x**5/5), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

$$\int x^3(A+Bx)\sqrt{a+bx^2}\,dx = \frac{(bx^2+a)^{\frac{3}{2}}Bx^3}{6\,b} + \frac{(bx^2+a)^{\frac{3}{2}}Ax^2}{5\,b} - \frac{(bx^2+a)^{\frac{3}{2}}Bax}{8\,b^2} + \frac{\sqrt{bx^2+a}Ba^2x}{16\,b^2} + \frac{Ba^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}} - \frac{2\,(bx^2+a)^{\frac{3}{2}}Aa}{15\,b^2}$$

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(3/2)*B*x^3/b + 1/5*(b*x^2 + a)^(3/2)*A*x^2/b - 1/8*(b*x^2
+ a)^(3/2)*B*a*x/b^2 + 1/16*sqrt(b*x^2 + a)*B*a^2*x/b^2 + 1/16*B*a^3*arcsin
h(b*x/sqrt(a*b))/b^(5/2) - 2/15*(b*x^2 + a)^(3/2)*A*a/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.73

$$\int x^3(A+Bx)\sqrt{a+bx^2}\,dx$$
$$= -\frac{Ba^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}} + \frac{1}{240}\sqrt{bx^2+a}\left(\left(2\left(\left(4\,(5\,Bx+6\,A)x+\frac{5\,Ba}{b}\right)x+\frac{8\,Aa}{b}\right)x-\frac{15\,Ba^2}{b^2}\right)x-\frac{32\,Aa^2}{b^2}\right)$$

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/16*B*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) + 1/240*sqrt(b*x
^2 + a)*((2*((4*(5*B*x + 6*A)*x + 5*B*a/b)*x + 8*A*a/b)*x - 15*B*a^2/b^2)*x
 - 32*A*a^2/b^2)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(A+Bx)\sqrt{a+bx^2}\,dx = \int x^3\,\sqrt{b\,x^2+a}\,(A+B\,x)\,dx$$

```
[In] int(x^3*(a + b*x^2)^(1/2)*(A + B*x),x)
```

```
[Out] int(x^3*(a + b*x^2)^(1/2)*(A + B*x), x)
```

3.2 $\int x^2(A+Bx)\sqrt{a+bx^2}\,dx$

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Mupad [F(-1)] . . . 85

Optimal result

Integrand size = 20, antiderivative size = 104

$$\int x^2(A+Bx)\sqrt{a+bx^2}\,dx=-\frac{aAx\sqrt{a+bx^2}}{8b}+\frac{Bx^2(a+bx^2)^{3/2}}{5b}-\frac{(8aB-15Abx)\,(a+bx^2)^{3/2}}{60b^2}-\frac{a^2A\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

```
[Out] 1/5*B*x^2*(b*x^2+a)^(3/2)/b-1/60*(-15*A*b*x+8*B*a)*(b*x^2+a)^(3/2)/b^2-1/8*
a^2*A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)-1/8*a*A*x*(b*x^2+a)^(1/2)/
b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {847, 794, 201, 223, 212}

$$\int x^2(A+Bx)\sqrt{a+bx^2}\,dx=-\frac{a^2A\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}-\frac{(a+bx^2)^{3/2}\,(8aB-15Abx)}{60b^2}-\frac{aAx\sqrt{a+bx^2}}{8b}+\frac{Bx^2(a+bx^2)^{3/2}}{5b}$$

```
[In] Int[x^2*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] -1/8*(a*A*x*Sqrt[a + b*x^2])/b + (B*x^2*(a + b*x^2)^(3/2))/(5*b) - ((8*a*B
- 15*A*b*x)*(a + b*x^2)^(3/2))/(60*b^2) - (a^2*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/(8*b^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{Bx^2(a+bx^2)^{3/2}}{5b} + \frac{\int x(-2aB+5Abx)\sqrt{a+bx^2}\,dx}{5b} \\
&= \frac{Bx^2(a+bx^2)^{3/2}}{5b} - \frac{(8aB-15Abx)\,(a+bx^2)^{3/2}}{60b^2} - \frac{(aA)\int\sqrt{a+bx^2}\,dx}{4b} \\
&= -\frac{aAx\sqrt{a+bx^2}}{8b} + \frac{Bx^2(a+bx^2)^{3/2}}{5b} - \frac{(8aB-15Abx)\,(a+bx^2)^{3/2}}{60b^2} - \frac{(a^2A)\int\frac{1}{\sqrt{a+bx^2}}\,dx}{8b}
\end{aligned}$$

$$= -\frac{aAx\sqrt{a+bx^2}}{8b} + \frac{Bx^2(a+bx^2)^{3/2}}{5b} - \frac{(8aB-15Abx)(a+bx^2)^{3/2}}{60b^2} - \frac{(a^2A)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{8b}$$

$$= -\frac{aAx\sqrt{a+bx^2}}{8b} + \frac{Bx^2(a+bx^2)^{3/2}}{5b} - \frac{(8aB-15Abx)(a+bx^2)^{3/2}}{60b^2} - \frac{a^2A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int x^2(A+Bx)\sqrt{a+bx^2}\,dx$$
$$= \frac{\sqrt{a+bx^2}(-16a^2B+6b^2x^3(5A+4Bx)+abx(15A+8Bx))+15a^2A\sqrt{b}\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{120b^2}$$

```
[In] Integrate[x^2*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-16*a^2*B + 6*b^2*x^3*(5*A + 4*B*x) + a*b*x*(15*A + 8*B*x
)) + 15*a^2*A*Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(120*b^2)
```

Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{(24b^2Bx^4+30Ab^2x^3+8Babx^2+15aAbx-16a^2B)\sqrt{bx^2+a}}{120b^2} - \frac{a^2A\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{8b^{\frac{3}{2}}}$	80
default	$B\left(\frac{x^2(bx^2+a)^{\frac{3}{2}}}{5b} - \frac{2a(bx^2+a)^{\frac{3}{2}}}{15b^2}\right) + A\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4b} - \frac{a\left(\frac{x\sqrt{bx^2+a}}{2} + \frac{a\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{2\sqrt{b}}\right)}{4b}\right)$	96

```
[In] int(x^2*(B*x+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*(24*B*b^2*x^4+30*A*b^2*x^3+8*B*a*b*x^2+15*A*a*b*x-16*B*a^2)/b^2*(b*x^
2+a)^(1/2)-1/8*a^2*A/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.68

$$\int x^2(A+Bx)\sqrt{a+bx^2}\,dx = \left[\frac{15\,Aa^2\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(24\,Bb^2x^4+30\,Ab^2x^3+8\,Babx^2+15\,Aabx-16\,Ba^2)}{240\,b^2}\right.$$

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/240*(15*A*a^2*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(24*B*b^2*x^4 + 30*A*b^2*x^3 + 8*B*a*b*x^2 + 15*A*a*b*x - 16*B*a^2)*sqrt(
b*x^2 + a))/b^2, 1/120*(15*A*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)
) + (24*B*b^2*x^4 + 30*A*b^2*x^3 + 8*B*a*b*x^2 + 15*A*a*b*x - 16*B*a^2)*sqr
t(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.17

$$\int x^2(A+Bx)\sqrt{a+bx^2}\,dx = \begin{cases} -\frac{Aa^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b}+\sqrt{a+bx^2}\left(\frac{Aax}{8b}+\frac{Ax^3}{4}-\frac{2Ba^2}{15b^2}+\frac{Bax^2}{15b}+\frac{Bx^4}{5}\right) & \text{for}\ b\neq 0\\ \sqrt{a}\left(\frac{Ax^3}{3}+\frac{Bx^4}{4}\right) & \text{otherwise}\end{cases}$$

```
[In] integrate(x**2*(B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-A*a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(
b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b) + sqrt(a + b*x**2)*(A*a
*x/(8*b) + A*x**3/4 - 2*B*a**2/(15*b**2) + B*a*x**2/(15*b) + B*x**4/5), Ne(
b, 0)), (sqrt(a)*(A*x**3/3 + B*x**4/4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int x^2(A+Bx)\sqrt{a+bx^2}\,dx = \frac{(bx^2+a)^{\frac{3}{2}}Bx^2}{5\,b} + \frac{(bx^2+a)^{\frac{3}{2}}Ax}{4\,b} - \frac{\sqrt{bx^2+a}Aax}{8\,b} - \frac{Aa^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} - \frac{2\,(bx^2+a)^{\frac{3}{2}}Ba}{15\,b^2}$$

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(b*x^2 + a)^(3/2)*B*x^2/b + 1/4*(b*x^2 + a)^(3/2)*A*x/b - 1/8*sqrt(b*x^
2 + a)*A*a*x/b - 1/8*A*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2/15*(b*x^2 + a
)^(3/2)*B*a/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.78

$$\int x^2(A+Bx)\sqrt{a+bx^2}\,dx = \frac{Aa^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}} + \frac{1}{120}\sqrt{bx^2+a}\left(\left(2\left(3\,(4\,Bx+5\,A)x+\frac{4\,Ba}{b}\right)x+\frac{15\,Aa}{b}\right)x-\frac{16\,Ba^2}{b^2}\right)$$

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*A*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/120*sqrt(b*x^2
 + a)*((2*(3*(4*B*x + 5*A)*x + 4*B*a/b)*x + 15*A*a/b)*x - 16*B*a^2/b^2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(A+Bx)\sqrt{a+bx^2}\,dx = \int x^2\,\sqrt{b\,x^2+a}\,(A+B\,x)\,dx$$

```
[In] int(x^2*(a + b*x^2)^(1/2)*(A + B*x),x)
```

```
[Out] int(x^2*(a + b*x^2)^(1/2)*(A + B*x), x)
```

3.3 $\int x(A+Bx)\sqrt{a+bx^2}\,dx$

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Maple [A] (verified) 88
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Sympy [A] (verification not implemented) 89
Maxima [A] (verification not implemented) 89
Giac [A] (verification not implemented) 90
Mupad [F(-1)] 90

Optimal result

Integrand size = 18, antiderivative size = 80

$$\int x(A+Bx)\sqrt{a+bx^2}\,dx=-\frac{aBx\sqrt{a+bx^2}}{8b}+\frac{(4A+3Bx)\left(a+bx^2\right)^{3/2}}{12b}-\frac{a^2B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

```
[Out] 1/12*(3*B*x+4*A)*(b*x^2+a)^(3/2)/b-1/8*a^2*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1
/2))/b^(3/2)-1/8*a*B*x*(b*x^2+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {794, 201, 223, 212}

$$\int x(A+Bx)\sqrt{a+bx^2}\,dx=-\frac{a^2B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{\left(a+bx^2\right)^{3/2}(4A+3Bx)}{12b}-\frac{aBx\sqrt{a+bx^2}}{8b}$$

```
[In] Int[x*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] -1/8*(a*B*x*Sqrt[a + b*x^2])/b + ((4*A + 3*B*x)*(a + b*x^2)^(3/2))/(12*b) -
 (a^2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(4A+3Bx)\left(a+bx^2\right)^{3/2}}{12b}-\frac{(aB)\int\sqrt{a+bx^2}\,dx}{4b}\\
&= -\frac{aBx\sqrt{a+bx^2}}{8b}+\frac{(4A+3Bx)\left(a+bx^2\right)^{3/2}}{12b}-\frac{\left(a^2B\right)\int\frac{1}{\sqrt{a+bx^2}}\,dx}{8b}\\
&= -\frac{aBx\sqrt{a+bx^2}}{8b}+\frac{(4A+3Bx)\left(a+bx^2\right)^{3/2}}{12b}-\frac{\left(a^2B\right)\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{8b}\\
&= -\frac{aBx\sqrt{a+bx^2}}{8b}+\frac{(4A+3Bx)\left(a+bx^2\right)^{3/2}}{12b}-\frac{a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int x(A+Bx)\sqrt{a+bx^2}\,dx=\frac{\sqrt{a+bx^2}(8aA+3aBx+8Abx^2+6bBx^3)}{24b}+\frac{a^2B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{3/2}}$$

```
[In] Integrate[x*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*a*A + 3*a*B*x + 8*A*b*x^2 + 6*b*B*x^3))/(24*b) + (a^2*B
*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{(6bB\,x^3+8Ab\,x^2+3Bax+8Aa)\sqrt{b\,x^2+a}}{24b}-\frac{B\,a^2\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{8b^{\frac{3}{2}}}$	65
default	$B\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4b}\right)+\frac{A(b\,x^2+a)^{\frac{3}{2}}}{3b}$	76

```
[In] int(x*(B*x+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(6*B*b*x^3+8*A*b*x^2+3*B*a*x+8*A*a)/b*(b*x^2+a)^(1/2)-1/8*B*a^2/b^(3/2
)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.96

$$\int x(A+Bx)\sqrt{a+bx^2}\,dx$$
$$=\left[\frac{3\,Ba^2\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(6\,Bb^2x^3+8\,Ab^2x^2+3\,Babx+8\,Aab)\sqrt{bx^2+a}}{48\,b^2},\frac{3\,Ba^2\cdots}{\cdots}\right.$$

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*B*a^2*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*
(6*B*b^2*x^3 + 8*A*b^2*x^2 + 3*B*a*b*x + 8*A*a*b)*sqrt(b*x^2 + a))/b^2, 1/2
4*(3*B*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (6*B*b^2*x^3 + 8*A
*b^2*x^2 + 3*B*a*b*x + 8*A*a*b)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.34

$$\int x(A+Bx)\sqrt{a+bx^2}\,dx = \begin{cases} -\frac{Ba^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b} + \sqrt{a+bx^2}\left(\frac{Aa}{3b}+\frac{Ax^2}{3}+\frac{Bax}{8b}+\frac{Bx^3}{4}\right) & \text{for } b\neq 0\\ \sqrt{a}\left(\frac{Ax^2}{2}+\frac{Bx^3}{3}\right) & \text{otherwise}\end{cases}$$

```
[In] integrate(x*(B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-B*a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(
b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b) + sqrt(a + b*x**2)*(A*a
/(3*b) + A*x**2/3 + B*a*x/(8*b) + B*x**3/4), Ne(b, 0)), (sqrt(a)*(A*x**2/2
+ B*x**3/3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

$$\int x(A+Bx)\sqrt{a+bx^2}\,dx = \frac{(bx^2+a)^{\frac{3}{2}}Bx}{4\,b} - \frac{\sqrt{bx^2+a}Bax}{8\,b} - \frac{Ba^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} + \frac{(bx^2+a)^{\frac{3}{2}}A}{3\,b}$$

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*B*x/b - 1/8*sqrt(b*x^2 + a)*B*a*x/b - 1/8*B*a^2*arcsi
nh(b*x/sqrt(a*b))/b^(3/2) + 1/3*(b*x^2 + a)^(3/2)*A/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int x(A+Bx)\sqrt{a+bx^2}\,dx=\frac{Ba^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}+\frac{1}{24}\sqrt{bx^2+a}\left(\left(2\,(3\,Bx+4\,A)x+\frac{3\,Ba}{b}\right)x+\frac{8\,Aa}{b}\right)$$

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] 1/8*B*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/24*sqrt(b*x^2
+ a)*((2*(3*B*x + 4*A)*x + 3*B*a/b)*x + 8*A*a/b)
```

Mupad [F(-1)]

Timed out.

$$\int x(A+Bx)\sqrt{a+bx^2}\,dx=\int x\,\sqrt{b\,x^2+a}\,(A+B\,x)\,dx$$

```
[In] int(x*(a + b*x^2)^(1/2)*(A + B*x),x)
[Out] int(x*(a + b*x^2)^(1/2)*(A + B*x), x)
```

3.4 $\int (A+Bx)\sqrt{a+bx^2}\,dx$

Optimal result 91
Rubi [A] (verified) 91
Mathematica [A] (verified) 92
Maple [A] (verified) 93
Fricas [A] (verification not implemented) 93
Sympy [A] (verification not implemented) 94
Maxima [A] (verification not implemented) 94
Giac [A] (verification not implemented) 94
Mupad [B] (verification not implemented) 95

Optimal result

Integrand size = 17, antiderivative size = 67

$$\int (A+Bx)\sqrt{a+bx^2}\,dx=\frac{1}{2}Ax\sqrt{a+bx^2}+\frac{B(a+bx^2)^{3/2}}{3b}+\frac{aA\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

```
[Out] 1/3*B*(b*x^2+a)^(3/2)/b+1/2*a*A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+
1/2*A*x*(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {655, 201, 223, 212}

$$\int (A+Bx)\sqrt{a+bx^2}\,dx=\frac{aA\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}Ax\sqrt{a+bx^2}+\frac{B(a+bx^2)^{3/2}}{3b}$$

```
[In] Int[(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (A*x*Sqrt[a + b*x^2])/2 + (B*(a + b*x^2)^(3/2))/(3*b) + (a*A*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

<u>Rule 201</u>

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{B(a+bx^2)^{3/2}}{3b} + A\int \sqrt{a+bx^2}\,dx\\ &= \frac{1}{2}Ax\sqrt{a+bx^2} + \frac{B(a+bx^2)^{3/2}}{3b} + \frac{1}{2}(aA)\int \frac{1}{\sqrt{a+bx^2}}\,dx\\ &= \frac{1}{2}Ax\sqrt{a+bx^2} + \frac{B(a+bx^2)^{3/2}}{3b} + \frac{1}{2}(aA)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{1}{2}Ax\sqrt{a+bx^2} + \frac{B(a+bx^2)^{3/2}}{3b} + \frac{aA\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int (A+Bx)\sqrt{a+bx^2}\,dx = \frac{\sqrt{a+bx^2}(2aB+3Abx+2bBx^2)}{6b} - \frac{aA\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2\sqrt{b}}$$

```
[In] Integrate[(A + B*x)*Sqrt[a + b*x^2],x]
[Out] (Sqrt[a + b*x^2]*(2*a*B + 3*A*b*x + 2*b*B*x^2))/(6*b) - (a*A*Log[-(Sqrt[b]*
x) + Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

method	result	size
default	$A\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+\frac{B\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3b}$	54
risch	$\frac{\left(2bB\,x^{2}+3Abx+2Ba\right)\sqrt{b\,x^{2}+a}}{6b}+\frac{Aa\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}$	56

```
[In] int((B*x+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))+1/3*B
*(b*x^2+a)^(3/2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.91

$$\int (A+Bx)\sqrt{a+bx^2}\,dx$$
$$=\left[\frac{3\,Aa\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(2\,Bbx^2+3\,Abx+2\,Ba)\sqrt{bx^2+a}}{12\,b},\right.$$
$$\left.-\frac{3\,Aa\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,Bbx^2+3\,Abx+2\,Ba)\sqrt{bx^2+a}}{6\,b}\right]$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*A*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*B*b*x^2 + 3*A*b*x + 2*B*a)*sqrt(b*x^2 + a))/b, -1/6*(3*A*a*sqrt(-b)*arctan
(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*B*b*x^2 + 3*A*b*x + 2*B*a)*sqrt(b*x^2 + a
))/b]
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.34

$$\int (A+Bx)\sqrt{a+bx^2}\,dx = \begin{cases} \frac{Aa\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2} + \sqrt{a+bx^2}\left(\frac{Ax}{2}+\frac{Ba}{3b}+\frac{Bx^2}{3}\right) & \text{for } b\neq 0 \\ \sqrt{a}\left(Ax+\frac{Bx^2}{2}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate((B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((A*a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b),
Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + sqrt(a + b*x**2)*(A*x/2 + B*a
/(3*b) + B*x**2/3), Ne(b, 0)), (sqrt(a)*(A*x + B*x**2/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.67

$$\int (A+Bx)\sqrt{a+bx^2}\,dx = \frac{1}{2}\sqrt{bx^2+a}Ax + \frac{Aa\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} + \frac{(bx^2+a)^{\frac{3}{2}}B}{3\,b}$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*A*x + 1/2*A*a*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 1/3*(b*x
^2 + a)^(3/2)*B/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int (A+Bx)\sqrt{a+bx^2}\,dx = -\frac{Aa\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}} + \frac{1}{6}\sqrt{bx^2+a}\left((2\,Bx+3\,A)x+\frac{2\,Ba}{b}\right)$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*A*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/6*sqrt(b*x^2 +
a)*((2*B*x + 3*A)*x + 2*B*a/b)
```

Mupad [B] (verification not implemented)

Time = 5.72 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int (A+Bx)\sqrt{a+bx^2}\,dx = \frac{B\,(b\,x^2+a)^{3/2}}{3\,b}+\frac{A\,x\,\sqrt{b\,x^2+a}}{2}+\frac{A\,a\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\,\sqrt{b}}$$

```
[In] int((a + b*x^2)^(1/2)*(A + B*x),x)
[Out] (B*(a + b*x^2)^(3/2))/(3*b) + (A*x*(a + b*x^2)^(1/2))/2 + (A*a*log(b^(1/2)*
x + (a + b*x^2)^(1/2)))/(2*b^(1/2))
```

3.5 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx$

Optimal result 96
Rubi [A] (verified) 96
Mathematica [A] (verified) 98
Maple [A] (verified) 99
Fricas [A] (verification not implemented) 99
Sympy [A] (verification not implemented) 100
Maxima [A] (verification not implemented) 100
Giac [F(-2)] 101
Mupad [B] (verification not implemented) 101

Optimal result

Integrand size = 20, antiderivative size = 79

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx = \frac{1}{2}(2A+Bx)\sqrt{a+bx^2}+\frac{aB\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}} - \sqrt{a}A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] -A*arctanh((b*x^2+a)^(1/2)/a^(1/2))*a^(1/2)+1/2*a*B*arctanh(x*b^(1/2)/(b*x^
2+a)^(1/2))/b^(1/2)+1/2*(B*x+2*A)*(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {829, 858, 223, 212, 272, 65, 214}

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx = -\sqrt{a}A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) + \frac{1}{2}\sqrt{a+bx^2}(2A+Bx)+\frac{aB\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

```
[In] Int[((A + B*x)*Sqrt[a + b*x^2])/x,x]
```

```
[Out] ((2*A + B*x)*Sqrt[a + b*x^2])/2 + (a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
)/(2*Sqrt[b]) - Sqrt[a]*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}(2A+Bx)\sqrt{a+bx^2}+\frac{\int \frac{2aAb+abBx}{x\sqrt{a+bx^2}}\,dx}{2b}\\
&= \frac{1}{2}(2A+Bx)\sqrt{a+bx^2}+(aA)\int \frac{1}{x\sqrt{a+bx^2}}\,dx+\frac{1}{2}(aB)\int \frac{1}{\sqrt{a+bx^2}}\,dx\\
&= \frac{1}{2}(2A+Bx)\sqrt{a+bx^2}+\frac{1}{2}(aA)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)\\
&\quad +\frac{1}{2}(aB)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\
&= \frac{1}{2}(2A+Bx)\sqrt{a+bx^2}+\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{(aA)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{b}\\
&= \frac{1}{2}(2A+Bx)\sqrt{a+bx^2}+\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}-\sqrt{a}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx=\frac{1}{2}\left((2A+Bx)\sqrt{a+bx^2}+4\sqrt{a}A\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{aB\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}\right)$$

```
[In] Integrate[((A + B*x)*Sqrt[a + b*x^2])/x,x]
[Out] ((2*A + B*x)*Sqrt[a + b*x^2] + 4*Sqrt[a]*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*
x^2])/Sqrt[a]] - (a*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b])/2
```

Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

method	result	size
default	$B\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+A\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)$	79

```
[In] int((B*x+A)*(b*x^2+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))+A*((b
*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 341, normalized size of antiderivative = 4.32

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx$$

$$=\left[\frac{Ba\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,A\sqrt{a}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(Bbx+2\,Ab)\sqrt{bx^2+\cdots}}{4\,b}\right.$$

$$-\frac{Ba\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-A\sqrt{a}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-(Bbx+2\,Ab)\sqrt{bx^2+a}}{2\,b},\ \frac{4\,A\sqrt{-a}b\,\mathrm{arcta}\cdots}{\cdots}$$

$$\left.-\frac{Ba\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-2\,A\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-(Bbx+2\,Ab)\sqrt{bx^2+a}}{2\,b}\right]$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(B*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*A*sqr
t(a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(B*b*x + 2*A
*b)*sqrt(b*x^2 + a))/b, -1/2*(B*a*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a
)) - A*sqrt(a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - (B*b
*x + 2*A*b)*sqrt(b*x^2 + a))/b, 1/4*(4*A*sqrt(-a)*b*arctan(sqrt(-a)/sqrt(b*
x^2 + a)) + B*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
*(B*b*x + 2*A*b)*sqrt(b*x^2 + a))/b, -1/2*(B*a*sqrt(-b)*arctan(sqrt(-b)*x/s
qrt(b*x^2 + a)) - 2*A*sqrt(-a)*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (B*b*x
+ 2*A*b)*sqrt(b*x^2 + a))/b]
```

Sympy [A] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.63

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx$$
$$= -A\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)+\frac{Aa}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}}+\frac{A\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}$$
$$+B\left(\begin{cases}\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x\sqrt{a+bx^2}}{2} & \text{for } b\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((B*x+A)*(b*x**2+a)**(1/2)/x,x)
```

```
[Out] -A*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x)) + A*a/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)
) + A*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + B*Piecewise((a*Piecewise((log(2*sqrt
(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), T
rue))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx = \frac{1}{2}\sqrt{bx^2+a}Bx+\frac{Ba\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} - A\sqrt{a}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)+\sqrt{bx^2+a}A$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*x + 1/2*B*a*arcsinh(b*x/sqrt(a*b))/sqrt(b) - A*sqrt(a
)*arcsinh(a/(sqrt(a*b)*abs(x))) + sqrt(b*x^2 + a)*A
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 5.98 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx = A\sqrt{b\,x^2+a} - A\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right) + \frac{B\,x\,\sqrt{b\,x^2+a}}{2} + \frac{B\,a\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\,\sqrt{b}}$$

```
[In] int(((a + b*x^2)^(1/2)*(A + B*x))/x,x)
```

```
[Out] A*(a + b*x^2)^(1/2) - A*a^(1/2)*atanh((a + b*x^2)^(1/2)/a^(1/2)) + (B*x*(a
+ b*x^2)^(1/2))/2 + (B*a*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/(2*b^(1/2))
```

3.6 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2}\,dx$

Optimal result 102
Rubi [A] (verified) 102
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Mupad [B] (verification not implemented) 107

Optimal result

Integrand size = 20, antiderivative size = 75

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2}\,dx = -\frac{(A-Bx)\sqrt{a+bx^2}}{x} + A\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \sqrt{a}B\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] -B*arctanh((b*x^2+a)^(1/2)/a^(1/2))*a^(1/2)+A*arctanh(x*b^(1/2)/(b*x^2+a)^(
1/2))*b^(1/2)-(-B*x+A)*(b*x^2+a)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {827, 858, 223, 212, 272, 65, 214}

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2}\,dx = A\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \frac{\sqrt{a+bx^2}(A-Bx)}{x} - \sqrt{a}B\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[In] Int[((A + B*x)*Sqrt[a + b*x^2])/x^2,x]
```

```
[Out] -(((A - B*x)*Sqrt[a + b*x^2])/x) + A*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b
*x^2]] - Sqrt[a]*B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(A-Bx)\sqrt{a+bx^2}}{x} - \frac{1}{2}\int \frac{-2aB-2Abx}{x\sqrt{a+bx^2}}\,dx \\
&= -\frac{(A-Bx)\sqrt{a+bx^2}}{x} + (Ab)\int \frac{1}{\sqrt{a+bx^2}}\,dx + (aB)\int \frac{1}{x\sqrt{a+bx^2}}\,dx \\
&= -\frac{(A-Bx)\sqrt{a+bx^2}}{x} + (Ab)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right) \\
&\quad + \frac{1}{2}(aB)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right) \\
&= -\frac{(A-Bx)\sqrt{a+bx^2}}{x} + A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) + \frac{(aB)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b} \\
&= -\frac{(A-Bx)\sqrt{a+bx^2}}{x} + A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \sqrt{a}B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2}\,dx = \frac{(-A+Bx)\sqrt{a+bx^2}}{x} + 2\sqrt{a}B\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right) - A\sqrt{b}\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

```
[In] Integrate[((A + B*x)*Sqrt[a + b*x^2])/x^2,x]
[Out] ((-A + B*x)*Sqrt[a + b*x^2])/x + 2*Sqrt[a]*B*ArcTanh[(Sqrt[b]*x - Sqrt[a +
b*x^2])/Sqrt[a]] - A*Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

Maple [A] (verified)

Time = 3.40 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04

method	result	size
risch	$-\frac{A\sqrt{b\,x^{2}+a}}{x}+A\sqrt{b}\,\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)+\sqrt{b\,x^{2}+a}\,B-B\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)$	78
default	$B\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)+A\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{ax}+\frac{2b\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{a}\right)$	103

[In] int((B*x+A)*(b*x^2+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)

[Out] -A*(b*x^2+a)^(1/2)/x+A*b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+(b*x^2+a)^(1/2
)*B-B*a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 333, normalized size of antiderivative = 4.44

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2}\,dx$$
$$=\left[\frac{A\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+B\sqrt{a}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,\sqrt{bx^2+a}(Bx-A)}{2\,x},\right.$$
$$-\frac{2\,A\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-B\sqrt{a}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,\sqrt{bx^2+a}(Bx-A)}{2\,x},\ \frac{2\,B\sqrt{-a}x\arctan\ldots}{}$$
$$\left.-\frac{A\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-B\sqrt{-a}x\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}(Bx-A)}{x}\right]$$

[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="fricas")

[Out] [1/2*(A*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + B*sqrt(
a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*sqrt(b*x^2 + a
)*(B*x - A))/x, -1/2*(2*A*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - B
*sqrt(a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*sqrt(b*x
^2 + a)*(B*x - A))/x, 1/2*(2*B*sqrt(-a)*x*arctan(sqrt(-a)/sqrt(b*x^2 + a))
+ A*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*sqrt(b*x^
2 + a)*(B*x - A))/x, -(A*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - B*
sqrt(-a)*x*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - sqrt(b*x^2 + a)*(B*x - A))/x]

Sympy [A] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.65

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2}\,dx = -\frac{A\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}} + A\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{Abx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} - B\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right) + \frac{Ba}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} + \frac{B\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}$$

```
[In] integrate((B*x+A)*(b*x**2+a)**(1/2)/x**2,x)
```

```
[Out] -A*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + A*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) - A*
b*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - B*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x)) + B
*a/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + B*sqrt(b)*x/sqrt(a/(b*x**2) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2}\,dx = A\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - B\sqrt{a}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \sqrt{bx^2+a}B - \frac{\sqrt{bx^2+a}A}{x}$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] A*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - B*sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(x)))
+ sqrt(b*x^2 + a)*B - sqrt(b*x^2 + a)*A/x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.36

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2}\,dx = \frac{2\,Ba\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - A\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right) + \sqrt{bx^2+a}B + \frac{2\,Aa\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*B*a*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - A*sqrt(b)*
log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + sqrt(b*x^2 + a)*B + 2*A*a*sqrt(b)/
((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

Mupad [B] (verification not implemented)

Time = 6.49 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2}\,dx = B\sqrt{b\,x^2+a} - \frac{A\sqrt{b\,x^2+a}}{x} - B\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right) - \frac{A\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x\,1\mathrm{i}}{\sqrt{a}}\right)\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}\sqrt{\frac{b\,x^2}{a}+1}}$$

```
[In] int(((a + b*x^2)^(1/2)*(A + B*x))/x^2,x)
```

```
[Out] B*(a + b*x^2)^(1/2) - (A*(a + b*x^2)^(1/2))/x - B*a^(1/2)*atanh((a + b*x^2)
^(1/2)/a^(1/2)) - (A*b^(1/2)*asin((b^(1/2)*x*1i)/a^(1/2))*(a + b*x^2)^(1/2)
*1i)/(a^(1/2)*((b*x^2)/a + 1)^(1/2))
```

3.7 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3} dx$

Optimal result 108
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Optimal result

Integrand size = 20, antiderivative size = 80

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3} dx = -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2} + \sqrt{b}B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \frac{Ab\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*A*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)+B*arctanh(x*b^(1/2)/(b*x^
2+a)^(1/2))*b^(1/2)-1/2*(2*B*x+A)*(b*x^2+a)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {825, 858, 223, 212, 272, 65, 214}

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3} dx = -\frac{Ab\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt{a+bx^2}(A+2Bx)}{2x^2} + \sqrt{b}B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

```
[In] Int[((A + B*x)*Sqrt[a + b*x^2])/x^3,x]
```

```
[Out] -1/2*((A + 2*B*x)*Sqrt[a + b*x^2])/x^2 + Sqrt[b]*B*ArcTanh[(Sqrt[b]*x)/Sqrt
[a + b*x^2]] - (A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*Sqrt[a])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2} - \frac{\int \frac{-2aAb-4abBx}{x\sqrt{a+bx^2}}\,dx}{4a} \\
&= -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2} + \frac{1}{2}(Ab)\int \frac{1}{x\sqrt{a+bx^2}}\,dx + (bB)\int \frac{1}{\sqrt{a+bx^2}}\,dx \\
&= -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2} + \frac{1}{4}(Ab)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right) \\
&\quad + (bB)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right) \\
&= -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2} + \sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) \\
&\quad + \frac{1}{2}A\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right) \\
&= -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2} + \sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.12

$$
\begin{aligned}
\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3}\,dx = &-\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2} + \frac{Ab\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}} \\
&- \sqrt{b}B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)
\end{aligned}
$$

```
[In] Integrate[((A + B*x)*Sqrt[a + b*x^2])/x^3,x]
```

```
[Out] -1/2*((A + 2*B*x)*Sqrt[a + b*x^2])/x^2 + (A*b*ArcTanh[(Sqrt[b]*x - Sqrt[a +
 b*x^2])/Sqrt[a]])/Sqrt[a] - Sqrt[b]*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{(2Bx+A)\sqrt{b\,x^2+a}}{2x^2}+\sqrt{b}\,B\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)-\frac{bA\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{2\sqrt{a}}$
default	$B\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{ax}+\frac{2b\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{a}\right)+A\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{2a\,x^2}+\frac{b\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right.}{2a}$

```
[In] int((B*x+A)*(b*x^2+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(2*B*x+A)*(b*x^2+a)^(1/2)/x^2+b^(1/2)*B*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-
1/2*b*A/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 377, normalized size of antiderivative = 4.71

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3}\,dx$$

$$= \left[\frac{2\,Ba\sqrt{b}x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+A\sqrt{a}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(2\,Bax+Aa)\sqrt{}}{4\,ax^2}\right.$$

$$-\frac{4\,Ba\sqrt{-b}x^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-A\sqrt{a}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(2\,Bax+Aa)\sqrt{bx^2+a}}{4\,ax^2},\ A\sqrt{-a}$$

$$\left.-\frac{2\,Ba\sqrt{-b}x^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-A\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(2\,Bax+Aa)\sqrt{bx^2+a}}{2\,ax^2}\right]$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*B*a*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + A
*sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(2*B
*a*x + A*a)*sqrt(b*x^2 + a))/(a*x^2), -1/4*(4*B*a*sqrt(-b)*x^2*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) - A*sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(a) + 2*a)/x^2) + 2*(2*B*a*x + A*a)*sqrt(b*x^2 + a))/(a*x^2), 1/2*(A*sqrt
(-a)*b*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + B*a*sqrt(b)*x^2*log(-2*b*x^2
- 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - (2*B*a*x + A*a)*sqrt(b*x^2 + a))/(a*x^
2), -1/2*(2*B*a*sqrt(-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - A*sqrt(-a
)*b*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (2*B*a*x + A*a)*sqrt(b*x^2 + a))
/(a*x^2)]
```

Sympy [A] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3}\,dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}} - \frac{B\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}} + B\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{Bbx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

```
[In] integrate((B*x+A)*(b*x**2+a)**(1/2)/x**3,x)
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) - A*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*s
qrt(a)) - B*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + B*sqrt(b)*asinh(sqrt(b)*x/sqrt
(a)) - B*b*x/(sqrt(a)*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3}\,dx = B\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{Ab\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\sqrt{a}} + \frac{\sqrt{bx^2+a}Ab}{2a} - \frac{\sqrt{bx^2+a}B}{x} - \frac{(bx^2+a)^{\frac{3}{2}}A}{2ax^2}$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="maxima")
[Out] B*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - 1/2*A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/sq
rt(a) + 1/2*sqrt(b*x^2 + a)*A*b/a - sqrt(b*x^2 + a)*B/x - 1/2*(b*x^2 + a)^(
3/2)*A/(a*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(62) = 124.

Time = 0.31 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.04

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3}\,dx = \frac{Ab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - B\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right) + \frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab + 2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b} + \left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aab - 2Ba^2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2}$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] A*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - B*sqrt(b)*lo
g(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b
 + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2
+ a))*A*a*b - 2*B*a^2*sqrt(b))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2
```

Mupad [B] (verification not implemented)

Time = 6.45 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3}\,dx = -\frac{A\sqrt{b\,x^2+a}}{2\,x^2} - \frac{B\sqrt{b\,x^2+a}}{x} - \frac{A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,\sqrt{a}} - \frac{B\,\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}\,\sqrt{\frac{b\,x^2}{a}+1}}$$

```
[In] int(((a + b*x^2)^(1/2)*(A + B*x))/x^3,x)
```

```
[Out] - (A*(a + b*x^2)^(1/2))/(2*x^2) - (B*(a + b*x^2)^(1/2))/x - (A*b*atanh((a +
 b*x^2)^(1/2)/a^(1/2)))/(2*a^(1/2)) - (B*b^(1/2)*asin((b^(1/2)*x*1i)/a^(1/2
))*(a + b*x^2)^(1/2)*1i)/(a^(1/2)*((b*x^2)/a + 1)^(1/2))
```

3.8 $\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 150

$$\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\frac{3a^3Bx\sqrt{a+bx^2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2}+\frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}-\frac{a(32A+35Bx)\left(a+bx^2\right)^{5/2}}{560b^2}+\frac{3a^4B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}$$

```
[Out] 1/64*a^2*B*x*(b*x^2+a)^(3/2)/b^2+1/7*A*x^2*(b*x^2+a)^(5/2)/b+1/8*B*x^3*(b*x
^2+a)^(5/2)/b-1/560*a*(35*B*x+32*A)*(b*x^2+a)^(5/2)/b^2+3/128*a^4*B*arctanh
(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)+3/128*a^3*B*x*(b*x^2+a)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {847, 794, 201, 223, 212}

$$\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\frac{3a^4B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}+\frac{3a^3Bx\sqrt{a+bx^2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2}-\frac{a\left(a+bx^2\right)^{5/2}(32A+35Bx)}{560b^2}+\frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}$$

```
[In] Int[x^3*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (3*a^3*B*x*Sqrt[a + b*x^2])/(128*b^2) + (a^2*B*x*(a + b*x^2)^(3/2))/(64*b^2
) + (A*x^2*(a + b*x^2)^(5/2))/(7*b) + (B*x^3*(a + b*x^2)^(5/2))/(8*b) - (a*
(32*A + 35*B*x)*(a + b*x^2)^(5/2))/(560*b^2) + (3*a^4*B*ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]])/(128*b^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{Bx^3(a+bx^2)^{5/2}}{8b} + \frac{\int x^2(-3aB+8Abx)\,(a+bx^2)^{3/2}\,dx}{8b} \\
&= \frac{Ax^2(a+bx^2)^{5/2}}{7b} + \frac{Bx^3(a+bx^2)^{5/2}}{8b} + \frac{\int x(-16aAb-21abBx)\,(a+bx^2)^{3/2}\,dx}{56b^2} \\
&= \frac{Ax^2(a+bx^2)^{5/2}}{7b} + \frac{Bx^3(a+bx^2)^{5/2}}{8b} - \frac{a(32A+35Bx)\,(a+bx^2)^{5/2}}{560b^2} + \frac{(a^2B)\int (a+bx^2)^{3/2}\,dx}{16b^2}
\end{aligned}$$

$$= \frac{a^2Bx(a+bx^2)^{3/2}}{64b^2} + \frac{Ax^2(a+bx^2)^{5/2}}{7b} + \frac{Bx^3(a+bx^2)^{5/2}}{8b} - \frac{a(32A+35Bx)\left(a+bx^2\right)^{5/2}}{560b^2} + \frac{(3a^3B)\int \sqrt{a+bx^2}\,dx}{64b^2}$$

$$= \frac{3a^3Bx\sqrt{a+bx^2}}{128b^2} + \frac{a^2Bx(a+bx^2)^{3/2}}{64b^2} + \frac{Ax^2(a+bx^2)^{5/2}}{7b} + \frac{Bx^3(a+bx^2)^{5/2}}{8b} - \frac{a(32A+35Bx)\left(a+bx^2\right)^{5/2}}{560b^2} + \frac{(3a^4B)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{128b^2}$$

$$= \frac{3a^3Bx\sqrt{a+bx^2}}{128b^2} + \frac{a^2Bx(a+bx^2)^{3/2}}{64b^2} + \frac{Ax^2(a+bx^2)^{5/2}}{7b} + \frac{Bx^3(a+bx^2)^{5/2}}{8b} - \frac{a(32A+35Bx)\left(a+bx^2\right)^{5/2}}{560b^2} + \frac{(3a^4B)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{128b^2}$$

$$= \frac{3a^3Bx\sqrt{a+bx^2}}{128b^2} + \frac{a^2Bx(a+bx^2)^{3/2}}{64b^2} + \frac{Ax^2(a+bx^2)^{5/2}}{7b} + \frac{Bx^3(a+bx^2)^{5/2}}{8b} - \frac{a(32A+35Bx)\left(a+bx^2\right)^{5/2}}{560b^2} + \frac{3a^4B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.79

$$\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}\,dx = \frac{\sqrt{b}\sqrt{a+bx^2}(80b^3x^6(8A+7Bx)+2a^2bx^2(64A+35Bx)+8ab^2x^4(128A+105Bx)-a^3(256A}{4480b^{5/2}}$$

```
[In] Integrate[x^3*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[a + b*x^2]*(80*b^3*x^6*(8*A + 7*B*x) + 2*a^2*b*x^2*(64*A + 35
*B*x) + 8*a*b^2*x^4*(128*A + 105*B*x) - a^3*(256*A + 105*B*x)) - 105*a^4*B*
Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(4480*b^(5/2))
```

Maple [A] (verified)

Time = 3.45 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{\left(-560 b^{3} B \,x^{7}-640 x^{6} b^{3} A -840 B a \,b^{2} x^{5}-1024 a A \,b^{2} x^{4}-70 B \,a^{2} b \,x^{3}-128 a^{2} A b \,x^{2}+105 a^{3} B x +256 a^{3} A \right) \sqrt{b \,x^{2}+a}}{4480 b^{2}}+\frac{3 B \,a^{4} \ln \left(x \sqrt{b}\right.}{128 b}$
default	$B \left(\frac{x^{3} \left(b \,x^{2}+a \right)^{\frac{5}{2}}}{8 b}-\frac{3 a \left(\frac{x \left(b \,x^{2}+a \right)^{\frac{5}{2}}}{6 b}-\frac{a \left(\frac{x \left(b \,x^{2}+a \right)^{\frac{3}{2}}}{4}+\frac{3 a \left(\frac{x \sqrt{b \,x^{2}+a}}{2}+\frac{a \ln \left(x \sqrt{b}+\sqrt{b \,x^{2}+a}\right)}{2 \sqrt{b}}\right)}{4}\right)}{6 b}\right)}{8 b}\right)+A \left(\frac{x^{2} \left(b \,x^{2}+a \right)^{\frac{5}{2}}}{7 b}-\right.$

```
[In] int(x^3*(B*x+A)*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4480*(-560*B*b^3*x^7-640*A*b^3*x^6-840*B*a*b^2*x^5-1024*A*a*b^2*x^4-70*B
*a^2*b*x^3-128*A*a^2*b*x^2+105*B*a^3*x+256*A*a^3)/b^2*(b*x^2+a)^(1/2)+3/128
*B*a^4/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.69

$$\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\left[\frac{105\,Ba^4\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(560\,Bb^4x^7+640\,Ab^4x^6+840\,Bab^3x^5}{8960\,b^3}\right.$$
$$-\frac{105\,Ba^4\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(560\,Bb^4x^7+640\,Ab^4x^6+840\,Bab^3x^5+1024\,Aab^3x^4+70\,Ba^2b^2x^3+12}{4480\,b^3}$$

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8960*(105*B*a^4*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(560*B*b^4*x^7 + 640*A*b^4*x^6 + 840*B*a*b^3*x^5 + 1024*A*a*b^3*x^4 + 7
```

```
0*B*a^2*b^2*x^3 + 128*A*a^2*b^2*x^2 - 105*B*a^3*b*x - 256*A*a^3*b)*sqrt(b*x
^2 + a))/b^3, -1/4480*(105*B*a^4*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)
) - (560*B*b^4*x^7 + 640*A*b^4*x^6 + 840*B*a*b^3*x^5 + 1024*A*a*b^3*x^4 + 7
0*B*a^2*b^2*x^3 + 128*A*a^2*b^2*x^2 - 105*B*a^3*b*x - 256*A*a^3*b)*sqrt(b*x
^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.12

$$\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\begin{cases}\frac{3Ba^4\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{128b^2}+\sqrt{a+bx^2}\left(-\frac{2Aa^3}{35b^2}+\frac{Aa^2x^2}{35b}+\frac{8Aax^4}{35}+\frac{Abx^6}{7}-\frac{3Ba^3x}{128b^2}\cdot\right. \\ a^{\frac{3}{2}}\left(\frac{Ax^4}{4}+\frac{Bx^5}{5}\right)\end{cases}$$

```
[In] integrate(x**3*(B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((3*B*a**4*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt
(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(128*b**2) + sqrt(a + b*x**2
)*(-2*A*a**3/(35*b**2) + A*a**2*x**2/(35*b) + 8*A*a*x**4/35 + A*b*x**6/7 -
3*B*a**3*x/(128*b**2) + B*a**2*x**3/(64*b) + 3*B*a*x**5/16 + B*b*x**7/8), N
e(b, 0)), (a**(3/2)*(A*x**4/4 + B*x**5/5), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.84

$$\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\frac{\left(bx^2+a\right)^{\frac{5}{2}}Bx^3}{8\,b}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}Ax^2}{7\,b}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}Bax}{16\,b^2}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}Ba^2x}{64\,b^2}+\frac{3\,\sqrt{bx^2+a}Ba^3x}{128\,b^2}+\frac{3\,Ba^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}-\frac{2\left(bx^2+a\right)^{\frac{5}{2}}Aa}{35\,b^2}$$

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^(5/2)*B*x^3/b + 1/7*(b*x^2 + a)^(5/2)*A*x^2/b - 1/16*(b*x^2
 + a)^(5/2)*B*a*x/b^2 + 1/64*(b*x^2 + a)^(3/2)*B*a^2*x/b^2 + 3/128*sqrt(b*x
^2 + a)*B*a^3*x/b^2 + 3/128*B*a^4*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 2/35*(b*
x^2 + a)^(5/2)*A*a/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.77

$$\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}\,dx = -\frac{3\,Ba^4\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}} - \frac{1}{4480}\sqrt{bx^2+a}\left(\frac{256\,Aa^3}{b^2}+\left(\frac{105\,Ba^3}{b^2}-2\left(\frac{64\,Aa^2}{b}+\left(\frac{35\,Ba^2}{b}+4\,(128\,Aa+5\,(21\,Ba+2\,(7\,Bbx+8\,Ab$$

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] -3/128*B*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) - 1/4480*sqrt(b
*x^2 + a)*(256*A*a^3/b^2 + (105*B*a^3/b^2 - 2*(64*A*a^2/b + (35*B*a^2/b + 4
*(128*A*a + 5*(21*B*a + 2*(7*B*b*x + 8*A*b)*x)*x)*x)*x)*x)*x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}\,dx = \int x^3\left(b\,x^2+a\right)^{3/2}(A+B\,x)\,dx$$

```
[In] int(x^3*(a + b*x^2)^(3/2)*(A + B*x),x)
[Out] int(x^3*(a + b*x^2)^(3/2)*(A + B*x), x)
```

3.9 $\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

Optimal result 120
Rubi [A] (verified) 120
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Optimal result

Integrand size = 20, antiderivative size = 127

$$\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=-\frac{a^2Ax\sqrt{a+bx^2}}{16b}-\frac{aAx(a+bx^2)^{3/2}}{24b}+\frac{Bx^2(a+bx^2)^{5/2}}{7b}-\frac{(12aB-35Abx)\left(a+bx^2\right)^{5/2}}{210b^2}-\frac{a^3A\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

```
[Out] -1/24*a*A*x*(b*x^2+a)^(3/2)/b+1/7*B*x^2*(b*x^2+a)^(5/2)/b-1/210*(-35*A*b*x+
12*B*a)*(b*x^2+a)^(5/2)/b^2-1/16*a^3*A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b
^(3/2)-1/16*a^2*A*x*(b*x^2+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {847, 794, 201, 223, 212}

$$\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=-\frac{a^3A\operatorname{Arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}-\frac{a^2Ax\sqrt{a+bx^2}}{16b}-\frac{\left(a+bx^2\right)^{5/2}(12aB-35Abx)}{210b^2}-\frac{aAx(a+bx^2)^{3/2}}{24b}+\frac{Bx^2(a+bx^2)^{5/2}}{7b}$$

```
[In] Int[x^2*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] -1/16*(a^2*A*x*Sqrt[a + b*x^2])/b - (a*A*x*(a + b*x^2)^(3/2))/(24*b) + (B*x
^2*(a + b*x^2)^(5/2))/(7*b) - ((12*a*B - 35*A*b*x)*(a + b*x^2)^(5/2))/(210*
b^2) - (a^3*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\text{integral} = \frac{Bx^2(a+bx^2)^{5/2}}{7b} + \frac{\int x(-2aB+7Abx)\left(a+bx^2\right)^{3/2}\,dx}{7b}$$

$$= \frac{Bx^2(a+bx^2)^{5/2}}{7b} - \frac{(12aB-35Abx)\left(a+bx^2\right)^{5/2}}{210b^2} - \frac{(aA)\int\left(a+bx^2\right)^{3/2}\,dx}{6b}$$

$$= -\frac{aAx(a+bx^2)^{3/2}}{24b} + \frac{Bx^2(a+bx^2)^{5/2}}{7b} - \frac{(12aB-35Abx)\left(a+bx^2\right)^{5/2}}{210b^2} - \frac{(a^2A)\int\sqrt{a+bx^2}\,dx}{8b}$$

$$= -\frac{a^2Ax\sqrt{a+bx^2}}{16b} - \frac{aAx(a+bx^2)^{3/2}}{24b} + \frac{Bx^2(a+bx^2)^{5/2}}{7b} - \frac{(12aB-35Abx)(a+bx^2)^{5/2}}{210b^2} - \frac{(a^3A)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{16b}$$

$$= -\frac{a^2Ax\sqrt{a+bx^2}}{16b} - \frac{aAx(a+bx^2)^{3/2}}{24b} + \frac{Bx^2(a+bx^2)^{5/2}}{7b} - \frac{(12aB-35Abx)(a+bx^2)^{5/2}}{210b^2} - \frac{(a^3A)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{16b}$$

$$= -\frac{a^2Ax\sqrt{a+bx^2}}{16b} - \frac{aAx(a+bx^2)^{3/2}}{24b} + \frac{Bx^2(a+bx^2)^{5/2}}{7b} - \frac{(12aB-35Abx)(a+bx^2)^{5/2}}{210b^2} - \frac{a^3A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

$$\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx = \frac{\sqrt{a+bx^2}(-96a^3B+40b^3x^5(7A+6Bx)+3a^2bx(35A+16Bx)+2ab^2x^3(245A+192Bx))+}{1680b^2}$$

```
[In] Integrate[x^2*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-96*a^3*B + 40*b^3*x^5*(7*A + 6*B*x) + 3*a^2*b*x*(35*A +
16*B*x) + 2*a*b^2*x^3*(245*A + 192*B*x)) + 105*a^3*A*Sqrt[b]*Log[-(Sqrt[b]*
x) + Sqrt[a + b*x^2]])/(1680*b^2)
```

Maple [A] (verified)

Time = 3.51 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.82

method	result	Size
risch	$\frac{\left(240b^3Bx^6+280Ab^3x^5+384Bab^2x^4+490aAb^2x^3+48Ba^2bx^2+105a^2Abx-96a^3B\right)\sqrt{bx^2+a}}{1680b^2}-\frac{Aa^3\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{16b^{\frac{3}{2}}}$	104
default	$B\left(\frac{x^2\left(bx^2+a\right)^{\frac{5}{2}}}{7b}-\frac{2a\left(bx^2+a\right)^{\frac{5}{2}}}{35b^2}\right)+A\left(\frac{x\left(bx^2+a\right)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x\left(bx^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6b}\right)$	112

```
[In] int(x^2*(B*x+A)*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1680*(240*B*b^3*x^6+280*A*b^3*x^5+384*B*a*b^2*x^4+490*A*a*b^2*x^3+48*B*a^
2*b*x^2+105*A*a^2*b*x-96*B*a^3)/b^2*(b*x^2+a)^(1/2)-1/16*A*a^3/b^(3/2)*ln(x
*b^(1/2)+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.76

$$\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\left[\frac{105\,Aa^3\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(240\,Bb^3x^6+280\,Ab^3x^5+384\,Bab^2x^4+}{3360\,b^2}\right.$$

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/3360*(105*A*a^3*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(240*B*b^3*x^6 + 280*A*b^3*x^5 + 384*B*a*b^2*x^4 + 490*A*a*b^2*x^3 + 48
*B*a^2*b*x^2 + 105*A*a^2*b*x - 96*B*a^3)*sqrt(b*x^2 + a))/b^2, 1/1680*(105*
A*a^3*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (240*B*b^3*x^6 + 280*A*
b^3*x^5 + 384*B*a*b^2*x^4 + 490*A*a*b^2*x^3 + 48*B*a^2*b*x^2 + 105*A*a^2*b*
x - 96*B*a^3)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.18

$$\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\begin{cases}-\frac{Aa^3\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{16b}+\sqrt{a+bx^2}\left(\frac{Aa^2x}{16b}+\frac{7Aax^3}{24}+\frac{Abx^5}{6}-\frac{2Ba^3}{35b^2}+\frac{Ba^2x^2}{35b}\right. \\ a^{\frac{3}{2}}\left(\frac{Ax^3}{3}+\frac{Bx^4}{4}\right)\end{cases}$$

```
[In] integrate(x**2*(B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-A*a**3*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(
b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(16*b) + sqrt(a + b*x**2)*(A*
a**2*x/(16*b) + 7*A*a*x**3/24 + A*b*x**5/6 - 2*B*a**3/(35*b**2) + B*a**2*x*
*2/(35*b) + 8*B*a*x**4/35 + B*b*x**6/7), Ne(b, 0)), (a**(3/2)*(A*x**3/3 + B
*x**4/4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.83

$$\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx = \frac{(bx^2+a)^{\frac{5}{2}}Bx^2}{7\,b} + \frac{(bx^2+a)^{\frac{5}{2}}Ax}{6\,b} - \frac{(bx^2+a)^{\frac{3}{2}}Aax}{24\,b} - \frac{\sqrt{bx^2+a}Aa^2x}{16\,b} - \frac{Aa^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}} - \frac{2\,(bx^2+a)^{\frac{5}{2}}Ba}{35\,b^2}$$

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/7*(b*x^2 + a)^(5/2)*B*x^2/b + 1/6*(b*x^2 + a)^(5/2)*A*x/b - 1/24*(b*x^2 +
 a)^(3/2)*A*a*x/b - 1/16*sqrt(b*x^2 + a)*A*a^2*x/b - 1/16*A*a^3*arcsinh(b*x
/sqrt(a*b))/b^(3/2) - 2/35*(b*x^2 + a)^(5/2)*B*a/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.81

$$\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx = \frac{Aa^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}} - \frac{1}{1680}\sqrt{bx^2+a}\left(\frac{96\,Ba^3}{b^2} - \left(\frac{105\,Aa^2}{b} + 2\left(\frac{24\,Ba^2}{b} + (245\,Aa + 4\,(48\,Ba + 5\,(6\,Bbx + 7\,Ab)x)x)x\right)x\right)x\right)$$

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*A*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) - 1/1680*sqrt(b*x
^2 + a)*(96*B*a^3/b^2 - (105*A*a^2/b + 2*(24*B*a^2/b + (245*A*a + 4*(48*B*a
 + 5*(6*B*b*x + 7*A*b)*x)*x)*x)*x)*x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx = \int x^2\,\left(b\,x^2+a\right)^{3/2}\,\left(A+B\,x\right)\,dx$$

```
[In] int(x^2*(a + b*x^2)^(3/2)*(A + B*x),x)
```

```
[Out] int(x^2*(a + b*x^2)^(3/2)*(A + B*x), x)
```

3.10 $\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

Optimal result . . . 125
Rubi [A] (verified) . . . 125
Mathematica [A] (verified) . . . 127
Maple [A] (verified) . . . 127
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Sympy [A] (verification not implemented) . . . 128
Maxima [A] (verification not implemented) . . . 129
Giac [A] (verification not implemented) . . . 129
Mupad [F(-1)] . . . 129

Optimal result

Integrand size = 18, antiderivative size = 103

$$\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=-\frac{a^2Bx\sqrt{a+bx^2}}{16b}-\frac{aBx(a+bx^2)^{3/2}}{24b}+\frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b}-\frac{a^3B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

```
[Out] -1/24*a*B*x*(b*x^2+a)^(3/2)/b+1/30*(5*B*x+6*A)*(b*x^2+a)^(5/2)/b-1/16*a^3*B
*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)-1/16*a^2*B*x*(b*x^2+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {794, 201, 223, 212}

$$\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=-\frac{a^3B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}-\frac{a^2Bx\sqrt{a+bx^2}}{16b}+\frac{\left(a+bx^2\right)^{5/2}(6A+5Bx)}{30b}-\frac{aBx(a+bx^2)^{3/2}}{24b}$$

```
[In] Int[x*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] -1/16*(a^2*B*x*Sqrt[a + b*x^2])/b - (a*B*x*(a + b*x^2)^(3/2))/(24*b) + ((6*
A + 5*B*x)*(a + b*x^2)^(5/2))/(30*b) - (a^3*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/(16*b^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b}-\frac{(aB)\int \left(a+bx^2\right)^{3/2}\,dx}{6b}\\
&= -\frac{aBx(a+bx^2)^{3/2}}{24b}+\frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b}-\frac{\left(a^2B\right)\int \sqrt{a+bx^2}\,dx}{8b}\\
&= -\frac{a^2Bx\sqrt{a+bx^2}}{16b}-\frac{aBx(a+bx^2)^{3/2}}{24b}+\frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b}-\frac{\left(a^3B\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{16b}\\
&= -\frac{a^2Bx\sqrt{a+bx^2}}{16b}-\frac{aBx(a+bx^2)^{3/2}}{24b}+\frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b}\\
&\quad -\frac{\left(a^3B\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{16b}\\
&= -\frac{a^2Bx\sqrt{a+bx^2}}{16b}-\frac{aBx(a+bx^2)^{3/2}}{24b}+\frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b}-\frac{a^3B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

$$\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\frac{\sqrt{a+bx^2}(48a^2A+15a^2Bx+96aAbx^2+70abBx^3+48Ab^2x^4+40b^2Bx^5)}{240b}+\frac{a^3B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{16b^{3/2}}$$

```
[In] Integrate[x*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(48*a^2*A + 15*a^2*B*x + 96*a*A*b*x^2 + 70*a*b*B*x^3 + 48*
A*b^2*x^4 + 40*b^2*B*x^5))/(240*b) + (a^3*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x
^2]])/(16*b^(3/2))
```

Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{(40b^2B\,x^5+48A\,b^2x^4+70Bab\,x^3+96aAb\,x^2+15a^2Bx+48a^2A)\sqrt{b\,x^2+a}}{240b}-\frac{B\,a^3\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{16b^{\frac{3}{2}}}$	89
default	$B\left(\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6b}\right)+\frac{A(b\,x^2+a)^{\frac{5}{2}}}{5b}$	92

```
[In] int(x*(B*x+A)*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/240*(40*B*b^2*x^5+48*A*b^2*x^4+70*B*a*b*x^3+96*A*a*b*x^2+15*B*a^2*x+48*A*
a^2)/b*(b*x^2+a)^(1/2)-1/16*B*a^3/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.99

$$\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\left[\frac{15\,Ba^3\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(40\,Bb^3x^5+48\,Ab^3x^4+70\,Bab^2x^3+96\,A}{480\,b^2}\right.$$

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/480*(15*B*a^3*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(40*B*b^3*x^5 + 48*A*b^3*x^4 + 70*B*a*b^2*x^3 + 96*A*a*b^2*x^2 + 15*B*a^2
*b*x + 48*A*a^2*b)*sqrt(b*x^2 + a))/b^2, 1/240*(15*B*a^3*sqrt(-b)*arctan(sq
rt(-b)*x/sqrt(b*x^2 + a)) + (40*B*b^3*x^5 + 48*A*b^3*x^4 + 70*B*a*b^2*x^3 +
 96*A*a*b^2*x^2 + 15*B*a^2*b*x + 48*A*a^2*b)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.30

$$\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\begin{cases}-\frac{Ba^3\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{16b}+\sqrt{a+bx^2}\left(\frac{Aa^2}{5b}+\frac{2Aax^2}{5}+\frac{Abx^4}{5}+\frac{Ba^2x}{16b}+\frac{7Bax^3}{24}+\right. \\ a^{\frac{3}{2}}\left(\frac{Ax^2}{2}+\frac{Bx^3}{3}\right)\end{cases}$$

```
[In] integrate(x*(B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((-B*a**3*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(
b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(16*b) + sqrt(a + b*x**2)*(A*
a**2/(5*b) + 2*A*a*x**2/5 + A*b*x**4/5 + B*a**2*x/(16*b) + 7*B*a*x**3/24 +
B*b*x**5/6), Ne(b, 0)), (a**(3/2)*(A*x**2/2 + B*x**3/3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx = \frac{(bx^2+a)^{\frac{5}{2}}Bx}{6\,b} - \frac{(bx^2+a)^{\frac{3}{2}}Bax}{24\,b} - \frac{\sqrt{bx^2+a}Ba^2x}{16\,b} - \frac{Ba^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}} + \frac{(bx^2+a)^{\frac{5}{2}}A}{5\,b}$$

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] 1/6*(b*x^2 + a)^(5/2)*B*x/b - 1/24*(b*x^2 + a)^(3/2)*B*a*x/b - 1/16*sqrt(b*
x^2 + a)*B*a^2*x/b - 1/16*B*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 1/5*(b*x^2
 + a)^(5/2)*A/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx = \frac{Ba^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}} + \frac{1}{240}\sqrt{bx^2+a}\left(\frac{48\,Aa^2}{b} + \left(\frac{15\,Ba^2}{b} + 2\,(48\,Aa + (35\,Ba + 4\,(5\,Bbx + 6\,Ab)x)x)x\right)x\right)$$

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] 1/16*B*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/240*sqrt(b*x^
2 + a)*(48*A*a^2/b + (15*B*a^2/b + 2*(48*A*a + (35*B*a + 4*(5*B*b*x + 6*A*b
)*x)*x)*x)*x)
```

Mupad [F(-1)]

Timed out.

$$\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx = \int x\,\left(b\,x^2+a\right)^{3/2}\,\left(A+B\,x\right)\,dx$$

```
[In] int(x*(a + b*x^2)^(3/2)*(A + B*x),x)
[Out] int(x*(a + b*x^2)^(3/2)*(A + B*x), x)
```

3.11 $\int (A+Bx) \left(a+bx^2\right)^{3/2} \, dx$

Optimal result 130
Rubi [A] (verified) 130
Mathematica [A] (verified) 132
Maple [A] (verified) 132
Fricas [A] (verification not implemented) 132
Sympy [A] (verification not implemented) 133
Maxima [A] (verification not implemented) 133
Giac [A] (verification not implemented) 134
Mupad [B] (verification not implemented) 134

Optimal result

Integrand size = 17, antiderivative size = 87

$$\int (A+Bx) \left(a+bx^2\right)^{3/2} \, dx = \frac{3}{8}aAx\sqrt{a+bx^2} + \frac{1}{4}Ax\left(a+bx^2\right)^{3/2} + \frac{B(a+bx^2)^{5/2}}{5b} + \frac{3a^2A\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}$$

```
[Out] 1/4*A*x*(b*x^2+a)^(3/2)+1/5*B*(b*x^2+a)^(5/2)/b+3/8*a^2*A*arctanh(x*b^(1/2)
/(b*x^2+a)^(1/2))/b^(1/2)+3/8*a*A*x*(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {655, 201, 223, 212}

$$\int (A+Bx) \left(a+bx^2\right)^{3/2} \, dx = \frac{3a^2A\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}} + \frac{1}{4}Ax\left(a+bx^2\right)^{3/2} + \frac{3}{8}aAx\sqrt{a+bx^2} + \frac{B(a+bx^2)^{5/2}}{5b}$$

```
[In] Int[(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (3*a*A*x*Sqrt[a + b*x^2])/8 + (A*x*(a + b*x^2)^(3/2))/4 + (B*(a + b*x^2)^(5
/2))/(5*b) + (3*a^2*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{B\left(a+bx^2\right)^{5/2}}{5b} + A\int \left(a+bx^2\right)^{3/2}\,dx \\
&= \frac{1}{4}Ax\left(a+bx^2\right)^{3/2} + \frac{B\left(a+bx^2\right)^{5/2}}{5b} + \frac{1}{4}(3aA)\int \sqrt{a+bx^2}\,dx \\
&= \frac{3}{8}aAx\sqrt{a+bx^2} + \frac{1}{4}Ax\left(a+bx^2\right)^{3/2} + \frac{B\left(a+bx^2\right)^{5/2}}{5b} + \frac{1}{8}\left(3a^2A\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx \\
&= \frac{3}{8}aAx\sqrt{a+bx^2} + \frac{1}{4}Ax\left(a+bx^2\right)^{3/2} + \frac{B\left(a+bx^2\right)^{5/2}}{5b} \\
&\quad + \frac{1}{8}\left(3a^2A\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right) \\
&= \frac{3}{8}aAx\sqrt{a+bx^2} + \frac{1}{4}Ax\left(a+bx^2\right)^{3/2} + \frac{B\left(a+bx^2\right)^{5/2}}{5b} + \frac{3a^2A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\frac{\sqrt{a+bx^2}(8a^2B+2b^2x^3(5A+4Bx)+abx(25A+16Bx))-15a^2A\sqrt{b}\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{40b}$$

```
[In] Integrate[(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*a^2*B + 2*b^2*x^3*(5*A + 4*B*x) + a*b*x*(25*A + 16*B*x)
) - 15*a^2*A*Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(40*b)
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

method	result	size
default	$A\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+\frac{B(b\,x^2+a)^{\frac{5}{2}}}{5b}$	70
risch	$\frac{(8b^2B\,x^4+10A\,b^2x^3+16Bab\,x^2+25aAbx+8a^2B)\sqrt{b\,x^2+a}}{40b}+\frac{3a^2A\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{8\sqrt{b}}$	80

```
[In] int((B*x+A)*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^
(1/2)+(b*x^2+a)^(1/2))))+1/5*B*(b*x^2+a)^(5/2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.02

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\left[\frac{15\,Aa^2\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(8\,Bb^2x^4+10\,Ab^2x^3+16\,Babx^2+25\,Aab\right.$$

$$\left.-\frac{15\,Aa^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(8\,Bb^2x^4+10\,Ab^2x^3+16\,Babx^2+25\,Aabx+8\,Ba^2)\sqrt{bx^2+a}}{40\,b}\right]$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/80*(15*A*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
*(8*B*b^2*x^4 + 10*A*b^2*x^3 + 16*B*a*b*x^2 + 25*A*a*b*x + 8*B*a^2)*sqrt(b*
x^2 + a))/b, -1/40*(15*A*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) -
(8*B*b^2*x^4 + 10*A*b^2*x^3 + 16*B*a*b*x^2 + 25*A*a*b*x + 8*B*a^2)*sqrt(b*x
^2 + a))/b]
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.37

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\begin{cases}\frac{3Aa^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log\left(x\right)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8}+\sqrt{a+bx^2}\cdot\left(\frac{5Aax}{8}+\frac{Abx^3}{4}+\frac{Ba^2}{5b}+\frac{2Bax^2}{5}+\frac{Bbx^4}{5}\right)\\ a^{\frac{3}{2}}\left(Ax+\frac{Bx^2}{2}\right)\end{cases}$$

```
[In] integrate((B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] Piecewise((3*A*a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt
(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/8 + sqrt(a + b*x**2)*(5*A*a*
x/8 + A*b*x**3/4 + B*a**2/(5*b) + 2*B*a*x**2/5 + B*b*x**4/5), Ne(b, 0)), (a
**(3/2)*(A*x + B*x**2/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Ax+\frac{3}{8}\sqrt{bx^2+a}Aax+\frac{3\,Aa^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{b}}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}B}{5\,b}$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*A*x + 3/8*sqrt(b*x^2 + a)*A*a*x + 3/8*A*a^2*arcsinh(b
*x/sqrt(a*b))/sqrt(b) + 1/5*(b*x^2 + a)^(5/2)*B/b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx = -\frac{3\,Aa^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}} + \frac{1}{40}\,\sqrt{bx^2+a}\left(\frac{8\,Ba^2}{b}+(25\,Aa+2\,(8\,Ba+(4\,Bbx+5\,Ab)x)x)x\right)$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -3/8*A*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/40*sqrt(b*x^2
 + a)*(8*B*a^2/b + (25*A*a + 2*(8*B*a + (4*B*b*x + 5*A*b)*x)*x)*x)
```

Mupad [B] (verification not implemented)

Time = 6.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.62

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx = \frac{B\,(b\,x^2+a)^{5/2}}{5\,b} + \frac{A\,x\,(b\,x^2+a)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

```
[In] int((a + b*x^2)^(3/2)*(A + B*x),x)
```

```
[Out] (B*(a + b*x^2)^(5/2))/(5*b) + (A*x*(a + b*x^2)^(3/2)*hypergeom([-3/2, 1/2],
 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^(3/2)
```

3.12 $\int \frac{(A+Bx)(a+bx^2)^{3/2}}{x} \, dx$

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Optimal result

Integrand size = 20, antiderivative size = 106

$$\int \frac{(A+Bx)(a+bx^2)^{3/2}}{x} \, dx = \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2} + \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \frac{3a^2B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}} - a^{3/2}A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] 1/12*(3*B*x+4*A)*(b*x^2+a)^(3/2)-a^(3/2)*A*arctanh((b*x^2+a)^(1/2)/a^(1/2))
+3/8*a^2*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+1/8*a*(3*B*x+8*A)*(b*
x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {829, 858, 223, 212, 272, 65, 214}

$$\int \frac{(A+Bx)(a+bx^2)^{3/2}}{x} \, dx = -a^{3/2}A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) + \frac{3a^2B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}} + \frac{1}{8}a\sqrt{a+bx^2}(8A+3Bx) + \frac{1}{12}\left(a+bx^2\right)^{3/2}(4A+3Bx)$$

```
[In] Int[((A + B*x)*(a + b*x^2)^(3/2))/x,x]
```

```
[Out] (a*(8*A + 3*B*x)*Sqrt[a + b*x^2])/8 + ((4*A + 3*B*x)*(a + b*x^2)^(3/2))/12
+ (3*a^2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b]) - a^(3/2)*A*Ar
cTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2}+\frac{\int \frac{(4aAb+3abBx)\sqrt{a+bx^2}}{x}\,dx}{4b}\\
&= \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2}+\frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2}+\frac{\int \frac{8a^2Ab^2+3a^2b^2Bx}{x\sqrt{a+bx^2}}\,dx}{8b^2}\\
&= \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2}+\frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2}\\
&\quad+\left(a^2A\right)\int \frac{1}{x\sqrt{a+bx^2}}\,dx+\frac{1}{8}\left(3a^2B\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx\\
&= \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2}+\frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2}\\
&\quad+\frac{1}{2}\left(a^2A\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)\\
&\quad+\frac{1}{8}\left(3a^2B\right)\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\
&= \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2}+\frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2}\\
&\quad+\frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{\left(a^2A\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{b}\\
&= \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2}+\frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2}\\
&\quad+\frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}-a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.04

$$
\begin{aligned}
\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x}\,dx &= 2a^{3/2}A\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)\\
&\quad+\frac{1}{24}\left(\sqrt{a+bx^2}\left(32aA+15aBx+8Abx^2+6bBx^3\right)-\frac{9a^2B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}\right)
\end{aligned}
$$

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(3/2))/x,x]
[Out] 2*a^(3/2)*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] + (Sqrt[a + b*x^
2]*(32*a*A + 15*a*B*x + 8*A*b*x^2 + 6*b*B*x^3) - (9*a^2*B*Log[-(Sqrt[b]*x)
+ Sqrt[a + b*x^2]])/Sqrt[b])/24
```

Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.03

method	result
default	$B\left(\frac{x(b\,x^{2}+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)+A\left(\frac{(b\,x^{2}+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x}}{x}\right.\right.\right.$

```
[In] int((B*x+A)*(b*x^2+a)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^
(1/2)+(b*x^2+a)^(1/2))))+A*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*
ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 439, normalized size of antiderivative = 4.14

$$\int\frac{(A+Bx)\,(a+bx^2)^{3/2}}{x}\,dx=\left[\frac{9\,Ba^2\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+24\,Aa^{\frac{3}{2}}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}}{x^2}\right.}{48\,b}\right.$$

$$-\frac{9\,Ba^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-12\,Aa^{\frac{3}{2}}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-(6\,Bb^2x^3+8\,Ab^2x^2+15\,Babx+32\,Aa}{24\,b}$$

$$-\frac{9\,Ba^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-24\,A\sqrt{-a}ab\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-(6\,Bb^2x^3+8\,Ab^2x^2+15\,Babx+32\,Aab)\sqrt{bx}}{24\,b}$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/48*(9*B*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 24
*A*a^(3/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(6*B*b
^2*x^3 + 8*A*b^2*x^2 + 15*B*a*b*x + 32*A*a*b)*sqrt(b*x^2 + a))/b, -1/24*(9*
B*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 12*A*a^(3/2)*b*log(-(b*
x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - (6*B*b^2*x^3 + 8*A*b^2*x^2 +
15*B*a*b*x + 32*A*a*b)*sqrt(b*x^2 + a))/b, 1/48*(48*A*sqrt(-a)*a*b*arctan(s
qrt(-a)/sqrt(b*x^2 + a)) + 9*B*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)
*sqrt(b)*x - a) + 2*(6*B*b^2*x^3 + 8*A*b^2*x^2 + 15*B*a*b*x + 32*A*a*b)*sqr
t(b*x^2 + a))/b, -1/24*(9*B*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a))
 - 24*A*sqrt(-a)*a*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (6*B*b^2*x^3 + 8*A*
b^2*x^2 + 15*B*a*b*x + 32*A*a*b)*sqrt(b*x^2 + a))/b]
```

Sympy [A] (verification not implemented)

Time = 4.26 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.58

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x}\,dx = -Aa^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)+\frac{Aa^2}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} + \frac{Aa\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}+Ab\left(\begin{cases}\frac{a\sqrt{a+bx^2}}{3b}+\frac{x^2\sqrt{a+bx^2}}{3} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^2}{2} & \text{otherwise}\end{cases}\right) + Ba\left(\begin{cases}\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x\sqrt{a+bx^2}}{2} & \text{for } b\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right) + Bb\left(\begin{cases}-\frac{a^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b}+\frac{ax\sqrt{a+bx^2}}{8b}+\frac{x^3\sqrt{a+bx^2}}{4} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^3}{3} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((B*x+A)*(b*x**2+a)**(3/2)/x,x)
[Out] -A*a**(3/2)*asinh(sqrt(a)/(sqrt(b)*x)) + A*a**2/(sqrt(b)*x*sqrt(a/(b*x**2)
+ 1)) + A*a*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + A*b*Piecewise((a*sqrt(a + b*x*
*2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True)) + B
*a*Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b),
 Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b,
0)), (sqrt(a)*x, True)) + B*b*Piecewise((-a**2*Piecewise((log(2*sqrt(b)*sqr
t(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(
8*b) + a*x*sqrt(a + b*x**2)/(8*b) + x**3*sqrt(a + b*x**2)/4, Ne(b, 0)), (sq
rt(a)*x**3/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.83

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x}\,dx = \frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Bx+\frac{3}{8}\sqrt{bx^2+a}Bax + \frac{3\,Ba^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{b}}-Aa^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)+\frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}A+\sqrt{bx^2+a}Aa$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*B*x + 3/8*sqrt(b*x^2 + a)*B*a*x + 3/8*B*a^2*arcsinh(b
*x/sqrt(a*b))/sqrt(b) - A*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/3*(b*x^
2 + a)^(3/2)*A + sqrt(b*x^2 + a)*A*a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 6.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x}\,dx = \frac{A\left(b\,x^2+a\right)^{3/2}}{3} - A\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right) + A\,a\,\sqrt{b\,x^2+a} + \frac{B\,x\left(b\,x^2+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

```
[In] int(((a + b*x^2)^(3/2)*(A + B*x))/x,x)
```

```
[Out] (A*(a + b*x^2)^(3/2))/3 - A*a^(3/2)*atanh((a + b*x^2)^(1/2)/a^(1/2)) + A*a*
(a + b*x^2)^(1/2) + (B*x*(a + b*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(b*
x^2)/a))/((b*x^2)/a + 1)^(3/2)
```

3.13 $\int \frac{(A+Bx)(a+bx^2)^{3/2}}{x^2}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 108

$$\int \frac{(A+Bx)(a+bx^2)^{3/2}}{x^2}\,dx = \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2}-\frac{(3A-Bx)(a+bx^2)^{3/2}}{3x} + \frac{3}{2}aA\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-a^{3/2}B\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] -1/3*(-B*x+3*A)*(b*x^2+a)^(3/2)/x-a^(3/2)*B*arctanh((b*x^2+a)^(1/2)/a^(1/2)
)+3/2*a*A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)+1/2*(3*A*b*x+2*B*a)*(b
*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {827, 829, 858, 223, 212, 272, 65, 214}

$$\int \frac{(A+Bx)(a+bx^2)^{3/2}}{x^2}\,dx = a^{3/2}(-B)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) + \frac{3}{2}aA\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{(a+bx^2)^{3/2}(3A-Bx)}{3x}+\frac{1}{2}\sqrt{a+bx^2}(2aB+3Abx)$$

```
[In] Int[((A + B*x)*(a + b*x^2)^(3/2))/x^2,x]
```

```
[Out] ((2*a*B + 3*A*b*x)*Sqrt[a + b*x^2])/2 - ((3*A - B*x)*(a + b*x^2)^(3/2))/(3*
x) + (3*a*A*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2 - a^(3/2)*B*Arc
Tanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
```

```
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} - \frac{1}{2}\int \frac{(-2aB-6Abx)\sqrt{a+bx^2}}{x}\,dx$$

$$= \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2} - \frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} - \frac{\int \frac{-4a^2bB-6aAb^2x}{x\sqrt{a+bx^2}}\,dx}{4b}$$

$$= \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2} - \frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} + \frac{1}{2}(3aAb)\int \frac{1}{\sqrt{a+bx^2}}\,dx + \left(a^2B\right)\int \frac{1}{x\sqrt{a+bx^2}}\,dx$$

$$= \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2} - \frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} + \frac{1}{2}(3aAb)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right) + \frac{1}{2}\left(a^2B\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2} - \frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} + \frac{3}{2}aA\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) + \frac{\left(a^2B\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b}$$

$$= \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2} - \frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} + \frac{3}{2}aA\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - a^{3/2}B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx=\frac{\sqrt{a+bx^2}\left(bx^2(3A+2Bx)+a(-6A+8Bx)\right)}{6x}+2a^{3/2}B\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{3}{2}aA\sqrt{b}\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(3/2))/x^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*(b*x^2*(3*A + 2*B*x) + a*(-6*A + 8*B*x)))/(6*x) + 2*a^(3/2
)*B*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] - (3*a*A*Sqrt[b]*Log[-(S
qrt[b]*x) + Sqrt[a + b*x^2]])/2
```

Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{aA\sqrt{b\,x^2+a}}{x}+\frac{Bb\,x^2\sqrt{b\,x^2+a}}{3}+\frac{4aB\sqrt{b\,x^2+a}}{3}+\frac{bAx\sqrt{b\,x^2+a}}{2}+\frac{3a\sqrt{b}\,A\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2}-a^{\frac{3}{2}}B\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2}}{x}\right.$
default	$B\left(\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)\right)+A\left(-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{ax}+\frac{4b\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2}}{2}\right.}{}\right.}{}\right.$

```
[In] int((B*x+A)*(b*x^2+a)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a*A*(b*x^2+a)^(1/2)/x+1/3*B*b*x^2*(b*x^2+a)^(1/2)+4/3*a*B*(b*x^2+a)^(1/2)+
1/2*b*A*x*(b*x^2+a)^(1/2)+3/2*a*b^(1/2)*A*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-a^(
3/2)*B*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 411, normalized size of antiderivative = 3.81

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx = \left[\frac{9\,Aa\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+6\,Ba^{\frac{3}{2}}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{\cdots}}{x^2}\right.}{12\,x}\right.$$

$$-\frac{9\,Aa\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-3\,Ba^{\frac{3}{2}}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-\left(2\,Bbx^3+3\,Abx^2+8\,Bax-6\,Aa\right)\sqrt{b\cdots}}{6\,x}$$

$$-\frac{9\,Aa\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-6\,B\sqrt{-a}ax\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(2\,Bbx^3+3\,Abx^2+8\,Bax-6\,Aa\right)\sqrt{bx^2+\cdots}}{6\,x}$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/12*(9*A*a*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 6*
B*a^(3/2)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*B*b*
x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*sqrt(b*x^2 + a))/x, -1/6*(9*A*a*sqrt(-b)
*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*B*a^(3/2)*x*log(-(b*x^2 - 2*sqrt(
b*x^2 + a)*sqrt(a) + 2*a)/x^2) - (2*B*b*x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*
sqrt(b*x^2 + a))/x, 1/12*(12*B*sqrt(-a)*a*x*arctan(sqrt(-a)/sqrt(b*x^2 + a)
) + 9*A*a*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*
B*b*x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*sqrt(b*x^2 + a))/x, -1/6*(9*A*a*sqrt
(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 6*B*sqrt(-a)*a*x*arctan(sqrt(-a
)/sqrt(b*x^2 + a)) - (2*B*b*x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*sqrt(b*x^2 +
 a))/x]
```

Sympy [A] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.25

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx = -\frac{Aa^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{A\sqrt{a}bx}{\sqrt{1+\frac{bx^2}{a}}} + Aa\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + Ab\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for}\ b\neq 0 \\ \sqrt{a}x & \text{otherwise} \end{cases}\right) - Ba^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right) + \frac{Ba^2}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} + \frac{Ba\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}} + Bb\left(\begin{cases} \frac{a\sqrt{a+bx^2}}{3b} + \frac{x^2\sqrt{a+bx^2}}{3} & \text{for}\ b\neq 0 \\ \frac{\sqrt{a}x^2}{2} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((B*x+A)*(b*x**2+a)**(3/2)/x**2,x)
```

```
[Out] -A*a**(3/2)/(x*sqrt(1 + b*x**2/a)) - A*sqrt(a)*b*x/sqrt(1 + b*x**2/a) + A*a
*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) + A*b*Piecewise((a*Piecewise((log(2*sqrt(
b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), Tr
ue))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) - B*a**(3/2)*a
sinh(sqrt(a)/(sqrt(b)*x)) + B*a**2/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + B*a*s
qrt(b)*x/sqrt(a/(b*x**2) + 1) + B*b*Piecewise((a*sqrt(a + b*x**2)/(3*b) + x
**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx = \frac{3}{2}\sqrt{bx^2+a}Abx + \frac{3}{2}Aa\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - Ba^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}B + \sqrt{bx^2+a}Ba - \frac{\left(bx^2+a\right)^{\frac{3}{2}}A}{x}$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(b*x^2 + a)*A*b*x + 3/2*A*a*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - B*a^(3
/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/3*(b*x^2 + a)^(3/2)*B + sqrt(b*x^2 +
a)*B*a - (b*x^2 + a)^(3/2)*A/x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx = \frac{2\,Ba^2\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{3}{2}\,Aa\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right) + \frac{2\,Aa^2\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a} + \frac{1}{6}\,\sqrt{bx^2+a}(8\,Ba+(2\,Bbx+3\,Ab)x)$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*B*a^2*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 3/2*A*a*
sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 2*A*a^2*sqrt(b)/((sqrt(b)*
x - sqrt(b*x^2 + a))^2 - a) + 1/6*sqrt(b*x^2 + a)*(8*B*a + (2*B*b*x + 3*A*b
)*x)
```

Mupad [B] (verification not implemented)

Time = 6.73 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx = \frac{B\,\left(b\,x^2+a\right)^{3/2}}{3} - B\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right) + B\,a\,\sqrt{b\,x^2+a} - \frac{A\,\left(b\,x^2+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

```
[In] int(((a + b*x^2)^(3/2)*(A + B*x))/x^2,x)
```

```
[Out] (B*(a + b*x^2)^(3/2))/3 - B*a^(3/2)*atanh((a + b*x^2)^(1/2)/a^(1/2)) + B*a*
(a + b*x^2)^(1/2) - (A*(a + b*x^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -(b*x
^2)/a))/(x*((b*x^2)/a + 1)^(3/2))
```

3.14 $\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^3}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 111

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^3}\,dx = -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x} - \frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2} + \frac{3}{2}a\sqrt{b}B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \frac{3}{2}\sqrt{a}Ab\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] -1/2*(-B*x+A)*(b*x^2+a)^(3/2)/x^2-3/2*A*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))*
a^(1/2)+3/2*a*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)-3/2*(-A*b*x+B*a)
*(b*x^2+a)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {827, 858, 223, 212, 272, 65, 214}

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^3}\,dx = -\frac{3}{2}\sqrt{a}Ab\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right) - \frac{\left(a+bx^2\right)^{3/2}(A-Bx)}{2x^2} - \frac{3\sqrt{a+bx^2}(aB-Abx)}{2x} + \frac{3}{2}a\sqrt{b}B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

```
[In] Int[((A + B*x)*(a + b*x^2)^(3/2))/x^3,x]
```

```
[Out] (-3*(a*B - A*b*x)*Sqrt[a + b*x^2])/(2*x) - ((A - B*x)*(a + b*x^2)^(3/2))/(2
*x^2) + (3*a*Sqrt[b]*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2 - (3*Sqrt[a]
*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2} - \frac{3}{8}\int \frac{(-4aB-4Abx)\sqrt{a+bx^2}}{x^2}\,dx$$
$$= -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x} - \frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2} + \frac{3}{16}\int \frac{8aAb+8abBx}{x\sqrt{a+bx^2}}\,dx$$
$$= -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x} - \frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2} + \frac{1}{2}(3aAb)\int \frac{1}{x\sqrt{a+bx^2}}\,dx + \frac{1}{2}(3abB)\int \frac{1}{\sqrt{a+bx^2}}\,dx$$
$$= -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x} - \frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2} + \frac{1}{4}(3aAb)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right) + \frac{1}{2}(3abB)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)$$
$$= -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x} - \frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2} + \frac{3}{2}a\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) + \frac{1}{2}(3aA)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)$$
$$= -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x} - \frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2} + \frac{3}{2}a\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \frac{3}{2}\sqrt{a}Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^3}\,dx = \frac{1}{2}\left(\frac{\sqrt{a+bx^2}\left(bx^2(2A+Bx)-a(A+2Bx)\right)}{x^2} + 6\sqrt{a}Ab\,\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right) - 3a\sqrt{b}B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)\right)$$

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(3/2))/x^3,x]
[Out] ((Sqrt[a + b*x^2]*(b*x^2*(2*A + B*x) - a*(A + 2*B*x)))/x^2 + 6*Sqrt[a]*A*b*
ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] - 3*a*Sqrt[b]*B*Log[-(Sqrt[b
]*x) + Sqrt[a + b*x^2]])/2
```

Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{a\sqrt{b\,x^{2}+a}\,(2Bx+A)}{2x^{2}}+\frac{3\sqrt{b}\,Ba\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2}+\frac{bBx\sqrt{b\,x^{2}+a}}{2}+bA\sqrt{b\,x^{2}+a}-\frac{3bA\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2}$
default	$B\left(-\frac{(b\,x^{2}+a)^{\frac{5}{2}}}{ax}+\frac{4b\left(\frac{x(b\,x^{2}+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{a}\right)+A\left(-\frac{(b\,x^{2}+a)^{\frac{5}{2}}}{2a\,x^{2}}+\frac{3b\left(\frac{(b\,x^{2}+a)^{\frac{3}{2}}}{3}+a\left(\right.\right.}{}\right.$

```
[In] int((B*x+A)*(b*x^2+a)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a*(b*x^2+a)^(1/2)*(2*B*x+A)/x^2+3/2*b^(1/2)*B*a*ln(x*b^(1/2)+(b*x^2+a)
^(1/2))+1/2*b*B*x*(b*x^2+a)^(1/2)+b*A*(b*x^2+a)^(1/2)-3/2*b*A*a^(1/2)*ln((2
*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 425, normalized size of antiderivative = 3.83

$$\int\frac{(A+Bx)\,(a+bx^2)^{3/2}}{x^3}\,dx=\left[\frac{3\,Ba\sqrt{b}x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+3\,A\sqrt{a}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2}}{x}\right.}{4\,x^2}\right.$$

$$-\frac{6\,Ba\sqrt{-b}x^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-3\,A\sqrt{a}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(Bbx^3+2\,Abx^2-2\,Bax-Aa)\sqrt{}}{4\,x^2}$$

$$-\frac{3\,Ba\sqrt{-b}x^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-3\,A\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-(Bbx^3+2\,Abx^2-2\,Bax-Aa)\sqrt{bx^2+a}}{2\,x^2}$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*B*a*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 3
*A*sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(B
*b*x^3 + 2*A*b*x^2 - 2*B*a*x - A*a)*sqrt(b*x^2 + a))/x^2, -1/4*(6*B*a*sqrt(
-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*A*sqrt(a)*b*x^2*log(-(b*x^2
- 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(B*b*x^3 + 2*A*b*x^2 - 2*B*a*x
- A*a)*sqrt(b*x^2 + a))/x^2, 1/4*(6*A*sqrt(-a)*b*x^2*arctan(sqrt(-a)/sqrt(b
*x^2 + a)) + 3*B*a*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
```

```
a) + 2*(B*b*x^3 + 2*A*b*x^2 - 2*B*a*x - A*a)*sqrt(b*x^2 + a))/x^2, -1/2*(3
*B*a*sqrt(-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*A*sqrt(-a)*b*x^2*a
rctan(sqrt(-a)/sqrt(b*x^2 + a)) - (B*b*x^3 + 2*A*b*x^2 - 2*B*a*x - A*a)*sqr
t(b*x^2 + a))/x^2]
```

Sympy [A] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.02

$$\int \frac{(A+Bx)(a+bx^2)^{3/2}}{x^3}\,dx = -\frac{3A\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2} - \frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x} + \frac{Aa\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}} + \frac{Ab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}} - \frac{Ba^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{B\sqrt{a}bx}{\sqrt{1+\frac{bx^2}{a}}} + Ba\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + Bb\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a \neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for } b \neq 0 \\ \sqrt{a}x & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((B*x+A)*(b*x**2+a)**(3/2)/x**3,x)
[Out] -3*A*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - A*a*sqrt(b)*sqrt(a/(b*x**2) +
 1)/(2*x) + A*a*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + A*b**(3/2)*x/sqrt(a/(b*x
**2) + 1) - B*a**(3/2)/(x*sqrt(1 + b*x**2/a)) - B*sqrt(a)*b*x/sqrt(1 + b*x*
*2/a) + B*a*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) + B*b*Piecewise((a*Piecewise((
log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(
b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx)(a+bx^2)^{3/2}}{x^3}\,dx = \frac{3}{2}\sqrt{bx^2+a}Bbx + \frac{3}{2}Ba\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{3}{2}A\sqrt{a}b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{3}{2}\sqrt{bx^2+a}Ab + \frac{(bx^2+a)^{\frac{3}{2}}Ab}{2a} - \frac{(bx^2+a)^{\frac{3}{2}}B}{x} - \frac{(bx^2+a)^{\frac{5}{2}}A}{2ax^2}$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(b*x^2 + a)*B*b*x + 3/2*B*a*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - 3/2*A*
sqrt(a)*b*arcsinh(a/(sqrt(a*b)*abs(x))) + 3/2*sqrt(b*x^2 + a)*A*b + 1/2*(b*
x^2 + a)^(3/2)*A*b/a - (b*x^2 + a)^(3/2)*B/x - 1/2*(b*x^2 + a)^(5/2)*A/(a*x
^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(88) = 176$.

Time = 0.31 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.72

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^3}\,dx = \frac{3\,Aab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{3}{2}\,Ba\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right) + \frac{1}{2}\,(Bbx+2\,Ab)\sqrt{bx^2+a} + \frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3 Aab + 2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 Ba^2\sqrt{b} + \left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aa^2b - 2\,Ba^3\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2}$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 3*A*a*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 3/2*B*a*
sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 1/2*(B*b*x + 2*A*b)*sqrt(b
*x^2 + a) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a*b + 2*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*B*a^2*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a^2*b - 2*B*a^3
*sqrt(b))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2
```

Mupad [B] (verification not implemented)

Time = 7.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^3}\,dx = A\,b\,\sqrt{b\,x^2+a} - \frac{A\,a\,\sqrt{b\,x^2+a}}{2\,x^2} - \frac{3\,A\,\sqrt{a}\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2} - \frac{B\,\left(b\,x^2+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

```
[In] int(((a + b*x^2)^(3/2)*(A + B*x))/x^3,x)
```

```
[Out] A*b*(a + b*x^2)^(1/2) - (A*a*(a + b*x^2)^(1/2))/(2*x^2) - (3*A*a^(1/2)*b*at
anh((a + b*x^2)^(1/2)/a^(1/2)))/2 - (B*(a + b*x^2)^(3/2)*hypergeom([-3/2, -
1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/a + 1)^(3/2))
```

3.15 $\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 173

$$\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\frac{3a^4Bx\sqrt{a+bx^2}}{256b^2}+\frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2}+\frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b}+\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}-\frac{a(160A+189Bx)\left(a+bx^2\right)^{7/2}}{5040b^2}+\frac{3a^5B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}$$

```
[Out] 1/128*a^3*B*x*(b*x^2+a)^(3/2)/b^2+1/160*a^2*B*x*(b*x^2+a)^(5/2)/b^2+1/9*A*x
^2*(b*x^2+a)^(7/2)/b+1/10*B*x^3*(b*x^2+a)^(7/2)/b-1/5040*a*(189*B*x+160*A)*
(b*x^2+a)^(7/2)/b^2+3/256*a^5*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)+
3/256*a^4*B*x*(b*x^2+a)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {847, 794, 201, 223, 212}

$$\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\frac{3a^5B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}+\frac{3a^4Bx\sqrt{a+bx^2}}{256b^2}+\frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2}-\frac{a\left(a+bx^2\right)^{7/2}(160A+189Bx)}{5040b^2}+\frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b}+\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}$$

```
[In] Int[x^3*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (3*a^4*B*x*Sqrt[a + b*x^2])/(256*b^2) + (a^3*B*x*(a + b*x^2)^(3/2))/(128*b^
2) + (a^2*B*x*(a + b*x^2)^(5/2))/(160*b^2) + (A*x^2*(a + b*x^2)^(7/2))/(9*b
) + (B*x^3*(a + b*x^2)^(7/2))/(10*b) - (a*(160*A + 189*B*x)*(a + b*x^2)^(7/
2))/(5040*b^2) + (3*a^5*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(5/2
))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\text{integral} = \frac{Bx^3(a+bx^2)^{7/2}}{10b} + \frac{\int x^2(-3aB+10Abx)\left(a+bx^2\right)^{5/2}\,dx}{10b}$$

$$= \frac{Ax^2(a+bx^2)^{7/2}}{9b} + \frac{Bx^3(a+bx^2)^{7/2}}{10b} + \frac{\int x(-20aAb-27abBx)\left(a+bx^2\right)^{5/2}\,dx}{90b^2}$$

$$= \frac{Ax^2(a+bx^2)^{7/2}}{9b} + \frac{Bx^3(a+bx^2)^{7/2}}{10b} - \frac{a(160A+189Bx)\left(a+bx^2\right)^{7/2}}{5040b^2} + \frac{(3a^2B)\int\left(a+bx^2\right)^{5/2}\,dx}{80b^2}$$

$$= \frac{a^2Bx(a+bx^2)^{5/2}}{160b^2} + \frac{Ax^2(a+bx^2)^{7/2}}{9b} + \frac{Bx^3(a+bx^2)^{7/2}}{10b} - \frac{a(160A+189Bx)\left(a+bx^2\right)^{7/2}}{5040b^2} + \frac{(a^3B)\int\left(a+bx^2\right)^{3/2}\,dx}{32b^2}$$

$$= \frac{a^3Bx(a+bx^2)^{3/2}}{128b^2} + \frac{a^2Bx(a+bx^2)^{5/2}}{160b^2} + \frac{Ax^2(a+bx^2)^{7/2}}{9b} + \frac{Bx^3(a+bx^2)^{7/2}}{10b} - \frac{a(160A+189Bx)\left(a+bx^2\right)^{7/2}}{5040b^2} + \frac{(3a^4B)\int\sqrt{a+bx^2}\,dx}{128b^2}$$

$$= \frac{3a^4Bx\sqrt{a+bx^2}}{256b^2} + \frac{a^3Bx(a+bx^2)^{3/2}}{128b^2} + \frac{a^2Bx(a+bx^2)^{5/2}}{160b^2} + \frac{Ax^2(a+bx^2)^{7/2}}{9b} + \frac{Bx^3(a+bx^2)^{7/2}}{10b} - \frac{a(160A+189Bx)\left(a+bx^2\right)^{7/2}}{5040b^2} + \frac{(3a^5B)\int\frac{1}{\sqrt{a+bx^2}}\,dx}{256b^2}$$

$$= \frac{3a^4Bx\sqrt{a+bx^2}}{256b^2} + \frac{a^3Bx(a+bx^2)^{3/2}}{128b^2} + \frac{a^2Bx(a+bx^2)^{5/2}}{160b^2} + \frac{Ax^2(a+bx^2)^{7/2}}{9b} + \frac{Bx^3(a+bx^2)^{7/2}}{10b} - \frac{a(160A+189Bx)\left(a+bx^2\right)^{7/2}}{5040b^2} + \frac{(3a^5B)\,\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{256b^2}$$

$$= \frac{3a^4Bx\sqrt{a+bx^2}}{256b^2} + \frac{a^3Bx(a+bx^2)^{3/2}}{128b^2} + \frac{a^2Bx(a+bx^2)^{5/2}}{160b^2} + \frac{Ax^2(a+bx^2)^{7/2}}{9b} + \frac{Bx^3(a+bx^2)^{7/2}}{10b} - \frac{a(160A+189Bx)\left(a+bx^2\right)^{7/2}}{5040b^2} + \frac{3a^5B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.79

$$\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\frac{\sqrt{b}\sqrt{a+bx^2}(896b^4x^8(10A+9Bx)+10a^3bx^2(128A+63Bx)-5a^4(512A+189Bx)+24a^2b}{80640b^5}$$

```
[In] Integrate[x^3*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[a + b*x^2]*(896*b^4*x^8*(10*A + 9*B*x) + 10*a^3*b*x^2*(128*A
+ 63*B*x) - 5*a^4*(512*A + 189*B*x) + 24*a^2*b^2*x^4*(800*A + 651*B*x) + 16
*a*b^3*x^6*(1520*A + 1323*B*x)) - 945*a^5*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x
^2]])/(80640*b^(5/2))
```

Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{\left(-8064 B\, b^{4} x^{9}-8960 A\, b^{4} x^{8}-21168 B a\, b^{3} x^{7}-24320 A a\, b^{3} x^{6}-15624 B\, a^{2} b^{2} x^{5}-19200 A\, a^{2} b^{2} x^{4}-630 B\, a^{3} b\, x^{3}-1280 A\, a^{3} b\, x^{2}+945 B\, a\right.}{80640 b^{2}}$
default	$B\left(\frac{x^{3}\left(b\, x^{2}+a\right)^{\frac{7}{2}}}{10 b}-\frac{3 a\left(\frac{x\left(b\, x^{2}+a\right)^{\frac{7}{2}}}{8 b}-\frac{a\left(\frac{x\left(b\, x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5 a\left(\frac{x\left(b\, x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3 a\left(\frac{x\sqrt{b\, x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\, x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)}{8 b}\right)}{10 b}\right)+A\left(\frac{x}{\right.}$

[In] int(x^3*(B*x+A)*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/80640*(-8064*B*b^4*x^9-8960*A*b^4*x^8-21168*B*a*b^3*x^7-24320*A*a*b^3*x^6-15624*B*a^2*b^2*x^5-19200*A*a^2*b^2*x^4-630*B*a^3*b*x^3-1280*A*a^3*b*x^2+945*B*a^4*x+2560*A*a^4)/b^2*(b*x^2+a)^(1/2)+3/256*a^5*B/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.75

$$\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\left[\frac{945\,Ba^5\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(8064\,Bb^5x^9+8960\,Ab^5x^8+21168\,Bab\right.$$

$$-\frac{945\,Ba^5\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(8064\,Bb^5x^9+8960\,Ab^5x^8+21168\,Bab^4x^7+24320\,Aab^4x^6+15624\,Ba^2}{80640\,b^3}$$

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/161280*(945*B*a^5*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a
) + 2*(8064*B*b^5*x^9 + 8960*A*b^5*x^8 + 21168*B*a*b^4*x^7 + 24320*A*a*b^4*
x^6 + 15624*B*a^2*b^3*x^5 + 19200*A*a^2*b^3*x^4 + 630*B*a^3*b^2*x^3 + 1280*
A*a^3*b^2*x^2 - 945*B*a^4*b*x - 2560*A*a^4*b)*sqrt(b*x^2 + a))/b^3, -1/8064
0*(945*B*a^5*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8064*B*b^5*x^9
+ 8960*A*b^5*x^8 + 21168*B*a*b^4*x^7 + 24320*A*a*b^4*x^6 + 15624*B*a^2*b^3*
x^5 + 19200*A*a^2*b^3*x^4 + 630*B*a^3*b^2*x^3 + 1280*A*a^3*b^2*x^2 - 945*B*
a^4*b*x - 2560*A*a^4*b)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.15

$$\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\begin{cases}\frac{3Ba^5\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{256b^2}+\sqrt{a+bx^2}\left(-\frac{2Aa^4}{63b^2}+\frac{Aa^3x^2}{63b}+\frac{5Aa^2x^4}{21}+\frac{19Aabx^6}{63}+A\right. \\ a^{\frac{5}{2}}\left(\frac{Ax^4}{4}+\frac{Bx^5}{5}\right)\end{cases}$$

```
[In] integrate(x**3*(B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((3*B*a**5*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt
(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(256*b**2) + sqrt(a + b*x**2
)*(-2*A*a**4/(63*b**2) + A*a**3*x**2/(63*b) + 5*A*a**2*x**4/21 + 19*A*a*b*x
**6/63 + A*b**2*x**8/9 - 3*B*a**4*x/(256*b**2) + B*a**3*x**3/(128*b) + 31*B
*a**2*x**5/160 + 21*B*a*b*x**7/80 + B*b**2*x**9/10), Ne(b, 0)), (a**(5/2)*(
A*x**4/4 + B*x**5/5), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.84

$$\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\frac{(bx^2+a)^{\frac{7}{2}}Bx^3}{10\,b}+\frac{(bx^2+a)^{\frac{7}{2}}Ax^2}{9\,b}-\frac{3\,(bx^2+a)^{\frac{7}{2}}Bax}{80\,b^2}+\frac{(bx^2+a)^{\frac{5}{2}}Ba^2x}{160\,b^2}+\frac{(bx^2+a)^{\frac{3}{2}}Ba^3x}{128\,b^2}+\frac{3\,\sqrt{bx^2+a}Ba^4x}{256\,b^2}+\frac{3\,Ba^5\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{5}{2}}}-\frac{2\,(bx^2+a)^{\frac{7}{2}}Aa}{63\,b^2}$$

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^2 + a)^(7/2)*B*x^3/b + 1/9*(b*x^2 + a)^(7/2)*A*x^2/b - 3/80*(b*x^
2 + a)^(7/2)*B*a*x/b^2 + 1/160*(b*x^2 + a)^(5/2)*B*a^2*x/b^2 + 1/128*(b*x^2
 + a)^(3/2)*B*a^3*x/b^2 + 3/256*sqrt(b*x^2 + a)*B*a^4*x/b^2 + 3/256*B*a^5*a
rcsinh(b*x/sqrt(a*b))/b^(5/2) - 2/63*(b*x^2 + a)^(7/2)*A*a/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.81

$$\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=-\frac{3\,Ba^5\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{5}{2}}}-\frac{1}{80640}\left(\frac{2560\,Aa^4}{b^2}+\left(\frac{945\,Ba^4}{b^2}-2\left(\frac{640\,Aa^3}{b}+\left(\frac{315\,Ba^3}{b}+4\left(2400\,Aa^2+\left(1953\,Ba^2+2\left(1520\,Aab+7\right.\right.\right.\right.\right.\right.\right.$$

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -3/256*B*a^5*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) - 1/80640*(2560
*A*a^4/b^2 + (945*B*a^4/b^2 - 2*(640*A*a^3/b + (315*B*a^3/b + 4*(2400*A*a^2
 + (1953*B*a^2 + 2*(1520*A*a*b + 7*(189*B*a*b + 8*(9*B*b^2*x + 10*A*b^2)*x)
*x)*x)*x)*x)*x)*x)*x)*sqrt(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\int x^3\,\left(b\,x^2+a\right)^{5/2}\,(A+B\,x)\;dx$$

```
[In] int(x^3*(a + b*x^2)^(5/2)*(A + B*x),x)
[Out] int(x^3*(a + b*x^2)^(5/2)*(A + B*x), x)
```

3.16 $\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 150

$$\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=-\frac{5a^3Ax\sqrt{a+bx^2}}{128b}-\frac{5a^2Ax(a+bx^2)^{3/2}}{192b}-\frac{aAx(a+bx^2)^{5/2}}{48b}+\frac{Bx^2(a+bx^2)^{7/2}}{9b}-\frac{(16aB-63Abx)\left(a+bx^2\right)^{7/2}}{504b^2}-\frac{5a^4A\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}$$

```
[Out] -5/192*a^2*A*x*(b*x^2+a)^(3/2)/b-1/48*a*A*x*(b*x^2+a)^(5/2)/b+1/9*B*x^2*(b*
x^2+a)^(7/2)/b-1/504*(-63*A*b*x+16*B*a)*(b*x^2+a)^(7/2)/b^2-5/128*a^4*A*arc
tanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)-5/128*a^3*A*x*(b*x^2+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {847, 794, 201, 223, 212}

$$\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=-\frac{5a^4A\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}-\frac{5a^3Ax\sqrt{a+bx^2}}{128b}-\frac{5a^2Ax(a+bx^2)^{3/2}}{192b}-\frac{\left(a+bx^2\right)^{7/2}(16aB-63Abx)}{504b^2}-\frac{aAx(a+bx^2)^{5/2}}{48b}+\frac{Bx^2(a+bx^2)^{7/2}}{9b}$$

```
[In] Int[x^2*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (-5*a^3*A*x*Sqrt[a + b*x^2])/(128*b) - (5*a^2*A*x*(a + b*x^2)^(3/2))/(192*b
) - (a*A*x*(a + b*x^2)^(5/2))/(48*b) + (B*x^2*(a + b*x^2)^(7/2))/(9*b) - ((
16*a*B - 63*A*b*x)*(a + b*x^2)^(7/2))/(504*b^2) - (5*a^4*A*ArcTanh[(Sqrt[b]
*x)/Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\text{integral} = \frac{Bx^2(a+bx^2)^{7/2}}{9b} + \frac{\int x(-2aB+9Abx)\,(a+bx^2)^{5/2}\,dx}{9b}$$

$$= \frac{Bx^2(a+bx^2)^{7/2}}{9b} - \frac{(16aB-63Abx)(a+bx^2)^{7/2}}{504b^2} - \frac{(aA)\int (a+bx^2)^{5/2}\,dx}{8b}$$

$$= -\frac{aAx(a+bx^2)^{5/2}}{48b} + \frac{Bx^2(a+bx^2)^{7/2}}{9b} - \frac{(16aB-63Abx)(a+bx^2)^{7/2}}{504b^2} - \frac{(5a^2A)\int (a+bx^2)^{3/2}\,dx}{48b}$$

$$= -\frac{5a^2Ax(a+bx^2)^{3/2}}{192b} - \frac{aAx(a+bx^2)^{5/2}}{48b} + \frac{Bx^2(a+bx^2)^{7/2}}{9b} - \frac{(16aB-63Abx)(a+bx^2)^{7/2}}{504b^2} - \frac{(5a^3A)\int \sqrt{a+bx^2}\,dx}{64b}$$

$$= -\frac{5a^3Ax\sqrt{a+bx^2}}{128b} - \frac{5a^2Ax(a+bx^2)^{3/2}}{192b} - \frac{aAx(a+bx^2)^{5/2}}{48b} + \frac{Bx^2(a+bx^2)^{7/2}}{9b} - \frac{(16aB-63Abx)(a+bx^2)^{7/2}}{504b^2} - \frac{(5a^4A)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{128b}$$

$$= -\frac{5a^3Ax\sqrt{a+bx^2}}{128b} - \frac{5a^2Ax(a+bx^2)^{3/2}}{192b} - \frac{aAx(a+bx^2)^{5/2}}{48b} + \frac{Bx^2(a+bx^2)^{7/2}}{9b} - \frac{(16aB-63Abx)(a+bx^2)^{7/2}}{504b^2} - \frac{(5a^4A)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{128b}$$

$$= -\frac{5a^3Ax\sqrt{a+bx^2}}{128b} - \frac{5a^2Ax(a+bx^2)^{3/2}}{192b} - \frac{aAx(a+bx^2)^{5/2}}{48b} + \frac{Bx^2(a+bx^2)^{7/2}}{9b} - \frac{(16aB-63Abx)(a+bx^2)^{7/2}}{504b^2} - \frac{5a^4A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.83

$$\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx = \frac{\sqrt{a+bx^2}(-256a^4B+112b^4x^7(9A+8Bx)+a^3bx(315A+128Bx)+8ab^3x^5(357A+304Bx)}{8064b^2}$$

```
[In] Integrate[x^2*(A + B*x)*(a + b*x^2)^(5/2),x]
[Out] (Sqrt[a + b*x^2]*(-256*a^4*B + 112*b^4*x^7*(9*A + 8*B*x) + a^3*b*x*(315*A +
 128*B*x) + 8*a*b^3*x^5*(357*A + 304*B*x) + 6*a^2*b^2*x^3*(413*A + 320*B*x)
) + 315*a^4*A*Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8064*b^2)
```

Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.85

method	result
default	$B\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{9b}-\frac{2a\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{63b^{2}}\right)+A\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{8b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}}\right.}{2\sqrt{b}}\right.}{4}\right.}{6}\right.}{8b}\right.$
risch	$\frac{\left(896B\,x^{8}b^{4}+1008A\,b^{4}x^{7}+2432B\,x^{6}a\,b^{3}+2856Aa\,b^{3}x^{5}+1920a^{2}B\,b^{2}x^{4}+2478A\,a^{2}b^{2}x^{3}+128B\,a^{3}b\,x^{2}+315A\,a^{3}bx-256B\,a^{4}\right)\sqrt{b\,x^{2}+a}}{8064b^{2}}$

[In] int(x^2*(B*x+A)*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)

[Out] B*(1/9*x^2*(b*x^2+a)^(7/2)/b-2/63*a/b^2*(b*x^2+a)^(7/2))+A*(1/8*x*(b*x^2+a)^(7/2)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.81

$$\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\left[\frac{315\,Aa^4\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(896\,Bb^4x^8+1008\,Ab^4x^7+2432\,Bab^3x}{1}\right.$$

[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="fricas")

[Out] [1/16128*(315*A*a^4*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(896*B*b^4*x^8 + 1008*A*b^4*x^7 + 2432*B*a*b^3*x^6 + 2856*A*a*b^3*x^5 + 1920*B*a^2*b^2*x^4 + 2478*A*a^2*b^2*x^3 + 128*B*a^3*b*x^2 + 315*A*a^3*b*x - 256*B*a^4)*sqrt(b*x^2 + a))/b^2, 1/8064*(315*A*a^4*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (896*B*b^4*x^8 + 1008*A*b^4*x^7 + 2432*B*a*b^3*x^6 + 2856*A*a*b^3*x^5 + 1920*B*a^2*b^2*x^4 + 2478*A*a^2*b^2*x^3 + 128*B*a^3*b*x^2 + 315*A*a^3*b*x - 256*B*a^4)*sqrt(b*x^2 + a))/b^2]

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.23

$$\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\begin{cases}-\frac{5Aa^4\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{128b}+\sqrt{a+bx^2}\cdot\left(\frac{5Aa^3x}{128b}+\frac{59Aa^2x^3}{192}+\frac{17Aabx^5}{48}+\frac{Ab^2x^7}{8}-\right. \\ a^{\frac{5}{2}}\left(\frac{Ax^3}{3}+\frac{Bx^4}{4}\right)\end{cases}$$

```
[In] integrate(x**2*(B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-5*A*a**4*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqr
t(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(128*b) + sqrt(a + b*x**2)*
(5*A*a**3*x/(128*b) + 59*A*a**2*x**3/192 + 17*A*a*b*x**5/48 + A*b**2*x**7/8
 - 2*B*a**4/(63*b**2) + B*a**3*x**2/(63*b) + 5*B*a**2*x**4/21 + 19*B*a*b*x*
*6/63 + B*b**2*x**8/9), Ne(b, 0)), (a**(5/2)*(A*x**3/3 + B*x**4/4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.83

$$\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\frac{\left(bx^2+a\right)^{\frac{7}{2}}Bx^2}{9\,b}+\frac{\left(bx^2+a\right)^{\frac{7}{2}}Ax}{8\,b}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}Aax}{48\,b}-\frac{5\left(bx^2+a\right)^{\frac{3}{2}}Aa^2x}{192\,b}-\frac{5\sqrt{bx^2+a}Aa^3x}{128\,b}-\frac{5\,Aa^4\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}}-\frac{2\left(bx^2+a\right)^{\frac{7}{2}}Ba}{63\,b^2}$$

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*(b*x^2 + a)^(7/2)*B*x^2/b + 1/8*(b*x^2 + a)^(7/2)*A*x/b - 1/48*(b*x^2 +
 a)^(5/2)*A*a*x/b - 5/192*(b*x^2 + a)^(3/2)*A*a^2*x/b - 5/128*sqrt(b*x^2 +
a)*A*a^3*x/b - 5/128*A*a^4*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2/63*(b*x^2 + a
)^(7/2)*B*a/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.85

$$\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\frac{5\,Aa^4\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}}$$
$$-\frac{1}{8064}\left(\frac{256\,Ba^4}{b^2}-\left(\frac{315\,Aa^3}{b}+2\left(\frac{64\,Ba^3}{b}+\left(1239\,Aa^2+4\left(240\,Ba^2+\left(357\,Aab+2\left(152\,Bab+7\left(8\,Bb^2\right.\right.\right.\right.\right.\right.\right.\right.$$

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 5/128*A*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) - 1/8064*(256*B*
a^4/b^2 - (315*A*a^3/b + 2*(64*B*a^3/b + (1239*A*a^2 + 4*(240*B*a^2 + (357*
A*a*b + 2*(152*B*a*b + 7*(8*B*b^2*x + 9*A*b^2)*x)*x)*x)*x)*x)*x)*x)*sqrt(b*
x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\int x^2\,\left(b\,x^2+a\right)^{5/2}\,\left(A+B\,x\right)\,dx$$

```
[In] int(x^2*(a + b*x^2)^(5/2)*(A + B*x),x)
```

```
[Out] int(x^2*(a + b*x^2)^(5/2)*(A + B*x), x)
```

3.17 $\int x(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

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Optimal result

Integrand size = 18, antiderivative size = 126

$$\int x(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=-\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx(a+bx^2)^{3/2}}{192b}-\frac{aBx(a+bx^2)^{5/2}}{48b}+\frac{(8A+7Bx)\left(a+bx^2\right)^{7/2}}{56b}-\frac{5a^4B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}$$

```
[Out] -5/192*a^2*B*x*(b*x^2+a)^(3/2)/b-1/48*a*B*x*(b*x^2+a)^(5/2)/b+1/56*(7*B*x+8
*A)*(b*x^2+a)^(7/2)/b-5/128*a^4*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2
)-5/128*a^3*B*x*(b*x^2+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {794, 201, 223, 212}

$$\int x(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=-\frac{5a^4B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}-\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx(a+bx^2)^{3/2}}{192b}+\frac{\left(a+bx^2\right)^{7/2}(8A+7Bx)}{56b}-\frac{aBx(a+bx^2)^{5/2}}{48b}$$

```
[In] Int[x*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (-5*a^3*B*x*Sqrt[a + b*x^2])/(128*b) - (5*a^2*B*x*(a + b*x^2)^(3/2))/(192*b
) - (a*B*x*(a + b*x^2)^(5/2))/(48*b) + ((8*A + 7*B*x)*(a + b*x^2)^(7/2))/(5
6*b) - (5*a^4*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(8A+7Bx)\left(a+bx^2\right)^{7/2}}{56b}-\frac{(aB)\int\left(a+bx^2\right)^{5/2}\,dx}{8b}\\
&= -\frac{aBx(a+bx^2)^{5/2}}{48b}+\frac{(8A+7Bx)\left(a+bx^2\right)^{7/2}}{56b}-\frac{(5a^2B)\int\left(a+bx^2\right)^{3/2}\,dx}{48b}\\
&= -\frac{5a^2Bx(a+bx^2)^{3/2}}{192b}-\frac{aBx(a+bx^2)^{5/2}}{48b}+\frac{(8A+7Bx)\left(a+bx^2\right)^{7/2}}{56b}-\frac{(5a^3B)\int\sqrt{a+bx^2}\,dx}{64b}\\
&= -\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx(a+bx^2)^{3/2}}{192b}-\frac{aBx(a+bx^2)^{5/2}}{48b}\\
&\quad+\frac{(8A+7Bx)\left(a+bx^2\right)^{7/2}}{56b}-\frac{(5a^4B)\int\frac{1}{\sqrt{a+bx^2}}\,dx}{128b}\\
&= -\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx(a+bx^2)^{3/2}}{192b}-\frac{aBx(a+bx^2)^{5/2}}{48b}\\
&\quad+\frac{(8A+7Bx)\left(a+bx^2\right)^{7/2}}{56b}-\frac{(5a^4B)\,\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{128b}
\end{aligned}
$$

$$= -\frac{5a^3Bx\sqrt{a+bx^2}}{128b} - \frac{5a^2Bx(a+bx^2)^{3/2}}{192b} - \frac{aBx(a+bx^2)^{5/2}}{48b} + \frac{(8A+7Bx)(a+bx^2)^{7/2}}{56b} - \frac{5a^4B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

$$\int x(A+Bx)\left(a+bx^2\right)^{5/2}\,dx = \frac{\sqrt{b}\sqrt{a+bx^2}(48b^3x^6(8A+7Bx)+3a^3(128A+35Bx)+8ab^2x^4(144A+119Bx)+2a^2bx^2(576}{2688b^{3/2}}$$

```
[In] Integrate[x*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[a + b*x^2]*(48*b^3*x^6*(8*A + 7*B*x) + 3*a^3*(128*A + 35*B*x)
 + 8*a*b^2*x^4*(144*A + 119*B*x) + 2*a^2*b*x^2*(576*A + 413*B*x)) + 105*a^4
*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2688*b^(3/2))
```

Maple [A] (verified)

Time = 3.53 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

method	result
default	$B\left(\frac{x(bx^2+a)^{\frac{7}{2}}}{8b}-\frac{a\left(\frac{x(bx^2+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)}{8b}\right)+\frac{A(bx^2+a)^{\frac{7}{2}}}{7b}$
risch	$\frac{(336b^3Bx^7+384x^6b^3A+952Bab^2x^5+1152aAb^2x^4+826Ba^2bx^3+1152a^2Abx^2+105a^3Bx+384a^3A)\sqrt{bx^2+a}}{2688b}-\frac{5Ba^4\ln\left(x\sqrt{b}+\sqrt{b\right.}}{128b^{\frac{3}{2}}}$

```
[In] int(x*(B*x+A)*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/8*x*(b*x^2+a)^(7/2)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x
^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+
a)^(1/2))))))+1/7*A*(b*x^2+a)^(7/2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.01

$$\int x(A+Bx)\left(a+bx^2\right)^{5/2}\,dx = \left[\frac{105\,Ba^4\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(336\,Bb^4x^7+384\,Ab^4x^6+952\,Bab^3x^5+\cdots}{5376\,b^2}\right.$$

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/5376*(105*B*a^4*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(336*B*b^4*x^7 + 384*A*b^4*x^6 + 952*B*a*b^3*x^5 + 1152*A*a*b^3*x^4 + 8
26*B*a^2*b^2*x^3 + 1152*A*a^2*b^2*x^2 + 105*B*a^3*b*x + 384*A*a^3*b)*sqrt(b
*x^2 + a))/b^2, 1/2688*(105*B*a^4*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a
)) + (336*B*b^4*x^7 + 384*A*b^4*x^6 + 952*B*a*b^3*x^5 + 1152*A*a*b^3*x^4 +
826*B*a^2*b^2*x^3 + 1152*A*a^2*b^2*x^2 + 105*B*a^3*b*x + 384*A*a^3*b)*sqrt(
b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.33

$$\int x(A+Bx)\left(a+bx^2\right)^{5/2}\,dx = \begin{cases} -\frac{5Ba^4\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{128b}+\sqrt{a+bx^2}\left(\frac{Aa^3}{7b}+\frac{3Aa^2x^2}{7}+\frac{3Aabx^4}{7}+\frac{Ab^2x^6}{7}+\frac{5Ba\cdots}{128\cdots}\right. \\ a^{\frac{5}{2}}\left(\frac{Ax^2}{2}+\frac{Bx^3}{3}\right) \end{cases}$$

```
[In] integrate(x*(B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((-5*B*a**4*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqr
t(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(128*b) + sqrt(a + b*x**2)*
(A*a**3/(7*b) + 3*A*a**2*x**2/7 + 3*A*a*b*x**4/7 + A*b**2*x**6/7 + 5*B*a**3
*x/(128*b) + 59*B*a**2*x**3/192 + 17*B*a*b*x**5/48 + B*b**2*x**7/8), Ne(b,
0)), (a**(5/2)*(A*x**2/2 + B*x**3/3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.83

$$\int x(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\frac{(bx^2+a)^{\frac{7}{2}}Bx}{8\,b}-\frac{(bx^2+a)^{\frac{5}{2}}Bax}{48\,b}-\frac{5\,(bx^2+a)^{\frac{3}{2}}Ba^2x}{192\,b}-\frac{5\,\sqrt{bx^2+a}Ba^3x}{128\,b}-\frac{5\,Ba^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}}+\frac{(bx^2+a)^{\frac{7}{2}}A}{7\,b}$$

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^(7/2)*B*x/b - 1/48*(b*x^2 + a)^(5/2)*B*a*x/b - 5/192*(b*x^2
 + a)^(3/2)*B*a^2*x/b - 5/128*sqrt(b*x^2 + a)*B*a^3*x/b - 5/128*B*a^4*arcsi
nh(b*x/sqrt(a*b))/b^(3/2) + 1/7*(b*x^2 + a)^(7/2)*A/b
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int x(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\frac{5\,Ba^4\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}}+\frac{1}{2688}\left(\frac{384\,Aa^3}{b}+\left(\frac{105\,Ba^3}{b}+2\left(576\,Aa^2+\left(413\,Ba^2+4\left(144\,Aab+\left(119\,Bab+6\left(7\,Bb^2x+8\,Ab^2\right)x\right)x\right)x\right.\right.\right.\right.$$

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 5/128*B*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/2688*(384*A*
a^3/b + (105*B*a^3/b + 2*(576*A*a^2 + (413*B*a^2 + 4*(144*A*a*b + (119*B*a*
b + 6*(7*B*b^2*x + 8*A*b^2)*x)*x)*x)*x)*x)*x)*sqrt(b*x^2 + a)
```

Mupad [F(-1)]

Timed out.

$$\int x(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\int x\,\left(b\,x^2+a\right)^{5/2}\,\left(A+B\,x\right)\,dx$$

```
[In] int(x*(a + b*x^2)^(5/2)*(A + B*x),x)
```

```
[Out] int(x*(a + b*x^2)^(5/2)*(A + B*x), x)
```

3.18 $\int (A+Bx)\left(a+bx^2\right)^{5/2} dx$

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Optimal result

Integrand size = 17, antiderivative size = 107

$$\int (A+Bx)\left(a+bx^2\right)^{5/2} dx = \frac{5}{16}a^2Ax\sqrt{a+bx^2} + \frac{5}{24}aAx\left(a+bx^2\right)^{3/2} + \frac{1}{6}Ax\left(a+bx^2\right)^{5/2} + \frac{B\left(a+bx^2\right)^{7/2}}{7b} + \frac{5a^3A\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}$$

```
[Out] 5/24*a*A*x*(b*x^2+a)^(3/2)+1/6*A*x*(b*x^2+a)^(5/2)+1/7*B*(b*x^2+a)^(7/2)/b+
5/16*a^3*A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+5/16*a^2*A*x*(b*x^2+a
)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {655, 201, 223, 212}

$$\int (A+Bx)\left(a+bx^2\right)^{5/2} dx = \frac{5a^3A\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}} + \frac{5}{16}a^2Ax\sqrt{a+bx^2} + \frac{1}{6}Ax\left(a+bx^2\right)^{5/2} + \frac{5}{24}aAx\left(a+bx^2\right)^{3/2} + \frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

```
[In] Int[(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (5*a^2*A*x*Sqrt[a + b*x^2])/16 + (5*a*A*x*(a + b*x^2)^(3/2))/24 + (A*x*(a +
 b*x^2)^(5/2))/6 + (B*(a + b*x^2)^(7/2))/(7*b) + (5*a^3*A*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{B(a+bx^2)^{7/2}}{7b} + A\int (a+bx^2)^{5/2}\,dx \\
&= \frac{1}{6}Ax(a+bx^2)^{5/2} + \frac{B(a+bx^2)^{7/2}}{7b} + \frac{1}{6}(5aA)\int (a+bx^2)^{3/2}\,dx \\
&= \frac{5}{24}aAx(a+bx^2)^{3/2} + \frac{1}{6}Ax(a+bx^2)^{5/2} + \frac{B(a+bx^2)^{7/2}}{7b} + \frac{1}{8}(5a^2A)\int \sqrt{a+bx^2}\,dx \\
&= \frac{5}{16}a^2Ax\sqrt{a+bx^2} + \frac{5}{24}aAx(a+bx^2)^{3/2} \\
&\quad + \frac{1}{6}Ax(a+bx^2)^{5/2} + \frac{B(a+bx^2)^{7/2}}{7b} + \frac{1}{16}(5a^3A)\int \frac{1}{\sqrt{a+bx^2}}\,dx \\
&= \frac{5}{16}a^2Ax\sqrt{a+bx^2} + \frac{5}{24}aAx(a+bx^2)^{3/2} + \frac{1}{6}Ax(a+bx^2)^{5/2} \\
&\quad + \frac{B(a+bx^2)^{7/2}}{7b} + \frac{1}{16}(5a^3A)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)
\end{aligned}
$$

$$= \frac{5}{16}a^2Ax\sqrt{a+bx^2} + \frac{5}{24}aAx(a+bx^2)^{3/2} + \frac{1}{6}Ax(a+bx^2)^{5/2} + \frac{B(a+bx^2)^{7/2}}{7b} + \frac{5a^3A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

$$\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx = \frac{\sqrt{a+bx^2}(48a^3B+8b^3x^5(7A+6Bx)+3a^2bx(77A+48Bx)+2ab^2x^3(91A+72Bx))-105a}{336b}$$

```
[In] Integrate[(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(48*a^3*B + 8*b^3*x^5*(7*A + 6*B*x) + 3*a^2*b*x*(77*A + 48
*B*x) + 2*a*b^2*x^3*(91*A + 72*B*x)) - 105*a^3*A*Sqrt[b]*Log[-(Sqrt[b]*x) +
 Sqrt[a + b*x^2]])/(336*b)
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

method	result	size
default	$A\left(\frac{x(bx^2+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)+\frac{B(bx^2+a)^{\frac{7}{2}}}{7b}$	86
risch	$\frac{(48b^3Bx^6+56Ab^3x^5+144Bab^2x^4+182aAb^2x^3+144Ba^2bx^2+231a^2Abx+48a^3B)\sqrt{bx^2+a}}{336b}+\frac{5a^3A\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{16\sqrt{b}}$	104

```
[In] int((B*x+A)*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a
)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))+1/7*B*(b*x^2+a)^(7/2
)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.09

$$\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\left[\frac{105\,Aa^3\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(48\,Bb^3x^6+56\,Ab^3x^5+144\,Bab^2x^4+18}{672\,b}\right.$$

$$-\frac{105\,Aa^3\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(48\,Bb^3x^6+56\,Ab^3x^5+144\,Bab^2x^4+182\,Aab^2x^3+144\,Ba^2bx^2+231\,Aa}{336\,b}$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/672*(105*A*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(48*B*b^3*x^6 + 56*A*b^3*x^5 + 144*B*a*b^2*x^4 + 182*A*a*b^2*x^3 + 144*B
*a^2*b*x^2 + 231*A*a^2*b*x + 48*B*a^3)*sqrt(b*x^2 + a))/b, -1/336*(105*A*a^
3*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (48*B*b^3*x^6 + 56*A*b^3*x^
5 + 144*B*a*b^2*x^4 + 182*A*a*b^2*x^3 + 144*B*a^2*b*x^2 + 231*A*a^2*b*x + 4
8*B*a^3)*sqrt(b*x^2 + a))/b]
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.40

$$\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\begin{cases}\frac{5Aa^3\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{16}+\sqrt{a+bx^2}\cdot\left(\frac{11Aa^2x}{16}+\frac{13Aabx^3}{24}+\frac{Ab^2x^5}{6}+\frac{Ba^3}{7b}+\frac{3Ba^2}{7}\right.\\ a^{\frac{5}{2}}\left(Ax+\frac{Bx^2}{2}\right)\end{cases}$$

```
[In] integrate((B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] Piecewise((5*A*a**3*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt
(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/16 + sqrt(a + b*x**2)*(11*A*
a**2*x/16 + 13*A*a*b*x**3/24 + A*b**2*x**5/6 + B*a**3/(7*b) + 3*B*a**2*x**2
/7 + 3*B*a*b*x**4/7 + B*b**2*x**6/7), Ne(b, 0)), (a**(5/2)*(A*x + B*x**2/2)
, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.72

$$\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx = \frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}Ax + \frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}Aax + \frac{5}{16}\sqrt{bx^2+a}Aa^2x + \frac{5\,Aa^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{b}} + \frac{\left(bx^2+a\right)^{\frac{7}{2}}B}{7\,b}$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*A*x + 5/24*(b*x^2 + a)^(3/2)*A*a*x + 5/16*sqrt(b*x^2
+ a)*A*a^2*x + 5/16*A*a^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 1/7*(b*x^2 + a)^
(7/2)*B/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx = -\frac{5\,Aa^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\sqrt{b}} + \frac{1}{336}\left(\frac{48\,Ba^3}{b} + \left(231\,Aa^2 + 2\left(72\,Ba^2 + \left(91\,Aab + 4\left(18\,Bab + \left(6\,Bb^2x + 7\,Ab^2\right)x\right)x\right)x\right)x\right)x\right)\sqrt{bx^2+a}$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/16*A*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/336*(48*B*a^
3/b + (231*A*a^2 + 2*(72*B*a^2 + (91*A*a*b + 4*(18*B*a*b + (6*B*b^2*x + 7*A
*b^2)*x)*x)*x)*x)*x)*sqrt(b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.50

$$\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx = \frac{B\left(b\,x^2+a\right)^{7/2}}{7\,b} + \frac{A\,x\left(b\,x^2+a\right)^{5/2}{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{5/2}}$$

```
[In] int((a + b*x^2)^(5/2)*(A + B*x),x)
```

```
[Out] (B*(a + b*x^2)^(7/2))/(7*b) + (A*x*(a + b*x^2)^(5/2)*hypergeom([-5/2, 1/2],
 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^(5/2)
```

3.19 $\int \frac{(A+Bx)(a+bx^2)^{5/2}}{x} \, dx$

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Maxima [A] (verification not implemented) 184
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Optimal result

Integrand size = 20, antiderivative size = 132

$$\int \frac{(A+Bx)(a+bx^2)^{5/2}}{x} \, dx = \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2}+\frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2} + \frac{1}{30}(6A+5Bx)\left(a+bx^2\right)^{5/2}+\frac{5a^3B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}-a^{5/2}A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] 1/24*a*(5*B*x+8*A)*(b*x^2+a)^(3/2)+1/30*(5*B*x+6*A)*(b*x^2+a)^(5/2)-a^(5/2)
*A*arctanh((b*x^2+a)^(1/2)/a^(1/2))+5/16*a^3*B*arctanh(x*b^(1/2)/(b*x^2+a)^
(1/2))/b^(1/2)+1/16*a^2*(5*B*x+16*A)*(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {829, 858, 223, 212, 272, 65, 214}

$$\int \frac{(A+Bx)(a+bx^2)^{5/2}}{x} \, dx = -a^{5/2}A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{5a^3B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}} +\frac{1}{16}a^2\sqrt{a+bx^2}(16A+5Bx)+\frac{1}{24}a\left(a+bx^2\right)^{3/2}(8A+5Bx)+\frac{1}{30}\left(a+bx^2\right)^{5/2}(6A+5Bx)$$

```
[In] Int[((A + B*x)*(a + b*x^2)^(5/2))/x,x]
```

```
[Out] (a^2*(16*A + 5*B*x)*Sqrt[a + b*x^2])/16 + (a*(8*A + 5*B*x)*(a + b*x^2)^(3/2
))/24 + ((6*A + 5*B*x)*(a + b*x^2)^(5/2))/30 + (5*a^3*B*ArcTanh[(Sqrt[b]*x)
```

```
/Sqrt[a + b*x^2]])/(16*Sqrt[b]) - a^(5/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{30}(6A+5Bx)\left(a+bx^2\right)^{5/2}+\frac{\int \frac{(6aAb+5abBx)\left(a+bx^2\right)^{3/2}}{x}\,dx}{6b}\\
&= \frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2}+\frac{1}{30}(6A+5Bx)\left(a+bx^2\right)^{5/2}+\frac{\int \frac{\left(24a^2Ab^2+15a^2b^2Bx\right)\sqrt{a+bx^2}}{x}\,dx}{24b^2}\\
&= \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2}+\frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2}\\
&\quad+\frac{1}{30}(6A+5Bx)\left(a+bx^2\right)^{5/2}+\frac{\int \frac{48a^3Ab^3+15a^3b^3Bx}{x\sqrt{a+bx^2}}\,dx}{48b^3}\\
&= \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2}+\frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2}\\
&\quad+\frac{1}{30}(6A+5Bx)\left(a+bx^2\right)^{5/2}+\left(a^3A\right)\int \frac{1}{x\sqrt{a+bx^2}}\,dx+\frac{1}{16}\left(5a^3B\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx\\
&= \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2}+\frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2}\\
&\quad+\frac{1}{30}(6A+5Bx)\left(a+bx^2\right)^{5/2}+\frac{1}{2}\left(a^3A\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)\\
&\quad+\frac{1}{16}\left(5a^3B\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\
&= \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2}+\frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2}\\
&\quad+\frac{1}{30}(6A+5Bx)\left(a+bx^2\right)^{5/2}+\frac{5a^3B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}\\
&\quad+\frac{\left(a^3A\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{b}\\
&= \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2}+\frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2}\\
&\quad+\frac{1}{30}(6A+5Bx)\left(a+bx^2\right)^{5/2}+\frac{5a^3B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}-a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x}\,dx = 2a^{5/2}A\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)$$
$$+\frac{1}{240}\left(\sqrt{a+bx^2}\left(8b^2x^4(6A+5Bx)+2abx^2(88A+65Bx)+a^2(368A+165Bx)\right)-\frac{75a^3B\log\left(-\sqrt{b}x+\sqrt{a+b}\right.}{\sqrt{b}}\right.$$

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(5/2))/x,x]
```

```
[Out] 2*a^(5/2)*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] + (Sqrt[a + b*x^
2]*(8*b^2*x^4*(6*A + 5*B*x) + 2*a*b*x^2*(88*A + 65*B*x) + a^2*(368*A + 165*
B*x)) - (75*a^3*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b])/240
```

Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05

method	result
default	$B\left(\frac{x\left(b\,x^2+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)+A\left(\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{5}+a\left(\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{3}+a\left(\right.\right.\right.$

```
[In] int((B*x+A)*(b*x^2+a)^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a
)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))+A*(1/5*(b*x^2+a)^(5/
2)+a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x
^2+a)^(1/2))/x))))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 539, normalized size of antiderivative = 4.08

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x}\,dx = \left[\frac{75\,Ba^3\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+240\,Aa^{\frac{5}{2}}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}}{x^2}\right.}{}\right.$$

$$-\frac{75\,Ba^3\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-120\,Aa^{\frac{5}{2}}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-\left(40\,Bb^3x^5+48\,Ab^3x^4+130\,Bab^2x^3\right.}{240\,b}$$

$$-\frac{75\,Ba^3\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-240\,A\sqrt{-a}a^2b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(40\,Bb^3x^5+48\,Ab^3x^4+130\,Bab^2x^3+17\right.}{240\,b}$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/480*(75*B*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
240*A*a^(5/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(40
*B*b^3*x^5 + 48*A*b^3*x^4 + 130*B*a*b^2*x^3 + 176*A*a*b^2*x^2 + 165*B*a^2*b
*x + 368*A*a^2*b)*sqrt(b*x^2 + a))/b, -1/240*(75*B*a^3*sqrt(-b)*arctan(sqrt
(-b)*x/sqrt(b*x^2 + a)) - 120*A*a^(5/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) - (40*B*b^3*x^5 + 48*A*b^3*x^4 + 130*B*a*b^2*x^3 + 176*A
*a*b^2*x^2 + 165*B*a^2*b*x + 368*A*a^2*b)*sqrt(b*x^2 + a))/b, 1/480*(480*A*
sqrt(-a)*a^2*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 75*B*a^3*sqrt(b)*log(-2*b
*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(40*B*b^3*x^5 + 48*A*b^3*x^4 +
130*B*a*b^2*x^3 + 176*A*a*b^2*x^2 + 165*B*a^2*b*x + 368*A*a^2*b)*sqrt(b*x^2
 + a))/b, -1/240*(75*B*a^3*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 24
0*A*sqrt(-a)*a^2*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (40*B*b^3*x^5 + 48*A*
b^3*x^4 + 130*B*a*b^2*x^3 + 176*A*a*b^2*x^2 + 165*B*a^2*b*x + 368*A*a^2*b)*
sqrt(b*x^2 + a))/b]
```

Sympy [A] (verification not implemented)

Time = 5.94 (sec) , antiderivative size = 474, normalized size of antiderivative = 3.59

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x}\,dx = -Aa^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right) + \frac{Aa^3}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} + \frac{Aa^2\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}} + 2Aab\left(\begin{cases}\frac{a\sqrt{a+bx^2}}{3b}+\frac{x^2\sqrt{a+bx^2}}{3} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^2}{2} & \text{otherwise}\end{cases}\right) + Ab^2\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^4}{4} & \text{otherwise}\end{cases}\right) + Ba^2\left(\begin{cases}\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x\sqrt{a+bx^2}}{2} & \text{for } b\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right) + 2Bab\left(\begin{cases}-\frac{a^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b}+\frac{ax\sqrt{a+bx^2}}{8b}+\frac{x^3\sqrt{a+bx^2}}{4} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^3}{3} & \text{otherwise}\end{cases}\right) + Bb^2\left(\begin{cases}\frac{a^3\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{16b^2}-\frac{a^2x\sqrt{a+bx^2}}{16b^2}+\frac{ax^3\sqrt{a+bx^2}}{24b}+\frac{x^5\sqrt{a+bx^2}}{6} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^5}{5} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((B*x+A)*(b*x**2+a)**(5/2)/x,x)
```

```
[Out] -A*a**(5/2)*asinh(sqrt(a)/(sqrt(b)*x)) + A*a**3/(sqrt(b)*x*sqrt(a/(b*x**2)
+ 1)) + A*a**2*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + 2*A*a*b*Piecewise((a*sqrt(a
 + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, Tru
e)) + A*b**2*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a
+ b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, Tru
e)) + B*a**2*Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x
)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)
/2, Ne(b, 0)), (sqrt(a)*x, True)) + 2*B*a*b*Piecewise((-a**2*Piecewise((log
(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x
**2), True))/(8*b) + a*x*sqrt(a + b*x**2)/(8*b) + x**3*sqrt(a + b*x**2)/4,
```

```
Ne(b, 0)), (sqrt(a)*x**3/3, True)) + B*b**2*Piecewise((a**3*Piecewise((log(
2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x*
*2), True))/(16*b**2) - a**2*x*sqrt(a + b*x**2)/(16*b**2) + a*x**3*sqrt(a +
 b*x**2)/(24*b) + x**5*sqrt(a + b*x**2)/6, Ne(b, 0)), (sqrt(a)*x**5/5, True
))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx)\,(a+bx^2)^{5/2}}{x}\,dx = \frac{1}{6}\,(bx^2+a)^{\frac{5}{2}}Bx + \frac{5}{24}\,(bx^2+a)^{\frac{3}{2}}Bax + \frac{5}{16}\,\sqrt{bx^2+a}Ba^2x + \frac{5\,Ba^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}} - Aa^{\frac{5}{2}}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{1}{5}\,(bx^2+a)^{\frac{5}{2}}A + \frac{1}{3}\,(bx^2+a)^{\frac{3}{2}}Aa + \sqrt{bx^2+a}Aa^2$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*B*x + 5/24*(b*x^2 + a)^(3/2)*B*a*x + 5/16*sqrt(b*x^2
+ a)*B*a^2*x + 5/16*B*a^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) - A*a^(5/2)*arcsin
h(a/(sqrt(a*b)*abs(x))) + 1/5*(b*x^2 + a)^(5/2)*A + 1/3*(b*x^2 + a)^(3/2)*A
*a + sqrt(b*x^2 + a)*A*a^2
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\,(a+bx^2)^{5/2}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.77

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x}\,dx=\frac{A\left(b\,x^2+a\right)^{5/2}}{5}+A\,a^2\,\sqrt{b\,x^2+a}+\frac{A\,a\,\left(b\,x^2+a\right)^{3/2}}{3}+\frac{B\,x\,\left(b\,x^2+a\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{5/2}}+A\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}$$

```
[In] int(((a + b*x^2)^(5/2)*(A + B*x))/x,x)
```

```
[Out] (A*(a + b*x^2)^(5/2))/5 + A*a^2*(a + b*x^2)^(1/2) + A*a^(5/2)*atan(((a + b*
x^2)^(1/2)*1i)/a^(1/2))*1i + (A*a*(a + b*x^2)^(3/2))/3 + (B*x*(a + b*x^2)^(
5/2)*hypergeom([-5/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^(5/2)
```

3.20 $\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^2} dx$

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Optimal result

Integrand size = 20, antiderivative size = 136

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^2}\,dx=\frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2}+\frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2}-\frac{(5A-Bx)\left(a+bx^2\right)^{5/2}}{5x}+\frac{15}{8}a^2A\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-a^{5/2}B\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] 1/12*(15*A*b*x+4*B*a)*(b*x^2+a)^(3/2)-1/5*(-B*x+5*A)*(b*x^2+a)^(5/2)/x-a^(5
/2)*B*arctanh((b*x^2+a)^(1/2)/a^(1/2))+15/8*a^2*A*arctanh(x*b^(1/2)/(b*x^2+
a)^(1/2))*b^(1/2)+1/8*a*(15*A*b*x+8*B*a)*(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {827, 829, 858, 223, 212, 272, 65, 214}

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^2}\,dx=a^{5/2}(-B)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{15}{8}a^2A\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)+\frac{1}{8}a\sqrt{a+bx^2}(8aB+15Abx)-\frac{\left(a+bx^2\right)^{5/2}(5A-Bx)}{5x}+\frac{1}{12}\left(a+bx^2\right)^{3/2}(4aB+15Abx)$$

```
[In] Int[((A + B*x)*(a + b*x^2)^(5/2))/x^2,x]
```

```
[Out] (a*(8*a*B + 15*A*b*x)*Sqrt[a + b*x^2])/8 + ((4*a*B + 15*A*b*x)*(a + b*x^2)^
(3/2))/12 - ((5*A - B*x)*(a + b*x^2)^(5/2))/(5*x) + (15*a^2*A*Sqrt[b]*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8 - a^(5/2)*B*ArcTanh[Sqrt[a + b*x^2]/Sqrt
[a]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
```

```
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(5A-Bx)\left(a+bx^2\right)^{5/2}}{5x}-\frac{1}{2}\int\frac{(-2aB-10Abx)\left(a+bx^2\right)^{3/2}}{x}\,dx\\
&= \frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2}-\frac{(5A-Bx)\left(a+bx^2\right)^{5/2}}{5x}-\frac{\int\frac{\left(-8a^2bB-30aAb^2x\right)\sqrt{a+bx^2}}{x}\,dx}{8b}\\
&= \frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2}+\frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2}\\
&\quad-\frac{(5A-Bx)\left(a+bx^2\right)^{5/2}}{5x}-\frac{\int\frac{-16a^3b^2B-30a^2Ab^3x}{x\sqrt{a+bx^2}}\,dx}{16b^2}\\
&= \frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2}+\frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2}\\
&\quad-\frac{(5A-Bx)\left(a+bx^2\right)^{5/2}}{5x}+\frac{1}{8}\left(15a^2Ab\right)\int\frac{1}{\sqrt{a+bx^2}}\,dx+\left(a^3B\right)\int\frac{1}{x\sqrt{a+bx^2}}\,dx\\
&= \frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2}+\frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2}\\
&\quad-\frac{(5A-Bx)\left(a+bx^2\right)^{5/2}}{5x}+\frac{1}{8}\left(15a^2Ab\right)\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\
&\quad+\frac{1}{2}\left(a^3B\right)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)
\end{aligned}$$

$$= \frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2}+\frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2}-\frac{(5A-Bx)\left(a+bx^2\right)^{5/2}}{5x} + \frac{15}{8}a^2A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)+\frac{(a^3B)\,\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{b}$$

$$= \frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2} + \frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2}-\frac{(5A-Bx)\left(a+bx^2\right)^{5/2}}{5x} + \frac{15}{8}a^2A\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-a^{5/2}B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.98

$$\int\frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^2}\,dx = \frac{\sqrt{a+bx^2}(-8a^2(15A-23Bx)+6b^2x^4(5A+4Bx)+abx^2(135A+88Bx))}{120x} + 2a^{5/2}B\,\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{15}{8}a^2A\sqrt{b}\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(5/2))/x^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^2*(15*A - 23*B*x) + 6*b^2*x^4*(5*A + 4*B*x) + a*b*x^
2*(135*A + 88*B*x)))/(120*x) + 2*a^(5/2)*B*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*
x^2])/Sqrt[a]] - (15*a^2*A*Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/8
```

Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{a^{2} A \sqrt{b\,x^{2}+a}}{x}+\frac{B\,b^{2} x^{4} \sqrt{b\,x^{2}+a}}{5}+\frac{11 B b a\,x^{2} \sqrt{b\,x^{2}+a}}{15}+\frac{23 a^{2} B \sqrt{b\,x^{2}+a}}{15}+\frac{b^{2} A\,x^{3} \sqrt{b\,x^{2}+a}}{4}+\frac{9 b A a x \sqrt{b\,x^{2}+a}}{8}+\frac{15 a^{2} \sqrt{b}\,A}{}$
default	$B\left(\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5}+a\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)\right)\right)+A\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{ax}+\frac{6b\left(\frac{x}{}\right)}{}\right.$

```
[In] int((B*x+A)*(b*x^2+a)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^2*A*(b*x^2+a)^(1/2)/x+1/5*B*b^2*x^4*(b*x^2+a)^(1/2)+11/15*B*b*a*x^2*(b*x
^2+a)^(1/2)+23/15*a^2*B*(b*x^2+a)^(1/2)+1/4*b^2*A*x^3*(b*x^2+a)^(1/2)+9/8*b
*A*a*x*(b*x^2+a)^(1/2)+15/8*a^2*b^(1/2)*A*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-a^(
5/2)*B*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 519, normalized size of antiderivative = 3.82

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^2}\,dx=\left[\frac{225\,Aa^2\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+120\,Ba^{\frac{5}{2}}x\log\left(-\frac{bx^2-2\,\sqrt{bx}}{}\right.}{}\right.$$

$$-\frac{225\,Aa^2\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-60\,Ba^{\frac{5}{2}}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-(24\,Bb^2x^5+30\,Ab^2x^4+88\,Babx^3+}{120\,x}$$

$$-\frac{225\,Aa^2\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-120\,B\sqrt{-a}a^2x\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-(24\,Bb^2x^5+30\,Ab^2x^4+88\,Babx^3+13}{120\,x}$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/240*(225*A*a^2*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + 120*B*a^(5/2)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*
(24*B*b^2*x^5 + 30*A*b^2*x^4 + 88*B*a*b*x^3 + 135*A*a*b*x^2 + 184*B*a^2*x -
 120*A*a^2)*sqrt(b*x^2 + a))/x, -1/120*(225*A*a^2*sqrt(-b)*x*arctan(sqrt(-b
)*x/sqrt(b*x^2 + a)) - 60*B*a^(5/2)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
a) + 2*a)/x^2) - (24*B*b^2*x^5 + 30*A*b^2*x^4 + 88*B*a*b*x^3 + 135*A*a*b*x^
2 + 184*B*a^2*x - 120*A*a^2)*sqrt(b*x^2 + a))/x, 1/240*(240*B*sqrt(-a)*a^2*
x*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 225*A*a^2*sqrt(b)*x*log(-2*b*x^2 - 2*s
qrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(24*B*b^2*x^5 + 30*A*b^2*x^4 + 88*B*a*b*x
^3 + 135*A*a*b*x^2 + 184*B*a^2*x - 120*A*a^2)*sqrt(b*x^2 + a))/x, -1/120*(2
25*A*a^2*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 120*B*sqrt(-a)*a^2
*x*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (24*B*b^2*x^5 + 30*A*b^2*x^4 + 88*B*a
*b*x^3 + 135*A*a*b*x^2 + 184*B*a^2*x - 120*A*a^2)*sqrt(b*x^2 + a))/x]
```

Sympy [A] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 420, normalized size of antiderivative = 3.09

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^2}\,dx = -\frac{Aa^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{Aa^{\frac{3}{2}}bx}{\sqrt{1+\frac{bx^2}{a}}} + Aa^2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + 2Aab\left(\begin{cases}\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for}\ b\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right) + Ab^2\left(\begin{cases}-\frac{a^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b} + \frac{ax\sqrt{a+bx^2}}{8b} + \frac{x^3\sqrt{a+bx^2}}{4} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^3}{3} & \text{otherwise}\end{cases}\right) - Ba^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right) + \frac{Ba^3}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} + \frac{Ba^2\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}} + 2Bab\left(\begin{cases}\frac{a\sqrt{a+bx^2}}{3b} + \frac{x^2\sqrt{a+bx^2}}{3} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^2}{2} & \text{otherwise}\end{cases}\right) + Bb^2\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2} + \frac{ax^2\sqrt{a+bx^2}}{15b} + \frac{x^4\sqrt{a+bx^2}}{5} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}x^4}{4} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((B*x+A)*(b*x**2+a)**(5/2)/x**2,x)
```

```
[Out] -A*a**(5/2)/(x*sqrt(1 + b*x**2/a)) - A*a**(3/2)*b*x/sqrt(1 + b*x**2/a) + A*
a**2*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) + 2*A*a*b*Piecewise((a*Piecewise((log
(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x
**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) + A*b*
*2*Piecewise((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt
(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b) + a*x*sqrt(a + b*x**2)
/(8*b) + x**3*sqrt(a + b*x**2)/4, Ne(b, 0)), (sqrt(a)*x**3/3, True)) - B*a*
*(5/2)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a**3/(sqrt(b)*x*sqrt(a/(b*x**2) + 1))
 + B*a**2*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + 2*B*a*b*Piecewise((a*sqrt(a + b*
x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True)) +
 B*b**2*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x
**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^2}\,dx = \frac{5}{4}\left(bx^2+a\right)^{\frac{3}{2}}Abx + \frac{15}{8}\sqrt{bx^2+a}Aabx + \frac{15}{8}Aa^2\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - Ba^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{1}{5}\left(bx^2+a\right)^{\frac{5}{2}}B + \frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}Ba + \sqrt{bx^2+a}Ba^2 - \frac{\left(bx^2+a\right)^{\frac{5}{2}}A}{x}$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 5/4*(b*x^2 + a)^(3/2)*A*b*x + 15/8*sqrt(b*x^2 + a)*A*a*b*x + 15/8*A*a^2*sqr
t(b)*arcsinh(b*x/sqrt(a*b)) - B*a^(5/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/5
*(b*x^2 + a)^(5/2)*B + 1/3*(b*x^2 + a)^(3/2)*B*a + sqrt(b*x^2 + a)*B*a^2 -
(b*x^2 + a)^(5/2)*A/x
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^2}\,dx = \frac{2\,Ba^3\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{15}{8}\,Aa^2\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right) + \frac{2\,Aa^3\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a} + \frac{1}{120}\left(184\,Ba^2+\left(135\,Aab+2\left(44\,Bab+3\left(4\,Bb^2x+5\,Ab^2\right)x\right)x\right)x\right)\sqrt{bx^2+a}$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*B*a^3*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 15/8*A*a
^2*sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 2*A*a^3*sqrt(b)/((sqrt(
b)*x - sqrt(b*x^2 + a))^2 - a) + 1/120*(184*B*a^2 + (135*A*a*b + 2*(44*B*a*
b + 3*(4*B*b^2*x + 5*A*b^2)*x)*x)*x)*sqrt(b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 6.72 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.76

$$\int \frac{(A+Bx)\,(a+bx^2)^{5/2}}{x^2}\,dx = \frac{B\,(b\,x^2+a)^{5/2}}{5} + B\,a^2\,\sqrt{b\,x^2+a} + \frac{B\,a\,(b\,x^2+a)^{3/2}}{3} - \frac{A\,(b\,x^2+a)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,\left(\frac{b\,x^2}{a}+1\right)^{5/2}} + B\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}$$

```
[In] int(((a + b*x^2)^(5/2)*(A + B*x))/x^2,x)
```

```
[Out] (B*(a + b*x^2)^(5/2))/5 + B*a^2*(a + b*x^2)^(1/2) + B*a^(5/2)*atan(((a + b*
x^2)^(1/2)*1i)/a^(1/2))*1i + (B*a*(a + b*x^2)^(3/2))/3 - (A*(a + b*x^2)^(5/
2)*hypergeom([-5/2, -1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/a + 1)^(5/2))
```

3.21 $\int \frac{(A+Bx)(a+bx^2)^{5/2}}{x^3}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 141

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^3}\,dx=\frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2}-\frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x}-\frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2}+\frac{15}{8}a^2\sqrt{b}B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{5}{2}a^{3/2}A\text{barctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] -5/12*(-2*A*b*x+3*B*a)*(b*x^2+a)^(3/2)/x-1/4*(-B*x+2*A)*(b*x^2+a)^(5/2)/x^2
-5/2*a^(3/2)*A*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))+15/8*a^2*B*arctanh(x*b^(1
/2)/(b*x^2+a)^(1/2))*b^(1/2)+5/8*a*b*(3*B*x+4*A)*(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {827, 829, 858, 223, 212, 272, 65, 214}

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^3}\,dx=-\frac{5}{2}a^{3/2}A\text{barctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{15}{8}a^2\sqrt{b}B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\left(a+bx^2\right)^{5/2}(2A-Bx)}{4x^2}-\frac{5\left(a+bx^2\right)^{3/2}(3aB-2Abx)}{12x}+\frac{5}{8}ab\sqrt{a+bx^2}(4A+3Bx)$$

```
[In] Int[((A + B*x)*(a + b*x^2)^(5/2))/x^3,x]
```

```
[Out] (5*a*b*(4*A + 3*B*x)*Sqrt[a + b*x^2])/8 - (5*(3*a*B - 2*A*b*x)*(a + b*x^2)^
(3/2))/(12*x) - ((2*A - B*x)*(a + b*x^2)^(5/2))/(4*x^2) + (15*a^2*Sqrt[b]*B
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8 - (5*a^(3/2)*A*b*ArcTanh[Sqrt[a +
b*x^2]/Sqrt[a]])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2} - \frac{5}{16}\int \frac{(-4aB-8Abx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx$$

$$= -\frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x} - \frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2} + \frac{5}{32}\int \frac{(16aAb+24abBx)\sqrt{a+bx^2}}{x}\,dx$$

$$= \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2} - \frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x} - \frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2} + \frac{5\int \frac{32a^2Ab^2+24a^2b^2Bx}{x\sqrt{a+bx^2}}\,dx}{64b}$$

$$= \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2} - \frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x} - \frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2} + \frac{1}{2}\left(5a^2Ab\right)\int \frac{1}{x\sqrt{a+bx^2}}\,dx + \frac{1}{8}\left(15a^2bB\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx$$

$$= \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2} - \frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x} - \frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2} + \frac{1}{4}\left(5a^2Ab\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right) + \frac{1}{8}\left(15a^2bB\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)$$

$$= \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2} - \frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x} - \frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2} + \frac{15}{8}a^2\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) + \frac{1}{2}\left(5a^2A\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)$$

$$= \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2} - \frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x} - \frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2} + \frac{15}{8}a^2\sqrt{b}B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \frac{5}{2}a^{3/2}Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^3}\,dx = 5a^{3/2}Ab\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right) + \frac{1}{24}\left(\frac{\sqrt{a+bx^2}\left(-12a^2(A+2Bx)+2b^2x^4(4A+3Bx)+abx^2(56A+27Bx)\right)}{x^2} - 45a^2\sqrt{b}B\log\left(-\sqrt{b}x+\sqrt{a-\right.\right.$$

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(5/2))/x^3,x]
```

```
[Out] 5*a^(3/2)*A*b*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] + ((Sqrt[a + b
*x^2]*(-12*a^2*(A + 2*B*x) + 2*b^2*x^4*(4*A + 3*B*x) + a*b*x^2*(56*A + 27*B
*x)))/x^2 - 45*a^2*Sqrt[b]*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/24
```

Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{a^{2}\sqrt{b\,x^{2}+a}\,(2Bx+A)}{2x^{2}}+\frac{15\sqrt{b}\,a^{2}B\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{8}+\frac{B\,b^{2}x^{3}\sqrt{b\,x^{2}+a}}{4}+\frac{9Bbax\sqrt{b\,x^{2}+a}}{8}+\frac{b^{2}A\,x^{2}\sqrt{b\,x^{2}+a}}{3}+\frac{7bAa\sqrt{b}}{3}$
default	$B\left(-\frac{(b\,x^{2}+a)^{\frac{7}{2}}}{ax}+\frac{6b\left(\frac{x(b\,x^{2}+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x(b\,x^{2}+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)}{a}\right)+A\left(-\frac{(b\,x^{2}+a)^{\frac{7}{2}}}{2a\,x^{2}}+-\right.$ 5b

```
[In] int((B*x+A)*(b*x^2+a)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2*(b*x^2+a)^(1/2)*(2*B*x+A)/x^2+15/8*b^(1/2)*a^2*B*ln(x*b^(1/2)+(b*x
^2+a)^(1/2))+1/4*B*b^2*x^3*(b*x^2+a)^(1/2)+9/8*B*b*a*x*(b*x^2+a)^(1/2)+1/3*
b^2*A*x^2*(b*x^2+a)^(1/2)+7/3*b*A*a*(b*x^2+a)^(1/2)-5/2*b*a^(3/2)*A*ln((2*a
+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 535, normalized size of antiderivative = 3.79

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^3}\,dx=\left[\frac{45\,Ba^2\sqrt{b}x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+60\,Aa^{\frac{3}{2}}bx^2\log\left(-\frac{bx^2-2\,\sqrt{b}}{}\right.}{}\right.$$

$$-\frac{45\,Ba^2\sqrt{-b}x^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-30\,Aa^{\frac{3}{2}}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-(6\,Bb^2x^5+8\,Ab^2x^4+27\,Babx^3+}{24\,x^2}$$

$$-\frac{45\,Ba^2\sqrt{-b}x^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-60\,A\sqrt{-a}abx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-(6\,Bb^2x^5+8\,Ab^2x^4+27\,Babx^3+56\,A}{24\,x^2}$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/48*(45*B*a^2*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + 60*A*a^(3/2)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) +
 2*(6*B*b^2*x^5 + 8*A*b^2*x^4 + 27*B*a*b*x^3 + 56*A*a*b*x^2 - 24*B*a^2*x -
12*A*a^2)*sqrt(b*x^2 + a))/x^2, -1/24*(45*B*a^2*sqrt(-b)*x^2*arctan(sqrt(-b
)*x/sqrt(b*x^2 + a)) - 30*A*a^(3/2)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) - (6*B*b^2*x^5 + 8*A*b^2*x^4 + 27*B*a*b*x^3 + 56*A*a*b*x
^2 - 24*B*a^2*x - 12*A*a^2)*sqrt(b*x^2 + a))/x^2, 1/48*(120*A*sqrt(-a)*a*b*
x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 45*B*a^2*sqrt(b)*x^2*log(-2*b*x^2 -
2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(6*B*b^2*x^5 + 8*A*b^2*x^4 + 27*B*a*b*
x^3 + 56*A*a*b*x^2 - 24*B*a^2*x - 12*A*a^2)*sqrt(b*x^2 + a))/x^2, -1/24*(45
*B*a^2*sqrt(-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 60*A*sqrt(-a)*a*b*
x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (6*B*b^2*x^5 + 8*A*b^2*x^4 + 27*B*a*
b*x^3 + 56*A*a*b*x^2 - 24*B*a^2*x - 12*A*a^2)*sqrt(b*x^2 + a))/x^2]
```

Sympy [A] (verification not implemented)

Time = 3.21 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.70

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^3}\,dx = -\frac{5Aa^{\frac{3}{2}}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2} - \frac{Aa^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x} + \frac{2Aa^2\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}} + \frac{2Aab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}} + Ab^2\left(\begin{cases}\frac{a\sqrt{a+bx^2}}{3b}+\frac{x^2\sqrt{a+bx^2}}{3} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^2}{2} & \text{otherwise}\end{cases}\right) - \frac{Ba^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{Ba^{\frac{3}{2}}bx}{\sqrt{1+\frac{bx^2}{a}}} + Ba^2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + 2Bab\left(\begin{cases}\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x\sqrt{a+bx^2}}{2} & \text{for } b\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right) + Bb^2\left(\begin{cases}-\frac{a^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b}+\frac{ax\sqrt{a+bx^2}}{8b}+\frac{x^3\sqrt{a+bx^2}}{4} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^3}{3} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((B*x+A)*(b*x**2+a)**(5/2)/x**3,x)
```

```
[Out] -5*A*a**(3/2)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - A*a**2*sqrt(b)*sqrt(a/(b*x**
2) + 1)/(2*x) + 2*A*a**2*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + 2*A*a*b**(3/2)*
x/sqrt(a/(b*x**2) + 1) + A*b**2*Piecewise((a*sqrt(a + b*x**2)/(3*b) + x**2*
```

```
sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True)) - B*a**(5/2)/(x*sqrt
(1 + b*x**2/a)) - B*a**(3/2)*b*x/sqrt(1 + b*x**2/a) + B*a**2*sqrt(b)*asinh(
sqrt(b)*x/sqrt(a)) + 2*B*a*b*Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a +
 b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*
sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) + B*b**2*Piecewise((-a**2
*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*
log(x)/sqrt(b*x**2), True))/(8*b) + a*x*sqrt(a + b*x**2)/(8*b) + x**3*sqrt(
a + b*x**2)/4, Ne(b, 0)), (sqrt(a)*x**3/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^3}\,dx = \frac{5}{4}\left(bx^2+a\right)^{\frac{3}{2}}Bbx + \frac{15}{8}\sqrt{bx^2+a}Babx + \frac{15}{8}Ba^2\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{5}{2}Aa^{\frac{3}{2}}b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{5}{6}\left(bx^2+a\right)^{\frac{3}{2}}Ab + \frac{\left(bx^2+a\right)^{\frac{5}{2}}Ab}{2a} + \frac{5}{2}\sqrt{bx^2+a}Aab - \frac{\left(bx^2+a\right)^{\frac{5}{2}}B}{x} - \frac{\left(bx^2+a\right)^{\frac{7}{2}}A}{2ax^2}$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] 5/4*(b*x^2 + a)^(3/2)*B*b*x + 15/8*sqrt(b*x^2 + a)*B*a*b*x + 15/8*B*a^2*sqr
t(b)*arcsinh(b*x/sqrt(a*b)) - 5/2*A*a^(3/2)*b*arcsinh(a/(sqrt(a*b)*abs(x)))
 + 5/6*(b*x^2 + a)^(3/2)*A*b + 1/2*(b*x^2 + a)^(5/2)*A*b/a + 5/2*sqrt(b*x^2
 + a)*A*a*b - (b*x^2 + a)^(5/2)*B/x - 1/2*(b*x^2 + a)^(7/2)*A/(a*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.55

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^3}\,dx = \frac{5\,Aa^2b\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{15}{8}Ba^2\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right) + \frac{1}{24}\left(56\,Aab+\left(27\,Bab+2\left(3\,Bb^2x+4\,Ab^2\right)x\right)x\right)\sqrt{bx^2+a} + \frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Aa^2b+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^3\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aa^3b-2\,Ba^4\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2}$$

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 5*A*a^2*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 15/8*B
*a^2*sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 1/24*(56*A*a*b + (27*
B*a*b + 2*(3*B*b^2*x + 4*A*b^2)*x)*x)*sqrt(b*x^2 + a) + ((sqrt(b)*x - sqrt(
b*x^2 + a))^3*A*a^2*b + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*sqrt(b) + (
sqrt(b)*x - sqrt(b*x^2 + a))*A*a^3*b - 2*B*a^4*sqrt(b))/((sqrt(b)*x - sqrt(
b*x^2 + a))^2 - a)^2
```

Mupad [B] (verification not implemented)

Time = 7.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.79

$$\int \frac{(A+Bx)\,(a+bx^2)^{5/2}}{x^3}\,dx = \frac{A\,b\,(b\,x^2+a)^{3/2}}{3} + 2\,A\,a\,b\,\sqrt{b\,x^2+a} - \frac{A\,a^2\,\sqrt{b\,x^2+a}}{2\,x^2} - \frac{B\,(b\,x^2+a)^{5/2}\;{}_2\mathrm{F}_1\!\left(-\frac{5}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,\left(\frac{b\,x^2}{a}+1\right)^{5/2}} + \frac{A\,a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{2}$$

```
[In] int(((a + b*x^2)^(5/2)*(A + B*x))/x^3,x)
```

```
[Out] (A*b*(a + b*x^2)^(3/2))/3 + 2*A*a*b*(a + b*x^2)^(1/2) - (A*a^2*(a + b*x^2)^
(1/2))/(2*x^2) + (A*a^(3/2)*b*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/2 -
(B*(a + b*x^2)^(5/2)*hypergeom([-5/2, -1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/
a + 1)^(5/2))
```

3.22 $\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 104

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx=\frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{a(16A+9Bx)\sqrt{a+bx^2}}{24b^2}+\frac{3a^2B\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

```
[Out] 3/8*a^2*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)+1/3*A*x^2*(b*x^2+a)^(1
/2)/b+1/4*B*x^3*(b*x^2+a)^(1/2)/b-1/24*a*(9*B*x+16*A)*(b*x^2+a)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {847, 794, 223, 212}

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx=\frac{3a^2B\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}-\frac{a\sqrt{a+bx^2}(16A+9Bx)}{24b^2}+\frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}$$

```
[In] Int[(x^3*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (A*x^2*Sqrt[a + b*x^2])/(3*b) + (B*x^3*Sqrt[a + b*x^2])/(4*b) - (a*(16*A +
9*B*x)*Sqrt[a + b*x^2])/(24*b^2) + (3*a^2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*
x^2]])/(8*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{Bx^3\sqrt{a+bx^2}}{4b} + \frac{\int \frac{x^2(-3aB+4Abx)}{\sqrt{a+bx^2}}\,dx}{4b} \\
&= \frac{Ax^2\sqrt{a+bx^2}}{3b} + \frac{Bx^3\sqrt{a+bx^2}}{4b} + \frac{\int \frac{x(-8aAb-9abBx)}{\sqrt{a+bx^2}}\,dx}{12b^2} \\
&= \frac{Ax^2\sqrt{a+bx^2}}{3b} + \frac{Bx^3\sqrt{a+bx^2}}{4b} - \frac{a(16A+9Bx)\sqrt{a+bx^2}}{24b^2} + \frac{(3a^2B)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8b^2} \\
&= \frac{Ax^2\sqrt{a+bx^2}}{3b} + \frac{Bx^3\sqrt{a+bx^2}}{4b} - \frac{a(16A+9Bx)\sqrt{a+bx^2}}{24b^2} \\
&\quad + \frac{(3a^2B)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{8b^2} \\
&= \frac{Ax^2\sqrt{a+bx^2}}{3b} + \frac{Bx^3\sqrt{a+bx^2}}{4b} - \frac{a(16A+9Bx)\sqrt{a+bx^2}}{24b^2} + \frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.74

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(-16aA-9aBx+8Abx^2+6bBx^3)}{24b^2} - \frac{3a^2B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{5/2}}$$

```
[In] Integrate[(x^3*(A + B*x))/Sqrt[a + b*x^2],x]
[Out] (Sqrt[a + b*x^2]*(-16*a*A - 9*a*B*x + 8*A*b*x^2 + 6*b*B*x^3))/(24*b^2) - (3
*a^2*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.62

method	result	size
risch	$-\frac{\left(-6bB\,x^3-8Ab\,x^2+9Bax+16Aa\right)\sqrt{b\,x^2+a}}{24b^2}+\frac{3a^2B\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{8b^{\frac{5}{2}}}$	65
default	$B\left(\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)+A\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)$	101

```
[In] int(x^3*(B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/24*(-6*B*b*x^3-8*A*b*x^2+9*B*a*x+16*A*a)/b^2*(b*x^2+a)^(1/2)+3/8*a^2*B/b
^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.52

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx = \left[\frac{9\,Ba^2\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(6\,Bb^2x^3+8\,Ab^2x^2-9\,Babx-16\,Aab)\sqrt{bx^2+a}}{48\,b^3}, -\frac{9\,Ba^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(6\,Bb^2x^3+8\,Ab^2x^2-9\,Babx-16\,Aab)\sqrt{bx^2+a}}{24\,b^3}\right]$$

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(9*B*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*
(6*B*b^2*x^3 + 8*A*b^2*x^2 - 9*B*a*b*x - 16*A*a*b)*sqrt(b*x^2 + a))/b^3, -1
/24*(9*B*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (6*B*b^2*x^3 + 8
*A*b^2*x^2 - 9*B*a*b*x - 16*A*a*b)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.16

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{3Ba^2\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{8b^2} + \sqrt{a+bx^2}\left(-\frac{2Aa}{3b^2}+\frac{Ax^2}{3b}-\frac{3Bax}{8b^2}+\frac{Bx^3}{4b}\right) & \text{for}\ b\neq 0 \\ \frac{\frac{Ax^4}{4}+\frac{Bx^5}{5}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**3*(B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((3*B*a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt
(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b**2) + sqrt(a + b*x**2)*
(-2*A*a/(3*b**2) + A*x**2/(3*b) - 3*B*a*x/(8*b**2) + B*x**3/(4*b)), Ne(b, 0
)), ((A*x**4/4 + B*x**5/5)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Bx^3}{4\,b} + \frac{\sqrt{bx^2+a}Ax^2}{3\,b} - \frac{3\,\sqrt{bx^2+a}Bax}{8\,b^2} + \frac{3\,Ba^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} - \frac{2\,\sqrt{bx^2+a}Aa}{3\,b^2}$$

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(b*x^2 + a)*B*x^3/b + 1/3*sqrt(b*x^2 + a)*A*x^2/b - 3/8*sqrt(b*x^2
+ a)*B*a*x/b^2 + 3/8*B*a^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 2/3*sqrt(b*x^2
+ a)*A*a/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.71

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{1}{24}\sqrt{bx^2+a}\left(\left(2\left(\frac{3\,Bx}{b}+\frac{4\,A}{b}\right)x-\frac{9\,Ba}{b^2}\right)x-\frac{16\,Aa}{b^2}\right) - \frac{3\,Ba^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] 1/24*sqrt(b*x^2 + a)*((2*(3*B*x/b + 4*A/b)*x - 9*B*a/b^2)*x - 16*A*a/b^2) -
 3/8*B*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx = \int \frac{x^3\,(A+B\,x)}{\sqrt{b\,x^2+a}}\,dx$$

```
[In] int((x^3*(A + B*x))/(a + b*x^2)^(1/2),x)
[Out] int((x^3*(A + B*x))/(a + b*x^2)^(1/2), x)
```

3.23 $\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}}\,dx=\frac{Bx^2\sqrt{a+bx^2}}{3b}-\frac{(4aB-3Abx)\sqrt{a+bx^2}}{6b^2}-\frac{aA\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

```
[Out] -1/2*a*A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+1/3*B*x^2*(b*x^2+a)^(1/
2)/b-1/6*(-3*A*b*x+4*B*a)*(b*x^2+a)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {847, 794, 223, 212}

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}}\,dx=-\frac{aA\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}-\frac{\sqrt{a+bx^2}(4aB-3Abx)}{6b^2}+\frac{Bx^2\sqrt{a+bx^2}}{3b}$$

```
[In] Int[(x^2*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (B*x^2*Sqrt[a + b*x^2])/(3*b) - ((4*a*B - 3*A*b*x)*Sqrt[a + b*x^2])/(6*b^2)
 - (a*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{Bx^2\sqrt{a+bx^2}}{3b} + \frac{\int \frac{x(-2aB+3Abx)}{\sqrt{a+bx^2}}\,dx}{3b} \\
&= \frac{Bx^2\sqrt{a+bx^2}}{3b} - \frac{(4aB-3Abx)\sqrt{a+bx^2}}{6b^2} - \frac{(aA)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b} \\
&= \frac{Bx^2\sqrt{a+bx^2}}{3b} - \frac{(4aB-3Abx)\sqrt{a+bx^2}}{6b^2} - \frac{(aA)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b} \\
&= \frac{Bx^2\sqrt{a+bx^2}}{3b} - \frac{(4aB-3Abx)\sqrt{a+bx^2}}{6b^2} - \frac{aA\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(-4aB+3Abx+2bBx^2)}{6b^2} - \frac{aA\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

```
[In] Integrate[(x^2*(A + B*x))/Sqrt[a + b*x^2],x]
[Out] (Sqrt[a + b*x^2]*(-4*a*B + 3*A*b*x + 2*b*B*x^2))/(6*b^2) - (a*A*ArcTanh[(Sq
rt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/b^(3/2)
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{(2bB\,x^2+3Abx-4Ba)\sqrt{b\,x^2+a}}{6b^2}-\frac{aA\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}$	56
default	$B\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)+A\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)$	77

```
[In] int(x^2*(B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*B*b*x^2+3*A*b*x-4*B*a)/b^2*(b*x^2+a)^(1/2)-1/2*a*A/b^(3/2)*ln(x*b^(1
/2)+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.57

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}}\,dx = \left[\frac{3\,Aa\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(2\,Bbx^2+3\,Abx-4\,Ba)\sqrt{bx^2+a}}{12\,b^2}, \frac{3\,Aa\sqrt{-b}\arctan\left(\cdots\right.}{}\right.$$

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*A*a*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*B*b*x^2 + 3*A*b*x - 4*B*a)*sqrt(b*x^2 + a))/b^2, 1/6*(3*A*a*sqrt(-b)*arcta
n(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*B*b*x^2 + 3*A*b*x - 4*B*a)*sqrt(b*x^2 +
a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.26

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}}\,dx = \begin{cases} -\frac{Aa\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2b}+\sqrt{a+bx^2}\left(\frac{Ax}{2b}-\frac{2Ba}{3b^2}+\frac{Bx^2}{3b}\right) & \text{for } b\neq 0 \\ \frac{\frac{Ax^3}{3}+\frac{Bx^4}{4}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*(B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-A*a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b),
 Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(2*b) + sqrt(a + b*x**2)*(A*x/(2
*b) - 2*B*a/(3*b**2) + B*x**2/(3*b)), Ne(b, 0)), ((A*x**3/3 + B*x**4/4)/sqr
t(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.83

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Bx^2}{3\,b} + \frac{\sqrt{bx^2+a}Ax}{2\,b} - \frac{Aa\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} - \frac{2\,\sqrt{bx^2+a}Ba}{3\,b^2}$$

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(b*x^2 + a)*B*x^2/b + 1/2*sqrt(b*x^2 + a)*A*x/b - 1/2*A*a*arcsinh(b
*x/sqrt(a*b))/b^(3/2) - 2/3*sqrt(b*x^2 + a)*B*a/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.75

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{1}{6}\,\sqrt{bx^2+a}\left(\left(\frac{2\,Bx}{b}+\frac{3\,A}{b}\right)x-\frac{4\,Ba}{b^2}\right)+\frac{Aa\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(b*x^2 + a)*((2*B*x/b + 3*A/b)*x - 4*B*a/b^2) + 1/2*A*a*log(abs(-sq
rt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 6.34 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.15

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{3\,B\,x^4+4\,A\,x^3}{12\,\sqrt{a}} & \text{if } b=0\\ \frac{A\,x\,\sqrt{b\,x^2+a}}{2\,b}-\frac{A\,a\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}}-\frac{B\,\sqrt{b\,x^2+a}\,\left(2\,a-b\,x^2\right)}{3\,b^2} & \text{if } b\neq 0 \end{cases}$$

```
[In] int((x^2*(A + B*x))/(a + b*x^2)^(1/2),x)
```

```
[Out] piecewise(b == 0, (4*A*x^3 + 3*B*x^4)/(12*a^(1/2)), b ~= 0, - (A*a*log(2*b^
(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2)) + (A*x*(a + b*x^2)^(1/2))/(2*b)
 - (B*(a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2))
```

3.24 $\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx$

Optimal result 211
Rubi [A] (verified) 211
Mathematica [A] (verified) 212
Maple [A] (verified) 212
Fricas [A] (verification not implemented) 213
Sympy [A] (verification not implemented) 213
Maxima [A] (verification not implemented) 214
Giac [A] (verification not implemented) 214
Mupad [B] (verification not implemented) 214

Optimal result

Integrand size = 18, antiderivative size = 56

$$\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{(2A+Bx)\sqrt{a+bx^2}}{2b} - \frac{aB\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

```
[Out] -1/2*a*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+1/2*(B*x+2*A)*(b*x^2+a)
^(1/2)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {794, 223, 212}

$$\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(2A+Bx)}{2b} - \frac{aB\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

```
[In] Int[(x*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] ((2*A + B*x)*Sqrt[a + b*x^2])/(2*b) - (a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/(2*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(2A+Bx)\sqrt{a+bx^2}}{2b} - \frac{(aB)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}\\ &= \frac{(2A+Bx)\sqrt{a+bx^2}}{2b} - \frac{(aB)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b}\\ &= \frac{(2A+Bx)\sqrt{a+bx^2}}{2b} - \frac{aB\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

$$\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{(2A+Bx)\sqrt{a+bx^2}}{2b} + \frac{aB\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{3/2}}$$

```
[In] Integrate[(x*(A + B*x))/Sqrt[a + b*x^2],x]
[Out] ((2*A + B*x)*Sqrt[a + b*x^2])/(2*b) + (a*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^
2]])/(2*b^(3/2))
```

Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

method	result	size
risch	$\frac{(Bx+2A)\sqrt{b\,x^{2}+a}}{2b}-\frac{aB\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}$	46
default	$B\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)+\frac{A\sqrt{b\,x^{2}+a}}{b}$	56

```
[In] int(x*(B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(B*x+2*A)*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*B*ln(x*b^(1/2)+(b*x^2+a)^(1/2
))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.95

$$\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx = \left[\frac{Ba\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(Bbx+2\,Ab)\sqrt{bx^2+a}}{4\,b^2}, \frac{Ba\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(Bbx\right.}{2\,b^2}$$

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] [1/4*(B*a*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(B*b*
x + 2*A*b)*sqrt(b*x^2 + a))/b^2, 1/2*(B*a*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) + (B*b*x + 2*A*b)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.55

$$\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx = \begin{cases} -\frac{Ba\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\: a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2b} + \sqrt{a+bx^2}\left(\frac{A}{b}+\frac{Bx}{2b}\right) & \text{for}\: b\neq 0 \\ \frac{\frac{Ax^2}{2}+\frac{Bx^3}{3}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*(B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-B*a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b),
 Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(2*b) + sqrt(a + b*x**2)*(A/b +
B*x/(2*b)), Ne(b, 0)), ((A*x**2/2 + B*x**3/3)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

$$\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Bx}{2\,b} - \frac{Ba\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} + \frac{\sqrt{bx^2+a}A}{b}$$

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*x/b - 1/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(3/2) + sqrt(b
*x^2 + a)*A/b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{1}{2}\sqrt{bx^2+a}\left(\frac{Bx}{b}+\frac{2\,A}{b}\right) + \frac{Ba\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*(B*x/b + 2*A/b) + 1/2*B*a*log(abs(-sqrt(b)*x + sqrt(b*x
^2 + a)))/b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 6.40 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.46

$$\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{2\,B\,x^3+3\,A\,x^2}{6\,\sqrt{a}} & \text{if } b=0 \\ \frac{A\,\sqrt{b\,x^2+a}}{b} - \frac{B\,a\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}} + \frac{B\,x\,\sqrt{b\,x^2+a}}{2\,b} & \text{if } b\neq 0 \end{cases}$$

```
[In] int((x*(A + B*x))/(a + b*x^2)^(1/2),x)
```

```
[Out] piecewise(b == 0, (3*A*x^2 + 2*B*x^3)/(6*a^(1/2)), b ~= 0, (A*(a + b*x^2)^(
1/2))/b - (B*a*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2)) + (B*x*(
a + b*x^2)^(1/2))/(2*b))
```

3.25 $\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx$

Optimal result . . . 215
Rubi [A] (verified) . . . 215
Mathematica [A] (verified) . . . 216
Maple [A] (verified) . . . 216
Fricas [A] (verification not implemented) . . . 217
Sympy [A] (verification not implemented) . . . 217
Maxima [A] (verification not implemented) . . . 217
Giac [A] (verification not implemented) . . . 218
Mupad [B] (verification not implemented) . . . 218

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx=\frac{B\sqrt{a+bx^2}}{b}+\frac{A\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+B*(b*x^2+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {655, 223, 212}

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx=\frac{A\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{B\sqrt{a+bx^2}}{b}$$

```
[In] Int[(A + B*x)/Sqrt[a + b*x^2],x]
```

```
[Out] (B*Sqrt[a + b*x^2])/b + (A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{B\sqrt{a+bx^2}}{b}+A\int\frac{1}{\sqrt{a+bx^2}}\,dx\\ &= \frac{B\sqrt{a+bx^2}}{b}+A\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{B\sqrt{a+bx^2}}{b}+\frac{A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int\frac{A+Bx}{\sqrt{a+bx^2}}\,dx=\frac{B\sqrt{a+bx^2}}{b}-\frac{A\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}$$

```
[In] Integrate[(A + B*x)/Sqrt[a + b*x^2],x]
[Out] (B*Sqrt[a + b*x^2])/b - (A*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b]
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{A\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+\frac{B\sqrt{b\,x^2+a}}{b}$	37
risch	$\frac{A\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+\frac{B\sqrt{b\,x^2+a}}{b}$	37

```
[In] int((B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] A*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)+B*(b*x^2+a)^(1/2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.14

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx = \left[\frac{A\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,\sqrt{bx^2+a}B}{2\,b},\ -\frac{A\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}B}{b}\right]$$

```
[In] integrate((B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] [1/2*(A*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*sqrt(b*
x^2 + a)*B)/b, -(A*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - sqrt(b*x^2
 + a)*B)/b]
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx = \begin{cases} A\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)+\frac{B\sqrt{a+bx^2}}{b} & \text{for } b\neq 0 \\ \frac{Ax+\frac{Bx^2}{2}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

```
[In] integrate((B*x+A)/(b*x**2+a)**(1/2),x)
[Out] Piecewise((A*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne
(a, 0)), (x*log(x)/sqrt(b*x**2), True)) + B*sqrt(a + b*x**2)/b, Ne(b, 0)),
((A*x + B*x**2/2)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx = \frac{A\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}+\frac{\sqrt{bx^2+a}B}{b}$$

```
[In] integrate((B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] A*arcsinh(b*x/sqrt(a*b))/sqrt(b) + sqrt(b*x^2 + a)*B/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx = -\frac{A\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}} + \frac{\sqrt{bx^2+a}B}{b}$$

```
[In] integrate((B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -A*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + sqrt(b*x^2 + a)*B/b
```

Mupad [B] (verification not implemented)

Time = 5.92 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx = \frac{B\sqrt{b\,x^2+a}}{b} + \frac{A\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

```
[In] int((A + B*x)/(a + b*x^2)^(1/2),x)
```

```
[Out] (B*(a + b*x^2)^(1/2))/b + (A*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2)
```

3.26 $\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx$

Optimal result . . . 219
Rubi [A] (verified) . . . 219
Mathematica [A] (verified) . . . 221
Maple [A] (verified) . . . 221
Fricas [A] (verification not implemented) . . . 221
Sympy [A] (verification not implemented) . . . 222
Maxima [A] (verification not implemented) . . . 222
Giac [F(-2)] . . . 223
Mupad [B] (verification not implemented) . . . 223

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx=\frac{B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)+B*arctanh(x*b^(1/2)/(b*x^2+a)^(
1/2))/b^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {858, 223, 212, 272, 65, 214}

$$\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx=\frac{B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] Int[(A + B*x)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (A*ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]])/Sqrt[a]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= A\int \frac{1}{x\sqrt{a+bx^2}}\,dx + B\int \frac{1}{\sqrt{a+bx^2}}\,dx \\
&= \frac{1}{2}A\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right) + B\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right) \\
&= \frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}} + \frac{A\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b} \\
&= \frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}} - \frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx=\frac{2A\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}$$

```
[In] Integrate[(A + B*x)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/Sqrt[a] - (B*Log[-(Sqr
t[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b]
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{B\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{A\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{\sqrt{a}}$	52

```
[In] int((B*x+A)/x/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)-A/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+
a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 273, normalized size of antiderivative = 5.15

$$\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx$$
$$=\left[\frac{Ba\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+A\sqrt{a}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)}{2\,ab},\right.$$
$$-\frac{2\,Ba\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-A\sqrt{a}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)}{2\,ab},\frac{2\,A\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+Ba\sqrt{b}\log}{2\,ab}$$
$$\left.-\frac{Ba\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-A\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)}{ab}\right]$$

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(B*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + A*sqrt(
a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a*b), -1/2*(2*B*
a*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - A*sqrt(a)*b*log(-(b*x^2 - 2
*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a*b), 1/2*(2*A*sqrt(-a)*b*arctan(sqr
t(-a)/sqrt(b*x^2 + a)) + B*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
b)*x - a))/(a*b), -(B*a*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - A*sqr
t(-a)*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)))/(a*b)]
```

Sympy [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx = -\frac{A\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}} + B\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \wedge b\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for } b\neq 0\\ \frac{x}{\sqrt{a}} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((B*x+A)/x/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + B*Piecewise((log(2*sqrt(b)*sqrt(a +
 b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0) & Ne(b, 0)), (x*log(x)/sqrt(b*x**2), Ne
(b, 0)), (x/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.62

$$\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx = \frac{B\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{A\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}}$$

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] B*arcsinh(b*x/sqrt(a*b))/sqrt(b) - A*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 6.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx = \frac{B\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}} - \frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] int((A + B*x)/(x*(a + b*x^2)^(1/2)),x)
```

```
[Out] (B*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2) - (A*atanh((a + b*x^2)^(1/2)
/a^(1/2)))/a^(1/2)
```

3.27 $\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx$

Optimal result 224
Rubi [A] (verified) 224
Mathematica [A] (verified) 225
Maple [A] (verified) 226
Fricas [A] (verification not implemented) 226
Sympy [A] (verification not implemented) 226
Maxima [A] (verification not implemented) 227
Giac [A] (verification not implemented) 227
Mupad [B] (verification not implemented) 227

Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx=-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -B*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)-A*(b*x^2+a)^(1/2)/a/x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {821, 272, 65, 214}

$$\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx=-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] Int[(A + B*x)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A\sqrt{a+bx^2}}{ax}+B\int \frac{1}{x\sqrt{a+bx^2}}\,dx \\
&= -\frac{A\sqrt{a+bx^2}}{ax}+\frac{1}{2}B\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right) \\
&= -\frac{A\sqrt{a+bx^2}}{ax}+\frac{B\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{b} \\
&= -\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx=-\frac{A\sqrt{a+bx^2}}{ax}+\frac{2B\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] Integrate[(A + B*x)/(x^2*Sqrt[a + b*x^2]),x]
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) + (2*B*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/S
qrt[a]])/Sqrt[a]
```

Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

method	result	size
default	$-\frac{B\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{\sqrt{a}}-\frac{A\sqrt{b\,x^2+a}}{ax}$	49
risch	$-\frac{B\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{\sqrt{a}}-\frac{A\sqrt{b\,x^2+a}}{ax}$	49

```
[In] int((B*x+A)/x^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -B/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-A*(b*x^2+a)^(1/2)/a/x
```

Fricas [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.15

$$\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx = \left[\frac{B\sqrt{a}x\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\sqrt{bx^2+a}A}{2ax}, \frac{B\sqrt{-a}x\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}A}{ax}\right]$$

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] [1/2*(B*sqrt(a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*s
qrt(b*x^2 + a)*A)/(a*x), (B*sqrt(-a)*x*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - s
qrt(b*x^2 + a)*A)/(a*x)]
```

Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}-\frac{B\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}$$

```
[In] integrate((B*x+A)/x**2/(b*x**2+a)**(1/2),x)
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/a - B*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx = -\frac{B\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} - \frac{\sqrt{bx^2+a}A}{ax}$$

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] -B*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) - sqrt(b*x^2 + a)*A/(a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx = \frac{2\,B\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2\,A\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] 2*B*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) + 2*A*sqrt(b)/
((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

Mupad [B] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx = -\frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{A\,\sqrt{b\,x^2+a}}{a\,x}$$

```
[In] int((A + B*x)/(x^2*(a + b*x^2)^(1/2)),x)
[Out] - (B*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2) - (A*(a + b*x^2)^(1/2))/(a*x
)
```

3.28 $\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx$

Optimal result 228
Rubi [A] (verified) 228
Mathematica [A] (verified) 230
Maple [A] (verified) 230
Fricas [A] (verification not implemented) 230
Sympy [A] (verification not implemented) 231
Maxima [A] (verification not implemented) 231
Giac [B] (verification not implemented) 231
Mupad [B] (verification not implemented) 232

Optimal result

Integrand size = 20, antiderivative size = 72

$$\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx=-\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}+\frac{Ab\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

```
[Out] 1/2*A*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)-1/2*A*(b*x^2+a)^(1/2)/a/x^
2-B*(b*x^2+a)^(1/2)/a/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {849, 821, 272, 65, 214}

$$\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx=\frac{Ab\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}$$

```
[In] Int[(A + B*x)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/2*(A*Sqrt[a + b*x^2])/(a*x^2) - (B*Sqrt[a + b*x^2])/(a*x) + (A*b*ArcTanh
[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{A\sqrt{a+bx^2}}{2ax^2} - \frac{\int \frac{-2aB+Abx}{x^2\sqrt{a+bx^2}}\,dx}{2a} \\
&= -\frac{A\sqrt{a+bx^2}}{2ax^2} - \frac{B\sqrt{a+bx^2}}{ax} - \frac{(Ab)\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{2a} \\
&= -\frac{A\sqrt{a+bx^2}}{2ax^2} - \frac{B\sqrt{a+bx^2}}{ax} - \frac{(Ab)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{4a} \\
&= -\frac{A\sqrt{a+bx^2}}{2ax^2} - \frac{B\sqrt{a+bx^2}}{ax} - \frac{A\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{2a} \\
&= -\frac{A\sqrt{a+bx^2}}{2ax^2} - \frac{B\sqrt{a+bx^2}}{ax} + \frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx=-\frac{(A+2Bx)\sqrt{a+bx^2}}{2ax^2}-\frac{Ab\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[In] Integrate[(A + B*x)/(x^3*Sqrt[a + b*x^2]),x]
[Out] -1/2*((A + 2*B*x)*Sqrt[a + b*x^2])/(a*x^2) - (A*b*ArcTanh[(Sqrt[b]*x - Sqrt
[a + b*x^2])/Sqrt[a]])/a^(3/2)
```

Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

method	result	size
risch	$-\frac{\sqrt{b\,x^{2}+a}\,(2Bx+A)}{2a\,x^{2}}+\frac{Ab\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2a^{\frac{3}{2}}}$	55
default	$-\frac{B\sqrt{b\,x^{2}+a}}{ax}+A\left(-\frac{\sqrt{b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)$	69

```
[In] int((B*x+A)/x^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(b*x^2+a)^(1/2)*(2*B*x+A)/a/x^2+1/2*A/a^(3/2)*b*ln((2*a+2*a^(1/2)*(b*x
^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.71

$$\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx=\left[\frac{A\sqrt{a}bx^2\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\,(2\,Bax+Aa)\sqrt{bx^2+a}}{4\,a^2x^2},\right.$$
$$\left.-\frac{A\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(2\,Bax+Aa)\sqrt{bx^2+a}}{2\,a^2x^2}\right]$$

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] [1/4*(A*sqrt(a)*b*x^2*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) -
 2*(2*B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*x^2), -1/2*(A*sqrt(-a)*b*x^2*arcta
n(sqrt(-a)/sqrt(b*x^2 + a)) + (2*B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*x^2)]
```

Sympy [A] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax} + \frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}$$

```
[In] integrate((B*x+A)/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + A*b*asinh(sqrt(a)/(sqrt(b)*x))/(2
*a**(3/2)) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx = \frac{Ab\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{\sqrt{bx^2+a}B}{ax} - \frac{\sqrt{bx^2+a}A}{2\,ax^2}$$

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - sqrt(b*x^2 + a)*B/(a*x) - 1
/2*sqrt(b*x^2 + a)*A/(a*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(58) = 116$.

Time = 0.30 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.03

$$\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx = -\frac{Ab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab + 2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b} + \left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aab - 2\,Ba^2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2a}$$

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -A*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a) + ((sqrt(
b)*x - sqrt(b*x^2 + a))^3*A*b + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(
b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b - 2*B*a^2*sqrt(b))/(((sqrt(b)*x -
sqrt(b*x^2 + a))^2 - a)^2*a)
```

Mupad [B] (verification not implemented)

Time = 5.81 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx=\frac{A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{B\,\sqrt{b\,x^2+a}}{a\,x}-\frac{A\,\sqrt{b\,x^2+a}}{2\,a\,x^2}$$

```
[In] int((A + B*x)/(x^3*(a + b*x^2)^(1/2)),x)
```

```
[Out] (A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2)) - (B*(a + b*x^2)^(1/2))/
(a*x) - (A*(a + b*x^2)^(1/2))/(2*a*x^2)
```

3.29 $\int \frac{x^3(A+Bx)}{\left(a+bx^2\right)^{3/2}} \, dx$

Optimal result 233
Rubi [A] (verified) 233
Mathematica [A] (verified) 235
Maple [A] (verified) 235
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Sympy [A] (verification not implemented) 236
Maxima [A] (verification not implemented) 236
Giac [A] (verification not implemented) 236
Mupad [F(-1)] 237

Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \frac{x^3(A+Bx)}{\left(a+bx^2\right)^{3/2}} \, dx = -\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}+\frac{(4A+3Bx)\sqrt{a+bx^2}}{2b^2}-\frac{3aB\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

```
[Out] -3/2*a*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)-x^2*(B*x+A)/b/(b*x^2+a)
^(1/2)+1/2*(3*B*x+4*A)*(b*x^2+a)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {833, 794, 223, 212}

$$\int \frac{x^3(A+Bx)}{\left(a+bx^2\right)^{3/2}} \, dx = \frac{\sqrt{a+bx^2}(4A+3Bx)}{2b^2}-\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}-\frac{3aB\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

```
[In] Int[(x^3*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((x^2*(A + B*x))/(b*Sqrt[a + b*x^2])) + ((4*A + 3*B*x)*Sqrt[a + b*x^2])/(2
*b^2) - (3*a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}+\frac{\int \frac{x(2aA+3aBx)}{\sqrt{a+bx^2}}\,dx}{ab}\\
&= -\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}+\frac{(4A+3Bx)\sqrt{a+bx^2}}{2b^2}-\frac{(3aB)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b^2}\\
&= -\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}+\frac{(4A+3Bx)\sqrt{a+bx^2}}{2b^2}-\frac{(3aB)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2b^2}\\
&= -\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}+\frac{(4A+3Bx)\sqrt{a+bx^2}}{2b^2}-\frac{3aB\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{3/2}}\,dx = \frac{4aA+3aBx+2Abx^2+bBx^3}{2b^2\sqrt{a+bx^2}} + \frac{3aB\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{5/2}}$$

```
[In] Integrate[(x^3*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] (4*a*A + 3*a*B*x + 2*A*b*x^2 + b*B*x^3)/(2*b^2*Sqrt[a + b*x^2]) + (3*a*B*Lo
g[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{(Bx+2A)\sqrt{b\,x^2+a}}{2b^2}+\frac{aBx}{b^2\sqrt{b\,x^2+a}}-\frac{3aB\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{5}{2}}}+\frac{aA}{b^2\sqrt{b\,x^2+a}}$	77
default	$B\left(\frac{x^3}{2b\sqrt{b\,x^2+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)+A\left(\frac{x^2}{\sqrt{b\,x^2+a}\,b}+\frac{2a}{b^2\sqrt{b\,x^2+a}}\right)$	98

```
[In] int(x^3*(B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(B*x+2*A)/b^2*(b*x^2+a)^(1/2)+a/b^2*B*x/(b*x^2+a)^(1/2)-3/2*a/b^(5/2)*B
*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+a/b^2*A/(b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.43

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{3/2}}\,dx = \left[\frac{3\left(Babx^2+Ba^2\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(Bb^2x^3+2\,Ab^2x^2+3\,B\right.}{4\left(b^4x^2+ab^3\right)}\right.$$

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(B*a*b*x^2 + B*a^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b
)*x - a) + 2*(B*b^2*x^3 + 2*A*b^2*x^2 + 3*B*a*b*x + 4*A*a*b)*sqrt(b*x^2 + a
))/(b^4*x^2 + a*b^3), 1/2*(3*(B*a*b*x^2 + B*a^2)*sqrt(-b)*arctan(sqrt(-b)*x
/sqrt(b*x^2 + a)) + (B*b^2*x^3 + 2*A*b^2*x^2 + 3*B*a*b*x + 4*A*a*b)*sqrt(b*
x^2 + a))/(b^4*x^2 + a*b^3)]
```

Sympy [A] (verification not implemented)

Time = 3.63 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.44

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{3/2}}\,dx = A\left(\begin{cases} \frac{2a}{b^2\sqrt{a+bx^2}}+\frac{x^2}{b\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + B\left(\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}}+\frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

```
[In] integrate(x**3*(B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(b,
 0)), (x**4/(4*a**(3/2)), True)) + B*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a
)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 +
 b*x**2/a)))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{3/2}}\,dx = \frac{Bx^3}{2\sqrt{bx^2+a}b}+\frac{Ax^2}{\sqrt{bx^2+a}b} + \frac{3\,Bax}{2\sqrt{bx^2+a}b^2}-\frac{3\,Ba\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}}+\frac{2\,Aa}{\sqrt{bx^2+a}b^2}$$

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*B*x^3/(sqrt(b*x^2 + a)*b) + A*x^2/(sqrt(b*x^2 + a)*b) + 3/2*B*a*x/(sqrt
(b*x^2 + a)*b^2) - 3/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 2*A*a/(sqrt(b*x
^2 + a)*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(\frac{Bx}{b}+\frac{2\,A}{b}\right)x+\frac{3\,Ba}{b^2}\right)x+\frac{4\,Aa}{b^2}}{2\sqrt{bx^2+a}}+\frac{3\,Ba\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(((B*x/b + 2*A/b)*x + 3*B*a/b^2)*x + 4*A*a/b^2)/sqrt(b*x^2 + a) + 3/2*B
*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{3/2}}\,dx = \int \frac{x^3\,(A+B\,x)}{(b\,x^2+a)^{3/2}}\,dx$$

```
[In] int((x^3*(A + B*x))/(a + b*x^2)^(3/2),x)
[Out] int((x^3*(A + B*x))/(a + b*x^2)^(3/2), x)
```

3.30 $\int \frac{x^2(A+Bx)}{\left(a+bx^2\right)^{3/2}} \, dx$

Optimal result 238
Rubi [A] (verified) 238
Mathematica [A] (verified) 239
Maple [A] (verified) 240
Fricas [A] (verification not implemented) 240
Sympy [A] (verification not implemented) 240
Maxima [A] (verification not implemented) 241
Giac [A] (verification not implemented) 241
Mupad [B] (verification not implemented) 241

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{x^2(A+Bx)}{\left(a+bx^2\right)^{3/2}} \, dx = -\frac{x(A+Bx)}{b\sqrt{a+bx^2}}+\frac{2B\sqrt{a+bx^2}}{b^2}+\frac{A\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

```
[Out] A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)-x*(B*x+A)/b/(b*x^2+a)^(1/2)+2*
B*(b*x^2+a)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {833, 655, 223, 212}

$$\int \frac{x^2(A+Bx)}{\left(a+bx^2\right)^{3/2}} \, dx = \frac{A\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{x(A+Bx)}{b\sqrt{a+bx^2}}+\frac{2B\sqrt{a+bx^2}}{b^2}$$

```
[In] Int[(x^2*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((x*(A + B*x))/(b*Sqrt[a + b*x^2])) + (2*B*Sqrt[a + b*x^2])/b^2 + (A*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x(A+Bx)}{b\sqrt{a+bx^2}}+\frac{\int \frac{aA+2aBx}{\sqrt{a+bx^2}}\,dx}{ab}\\
&= -\frac{x(A+Bx)}{b\sqrt{a+bx^2}}+\frac{2B\sqrt{a+bx^2}}{b^2}+\frac{A\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b}\\
&= -\frac{x(A+Bx)}{b\sqrt{a+bx^2}}+\frac{2B\sqrt{a+bx^2}}{b^2}+\frac{A\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{b}\\
&= -\frac{x(A+Bx)}{b\sqrt{a+bx^2}}+\frac{2B\sqrt{a+bx^2}}{b^2}+\frac{A\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

$$
\int \frac{x^2(A+Bx)}{(a+bx^2)^{3/2}}\,dx=\frac{2aB-Abx+bBx^2}{b^2\sqrt{a+bx^2}}+\frac{2A\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{3/2}}
$$

```
[In] Integrate[(x^2*(A + B*x))/(a + b*x^2)^(3/2),x]
[Out] (2*a*B - A*b*x + b*B*x^2)/(b^2*Sqrt[a + b*x^2]) + (2*A*ArcTanh[(Sqrt[b]*x)/
(-Sqrt[a] + Sqrt[a + b*x^2])])/b^(3/2)
```

Maple [A] (verified)

Time = 3.53 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

method	result	size
risch	$\frac{B\sqrt{b\,x^{2}+a}}{b^{2}}-\frac{Ax}{b\sqrt{b\,x^{2}+a}}+\frac{A\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}+\frac{aB}{b^{2}\sqrt{b\,x^{2}+a}}$	68
default	$B\left(\frac{x^{2}}{\sqrt{b\,x^{2}+a}\,b}+\frac{2a}{b^{2}\sqrt{b\,x^{2}+a}}\right)+A\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)$	74

```
[In] int(x^2*(B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] B*(b*x^2+a)^(1/2)/b^2-1/b*A*x/(b*x^2+a)^(1/2)+1/b^(3/2)*A*ln(x*b^(1/2)+(b*x
^2+a)^(1/2))+a/b^2*B/(b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.48

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{3/2}}\,dx = \left[\frac{(Abx^2+Aa)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(Bbx^2-Abx+2\,Ba)\sqrt{bx^2+}}{2\,(b^3x^2+ab^2)}\right.$$
$$\left.-\frac{(Abx^2+Aa)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(Bbx^2-Abx+2\,Ba)\sqrt{bx^2+a}}{b^3x^2+ab^2}\right]$$

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] [1/2*((A*b*x^2 + A*a)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
a) + 2*(B*b*x^2 - A*b*x + 2*B*a)*sqrt(b*x^2 + a))/(b^3*x^2 + a*b^2), -((A*b
*x^2 + A*a)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (B*b*x^2 - A*b*x
+ 2*B*a)*sqrt(b*x^2 + a))/(b^3*x^2 + a*b^2)]
```

Sympy [A] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.26

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{3/2}}\,dx = A\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$
$$+B\left(\begin{cases}\frac{2a}{b^2\sqrt{a+bx^2}}+\frac{x^2}{b\sqrt{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x**2*(B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) +
B*Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(b,
 0)), (x**4/(4*a**(3/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{3/2}}\,dx = \frac{Bx^2}{\sqrt{bx^2+ab}} - \frac{Ax}{\sqrt{bx^2+ab}} + \frac{A\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} + \frac{2\,Ba}{\sqrt{bx^2+ab^2}}$$

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] B*x^2/(sqrt(b*x^2 + a)*b) - A*x/(sqrt(b*x^2 + a)*b) + A*arcsinh(b*x/sqrt(a*
b))/b^(3/2) + 2*B*a/(sqrt(b*x^2 + a)*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\frac{Bx}{b}-\frac{A}{b}\right)x+\frac{2\,Ba}{b^2}}{\sqrt{bx^2+a}} - \frac{A\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}$$

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((B*x/b - A/b)*x + 2*B*a/b^2)/sqrt(b*x^2 + a) - A*log(abs(-sqrt(b)*x + sqrt
(b*x^2 + a)))/b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 6.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{3/2}}\,dx = \frac{A\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}} - \frac{A\,x}{b\,\sqrt{b\,x^2+a}} + \frac{B\,(b\,x^2+2\,a)}{b^2\,\sqrt{b\,x^2+a}}$$

```
[In] int((x^2*(A + B*x))/(a + b*x^2)^(3/2),x)
```

```
[Out] (A*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(3/2) - (A*x)/(b*(a + b*x^2)^(1/2)
) + (B*(2*a + b*x^2))/(b^2*(a + b*x^2)^(1/2))
```

3.31 $\int \frac{x(A+Bx)}{(a+bx^2)^{3/2}}\,dx$

Optimal result 242
Rubi [A] (verified) 242
Mathematica [A] (verified) 243
Maple [A] (verified) 243
Fricas [A] (verification not implemented) 244
Sympy [A] (verification not implemented) 244
Maxima [A] (verification not implemented) 245
Giac [A] (verification not implemented) 245
Mupad [B] (verification not implemented) 245

Optimal result

Integrand size = 18, antiderivative size = 48

$$\int \frac{x(A+Bx)}{(a+bx^2)^{3/2}}\,dx = -\frac{A+Bx}{b\sqrt{a+bx^2}} + \frac{B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

```
[Out] B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+(-B*x-A)/b/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {792, 223, 212}

$$\int \frac{x(A+Bx)}{(a+bx^2)^{3/2}}\,dx = \frac{B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}} - \frac{A+Bx}{b\sqrt{a+bx^2}}$$

```
[In] Int[(x*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((A + B*x)/(b*Sqrt[a + b*x^2])) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])
/b^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A+Bx}{b\sqrt{a+bx^2}}+\frac{B\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b}\\
&= -\frac{A+Bx}{b\sqrt{a+bx^2}}+\frac{B\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{b}\\
&= -\frac{A+Bx}{b\sqrt{a+bx^2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{x(A+Bx)}{(a+bx^2)^{3/2}}\,dx=\frac{-A-Bx}{b\sqrt{a+bx^2}}-\frac{B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}}$$

```
[In] Integrate[(x*(A + B*x))/(a + b*x^2)^(3/2),x]
[Out] (-A - B*x)/(b*Sqrt[a + b*x^2]) - (B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^
(3/2)
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

method	result	size
default	$B\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)-\frac{A}{b\sqrt{b\,x^2+a}}$	55

```
[In] int(x*(B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))-1/b*A/(b*x
^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 147, normalized size of antiderivative = 3.06

$$\int \frac{x(A+Bx)}{(a+bx^2)^{3/2}}\,dx = \left[\frac{(Bbx^2+Ba)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(Bbx+Ab)\sqrt{bx^2+a}}{2\,(b^3x^2+ab^2)},\right.$$
$$\left.-\frac{(Bbx^2+Ba)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(Bbx+Ab)\sqrt{bx^2+a}}{b^3x^2+ab^2}\right]$$

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((B*b*x^2 + B*a)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
a) - 2*(B*b*x + A*b)*sqrt(b*x^2 + a))/(b^3*x^2 + a*b^2), -((B*b*x^2 + B*a)*
sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (B*b*x + A*b)*sqrt(b*x^2 + a)
)/(b^3*x^2 + a*b^2)]
```

Sympy [A] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

$$\int \frac{x(A+Bx)}{(a+bx^2)^{3/2}}\,dx = A\left(\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for}\ b\neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + B\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

```
[In] integrate(x*(B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True))
 + B*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{x(A+Bx)}{(a+bx^2)^{3/2}}\,dx = -\frac{Bx}{\sqrt{bx^2+ab}} + \frac{B\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} - \frac{A}{\sqrt{bx^2+ab}}$$

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -B*x/(sqrt(b*x^2 + a)*b) + B*arcsinh(b*x/sqrt(a*b))/b^(3/2) - A/(sqrt(b*x^2
 + a)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{x(A+Bx)}{(a+bx^2)^{3/2}}\,dx = -\frac{\frac{Bx}{b}+\frac{A}{b}}{\sqrt{bx^2+a}} - \frac{B\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}$$

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(B*x/b + A/b)/sqrt(b*x^2 + a) - B*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b
^(3/2)
```

Mupad [B] (verification not implemented)

Time = 5.64 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{x(A+Bx)}{(a+bx^2)^{3/2}}\,dx = \frac{B\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}} - \frac{A}{b\,\sqrt{b\,x^2+a}} - \frac{B\,x}{b\,\sqrt{b\,x^2+a}}$$

```
[In] int((x*(A + B*x))/(a + b*x^2)^(3/2),x)
```

```
[Out] (B*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(3/2) - A/(b*(a + b*x^2)^(1/2)) -
(B*x)/(b*(a + b*x^2)^(1/2))
```

3.32 $\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx$

Optimal result 246
Rubi [A] (verified) 246
Mathematica [A] (verified) 247
Maple [A] (verified) 247
Fricas [A] (verification not implemented) 247
Sympy [A] (verification not implemented) 248
Maxima [A] (verification not implemented) 248
Giac [A] (verification not implemented) 248
Mupad [B] (verification not implemented) 249

Optimal result

Integrand size = 17, antiderivative size = 28

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx = -\frac{aB-Abx}{ab\sqrt{a+bx^2}}$$

```
[Out] (A*b*x-B*a)/a/b/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {651}

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx = -\frac{aB-Abx}{ab\sqrt{a+bx^2}}$$

```
[In] Int[(A + B*x)/(a + b*x^2)^(3/2),x]
```

```
[Out] -((a*B - A*b*x)/(a*b*Sqrt[a + b*x^2]))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\text{integral} = -\frac{aB-Abx}{ab\sqrt{a+bx^2}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx}{\left(a+bx^2\right)^{3/2}}\,dx=\frac{-aB+Abx}{ab\sqrt{a+bx^2}}$$

```
[In] Integrate[(A + B*x)/(a + b*x^2)^(3/2),x]
[Out] (-(a*B) + A*b*x)/(a*b*Sqrt[a + b*x^2])
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{Abx-Ba}{ab\sqrt{b\,x^2+a}}$	26
trager	$\frac{Abx-Ba}{ab\sqrt{b\,x^2+a}}$	26
default	$\frac{Ax}{a\sqrt{b\,x^2+a}}-\frac{B}{b\sqrt{b\,x^2+a}}$	32

```
[In] int((B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] (A*b*x-B*a)/a/b/(b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx}{\left(a+bx^2\right)^{3/2}}\,dx=\frac{(Abx-Ba)\sqrt{bx^2+a}}{ab^2x^2+a^2b}$$

```
[In] integrate((B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] (A*b*x - B*a)*sqrt(b*x^2 + a)/(a*b^2*x^2 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx = \frac{Ax}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}} + B\left(\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((B*x+A)/(b*x**2+a)**(3/2),x)
[Out] A*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*Piecewise((-1/(b*sqrt(a + b*x**2)), N
e(b, 0)), (x**2/(2*a**(3/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx = \frac{Ax}{\sqrt{bx^2+a}a} - \frac{B}{\sqrt{bx^2+a}b}$$

```
[In] integrate((B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] A*x/(sqrt(b*x^2 + a)*a) - B/(sqrt(b*x^2 + a)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx = \frac{\frac{Ax}{a}-\frac{B}{b}}{\sqrt{bx^2+a}}$$

```
[In] integrate((B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] (A*x/a - B/b)/sqrt(b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 6.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx = -\frac{\frac{B}{b}-\frac{A\,x}{a}}{\sqrt{b\,x^2+a}}$$

```
[In] int((A + B*x)/(a + b*x^2)^(3/2),x)
[Out] -(B/b - (A*x)/a)/(a + b*x^2)^(1/2)
```

3.33 $\int \frac{A+Bx}{x(a+bx^2)^{3/2}}\,dx$

Optimal result 250
Rubi [A] (verified) 250
Mathematica [A] (verified) 252
Maple [A] (verified) 252
Fricas [A] (verification not implemented) 252
Sympy [B] (verification not implemented) 253
Maxima [A] (verification not implemented) 253
Giac [A] (verification not implemented) 254
Mupad [B] (verification not implemented) 254

Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \frac{A+Bx}{x(a+bx^2)^{3/2}}\,dx = \frac{A+Bx}{a\sqrt{a+bx^2}} - \frac{A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)+(B*x+A)/a/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {837, 12, 272, 65, 214}

$$\int \frac{A+Bx}{x(a+bx^2)^{3/2}}\,dx = \frac{A+Bx}{a\sqrt{a+bx^2}} - \frac{A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[In] Int[(A + B*x)/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A + B*x)/(a*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{A+Bx}{a\sqrt{a+bx^2}}+\frac{\int \frac{aAb}{x\sqrt{a+bx^2}}\,dx}{a^2b}\\
&= \frac{A+Bx}{a\sqrt{a+bx^2}}+\frac{A\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{a}\\
&= \frac{A+Bx}{a\sqrt{a+bx^2}}+\frac{A\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a}\\
&= \frac{A+Bx}{a\sqrt{a+bx^2}}+\frac{A\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{ab}\\
&= \frac{A+Bx}{a\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx}{x\left(a+bx^2\right)^{3/2}}\,dx=\frac{A+Bx}{a\sqrt{a+bx^2}}+\frac{2A\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[In] Integrate[(A + B*x)/(x*(a + b*x^2)^(3/2)),x]
[Out] (A + B*x)/(a*Sqrt[a + b*x^2]) + (2*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/
Sqrt[a]])/a^(3/2)
```

Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.30

method	result	size
default	$\frac{Bx}{a\sqrt{b\,x^{2}+a}}+A\left(\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)$	61

```
[In] int((B*x+A)/x/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] B*x/a/(b*x^2+a)^(1/2)+A*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b
*x^2+a)^(1/2))/x))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.11

$$\int \frac{A+Bx}{x\left(a+bx^2\right)^{3/2}}\,dx=\left[\frac{(Abx^2+Aa)\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(Bax+Aa\right)\sqrt{bx^2+a}}{2\left(a^2bx^2+a^3\right)},\ \frac{(Abx^2+Aa)\sqrt{}}{}\right.$$

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] [1/2*((A*b*x^2 + A*a)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a
)/x^2) + 2*(B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*b*x^2 + a^3), ((A*b*x^2 + A*
a)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (B*a*x + A*a)*sqrt(b*x^2 + a
))/(a^2*b*x^2 + a^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(39) = 78$.

Time = 3.30 (sec) , antiderivative size = 206, normalized size of antiderivative = 4.38

$$\int \frac{A+Bx}{x(a+bx^2)^{3/2}}\,dx = A\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)+\frac{Bx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

```
[In] integrate((B*x+A)/x/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**3*log(b*
x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 + b*x**2/a) +
1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)/(2*a**(9/2)
 + 2*a**(7/2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/
2) + 2*a**(7/2)*b*x**2)) + B*x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx}{x(a+bx^2)^{3/2}}\,dx = \frac{Bx}{\sqrt{bx^2+a}a}-\frac{A\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}}+\frac{A}{\sqrt{bx^2+a}a}$$

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] B*x/(sqrt(b*x^2 + a)*a) - A*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + A/(sqrt
(b*x^2 + a)*a)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx}{x\left(a+bx^2\right)^{3/2}}\,dx = \frac{\frac{Bx}{a}+\frac{A}{a}}{\sqrt{bx^2+a}} + \frac{2\,A\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}$$

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] (B*x/a + A/a)/sqrt(b*x^2 + a) + 2*A*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/s
qrt(-a))/(sqrt(-a)*a)
```

Mupad [B] (verification not implemented)

Time = 6.49 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx}{x\left(a+bx^2\right)^{3/2}}\,dx = \frac{A}{a\,\sqrt{b\,x^2+a}} - \frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{B\,x}{a\,\sqrt{b\,x^2+a}}$$

```
[In] int((A + B*x)/(x*(a + b*x^2)^(3/2)),x)
```

```
[Out] A/(a*(a + b*x^2)^(1/2)) - (A*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(3/2) + (B
*x)/(a*(a + b*x^2)^(1/2))
```

3.34 $\int \frac{A+Bx}{x^2(a+bx^2)^{3/2}}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 70

$$\int \frac{A+Bx}{x^2(a+bx^2)^{3/2}}\,dx = \frac{A+Bx}{ax\sqrt{a+bx^2}} - \frac{2A\sqrt{a+bx^2}}{a^2x} - \frac{B\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -B*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)+(B*x+A)/a/x/(b*x^2+a)^(1/2)-2*A
*(b*x^2+a)^(1/2)/a^2/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {837, 821, 272, 65, 214}

$$\int \frac{A+Bx}{x^2(a+bx^2)^{3/2}}\,dx = -\frac{B\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2A\sqrt{a+bx^2}}{a^2x} + \frac{A+Bx}{ax\sqrt{a+bx^2}}$$

```
[In] Int[(A + B*x)/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A + B*x)/(a*x*Sqrt[a + b*x^2]) - (2*A*Sqrt[a + b*x^2])/(a^2*x) - (B*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{A+Bx}{ax\sqrt{a+bx^2}} - \frac{\int \frac{-2aAb-abBx}{x^2\sqrt{a+bx^2}}\,dx}{a^2b} \\
&= \frac{A+Bx}{ax\sqrt{a+bx^2}} - \frac{2A\sqrt{a+bx^2}}{a^2x} + \frac{B\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{a} \\
&= \frac{A+Bx}{ax\sqrt{a+bx^2}} - \frac{2A\sqrt{a+bx^2}}{a^2x} + \frac{B\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{2a} \\
&= \frac{A+Bx}{ax\sqrt{a+bx^2}} - \frac{2A\sqrt{a+bx^2}}{a^2x} + \frac{B\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{ab}
\end{aligned}
$$

$$= \frac{A+Bx}{ax\sqrt{a+bx^2}} - \frac{2A\sqrt{a+bx^2}}{a^2x} - \frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx}{x^2(a+bx^2)^{3/2}}\,dx = \frac{-aA+aBx-2Abx^2}{a^2x\sqrt{a+bx^2}} + \frac{2B\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[In] Integrate[(A + B*x)/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-(a*A) + a*B*x - 2*A*b*x^2)/(a^2*x*Sqrt[a + b*x^2]) + (2*B*ArcTanh[(Sqrt[b
]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(3/2)
```

Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.14

method	result	size
risch	$-\frac{A\sqrt{b\,x^2+a}}{a^2x} - \frac{Abx}{a^2\sqrt{b\,x^2+a}} + \frac{B}{a\sqrt{b\,x^2+a}} - \frac{B\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}$	80
default	$B\left(\frac{1}{a\sqrt{b\,x^2+a}} - \frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right) + A\left(-\frac{1}{ax\sqrt{b\,x^2+a}} - \frac{2bx}{a^2\sqrt{b\,x^2+a}}\right)$	82

```
[In] int((B*x+A)/x^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2*A*(b*x^2+a)^(1/2)/x-1/a^2*A*b*x/(b*x^2+a)^(1/2)+B/a/(b*x^2+a)^(1/2)-
1/a^(3/2)*B*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.41

$$\int \frac{A+Bx}{x^2(a+bx^2)^{3/2}}\,dx = \left[\frac{(Bbx^3+Bax)\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right) - 2(2Abx^2-Bax+Aa)\sqrt{bx^2+a}}{2(a^2bx^3+a^3x)}, -\right.$$

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((B*b*x^3 + B*a*x)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2
*a)/x^2) - 2*(2*A*b*x^2 - B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*b*x^3 + a^3*x)
, ((B*b*x^3 + B*a*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (2*A*b*x^2
 - B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*b*x^3 + a^3*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(60) = 120$.

Time = 4.15 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.36

$$\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{3/2}}\,dx = A\left(-\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2}+1}}-\frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right) + B\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} + \frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)$$

```
[In] integrate((B*x+A)/x**2/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x**
2) + 1))) + B*(2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) +
 a**3*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 +
b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)
/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) +
1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{3/2}}\,dx = -\frac{2\,Abx}{\sqrt{bx^2+a}a^2}-\frac{B\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}}+\frac{B}{\sqrt{bx^2+a}a}-\frac{A}{\sqrt{bx^2+a}ax}$$

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*A*b*x/(sqrt(b*x^2 + a)*a^2) - B*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) +
B/(sqrt(b*x^2 + a)*a) - A/(sqrt(b*x^2 + a)*a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.37

$$\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{3/2}}\,dx = -\frac{\frac{Abx}{a^2}-\frac{B}{a}}{\sqrt{bx^2+a}} + \frac{2\,B\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a}$$

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(A*b*x/a^2 - B/a)/sqrt(b*x^2 + a) + 2*B*arctan(-(sqrt(b)*x - sqrt(b*x^2 +
a))/sqrt(-a))/(sqrt(-a)*a) + 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2
- a)*a)
```

Mupad [B] (verification not implemented)

Time = 6.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{3/2}}\,dx = \frac{B}{a\sqrt{b\,x^2+a}} - \frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{A}{a\,x\sqrt{b\,x^2+a}} - \frac{2\,A\,b\,x}{a^2\sqrt{b\,x^2+a}}$$

```
[In] int((A + B*x)/(x^2*(a + b*x^2)^(3/2)),x)
```

```
[Out] B/(a*(a + b*x^2)^(1/2)) - (B*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(3/2) - A/
(a*x*(a + b*x^2)^(1/2)) - (2*A*b*x)/(a^2*(a + b*x^2)^(1/2))
```

3.35 $\int \frac{A+Bx}{x^3(a+bx^2)^{3/2}} dx$

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Optimal result

Integrand size = 20, antiderivative size = 95

$$\int \frac{A+Bx}{x^3(a+bx^2)^{3/2}}\,dx=\frac{A+Bx}{ax^2\sqrt{a+bx^2}}-\frac{3A\sqrt{a+bx^2}}{2a^2x^2}-\frac{2B\sqrt{a+bx^2}}{a^2x}+\frac{3Ab\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

```
[Out] 3/2*A*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)+(B*x+A)/a/x^2/(b*x^2+a)^(1
/2)-3/2*A*(b*x^2+a)^(1/2)/a^2/x^2-2*B*(b*x^2+a)^(1/2)/a^2/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {837, 849, 821, 272, 65, 214}

$$\int \frac{A+Bx}{x^3(a+bx^2)^{3/2}}\,dx=\frac{3Ab\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3A\sqrt{a+bx^2}}{2a^2x^2}-\frac{2B\sqrt{a+bx^2}}{a^2x}+\frac{A+Bx}{ax^2\sqrt{a+bx^2}}$$

```
[In] Int[(A + B*x)/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A + B*x)/(a*x^2*Sqrt[a + b*x^2]) - (3*A*Sqrt[a + b*x^2])/(2*a^2*x^2) - (2*
B*Sqrt[a + b*x^2])/(a^2*x) + (3*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^
(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$\text{integral} = \frac{A+Bx}{ax^2\sqrt{a+bx^2}} - \frac{\int \frac{-3aAb-2abBx}{x^3\sqrt{a+bx^2}}\,dx}{a^2b}$$

$$= \frac{A+Bx}{ax^2\sqrt{a+bx^2}} - \frac{3A\sqrt{a+bx^2}}{2a^2x^2} + \frac{\int \frac{4a^2bB-3aAb^2x}{x^2\sqrt{a+bx^2}}\,dx}{2a^3b}$$

$$= \frac{A+Bx}{ax^2\sqrt{a+bx^2}} - \frac{3A\sqrt{a+bx^2}}{2a^2x^2} - \frac{2B\sqrt{a+bx^2}}{a^2x} - \frac{(3Ab)\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{2a^2}$$

$$= \frac{A+Bx}{ax^2\sqrt{a+bx^2}} - \frac{3A\sqrt{a+bx^2}}{2a^2x^2} - \frac{2B\sqrt{a+bx^2}}{a^2x} - \frac{(3Ab)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{4a^2}$$

$$= \frac{A+Bx}{ax^2\sqrt{a+bx^2}} - \frac{3A\sqrt{a+bx^2}}{2a^2x^2} - \frac{2B\sqrt{a+bx^2}}{a^2x} - \frac{(3A)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{2a^2}$$

$$= \frac{A+Bx}{ax^2\sqrt{a+bx^2}} - \frac{3A\sqrt{a+bx^2}}{2a^2x^2} - \frac{2B\sqrt{a+bx^2}}{a^2x} + \frac{3Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{3/2}}\,dx = \frac{-a(A+2Bx)-bx^2(3A+4Bx)}{2a^2x^2\sqrt{a+bx^2}} - \frac{3Ab\,\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[In] Integrate[(A + B*x)/(x^3*(a + b*x^2)^(3/2)),x]
[Out] (-(a*(A + 2*B*x)) - b*x^2*(3*A + 4*B*x))/(2*a^2*x^2*Sqrt[a + b*x^2]) - (3*A
*b*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(5/2)
```

Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

method	result	size
risch	$-\frac{\sqrt{b\,x^{2}+a}\,(2Bx+A)}{2a^{2}x^{2}}-\frac{bBx}{a^{2}\sqrt{b\,x^{2}+a}}-\frac{Ab}{a^{2}\sqrt{b\,x^{2}+a}}+\frac{3bA\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2a^{\frac{5}{2}}}$	88
default	$B\left(-\frac{1}{ax\sqrt{b\,x^{2}+a}}-\frac{2bx}{a^{2}\sqrt{b\,x^{2}+a}}\right)+A\left(-\frac{1}{2a\,x^{2}\sqrt{b\,x^{2}+a}}-\frac{3b\left(\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2a}\right)$	106

```
[In] int((B*x+A)/x^3/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(b*x^2+a)^(1/2)*(2*B*x+A)/a^2/x^2-b/a^2*B*x/(b*x^2+a)^(1/2)-1/a^2*A/(b
*x^2+a)^(1/2)*b+3/2*b/a^(5/2)*A*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.22

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{3/2}}\,dx=\left[\frac{3\left(Ab^2x^4+Aabx^2\right)\sqrt{a}\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\left(4\,Babx^3+3\,Aabx^2+2\,Ba^2x\right.}{4\left(a^3bx^4+a^4x^2\right)}\right.$$
$$\left.-\frac{3\left(Ab^2x^4+Aabx^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(4\,Babx^3+3\,Aabx^2+2\,Ba^2x+Aa^2\right)\sqrt{bx^2+a}}{2\left(a^3bx^4+a^4x^2\right)}\right]$$

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(A*b^2*x^4 + A*a*b*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqr
t(a) + 2*a)/x^2) - 2*(4*B*a*b*x^3 + 3*A*a*b*x^2 + 2*B*a^2*x + A*a^2)*sqrt(b
*x^2 + a))/(a^3*b*x^4 + a^4*x^2), -1/2*(3*(A*b^2*x^4 + A*a*b*x^2)*sqrt(-a)*
arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (4*B*a*b*x^3 + 3*A*a*b*x^2 + 2*B*a^2*x +
 A*a^2)*sqrt(b*x^2 + a))/(a^3*b*x^4 + a^4*x^2)]
```

Sympy [A] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.31

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{3/2}}\,dx=A\left(-\frac{1}{2a\sqrt{b}x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}}+\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{5}{2}}}\right)$$
$$+B\left(-\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2}+1}}-\frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right)$$

```
[In] integrate((B*x+A)/x**3/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a/
(b*x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(5/2))) + B*(-1/(a*sq
rt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x**2) + 1)))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{3/2}}\,dx = -\frac{2\,Bbx}{\sqrt{bx^2+a}a^2} + \frac{3\,Ab\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{5}{2}}} - \frac{3\,Ab}{2\,\sqrt{bx^2+a}a^2} - \frac{B}{\sqrt{bx^2+a}ax} - \frac{A}{2\,\sqrt{bx^2+a}ax^2}$$

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*B*b*x/(sqrt(b*x^2 + a)*a^2) + 3/2*A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5
/2) - 3/2*A*b/(sqrt(b*x^2 + a)*a^2) - B/(sqrt(b*x^2 + a)*a*x) - 1/2*A/(sqrt
(b*x^2 + a)*a*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(79) = 158.

Time = 0.30 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.80

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{3/2}}\,dx = -\frac{\frac{Bbx}{a^2}+\frac{Ab}{a^2}}{\sqrt{bx^2+a}} - \frac{3\,Ab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aab-2\,Ba^2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2a^2}$$

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(B*b*x/a^2 + A*b/a^2)/sqrt(b*x^2 + a) - 3*A*b*arctan(-(sqrt(b)*x - sqrt(b*
x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b +
 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 +
a))*A*a*b - 2*B*a^2*sqrt(b))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 5.94 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{3/2}}\,dx = \frac{3\,A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{5/2}} - \frac{3\,A\,b}{2\,a^2\,\sqrt{b\,x^2+a}} - \frac{A}{2\,a\,x^2\,\sqrt{b\,x^2+a}} - \frac{\sqrt{b\,x^2+a}\,\left(\frac{B}{a}+\frac{2\,B\,b\,x^2}{a^2}\right)}{b\,x^3+a\,x}$$

```
[In] int((A + B*x)/(x^3*(a + b*x^2)^(3/2)),x)
```

```
[Out] (3*A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(5/2)) - (3*A*b)/(2*a^2*(a +
b*x^2)^(1/2)) - A/(2*a*x^2*(a + b*x^2)^(1/2)) - ((a + b*x^2)^(1/2)*(B/a + (
2*B*b*x^2)/a^2))/(a*x + b*x^3)
```

3.36 $\int \frac{x^3(A+Bx)}{(a+bx^2)^{5/2}}\,dx$

Optimal result 266
Rubi [A] (verified) 266
Mathematica [A] (verified) 268
Maple [A] (verified) 268
Fricas [A] (verification not implemented) 268
Sympy [A] (verification not implemented) 269
Maxima [A] (verification not implemented) 269
Giac [A] (verification not implemented) 270
Mupad [F(-1)] 270

Optimal result

Integrand size = 20, antiderivative size = 79

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{5/2}}\,dx = -\frac{x^2(A+Bx)}{3b\,(a+bx^2)^{3/2}} - \frac{2A+3Bx}{3b^2\sqrt{a+bx^2}} + \frac{B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

```
[Out] -1/3*x^2*(B*x+A)/b/(b*x^2+a)^(3/2)+B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(
5/2)+1/3*(-3*B*x-2*A)/b^2/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {833, 792, 223, 212}

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{5/2}}\,dx = -\frac{2A+3Bx}{3b^2\sqrt{a+bx^2}} - \frac{x^2(A+Bx)}{3b\,(a+bx^2)^{3/2}} + \frac{B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

```
[In] Int[(x^3*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/3*(x^2*(A + B*x))/(b*(a + b*x^2)^(3/2)) - (2*A + 3*B*x)/(3*b^2*Sqrt[a +
b*x^2]) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}+\frac{\int \frac{x(2aA+3aBx)}{\left(a+bx^2\right)^{3/2}}\,dx}{3ab}\\
&= -\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}-\frac{2A+3Bx}{3b^2\sqrt{a+bx^2}}+\frac{B\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b^2}\\
&= -\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}-\frac{2A+3Bx}{3b^2\sqrt{a+bx^2}}+\frac{B\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{b^2}\\
&= -\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}-\frac{2A+3Bx}{3b^2\sqrt{a+bx^2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{5/2}}\,dx = \frac{-2aA-3aBx-3Abx^2-4bBx^3}{3b^2\,(a+bx^2)^{3/2}} - \frac{B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{5/2}}$$

```
[In] Integrate[(x^3*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-2*a*A - 3*a*B*x - 3*A*b*x^2 - 4*b*B*x^3)/(3*b^2*(a + b*x^2)^(3/2)) - (B*L
og[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(5/2)
```

Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.23

method	result	size
default	$B\left(-\frac{x^3}{3b(b\,x^2+a)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}}{b}\right)+A\left(-\frac{x^2}{b(b\,x^2+a)^{\frac{3}{2}}}-\frac{2a}{3b^2(b\,x^2+a)^{\frac{3}{2}}}\right)$	97

```
[In] int(x^3*(B*x+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1
/2)+(b*x^2+a)^(1/2))))+A*(-x^2/b/(b*x^2+a)^(3/2)-2/3*a/b^2/(b*x^2+a)^(3/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.03

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{5/2}}\,dx = \left[\frac{3\,(Bb^2x^4+2\,Babx^2+Ba^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(4\,Bb^2x^3+3\,A}{6\,(b^5x^4+2\,ab^4x^2+a^2b^3)}\right.$$

$$\left.-\frac{3\,(Bb^2x^4+2\,Babx^2+Ba^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(4\,Bb^2x^3+3\,Ab^2x^2+3\,Babx+2\,Aab)\sqrt{bx^2+a}}{3\,(b^5x^4+2\,ab^4x^2+a^2b^3)}\right]$$

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(B*b^2*x^4 + 2*B*a*b*x^2 + B*a^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x
^2 + a)*sqrt(b)*x - a) - 2*(4*B*b^2*x^3 + 3*A*b^2*x^2 + 3*B*a*b*x + 2*A*a*b
)*sqrt(b*x^2 + a))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3), -1/3*(3*(B*b^2*x^4 +
```

```
2*B*a*b*x^2 + B*a^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (4*B*b^2
*x^3 + 3*A*b^2*x^2 + 3*B*a*b*x + 2*A*a*b)*sqrt(b*x^2 + a))/(b^5*x^4 + 2*a*b
^4*x^2 + a^2*b^3)]
```

Sympy [A] (verification not implemented)

Time = 5.33 (sec) , antiderivative size = 400, normalized size of antiderivative = 5.06

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{5/2}}\,dx = A\left(\begin{cases} -\frac{2a}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} - \frac{3bx^2}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right) + B\left(\frac{3a^{\frac{39}{2}}b^{11}\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} + \frac{3a^{\frac{37}{2}}b^{12}x^2\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} - \frac{3a^{19}b^{\frac{23}{2}}x}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} - \frac{4a^{18}b^{\frac{25}{2}}x^3}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}\right)$$

```
[In] integrate(x**3*(B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*Piecewise((-2*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)
) - 3*b*x**2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)), Ne
(b, 0)), (x**4/(4*a**(5/2)), True)) + B*(3*a**(39/2)*b**11*sqrt(1 + b*x**2/
a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a
**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) + 3*a**(37/2)*b**12*x**2*sqrt(1
 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**
2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 3*a**19*b**(23/2)*x
/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqr
t(1 + b*x**2/a)) - 4*a**18*b**(25/2)*x**3/(3*a**(39/2)*b**(27/2)*sqrt(1 + b
*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.29

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{5/2}}\,dx = -\frac{1}{3}\,Bx\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right) - \frac{Ax^2}{(bx^2+a)^{\frac{3}{2}}b} - \frac{Bx}{3\,\sqrt{bx^2+a}b^2} + \frac{B\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}} - \frac{2\,Aa}{3\,(bx^2+a)^{\frac{3}{2}}b^2}$$

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*B*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) - A*x^
2/((b*x^2 + a)^(3/2)*b) - 1/3*B*x/(sqrt(b*x^2 + a)*b^2) + B*arcsinh(b*x/sqr
t(a*b))/b^(5/2) - 2/3*A*a/((b*x^2 + a)^(3/2)*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{5/2}}\,dx = -\frac{\left(\left(\frac{4\,Bx}{b}+\frac{3\,A}{b}\right)x+\frac{3\,Ba}{b^2}\right)x+\frac{2\,Aa}{b^2}}{3\,(bx^2+a)^{\frac{3}{2}}} - \frac{B\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}}$$

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(((4*B*x/b + 3*A/b)*x + 3*B*a/b^2)*x + 2*A*a/b^2)/(b*x^2 + a)^(3/2) -
B*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{5/2}}\,dx = \int \frac{x^3\,(A+B\,x)}{(b\,x^2+a)^{5/2}}\,dx$$

```
[In] int((x^3*(A + B*x))/(a + b*x^2)^(5/2),x)
```

```
[Out] int((x^3*(A + B*x))/(a + b*x^2)^(5/2), x)
```

3.37 $\int \frac{x^2(A+Bx)}{(a+bx^2)^{5/2}} dx$

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Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{5/2}}\,dx = -\frac{x^2(aB-Abx)}{3ab\,(a+bx^2)^{3/2}} - \frac{2B}{3b^2\sqrt{a+bx^2}}$$

```
[Out] -1/3*x^2*(-A*b*x+B*a)/a/b/(b*x^2+a)^(3/2)-2/3*B/b^2/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {819, 267}

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{5/2}}\,dx = -\frac{x^2(aB-Abx)}{3ab\,(a+bx^2)^{3/2}} - \frac{2B}{3b^2\sqrt{a+bx^2}}$$

```
[In] Int[(x^2*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/3*(x^2*(a*B - A*b*x))/(a*b*(a + b*x^2)^(3/2)) - (2*B)/(3*b^2*Sqrt[a + b*
x^2])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
```

```
*(p + 1))), x] - Dist[m*((c*d*f + a*e*g)/(2*a*c*(p + 1))), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x^2(aB - Abx)}{3ab\left(a+bx^2\right)^{3/2}} + \frac{(2B)\int \frac{x}{\left(a+bx^2\right)^{3/2}}\,dx}{3b} \\ &= -\frac{x^2(aB - Abx)}{3ab\left(a+bx^2\right)^{3/2}} - \frac{2B}{3b^2\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \frac{x^2(A+Bx)}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{-2a^2B - 3abBx^2 + Ab^2x^3}{3ab^2\left(a+bx^2\right)^{3/2}}$$

```
[In] Integrate[(x^2*(A + B*x))/(a + b*x^2)^(5/2),x]
[Out] (-2*a^2*B - 3*a*b*B*x^2 + A*b^2*x^3)/(3*a*b^2*(a + b*x^2)^(3/2))
```

Maple [A] (verified)

Time = 3.40 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{A\,b^2x^3-3Bab\,x^2-2a^2B}{3\left(b\,x^2+a\right)^{\frac{3}{2}}a\,b^2}$	41
trager	$\frac{A\,b^2x^3-3Bab\,x^2-2a^2B}{3\left(b\,x^2+a\right)^{\frac{3}{2}}a\,b^2}$	41
default	$B\left(-\frac{x^2}{b\left(b\,x^2+a\right)^{\frac{3}{2}}}-\frac{2a}{3b^2\left(b\,x^2+a\right)^{\frac{3}{2}}}\right)+A\left(-\frac{x}{2b\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{a\left(\frac{x}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)}{2b}\right)$	92

```
[In] int(x^2*(B*x+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(A*b^2*x^3-3*B*a*b*x^2-2*B*a^2)/(b*x^2+a)^(3/2)/a/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{5/2}}\,dx = \frac{(Ab^2x^3 - 3\,Babx^2 - 2\,Ba^2)\sqrt{bx^2+a}}{3\,(ab^4x^4 + 2\,a^2b^3x^2 + a^3b^2)}$$

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(A*b^2*x^3 - 3*B*a*b*x^2 - 2*B*a^2)*sqrt(b*x^2 + a)/(a*b^4*x^4 + 2*a^2*
b^3*x^2 + a^3*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(46) = 92.

Time = 4.25 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.66

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{5/2}}\,dx = \frac{Ax^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}} + 3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + B\left(\begin{cases} -\frac{2a}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} - \frac{3bx^2}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate(x**2*(B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a
)) + B*Piecewise((-2*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*
x**2)) - 3*b*x**2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)
), Ne(b, 0)), (x**4/(4*a**(5/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{5/2}}\,dx = -\frac{Bx^2}{(bx^2+a)^{\frac{3}{2}}b} - \frac{Ax}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{Ax}{3\,\sqrt{bx^2+a}ab} - \frac{2\,Ba}{3\,(bx^2+a)^{\frac{3}{2}}b^2}$$

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -B*x^2/((b*x^2 + a)^(3/2)*b) - 1/3*A*x/((b*x^2 + a)^(3/2)*b) + 1/3*A*x/(sqr
t(b*x^2 + a)*a*b) - 2/3*B*a/((b*x^2 + a)^(3/2)*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\frac{Ax}{a} - \frac{3B}{b}\right)x^2 - \frac{2Ba}{b^2}}{3\,(bx^2+a)^{\frac{3}{2}}}$$

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
[Out] 1/3*((A*x/a - 3*B/b)*x^2 - 2*B*a/b^2)/(b*x^2 + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 5.46 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{5/2}}\,dx = \frac{B\,a^2 - 3\,B\,a\,(b\,x^2+a) + A\,b\,x\,(b\,x^2+a) - A\,a\,b\,x}{3\,a\,b^2\,(b\,x^2+a)^{3/2}}$$

```
[In] int((x^2*(A + B*x))/(a + b*x^2)^(5/2),x)
[Out] (B*a^2 - 3*B*a*(a + b*x^2) + A*b*x*(a + b*x^2) - A*a*b*x)/(3*a*b^2*(a + b*x
^2)^(3/2))
```

3.38 $\int \frac{x(A+Bx)}{(a+bx^2)^{5/2}} \, dx$

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Optimal result

Integrand size = 18, antiderivative size = 50

$$\int \frac{x(A+Bx)}{(a+bx^2)^{5/2}} \, dx = \frac{-A-Bx}{3b(a+bx^2)^{3/2}} + \frac{Bx}{3ab\sqrt{a+bx^2}}$$

```
[Out] 1/3*(-B*x-A)/b/(b*x^2+a)^(3/2)+1/3*B*x/a/b/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {792, 197}

$$\int \frac{x(A+Bx)}{(a+bx^2)^{5/2}} \, dx = \frac{Bx}{3ab\sqrt{a+bx^2}} - \frac{A+Bx}{3b(a+bx^2)^{3/2}}$$

```
[In] Int[(x*(A + B*x))/(a + b*x^2)^(5/2),x]
[Out] -1/3*(A + B*x)/(b*(a + b*x^2)^(3/2)) + (B*x)/(3*a*b*Sqrt[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
```

```
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{A+Bx}{3b\,(a+bx^2)^{3/2}}+\frac{B\int \frac{1}{(a+bx^2)^{3/2}}\,dx}{3b}\\ &= -\frac{A+Bx}{3b\,(a+bx^2)^{3/2}}+\frac{Bx}{3ab\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.64

$$\int \frac{x(A+Bx)}{(a+bx^2)^{5/2}}\,dx=\frac{-aA+bBx^3}{3ab\,(a+bx^2)^{3/2}}$$

```
[In] Integrate[(x*(A + B*x))/(a + b*x^2)^(5/2),x]
[Out] (-(a*A) + b*B*x^3)/(3*a*b*(a + b*x^2)^(3/2))
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.58

method	result	size
gosper	$-\frac{-bB\,x^3+Aa}{3(b\,x^2+a)^{\frac{3}{2}}\,ab}$	29
trager	$-\frac{-bB\,x^3+Aa}{3(b\,x^2+a)^{\frac{3}{2}}\,ab}$	29
default	$B\left(-\frac{x}{2b(b\,x^2+a)^{\frac{3}{2}}}+\frac{a\left(\frac{x}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)}{2b}\right)-\frac{A}{3b(b\,x^2+a)^{\frac{3}{2}}}$	72

```
[In] int(x*(B*x+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(-B*b*x^3+A*a)/(b*x^2+a)^(3/2)/a/b
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{x(A+Bx)}{(a+bx^2)^{5/2}}\,dx = \frac{(Bbx^3 - Aa)\sqrt{bx^2+a}}{3\,(ab^3x^4 + 2\,a^2b^2x^2 + a^3b)}$$

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
[Out] 1/3*(B*b*x^3 - A*a)*sqrt(b*x^2 + a)/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)
```

Sympy [A] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.90

$$\int \frac{x(A+Bx)}{(a+bx^2)^{5/2}}\,dx = A\left(\begin{cases} -\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right) + \frac{Bx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}} + 3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

```
[In] integrate(x*(B*x+A)/(b*x**2+a)**(5/2),x)
[Out] A*Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqrt(a + b*x**2)), Ne
(b, 0)), (x**2/(2*a**(5/2)), True)) + B*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a)
 + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \frac{x(A+Bx)}{(a+bx^2)^{5/2}}\,dx = -\frac{Bx}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{Bx}{3\,\sqrt{bx^2+a}ab} - \frac{A}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
[Out] -1/3*B*x/((b*x^2 + a)^(3/2)*b) + 1/3*B*x/(sqrt(b*x^2 + a)*a*b) - 1/3*A/((b*
x^2 + a)^(3/2)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.52

$$\int \frac{x(A+Bx)}{(a+bx^2)^{5/2}}\,dx = \frac{\frac{Bx^3}{a} - \frac{A}{b}}{3\,(bx^2+a)^{\frac{3}{2}}}$$

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(B*x^3/a - A/b)/(b*x^2 + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 5.41 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int \frac{x(A+Bx)}{(a+bx^2)^{5/2}}\,dx = \frac{B\,x^3}{3\,a\,(b\,x^2+a)^{3/2}} - \frac{A}{3\,b\,(b\,x^2+a)^{3/2}}$$

```
[In] int((x*(A + B*x))/(a + b*x^2)^(5/2),x)
```

```
[Out] (B*x^3)/(3*a*(a + b*x^2)^(3/2)) - A/(3*b*(a + b*x^2)^(3/2))
```

3.39 $\int \frac{A+Bx}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal result 279
Rubi [A] (verified) 279
Mathematica [A] (verified) 280
Maple [A] (verified) 280
Fricas [A] (verification not implemented) 281
Sympy [B] (verification not implemented) 281
Maxima [A] (verification not implemented) 281
Giac [A] (verification not implemented) 282
Mupad [B] (verification not implemented) 282

Optimal result

Integrand size = 17, antiderivative size = 51

$$\int \frac{A+Bx}{\left(a+bx^2\right)^{5/2}}\,dx=\frac{-aB+Abx}{3ab\left(a+bx^2\right)^{3/2}}+\frac{2Ax}{3a^2\sqrt{a+bx^2}}$$

```
[Out] 1/3*(A*b*x-B*a)/a/b/(b*x^2+a)^(3/2)+2/3*A*x/a^2/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {653, 197}

$$\int \frac{A+Bx}{\left(a+bx^2\right)^{5/2}}\,dx=\frac{2Ax}{3a^2\sqrt{a+bx^2}}-\frac{aB-Abx}{3ab\left(a+bx^2\right)^{3/2}}$$

```
[In] Int[(A + B*x)/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/3*(a*B - A*b*x)/(a*b*(a + b*x^2)^(3/2)) + (2*A*x)/(3*a^2*Sqrt[a + b*x^2]
)
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
```

```
Q[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{aB - Abx}{3ab\left(a + bx^2\right)^{3/2}} + \frac{(2A)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a} \\ &= -\frac{aB - Abx}{3ab\left(a + bx^2\right)^{3/2}} + \frac{2Ax}{3a^2\sqrt{a + bx^2}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{A + Bx}{\left(a + bx^2\right)^{5/2}}\,dx = \frac{-a^2B + 3aAbx + 2Ab^2x^3}{3a^2b\left(a + bx^2\right)^{3/2}}$$

```
[In] Integrate[(A + B*x)/(a + b*x^2)^(5/2),x]
[Out] (-(a^2*B) + 3*a*A*b*x + 2*A*b^2*x^3)/(3*a^2*b*(a + b*x^2)^(3/2))
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{2A\,b^2x^3+3aAbx-a^2B}{3\left(b\,x^2+a\right)^{\frac{3}{2}}a^2b}$	40
trager	$\frac{2A\,b^2x^3+3aAbx-a^2B}{3\left(b\,x^2+a\right)^{\frac{3}{2}}a^2b}$	40
default	$A\left(\frac{x}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)-\frac{B}{3b\left(b\,x^2+a\right)^{\frac{3}{2}}}$	50

```
[In] int((B*x+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(2*A*b^2*x^3+3*A*a*b*x-B*a^2)/(b*x^2+a)^(3/2)/a^2/b
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.22

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx = \frac{(2\,Ab^2x^3+3\,Aabx-Ba^2)\sqrt{bx^2+a}}{3\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)}$$

```
[In] integrate((B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*A*b^2*x^3 + 3*A*a*b*x - B*a^2)*sqrt(b*x^2 + a)/(a^2*b^3*x^4 + 2*a^3*
b^2*x^2 + a^4*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(44) = 88$.

Time = 3.59 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.86

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx = A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + \frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right) + B\left(\begin{cases} -\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for}\ b \neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate((B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2
/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 +
 b*x**2/a))) + B*Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqrt(a
 + b*x**2)), Ne(b, 0)), (x**2/(2*a**(5/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx = \frac{2\,Ax}{3\,\sqrt{bx^2+a}a^2} + \frac{Ax}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{B}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

```
[In] integrate((B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*A*x/(sqrt(b*x^2 + a)*a^2) + 1/3*A*x/((b*x^2 + a)^(3/2)*a) - 1/3*B/((b*x
^2 + a)^(3/2)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\frac{2\,Abx^2}{a^2}+\frac{3\,A}{a}\right)x-\frac{B}{b}}{3\,(bx^2+a)^{\frac{3}{2}}}$$

```
[In] integrate((B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
[Out] 1/3*((2*A*b*x^2/a^2 + 3*A/a)*x - B/b)/(b*x^2 + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx = \frac{2\,A\,b\,x\,(b\,x^2+a)-B\,a^2+A\,a\,b\,x}{3\,a^2\,b\,(b\,x^2+a)^{3/2}}$$

```
[In] int((A + B*x)/(a + b*x^2)^(5/2),x)
[Out] (2*A*b*x*(a + b*x^2) - B*a^2 + A*a*b*x)/(3*a^2*b*(a + b*x^2)^(3/2))
```

3.40 $\int \frac{A+Bx}{x\left(a+bx^2\right)^{5/2}}\,dx$

Optimal result 283
Rubi [A] (verified) 283
Mathematica [A] (verified) 285
Maple [A] (verified) 285
Fricas [A] (verification not implemented) 285
Sympy [B] (verification not implemented) 286
Maxima [A] (verification not implemented) 288
Giac [A] (verification not implemented) 289
Mupad [B] (verification not implemented) 289

Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \frac{A+Bx}{x\left(a+bx^2\right)^{5/2}}\,dx=\frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}+\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}-\frac{A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] 1/3*(B*x+A)/a/(b*x^2+a)^(3/2)-A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)+1/
3*(2*B*x+3*A)/a^2/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {837, 12, 272, 65, 214}

$$\int \frac{A+Bx}{x\left(a+bx^2\right)^{5/2}}\,dx=-\frac{A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}+\frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}$$

```
[In] Int[(A + B*x)/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] (A + B*x)/(3*a*(a + b*x^2)^(3/2)) + (3*A + 2*B*x)/(3*a^2*Sqrt[a + b*x^2]) -
 (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}-\frac{\int \frac{-3aAb-2abBx}{x\left(a+bx^2\right)^{3/2}}\,dx}{3a^2b}\\
&= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}+\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}+\frac{\int \frac{3a^2Ab^2}{x\sqrt{a+bx^2}}\,dx}{3a^4b^2}\\
&= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}+\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}+\frac{A\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{a^2}\\
&= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}+\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}+\frac{A\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a^2}\\
&= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}+\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}+\frac{A\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{a^2b}
\end{aligned}
$$

$$= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}+\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx}{x\left(a+bx^2\right)^{5/2}}\,dx = \frac{bx^2(3A+2Bx)+a(4A+3Bx)}{3a^2\left(a+bx^2\right)^{3/2}}+\frac{2A\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[In] Integrate[(A + B*x)/(x*(a + b*x^2)^(5/2)),x]
[Out] (b*x^2*(3*A + 2*B*x) + a*(4*A + 3*B*x))/(3*a^2*(a + b*x^2)^(3/2)) + (2*A*Ar
cTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(5/2)
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.29

method	result	size
default	$B\left(\frac{x}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)+A\left(\frac{1}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}\right)$	98

```
[In] int((B*x+A)/x/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] B*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))+A*(1/3/a/(b*x^2+a)^(3
/2)+1/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x
)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.14

$$\int \frac{A+Bx}{x\left(a+bx^2\right)^{5/2}}\,dx = \left[\frac{3\left(Ab^2x^4+2\,Aabx^2+Aa^2\right)\sqrt{a}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(2\,Babx^3+3\,Aabx^2+\right.}{6\left(a^3b^2x^4+2\,a^4bx^2+a^5\right)}\right.$$

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(A*b^2*x^4 + 2*A*a*b*x^2 + A*a^2)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^
2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*B*a*b*x^3 + 3*A*a*b*x^2 + 3*B*a^2*x + 4*A
*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5), 1/3*(3*(A*b^2*x^4
 + 2*A*a*b*x^2 + A*a^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (2*B*a*
b*x^3 + 3*A*a*b*x^2 + 3*B*a^2*x + 4*A*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^4 +
2*a^4*b*x^2 + a^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 840 vs. $2(65) = 130$.

Time = 8.43 (sec) , antiderivative size = 840, normalized size of antiderivative = 11.05

$$\int \frac{A+Bx}{x\left(a+bx^2\right)^{5/2}}\,dx = A\left(\frac{8a^7\sqrt{1+\frac{bx^2}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6} + \frac{3a^7\log\left(\frac{bx^2}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6} - \frac{6a^7\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6} + \frac{14a^6bx^2\sqrt{1+\frac{bx^2}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6} + \frac{9a^6bx^2\log\left(\frac{bx^2}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6} - \frac{18a^6bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6} + \frac{6a^5b^2x^4\sqrt{1+\frac{bx^2}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6} + \frac{9a^5b^2x^4\log\left(\frac{bx^2}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6} - \frac{18a^5b^2x^4\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6} + \frac{3a^4b^3x^6\log\left(\frac{bx^2}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6} - \frac{6a^4b^3x^6\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}\right) + B\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + \frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right)$$

```
[In] integrate((B*x+A)/x/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(8*a**7*sqrt(1 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15
/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**7*log(b*x**2/a)/(6*a**(19/2)
```

```
+ 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6
*a**7*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a
**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt(1 + b*x**
2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13
/2)*b**3*x**6) + 9*a**6*b*x**2*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*
x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**6*b*x**2*log
(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*
b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 6*a**5*b**2*x**4*sqrt(1 + b*x**2/a)/(6
*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**
3*x**6) + 9*a**5*b**2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2
 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**5*b**2*x**4*log(
sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b
**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**4*b**3*x**6*log(b*x**2/a)/(6*a**(1
9/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6
) - 6*a**4*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2
)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6)) + B*(3*a*x/(3*a
**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)) + 2*b*x*
*3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx}{x\,(a+bx^2)^{5/2}}\,dx = \frac{2\,Bx}{3\,\sqrt{bx^2+a}a^2} + \frac{Bx}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{A\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{5}{2}}} + \frac{A}{\sqrt{bx^2+a}a^2} + \frac{A}{3\,(bx^2+a)^{\frac{3}{2}}a}$$

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*B*x/(sqrt(b*x^2 + a)*a^2) + 1/3*B*x/((b*x^2 + a)^(3/2)*a) - A*arcsinh(a
/(sqrt(a*b)*abs(x)))/a^(5/2) + A/(sqrt(b*x^2 + a)*a^2) + 1/3*A/((b*x^2 + a)
^(3/2)*a)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx}{x\left(a+bx^2\right)^{5/2}}\,dx = \frac{\left(\left(\frac{2\,Bbx}{a^2}+\frac{3\,Ab}{a^2}\right)x+\frac{3\,B}{a}\right)x+\frac{4\,A}{a}}{3\left(bx^2+a\right)^{\frac{3}{2}}}+\frac{2\,A\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}$$

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(((2*B*b*x/a^2 + 3*A*b/a^2)*x + 3*B/a)*x + 4*A/a)/(b*x^2 + a)^(3/2) + 2
*A*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 6.37 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx}{x\left(a+bx^2\right)^{5/2}}\,dx = \frac{\frac{A}{3\,a}+\frac{A\,\left(b\,x^2+a\right)}{a^2}}{\left(b\,x^2+a\right)^{3/2}}+\frac{2\,B\,x\,\left(b\,x^2+a\right)+B\,a\,x}{3\,a^2\,\left(b\,x^2+a\right)^{3/2}}-\frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[In] int((A + B*x)/(x*(a + b*x^2)^(5/2)),x)
```

```
[Out] (A/(3*a) + (A*(a + b*x^2))/a^2)/(a + b*x^2)^(3/2) + (2*B*x*(a + b*x^2) + B*
a*x)/(3*a^2*(a + b*x^2)^(3/2)) - (A*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(5/
2)
```

3.41 $\int \frac{A+Bx}{x^2(a+bx^2)^{5/2}}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 104

$$\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{5/2}}\,dx=\frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}+\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}-\frac{8A\sqrt{a+bx^2}}{3a^3x}-\frac{B\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] 1/3*(B*x+A)/a/x/(b*x^2+a)^(3/2)-B*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)+
1/3*(3*B*x+4*A)/a^2/x/(b*x^2+a)^(1/2)-8/3*A*(b*x^2+a)^(1/2)/a^3/x
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {837, 821, 272, 65, 214}

$$\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{5/2}}\,dx=-\frac{B\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{8A\sqrt{a+bx^2}}{3a^3x}+\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}+\frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}$$

```
[In] Int[(A + B*x)/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] (A + B*x)/(3*a*x*(a + b*x^2)^(3/2)) + (4*A + 3*B*x)/(3*a^2*x*Sqrt[a + b*x^2
]) - (8*A*Sqrt[a + b*x^2])/(3*a^3*x) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])
/a^(5/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-4aAb-3abBx}{x^2\left(a+bx^2\right)^{3/2}}\,dx}{3a^2b}\\
&= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}} + \frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}} + \frac{\int \frac{8a^2Ab^2+3a^2b^2Bx}{x^2\sqrt{a+bx^2}}\,dx}{3a^4b^2}\\
&= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}} + \frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}} - \frac{8A\sqrt{a+bx^2}}{3a^3x} + \frac{B\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{a^2}
\end{aligned}$$

$$= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}+\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}-\frac{8A\sqrt{a+bx^2}}{3a^3x}+\frac{B\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a^2}$$

$$= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}+\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}-\frac{8A\sqrt{a+bx^2}}{3a^3x}+\frac{B\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{a^2b}$$

$$= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}+\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}-\frac{8A\sqrt{a+bx^2}}{3a^3x}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{5/2}}\,dx = \frac{-8Ab^2x^4+3abx^2(-4A+Bx)+a^2(-3A+4Bx)}{3a^3x\left(a+bx^2\right)^{3/2}} + \frac{2B\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[In] Integrate[(A + B*x)/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-8*A*b^2*x^4 + 3*a*b*x^2*(-4*A + B*x) + a^2*(-3*A + 4*B*x))/(3*a^3*x*(a +
b*x^2)^(3/2)) + (2*B*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(5/2
)
```

Maple [A] (verified)

Time = 3.44 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.17

method	result
default	$B\left(\frac{1}{3a(bx^2+a)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{bx^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\sqrt{bx^2+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}\right)+A\left(-\frac{1}{ax(bx^2+a)^{\frac{3}{2}}}-\frac{4b\left(\frac{x}{3a\left(bx^2+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{bx^2+a}}\right)}{a}\right)$
risch	$-\frac{A\sqrt{bx^2+a}}{a^3x}-\frac{\ln\left(\frac{2a+2\sqrt{a}\sqrt{bx^2+a}}{x}\right)B}{a^{\frac{5}{2}}}-\frac{5\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2b+2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}\,A}{6a^3\left(x-\frac{\sqrt{-ab}}{b}\right)}+\frac{7\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2b+2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}\,B}{12a^2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}$

```
[In] int((B*x+A)/x^2/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/3/a/(b*x^2+a)^(3/2)+1/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/
2)*(b*x^2+a)^(1/2))/x)))+A*(-1/a/x/(b*x^2+a)^(3/2)-4*b/a*(1/3*x/a/(b*x^2+a)
^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.54

$$\int \frac{A+Bx}{x^2(a+bx^2)^{5/2}}\,dx = \left[\frac{3\,(Bb^2x^5+2\,Babx^3+Ba^2x)\sqrt{a}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(8\,Ab^2x^4-3\,Babx^3}{6\,(a^3b^2x^5+2\,a^4bx^3+a^5x)}\right.$$

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
[Out] [1/6*(3*(B*b^2*x^5 + 2*B*a*b*x^3 + B*a^2*x)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*
x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(8*A*b^2*x^4 - 3*B*a*b*x^3 + 12*A*a*b*x^2
- 4*B*a^2*x + 3*A*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)
, 1/3*(3*(B*b^2*x^5 + 2*B*a*b*x^3 + B*a^2*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(
b*x^2 + a)) - (8*A*b^2*x^4 - 3*B*a*b*x^3 + 12*A*a*b*x^2 - 4*B*a^2*x + 3*A*a
^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. 2(88) = 176.

Time = 6.91 (sec) , antiderivative size = 910, normalized size of antiderivative = 8.75

$$\int \frac{A+Bx}{x^2(a+bx^2)^{5/2}}\,dx = A\left(-\frac{3a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{12ab^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{8b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}\right)+B\left(\frac{8a^7\sqrt{1+\frac{bx^2}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}+\frac{3a^7\log\left(\frac{bx^2}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}-\frac{6a^7\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}+\frac{14a^6bx^2\sqrt{1+\frac{bx^2}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}+\frac{9a^6bx^2\log\left(\frac{bx^2}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}-\frac{18a^6bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}+\frac{6a^5b^2x^4\sqrt{1+\frac{bx^2}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}+\frac{9a^5b^2x^4\log\left(\frac{bx^2}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}-\frac{18a^5b^2x^4\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}+\frac{3a^4b^3x^6\log\left(\frac{bx^2}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}-\frac{6a^4b^3x^6\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}\right)$$

```
[In] integrate((B*x+A)/x**2/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 +
3*a**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 +
 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(a/(b*x**2) +
1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4)) + B*(8*a**7*sqrt(1
+ b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6
*a**(13/2)*b**3*x**6) + 3*a**7*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*
x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6*a**7*log(sqrt(1
+ b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**
4 + 6*a**(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt(1 + b*x**2/a)/(6*a**(19/2)
 + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) +
9*a**6*b*x**2*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/
2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**6*b*x**2*log(sqrt(1 + b*x**2/
a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**
(13/2)*b**3*x**6) + 6*a**5*b**2*x**4*sqrt(1 + b*x**2/a)/(6*a**(19/2) + 18*a
**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 9*a**5*
b**2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b
**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**5*b**2*x**4*log(sqrt(1 + b*x**2/a
) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(
13/2)*b**3*x**6) + 3*a**4*b**3*x**6*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/
2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6*a**4*b**3*x
**6*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**
(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{5/2}}\,dx = -\frac{8\,Abx}{3\,\sqrt{bx^2+a}a^3} - \frac{4\,Abx}{3\,(bx^2+a)^{\frac{3}{2}}a^2} - \frac{B\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{5}{2}}} + \frac{B}{\sqrt{bx^2+a}a^2} + \frac{B}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{A}{(bx^2+a)^{\frac{3}{2}}ax}$$

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -8/3*A*b*x/(sqrt(b*x^2 + a)*a^3) - 4/3*A*b*x/((b*x^2 + a)^(3/2)*a^2) - B*ar
csinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + B/(sqrt(b*x^2 + a)*a^2) + 1/3*B/((b*x
^2 + a)^(3/2)*a) - A/((b*x^2 + a)^(3/2)*a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{5/2}}\,dx = -\frac{\left(\left(\frac{5\,Ab^2x}{a^3}-\frac{3\,Bb}{a^2}\right)x+\frac{6\,Ab}{a^2}\right)x-\frac{4\,B}{a}}{3\left(bx^2+a\right)^{\frac{3}{2}}} + \frac{2\,B\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^2}$$

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(((5*A*b^2*x/a^3 - 3*B*b/a^2)*x + 6*A*b/a^2)*x - 4*B/a)/(b*x^2 + a)^(3
/2) + 2*B*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) +
2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 6.65 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{5/2}}\,dx = \frac{\frac{B}{3\,a}+\frac{B\,\left(b\,x^2+a\right)}{a^2}}{\left(b\,x^2+a\right)^{3/2}} - \frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{A\,a^2-8\,A\,\left(b\,x^2+a\right)^2+4\,A\,a\,\left(b\,x^2+a\right)}{3\,a^3\,x\,\left(b\,x^2+a\right)^{3/2}}$$

```
[In] int((A + B*x)/(x^2*(a + b*x^2)^(5/2)),x)
```

```
[Out] (B/(3*a) + (B*(a + b*x^2))/a^2)/(a + b*x^2)^(3/2) - (B*atanh((a + b*x^2)^(1
/2)/a^(1/2)))/a^(5/2) + (A*a^2 - 8*A*(a + b*x^2)^2 + 4*A*a*(a + b*x^2))/(3*
a^3*x*(a + b*x^2)^(3/2))
```

3.42 $\int \frac{A+Bx}{x^3(a+bx^2)^{5/2}}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 129

$$\int \frac{A+Bx}{x^3(a+bx^2)^{5/2}}\,dx=\frac{A+Bx}{3ax^2(a+bx^2)^{3/2}}+\frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}}-\frac{5A\sqrt{a+bx^2}}{2a^3x^2}-\frac{8B\sqrt{a+bx^2}}{3a^3x}+\frac{5Ab\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}$$

```
[Out] 1/3*(B*x+A)/a/x^2/(b*x^2+a)^(3/2)+5/2*A*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/
a^(7/2)+1/3*(4*B*x+5*A)/a^2/x^2/(b*x^2+a)^(1/2)-5/2*A*(b*x^2+a)^(1/2)/a^3/x
^2-8/3*B*(b*x^2+a)^(1/2)/a^3/x
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {837, 849, 821, 272, 65, 214}

$$\int \frac{A+Bx}{x^3(a+bx^2)^{5/2}}\,dx=\frac{5Ab\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{5A\sqrt{a+bx^2}}{2a^3x^2}-\frac{8B\sqrt{a+bx^2}}{3a^3x}+\frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}}+\frac{A+Bx}{3ax^2(a+bx^2)^{3/2}}$$

```
[In] Int[(A + B*x)/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] (A + B*x)/(3*a*x^2*(a + b*x^2)^(3/2)) + (5*A + 4*B*x)/(3*a^2*x^2*Sqrt[a + b
*x^2]) - (5*A*Sqrt[a + b*x^2])/(2*a^3*x^2) - (8*B*Sqrt[a + b*x^2])/(3*a^3*x
) + (5*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(7/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

p])

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{A+Bx}{3ax^2(a+bx^2)^{3/2}} - \frac{\int \frac{-5aAb-4abBx}{x^3(a+bx^2)^{3/2}}\,dx}{3a^2b} \\
&= \frac{A+Bx}{3ax^2(a+bx^2)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} + \frac{\int \frac{15a^2Ab^2+8a^2b^2Bx}{x^3\sqrt{a+bx^2}}\,dx}{3a^4b^2} \\
&= \frac{A+Bx}{3ax^2(a+bx^2)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} - \frac{5A\sqrt{a+bx^2}}{2a^3x^2} - \frac{\int \frac{-16a^3b^2B+15a^2Ab^3x}{x^2\sqrt{a+bx^2}}\,dx}{6a^5b^2} \\
&= \frac{A+Bx}{3ax^2(a+bx^2)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} - \frac{5A\sqrt{a+bx^2}}{2a^3x^2} - \frac{8B\sqrt{a+bx^2}}{3a^3x} - \frac{(5Ab)\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{2a^3} \\
&= \frac{A+Bx}{3ax^2(a+bx^2)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} - \frac{5A\sqrt{a+bx^2}}{2a^3x^2} \\
&\quad - \frac{8B\sqrt{a+bx^2}}{3a^3x} - \frac{(5Ab)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{4a^3} \\
&= \frac{A+Bx}{3ax^2(a+bx^2)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} - \frac{5A\sqrt{a+bx^2}}{2a^3x^2} \\
&\quad - \frac{8B\sqrt{a+bx^2}}{3a^3x} - \frac{(5A)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{2a^3} \\
&= \frac{A+Bx}{3ax^2(a+bx^2)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} - \frac{5A\sqrt{a+bx^2}}{2a^3x^2} - \frac{8B\sqrt{a+bx^2}}{3a^3x} + \frac{5Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx}{x^3(a+bx^2)^{5/2}}\,dx = \frac{-3a^2(A+2Bx)-4abx^2(5A+6Bx)-b^2x^4(15A+16Bx)}{6a^3x^2(a+bx^2)^{3/2}} - \frac{5Ab\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{7/2}}$$

```
[In] Integrate[(A + B*x)/(x^3*(a + b*x^2)^(5/2)),x]
[Out] (-3*a^2*(A + 2*B*x) - 4*a*b*x^2*(5*A + 6*B*x) - b^2*x^4*(15*A + 16*B*x))/(6
*a^3*x^2*(a + b*x^2)^(3/2)) - (5*A*b*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/
Sqrt[a]])/a^(7/2)
```

Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.13

method	result
default	$B\left(-\frac{1}{ax(b\,x^2+a)^{\frac{3}{2}}}-\frac{4b\left(\frac{x}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)}{a}\right)+A\left(-\frac{1}{2a\,x^2(b\,x^2+a)^{\frac{3}{2}}}-\frac{5b\left(\frac{1}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(2a\right.}{}}{a}\right.}{2a}\right.$
risch	$-\frac{\sqrt{b\,x^2+a}\,(2Bx+A)}{2a^3x^2}+\frac{5\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)Ab}{2a^{\frac{7}{2}}}+\frac{13\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2b-2\sqrt{-ab}\,\left(x+\frac{\sqrt{-ab}}{b}\right)}\,Ab}{12a^3\sqrt{-ab}\,\left(x+\frac{\sqrt{-ab}}{b}\right)}-\frac{5\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2b-2\sqrt{-ab}}}{6a^3\left(x+\frac{\sqrt{-ab}}{b}\right.}$

[In] int((B*x+A)/x^3/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)

[Out] B*(-1/a/x/(b*x^2+a)^(3/2)-4*b/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))+A*(-1/2/a/x^2/(b*x^2+a)^(3/2)-5/2*b/a*(1/3/a/(b*x^2+a)^(3/2)+1/a* (1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.38

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{5/2}}\,dx=\left[\frac{15\left(Ab^3x^6+2\,Aab^2x^4+Aa^2bx^2\right)\sqrt{a}\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\left(16\,Bab^2x^5+15\ldots\right.}{12\left(a^4b^2x^6+2\,a^5bx^4+a\ldots\right.}\right.$$

$$-\frac{15\left(Ab^3x^6+2\,Aab^2x^4+Aa^2bx^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(16\,Bab^2x^5+15\,Aab^2x^4+24\,Ba^2bx^3+20\,Aa^2b\ldots\right.}{6\left(a^4b^2x^6+2\,a^5bx^4+a^6x^2\right)}$$

[In] integrate((B*x+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="fricas")

[Out] [1/12*(15*(A*b^3*x^6 + 2*A*a*b^2*x^4 + A*a^2*b*x^2)*sqrt(a)*log(-(b*x^2 + 2 *sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(16*B*a*b^2*x^5 + 15*A*a*b^2*x^4 + 24*B*a^2*b*x^3 + 20*A*a^2*b*x^2 + 6*B*a^3*x + 3*A*a^3)*sqrt(b*x^2 + a))/(a ^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2), -1/6*(15*(A*b^3*x^6 + 2*A*a*b^2*x^4 + A*a^2*b*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (16*B*a*b^2*x^5 + 15*A*a*b^2*x^4 + 24*B*a^2*b*x^3 + 20*A*a^2*b*x^2 + 6*B*a^3*x + 3*A*a^3)*sqr t(b*x^2 + a))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1034 vs. 2(122) = 244.

Time = 7.94 (sec) , antiderivative size = 1034, normalized size of antiderivative = 8.02

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{5/2}}\,dx = A\left(-\frac{6a^{17}\sqrt{1+\frac{bx^2}{a}}}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8} - \frac{46a^{16}bx^2\sqrt{1+\frac{bx^2}{a}}}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8} - \frac{15a^{16}bx^2\log\left(\frac{bx^2}{a}\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8} + \frac{30a^{16}bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8} - \frac{70a^{15}b^2x^4\sqrt{1+\frac{bx^2}{a}}}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8} - \frac{45a^{15}b^2x^4\log\left(\frac{bx^2}{a}\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8} + \frac{90a^{15}b^2x^4\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8} - \frac{30a^{14}b^3x^6\sqrt{1+\frac{bx^2}{a}}}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8} - \frac{45a^{14}b^3x^6\log\left(\frac{bx^2}{a}\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8} + \frac{90a^{14}b^3x^6\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8} - \frac{15a^{13}b^4x^8\log\left(\frac{bx^2}{a}\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8} + \frac{30a^{13}b^4x^8\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}\right) + B\left(-\frac{3a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4} - \frac{12ab^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4} - \frac{8b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}\right)$$

```
[In] integrate((B*x+A)/x**3/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-6*a**17*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 3
6*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 46*a**16*b*x**2*sqrt(1 +
b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6
 + 12*a**(33/2)*b**3*x**8) - 15*a**16*b*x**2*log(b*x**2/a)/(12*a**(39/2)*x*
*2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8)
 + 30*a**16*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(
37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 70*a**15*
b**2*x**4*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*
a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45*a**15*b**2*x**4*log(b*x*
*2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 1
2*a**(33/2)*b**3*x**8) + 90*a**15*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(12
*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/
2)*b**3*x**8) - 30*a**14*b**3*x**6*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 +
36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45
*a**14*b**3*x**6*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 3
6*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 90*a**14*b**3*x**6*log(sq
rt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/
2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 15*a**13*b**4*x**8*log(b*x**2/a)/(
12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(3
3/2)*b**3*x**8) + 30*a**13*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39
/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3
*x**8)) + B*(-3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b*
*5*x**2 + 3*a**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a
**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(a/(
b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{5/2}}\,dx = -\frac{8\,Bbx}{3\,\sqrt{bx^2+a}a^3} - \frac{4\,Bbx}{3\,(bx^2+a)^{\frac{3}{2}}a^2} + \frac{5\,Ab\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{7}{2}}} - \frac{5\,Ab}{2\,\sqrt{bx^2+a}a^3} - \frac{5\,Ab}{6\,(bx^2+a)^{\frac{3}{2}}a^2} - \frac{B}{(bx^2+a)^{\frac{3}{2}}ax} - \frac{A}{2\,(bx^2+a)^{\frac{3}{2}}ax^2}$$

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -8/3*B*b*x/(sqrt(b*x^2 + a)*a^3) - 4/3*B*b*x/((b*x^2 + a)^(3/2)*a^2) + 5/2*
A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) - 5/2*A*b/(sqrt(b*x^2 + a)*a^3) -
 5/6*A*b/((b*x^2 + a)^(3/2)*a^2) - B/((b*x^2 + a)^(3/2)*a*x) - 1/2*A/((b*x^
2 + a)^(3/2)*a*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.53

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{5/2}}\,dx = -\frac{\left(\left(\frac{5\,Bb^2x}{a^3}+\frac{6\,Ab^2}{a^3}\right)x+\frac{6\,Bb}{a^2}\right)x+\frac{7\,Ab}{a^2}}{3\left(bx^2+a\right)^{\frac{3}{2}}}-\frac{5\,Ab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3}+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aab-2\,Ba^2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2a^3}$$

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(((5*B*b^2*x/a^3 + 6*A*b^2/a^3)*x + 6*B*b/a^2)*x + 7*A*b/a^2)/(b*x^2 +
 a)^(3/2) - 5*A*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)
*a^3) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b + 2*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b - 2*B*a^2*sqrt(b))/
(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 6.74 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{5/2}}\,dx = \frac{B\,a^2-8\,B\left(b\,x^2+a\right)^2+4\,B\,a\left(b\,x^2+a\right)}{3\,a^3\,x\left(b\,x^2+a\right)^{3/2}}-\frac{10\,A\,b}{3\,a^2\left(b\,x^2+a\right)^{3/2}}-\frac{A}{2\,a\,x^2\left(b\,x^2+a\right)^{3/2}}+\frac{5\,A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{7/2}}-\frac{5\,A\,b^2\,x^2}{2\,a^3\left(b\,x^2+a\right)^{3/2}}$$

```
[In] int((A + B*x)/(x^3*(a + b*x^2)^(5/2)),x)
```

```
[Out] (B*a^2 - 8*B*(a + b*x^2)^2 + 4*B*a*(a + b*x^2))/(3*a^3*x*(a + b*x^2)^(3/2))
 - (10*A*b)/(3*a^2*(a + b*x^2)^(3/2)) - A/(2*a*x^2*(a + b*x^2)^(3/2)) + (5*
A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(7/2)) - (5*A*b^2*x^2)/(2*a^3*(a
 + b*x^2)^(3/2))
```

3.43 $\int \frac{(1-x)x}{\sqrt{1-x^2}} dx$

Optimal result 303
Rubi [A] (verified) 303
Mathematica [A] (verified) 304
Maple [A] (verified) 304
Fricas [A] (verification not implemented) 305
Sympy [A] (verification not implemented) 305
Maxima [A] (verification not implemented) 305
Giac [A] (verification not implemented) 305
Mupad [B] (verification not implemented) 306

Optimal result

Integrand size = 18, antiderivative size = 27

$$\int \frac{(1-x)x}{\sqrt{1-x^2}} dx = -\frac{1}{2}(2-x)\sqrt{1-x^2} - \frac{\arcsin(x)}{2}$$

```
[Out] -1/2*arcsin(x)-1/2*(2-x)*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {794, 222}

$$\int \frac{(1-x)x}{\sqrt{1-x^2}} dx = -\frac{\arcsin(x)}{2} - \frac{1}{2}\sqrt{1-x^2}(2-x)$$

```
[In] Int[((1 - x)*x)/Sqrt[1 - x^2],x]
```

```
[Out] -1/2*((2 - x)*Sqrt[1 - x^2]) - ArcSin[x]/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{2}(2-x)\sqrt{1-x^2} - \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\frac{1}{2}(2-x)\sqrt{1-x^2} - \frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}(2-x)\sqrt{1-x^2} + \arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

```
[In] Integrate[((1 - x)*x)/Sqrt[1 - x^2],x]
[Out] -1/2*((2 - x)*Sqrt[1 - x^2]) + ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

method	result	size
risch	$-\frac{(-2+x)(x^2-1)}{2\sqrt{-x^2+1}} - \frac{\arcsin(x)}{2}$	25
default	$\frac{x\sqrt{-x^2+1}}{2} - \frac{\arcsin(x)}{2} - \sqrt{-x^2+1}$	29
trager	$\left(-1+\frac{x}{2}\right)\sqrt{-x^2+1} + \frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{2}$	45
meijerg	$-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-x^2+1}}{2\sqrt{\pi}} - \frac{i\left(i\sqrt{\pi}\,x\sqrt{-x^2+1}-i\sqrt{\pi}\,\arcsin(x)\right)}{2\sqrt{\pi}}$	58

```
[In] int((1-x)*x/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(-2+x)*(x^2-1)/(-x^2+1)^(1/2)-1/2*arcsin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx = \frac{1}{2}\sqrt{-x^2+1}(x-2) + \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

```
[In] integrate((1-x)*x/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) + arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx = \frac{x\sqrt{1-x^2}}{2} - \sqrt{1-x^2} - \frac{\operatorname{asin}(x)}{2}$$

```
[In] integrate((1-x)*x/(-x**2+1)**(1/2),x)
[Out] x*sqrt(1 - x**2)/2 - sqrt(1 - x**2) - asin(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx = \frac{1}{2}\sqrt{-x^2+1}x - \sqrt{-x^2+1} - \frac{1}{2}\arcsin(x)$$

```
[In] integrate((1-x)*x/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1) - 1/2*arcsin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx = \frac{1}{2}\sqrt{-x^2+1}(x-2) - \frac{1}{2}\arcsin(x)$$

```
[In] integrate((1-x)*x/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) - 1/2*arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx = \left(\frac{x}{2}-1\right)\sqrt{1-x^2}-\frac{\operatorname{asin}(x)}{2}$$

```
[In] int(-(x*(x - 1))/(1 - x^2)^(1/2),x)
[Out] (x/2 - 1)*(1 - x^2)^(1/2) - asin(x)/2
```

3.44 $\int \frac{x-x^2}{\sqrt{1-x^2}} \, dx$

Optimal result 307
Rubi [A] (verified) 307
Mathematica [A] (verified) 308
Maple [A] (verified) 308
Fricas [A] (verification not implemented) 309
Sympy [A] (verification not implemented) 309
Maxima [A] (verification not implemented) 309
Giac [A] (verification not implemented) 309
Mupad [B] (verification not implemented) 310

Optimal result

Integrand size = 19, antiderivative size = 27

$$\int \frac{x-x^2}{\sqrt{1-x^2}} \, dx = -\frac{1}{2}(2-x)\sqrt{1-x^2} - \frac{\arcsin(x)}{2}$$

```
[Out] -1/2*arcsin(x)-1/2*(2-x)*(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1607, 794, 222}

$$\int \frac{x-x^2}{\sqrt{1-x^2}} \, dx = -\frac{\arcsin(x)}{2} - \frac{1}{2}\sqrt{1-x^2}(2-x)$$

```
[In] Int[(x - x^2)/Sqrt[1 - x^2],x]
```

```
[Out] -1/2*((2 - x)*Sqrt[1 - x^2]) - ArcSin[x]/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx\\ &= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{x-x^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}(2-x)\sqrt{1-x^2}+\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

```
[In] Integrate[(x - x^2)/Sqrt[1 - x^2],x]
[Out] -1/2*((2 - x)*Sqrt[1 - x^2]) + ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

method	result	size
risch	$-\frac{(-2+x)(x^2-1)}{2\sqrt{-x^2+1}}-\frac{\arcsin(x)}{2}$	25
default	$\frac{x\sqrt{-x^2+1}}{2}-\frac{\arcsin(x)}{2}-\sqrt{-x^2+1}$	29
trager	$\left(-1+\frac{x}{2}\right)\sqrt{-x^2+1}+\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{2}$	45
meijerg	$-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-x^2+1}}{2\sqrt{\pi}}-\frac{i\left(i\sqrt{\pi}\,x\sqrt{-x^2+1}-i\sqrt{\pi}\,\arcsin(x)\right)}{2\sqrt{\pi}}$	58

```
[In] int((-x^2+x)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(-2+x)*(x^2-1)/(-x^2+1)^(1/2)-1/2*arcsin(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{x-x^2}{\sqrt{1-x^2}}\,dx = \frac{1}{2}\sqrt{-x^2+1}(x-2)+\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

```
[In] integrate((-x^2+x)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) + arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x-x^2}{\sqrt{1-x^2}}\,dx = \frac{x\sqrt{1-x^2}}{2}-\sqrt{1-x^2}-\frac{\operatorname{asin}(x)}{2}$$

```
[In] integrate((-x**2+x)/(-x**2+1)**(1/2),x)
[Out] x*sqrt(1 - x**2)/2 - sqrt(1 - x**2) - asin(x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{x-x^2}{\sqrt{1-x^2}}\,dx = \frac{1}{2}\sqrt{-x^2+1}x-\sqrt{-x^2+1}-\frac{1}{2}\arcsin(x)$$

```
[In] integrate((-x^2+x)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1) - 1/2*arcsin(x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x-x^2}{\sqrt{1-x^2}}\,dx = \frac{1}{2}\sqrt{-x^2+1}(x-2)-\frac{1}{2}\arcsin(x)$$

```
[In] integrate((-x^2+x)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) - 1/2*arcsin(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x-x^2}{\sqrt{1-x^2}}\,dx = \left(\frac{x}{2}-1\right)\sqrt{1-x^2}-\frac{\operatorname{asin}(x)}{2}$$

```
[In] int((x - x^2)/(1 - x^2)^(1/2),x)
[Out] (x/2 - 1)*(1 - x^2)^(1/2) - asin(x)/2
```

3.45 $\int \frac{3+x^2}{-3+x^2}\,dx$

Optimal result 311
Rubi [A] (verified) 311
Mathematica [A] (verified) 312
Maple [A] (verified) 312
Fricas [A] (verification not implemented) 313
Sympy [A] (verification not implemented) 313
Maxima [A] (verification not implemented) 313
Giac [B] (verification not implemented) 313
Mupad [B] (verification not implemented) 314

Optimal result

Integrand size = 13, antiderivative size = 17

$$\int \frac{3+x^2}{-3+x^2}\,dx = x-2\sqrt{3}\text{arctanh}\left(\frac{x}{\sqrt{3}}\right)$$

```
[Out] x-2*arctanh(1/3*x*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {396, 213}

$$\int \frac{3+x^2}{-3+x^2}\,dx = x-2\sqrt{3}\text{arctanh}\left(\frac{x}{\sqrt{3}}\right)$$

```
[In] Int[(3 + x^2)/(-3 + x^2),x]
```

```
[Out] x - 2*Sqrt[3]*ArcTanh[x/Sqrt[3]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = x + 6\int \frac{1}{-3+x^2}\,dx$$
$$= x - 2\sqrt{3}\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int \frac{3+x^2}{-3+x^2}\,dx = x + \sqrt{3}\log\left(\sqrt{3}-x\right) - \sqrt{3}\log\left(\sqrt{3}+x\right)$$

```
[In] Integrate[(3 + x^2)/(-3 + x^2),x]
[Out] x + Sqrt[3]*Log[Sqrt[3] - x] - Sqrt[3]*Log[Sqrt[3] + x]
```

Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

method	result	size
default	$x-2\,\operatorname{arctanh}\left(\frac{\sqrt{3}\,x}{3}\right)\sqrt{3}$	15
risch	$x+\sqrt{3}\,\ln\left(x-\sqrt{3}\right)-\sqrt{3}\,\ln\left(x+\sqrt{3}\right)$	26
meijerg	$-\operatorname{arctanh}\left(\frac{\sqrt{3}\,x}{3}\right)\sqrt{3}-\frac{i\sqrt{3}\left(\frac{2i\sqrt{3}\,x}{3}-2i\operatorname{arctanh}\left(\frac{\sqrt{3}\,x}{3}\right)\right)}{2}$	38

```
[In] int((x^2+3)/(x^2-3),x,method=_RETURNVERBOSE)
[Out] x-2*arctanh(1/3*3^(1/2)*x)*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{3+x^2}{-3+x^2}\,dx=\sqrt{3}\log\left(\frac{x^2-2\sqrt{3}x+3}{x^2-3}\right)+x$$

```
[In] integrate((x^2+3)/(x^2-3),x, algorithm="fricas")
[Out] sqrt(3)*log((x^2 - 2*sqrt(3)*x + 3)/(x^2 - 3)) + x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{3+x^2}{-3+x^2}\,dx=x+\sqrt{3}\log\left(x-\sqrt{3}\right)-\sqrt{3}\log\left(x+\sqrt{3}\right)$$

```
[In] integrate((x**2+3)/(x**2-3),x)
[Out] x + sqrt(3)*log(x - sqrt(3)) - sqrt(3)*log(x + sqrt(3))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \frac{3+x^2}{-3+x^2}\,dx=\sqrt{3}\log\left(\frac{x-\sqrt{3}}{x+\sqrt{3}}\right)+x$$

```
[In] integrate((x^2+3)/(x^2-3),x, algorithm="maxima")
[Out] sqrt(3)*log((x - sqrt(3))/(x + sqrt(3))) + x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(14) = 28.

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \frac{3+x^2}{-3+x^2}\,dx=\sqrt{3}\log\left(\frac{|2\,x-2\,\sqrt{3}|}{|2\,x+2\,\sqrt{3}|}\right)+x$$

```
[In] integrate((x^2+3)/(x^2-3),x, algorithm="giac")
[Out] sqrt(3)*log(abs(2*x - 2*sqrt(3))/abs(2*x + 2*sqrt(3))) + x
```

Mupad [B] (verification not implemented)

Time = 5.41 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{3+x^2}{-3+x^2}\,dx = x-2\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,x}{3}\right)$$

```
[In] int((x^2 + 3)/(x^2 - 3),x)
[Out] x - 2*3^(1/2)*atanh((3^(1/2)*x)/3)
```

3.46 $\int \frac{-1+x^2}{1+x^2}\,dx$

Optimal result 315
Rubi [A] (verified) 315
Mathematica [A] (verified) 316
Maple [A] (verified) 316
Fricas [A] (verification not implemented) 317
Sympy [A] (verification not implemented) 317
Maxima [A] (verification not implemented) 317
Giac [A] (verification not implemented) 317
Mupad [B] (verification not implemented) 318

Optimal result

Integrand size = 13, antiderivative size = 6

$$\int \frac{-1+x^2}{1+x^2}\,dx = x-2\arctan(x)$$

```
[Out] x-2*arctan(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {396, 209}

$$\int \frac{-1+x^2}{1+x^2}\,dx = x-2\arctan(x)$$

```
[In] Int[(-1 + x^2)/(1 + x^2),x]
```

```
[Out] x - 2*ArcTan[x]
```

<u>Rule 209</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

<u>Rule 396</u>

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = x - 2\int \frac{1}{1+x^2}\,dx$$
$$= x - 2\tan^{-1}(x)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^2}{1+x^2}\,dx = x - 2\arctan(x)$$

```
[In] Integrate[(-1 + x^2)/(1 + x^2),x]
[Out] x - 2*ArcTan[x]
```

Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
default	$x - 2\arctan(x)$	7
meijerg	$x - 2\arctan(x)$	7
risch	$x - 2\arctan(x)$	7
parallelrisch	$x + i\ln(x - i) - i\ln(x + i)$	19

```
[In] int((x^2-1)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] x-2*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^2}{1+x^2}\,dx = x - 2\arctan(x)$$

```
[In] integrate((x^2-1)/(x^2+1),x, algorithm="fricas")
[Out] x - 2*arctan(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{-1+x^2}{1+x^2}\,dx = x - 2\operatorname{atan}(x)$$

```
[In] integrate((x**2-1)/(x**2+1),x)
[Out] x - 2*atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^2}{1+x^2}\,dx = x - 2\arctan(x)$$

```
[In] integrate((x^2-1)/(x^2+1),x, algorithm="maxima")
[Out] x - 2*arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^2}{1+x^2}\,dx = x - 2\arctan(x)$$

```
[In] integrate((x^2-1)/(x^2+1),x, algorithm="giac")
[Out] x - 2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^2}{1+x^2}\,dx = x - 2\,\mathrm{atan}(x)$$

```
[In] int((x^2 - 1)/(x^2 + 1),x)
[Out] x - 2*atan(x)
```

3.47 $\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

Optimal result . . . 319
Rubi [A] (verified) . . . 319
Mathematica [A] (verified) . . . 322
Maple [A] (verified) . . . 322
Fricas [A] (verification not implemented) . . . 323
Sympy [A] (verification not implemented) . . . 323
Maxima [B] (verification not implemented) . . . 326
Giac [A] (verification not implemented) . . . 327
Mupad [F(-1)] . . . 327

Optimal result

Integrand size = 25, antiderivative size = 213

$$\begin{aligned}\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = &-\frac{x^7(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}}\\ &-\frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\,(a+bx^2)^{5/2}}-\frac{x^3(35aB-6(Ab-8aC)x)}{105ab^3\,(a+bx^2)^{3/2}}\\ &-\frac{x(35aB-8(Ab-8aC)x)}{35ab^4\sqrt{a+bx^2}}-\frac{16(Ab-8aC)\sqrt{a+bx^2}}{35ab^5}+\frac{B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}\end{aligned}$$

```
[Out] -1/7*x^7*(B*a-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)-1/35*x^5*(7*B*a-(A*b-8*C*a)*
x)/a/b^2/(b*x^2+a)^(5/2)-1/105*x^3*(35*B*a-6*(A*b-8*C*a)*x)/a/b^3/(b*x^2+a)
^(3/2)+B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(9/2)-1/35*x*(35*B*a-8*(A*b-8
*C*a)*x)/a/b^4/(b*x^2+a)^(1/2)-16/35*(A*b-8*C*a)*(b*x^2+a)^(1/2)/a/b^5
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1818, 833, 655, 223, 212}

$$\begin{aligned}\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = &-\frac{16\sqrt{a+bx^2}(Ab-8aC)}{35ab^5}\\ &-\frac{x(35aB-8x(Ab-8aC))}{35ab^4\sqrt{a+bx^2}}-\frac{x^3(35aB-6x(Ab-8aC))}{105ab^3\,(a+bx^2)^{3/2}}\\ &-\frac{x^5(7aB-x(Ab-8aC))}{35ab^2\,(a+bx^2)^{5/2}}-\frac{x^7(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}+\frac{B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}\end{aligned}$$

```
[In] Int[(x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/7*(x^7*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) - (x^5*(7*a*B - (A
*b - 8*a*C)*x))/(35*a*b^2*(a + b*x^2)^(5/2)) - (x^3*(35*a*B - 6*(A*b - 8*a*
C)*x))/(105*a*b^3*(a + b*x^2)^(3/2)) - (x*(35*a*B - 8*(A*b - 8*a*C)*x))/(35
*a*b^4*Sqrt[a + b*x^2]) - (16*(A*b - 8*a*C)*Sqrt[a + b*x^2])/(35*a*b^5) + (
B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(9/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{x^7(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^6(-7aB+(Ab-8aC)x)}{(a+bx^2)^{7/2}}\,dx}{7ab}$$

$$= -\frac{x^7(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{\int \frac{x^4\left(-35a^2B+6a(Ab-8aC)x\right)}{(a+bx^2)^{5/2}}\,dx}{35a^2b^2}$$

$$= -\frac{x^7(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{x^3(35aB-6(Ab-8aC)x)}{105ab^3\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{x^2\left(-105a^3B+24a^2(Ab-8aC)x\right)}{(a+bx^2)^{3/2}}\,dx}{105a^3b^3}$$

$$= -\frac{x^7(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{x^3(35aB-6(Ab-8aC)x)}{105ab^3\left(a+bx^2\right)^{3/2}} - \frac{x(35aB-8(Ab-8aC)x)}{35ab^4\sqrt{a+bx^2}} - \frac{\int \frac{-105a^4B+48a^3(Ab-8aC)x}{\sqrt{a+bx^2}}\,dx}{105a^4b^4}$$

$$= -\frac{x^7(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{x^3(35aB-6(Ab-8aC)x)}{105ab^3\left(a+bx^2\right)^{3/2}} - \frac{x(35aB-8(Ab-8aC)x)}{35ab^4\sqrt{a+bx^2}} - \frac{16(Ab-8aC)\sqrt{a+bx^2}}{35ab^5} + \frac{B\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b^4}$$

$$= -\frac{x^7(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{x^3(35aB-6(Ab-8aC)x)}{105ab^3\left(a+bx^2\right)^{3/2}} - \frac{x(35aB-8(Ab-8aC)x)}{35ab^4\sqrt{a+bx^2}} - \frac{16(Ab-8aC)\sqrt{a+bx^2}}{35ab^5} + \frac{B\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{b^4}$$

$$= -\frac{x^7(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{x^3(35aB-6(Ab-8aC)x)}{105ab^3\left(a+bx^2\right)^{3/2}} - \frac{x(35aB-8(Ab-8aC)x)}{35ab^4\sqrt{a+bx^2}} - \frac{16(Ab-8aC)\sqrt{a+bx^2}}{35ab^5} + \frac{B\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.73

$$\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{384a^4C-3a^3b(16A+7x(5B-64Cx))+14a^2b^2x^2(-12A+5x(-5B+24Cx))}{}$$

[In] Integrate[(x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]

[Out] (384*a^4*C - 3*a^3*b*(16*A + 7*x*(5*B - 64*C*x)) + 14*a^2*b^2*x^2*(-12*A + 5*x*(-5*B + 24*C*x)) + 14*a*b^3*x^4*(-15*A + x*(-29*B + 60*C*x)) + b^4*x^6*(-105*A + x*(-176*B + 105*C*x)) - 105*Sqrt[b]*B*(a + b*x^2)^(7/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(105*b^5*(a + b*x^2)^(7/2))

Maple [A] (verified)

Time = 3.71 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.38

method	result
default	$C\left(\frac{x^8}{b(bx^2+a)^{\frac{7}{2}}}-\frac{8a\left(-\frac{x^6}{b(bx^2+a)^{\frac{7}{2}}}+\frac{6a\left(-\frac{x^4}{3b(bx^2+a)^{\frac{7}{2}}}+\frac{4a\left(-\frac{x^2}{5b(bx^2+a)^{\frac{7}{2}}}-\frac{2a}{35b^2(bx^2+a)^{\frac{7}{2}}}\right)}{3b}\right)}{b}\right)}{b}\right)+B\left(-\frac{x^7}{7b(bx^2+a)^{\frac{7}{2}}}\right.$
risch	Expression too large to display

[In] int(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)

[Out] C*(x^8/b/(b*x^2+a)^(7/2)-8*a/b*(-x^6/b/(b*x^2+a)^(7/2)+6*a/b*(-1/3*x^4/b/(b*x^2+a)^(7/2)+4/3*a/b*(-1/5*x^2/b/(b*x^2+a)^(7/2)-2/35*a/b^2/(b*x^2+a)^(7/2))))))+B*(-1/7*x^7/b/(b*x^2+a)^(7/2)+1/b*(-1/5*x^5/b/(b*x^2+a)^(5/2)+1/b*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(

```
b*x^2+a)^(1/2))))))+A*(-x^6/b/(b*x^2+a)^(7/2)+6*a/b*(-1/3*x^4/b/(b*x^2+a)^(
7/2)+4/3*a/b*(-1/5*x^2/b/(b*x^2+a)^(7/2)-2/35*a/b^2/(b*x^2+a)^(7/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 522, normalized size of antiderivative = 2.45

$$\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \left[\frac{105\,(Bb^4x^8+4\,Bab^3x^6+6\,Ba^2b^2x^4+4\,Ba^3bx^2+Ba^4)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{b}\right.}{}\right.$$

$$-\frac{105\,(Bb^4x^8+4\,Bab^3x^6+6\,Ba^2b^2x^4+4\,Ba^3bx^2+Ba^4)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(105\,Cb^4x^8-176\,Bb^4x^7}{10}$$

```
[In] integrate(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(B*b^4*x^8 + 4*B*a*b^3*x^6 + 6*B*a^2*b^2*x^4 + 4*B*a^3*b*x^2 +
B*a^4)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(105*C*b
^4*x^8 - 176*B*b^4*x^7 - 406*B*a*b^3*x^5 - 350*B*a^2*b^2*x^3 + 105*(8*C*a*b
^3 - A*b^4)*x^6 - 105*B*a^3*b*x + 384*C*a^4 - 48*A*a^3*b + 210*(8*C*a^2*b^2
 - A*a*b^3)*x^4 + 168*(8*C*a^3*b - A*a^2*b^2)*x^2)*sqrt(b*x^2 + a))/(b^9*x^
8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5), -1/105*(105*(B*
b^4*x^8 + 4*B*a*b^3*x^6 + 6*B*a^2*b^2*x^4 + 4*B*a^3*b*x^2 + B*a^4)*sqrt(-b)
*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (105*C*b^4*x^8 - 176*B*b^4*x^7 - 406*
B*a*b^3*x^5 - 350*B*a^2*b^2*x^3 + 105*(8*C*a*b^3 - A*b^4)*x^6 - 105*B*a^3*b
*x + 384*C*a^4 - 48*A*a^3*b + 210*(8*C*a^2*b^2 - A*a*b^3)*x^4 + 168*(8*C*a^
3*b - A*a^2*b^2)*x^2)*sqrt(b*x^2 + a))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x
^4 + 4*a^3*b^6*x^2 + a^4*b^5)]
```

Sympy [A] (verification not implemented)

Time = 34.30 (sec) , antiderivative size = 3806, normalized size of antiderivative = 17.87

$$\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x**7*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*Piecewise((-16*a**3/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*x**2*s
qrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqrt(a +
b*x**2)) - 56*a**2*b*x**2/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*x*
*2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqrt(
a + b*x**2)) - 70*a*b**2*x**4/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**
```

5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*s
qrt(a + b*x**2)) - 35*b**3*x**6/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b
5*x2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6
*sqrt(a + b*x**2)), Ne(b, 0)), (x**8/(8*a**(9/2)), True)) + B*(105*a**(205/
2)*b**45*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99
/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a)
+ 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(
105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b
*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193
/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 630*a**(203/2)*b**46*x**2*sqrt(1
 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*
x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/
2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqr
t(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*
a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*
x**12*sqrt(1 + b*x**2/a)) + 1575*a**(201/2)*b**47*x**4*sqrt(1 + b*x**2/a)*a
sinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*
a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*
x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a
) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**
(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 +
 b*x**2/a)) + 2100*a**(199/2)*b**48*x**6*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x
/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**
(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 +
b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(1
97/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*
sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) +
1575*a**(197/2)*b**49*x**8*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105
*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*s
qrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 21
00*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/
2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**
2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 630*a**(195/2)
*b**50*x**10*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b*
*(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2
/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*
b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1
 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**
(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 105*a**(193/2)*b**51*x**12*s
qrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1
 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**
(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**
6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) +
 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(11
1/2)*x**12*sqrt(1 + b*x**2/a)) - 105*a**102*b**(91/2)*x/(105*a**(205/2)*b**

```
(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/
a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b
**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1
+ b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(
193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 665*a**101*b**(93/2)*x**3/(10
5*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*
sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2
100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107
/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x*
*2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 1771*a**100*b
**(95/2)*x**5/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)
*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(
1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a
**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x*
*10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)
) - 2549*a**99*b**(97/2)*x**7/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a)
+ 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(1
03/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*
x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/
2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sq
rt(1 + b*x**2/a)) - 2096*a**98*b**(99/2)*x**9/(105*a**(205/2)*b**(99/2)*sqr
t(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*
a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*
x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a
) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**
(111/2)*x**12*sqrt(1 + b*x**2/a)) - 934*a**97*b**(101/2)*x**11/(105*a**(205
/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 +
b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(1
99/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*
sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 1
05*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 176*a**96*b**(103/2)*x
**13/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/
2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**
2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)
*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(
1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a))) + C*Pi
ecewise((128*a**4/(35*a**3*b**5*sqrt(a + b*x**2) + 105*a**2*b**6*x**2*sqrt(
a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**8*x**6*sqrt(a + b*x*
*2)) + 448*a**3*b*x**2/(35*a**3*b**5*sqrt(a + b*x**2) + 105*a**2*b**6*x**2*
sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**8*x**6*sqrt(a +
 b*x**2)) + 560*a**2*b**2*x**4/(35*a**3*b**5*sqrt(a + b*x**2) + 105*a**2*b*
*6*x**2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**8*x**6*
sqrt(a + b*x**2)) + 280*a*b**3*x**6/(35*a**3*b**5*sqrt(a + b*x**2) + 105*a*
*2*b**6*x**2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**8*
x**6*sqrt(a + b*x**2)) + 35*b**4*x**8/(35*a**3*b**5*sqrt(a + b*x**2) + 105*
```

```
a**2*b**6*x**2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**
8*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**10/(10*a**(9/2)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(189) = 378$.

Time = 0.22 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.04

$$\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{Cx^8}{(bx^2+a)^{\frac{7}{2}}b} - \frac{1}{35}\left(\frac{35\,x^6}{(bx^2+a)^{\frac{7}{2}}b} + \frac{70\,ax^4}{(bx^2+a)^{\frac{7}{2}}b^2} + \frac{56\,a^2x^2}{(bx^2+a)^{\frac{7}{2}}b^3} + \frac{16\,a^3}{(bx^2+a)^{\frac{7}{2}}b^4}\right)Bx + \frac{8\,Cax^6}{(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Ax^6}{(bx^2+a)^{\frac{7}{2}}b} - \frac{Bx\left(\frac{15\,x^4}{(bx^2+a)^{\frac{5}{2}}b} + \frac{20\,ax^2}{(bx^2+a)^{\frac{5}{2}}b^2} + \frac{8\,a^2}{(bx^2+a)^{\frac{5}{2}}b^3}\right)}{15\,b} - \frac{Bx\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{3\,b^2} + \frac{16\,Ca^2x^4}{(bx^2+a)^{\frac{7}{2}}b^3} - \frac{2\,Aax^4}{(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Bax^3}{(bx^2+a)^{\frac{5}{2}}b^3} + \frac{64\,Ca^3x^2}{5\,(bx^2+a)^{\frac{7}{2}}b^4} - \frac{8\,Aa^2x^2}{5\,(bx^2+a)^{\frac{7}{2}}b^3} + \frac{139\,Bx}{105\,\sqrt{bx^2+a}b^4} + \frac{17\,Bax}{105\,(bx^2+a)^{\frac{3}{2}}b^4} - \frac{29\,Ba^2x}{35\,(bx^2+a)^{\frac{5}{2}}b^4} + \frac{B\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{9}{2}}} + \frac{128\,Ca^4}{35\,(bx^2+a)^{\frac{7}{2}}b^5} - \frac{16\,Aa^3}{35\,(bx^2+a)^{\frac{7}{2}}b^4}$$

```
[In] integrate(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] C*x^8/((b*x^2 + a)^(7/2)*b) - 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4
/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*
x^2 + a)^(7/2)*b^4))*B*x + 8*C*a*x^6/((b*x^2 + a)^(7/2)*b^2) - A*x^6/((b*x^
2 + a)^(7/2)*b) - 1/15*B*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2
 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/3*B*x*(3*x^2/((b*x^
2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 16*C*a^2*x^4/((b*x^2 +
 a)^(7/2)*b^3) - 2*A*a*x^4/((b*x^2 + a)^(7/2)*b^2) - B*a*x^3/((b*x^2 + a)^(
5/2)*b^3) + 64/5*C*a^3*x^2/((b*x^2 + a)^(7/2)*b^4) - 8/5*A*a^2*x^2/((b*x^2
+ a)^(7/2)*b^3) + 139/105*B*x/(sqrt(b*x^2 + a)*b^4) + 17/105*B*a*x/((b*x^2
+ a)^(3/2)*b^4) - 29/35*B*a^2*x/((b*x^2 + a)^(5/2)*b^4) + B*arcsinh(b*x/sqr
t(a*b))/b^(9/2) + 128/35*C*a^4/((b*x^2 + a)^(7/2)*b^5) - 16/35*A*a^3/((b*x^
2 + a)^(7/2)*b^4)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.96

$$\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(\left(\left(\left(\left(\left(\frac{105\,Cx}{b}-\frac{176\,B}{b}\right)x+\frac{105\,(8\,Ca^4b^7-Aa^3b^8)}{a^3b^9}\right)x-\frac{406\,Ba}{b^2}\right)x+\frac{210\,(8\,Ca^5b^6-Aa^4b^7)}{a^3b^9}\cdots}{105\,(bx^2+a)\cdots}$$
$$-\frac{B\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}$$

```
[In] integrate(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((((((((105*C*x/b - 176*B/b)*x + 105*(8*C*a^4*b^7 - A*a^3*b^8)/(a^3*b
^9))*x - 406*B*a/b^2)*x + 210*(8*C*a^5*b^6 - A*a^4*b^7)/(a^3*b^9))*x - 350*
B*a^2/b^3)*x + 168*(8*C*a^6*b^5 - A*a^5*b^6)/(a^3*b^9))*x - 105*B*a^3/b^4)*
x + 48*(8*C*a^7*b^4 - A*a^6*b^5)/(a^3*b^9))/(b*x^2 + a)^(7/2) - B*log(abs(-
sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \int \frac{x^7\,(C\,x^2+B\,x+A)}{(b\,x^2+a)^{9/2}}\,dx$$

```
[In] int((x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] int((x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2), x)
```

3.48 $\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

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Optimal result

Integrand size = 25, antiderivative size = 150

$$\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{x^6(aB-(Ab-aC)x)}{7ab(a+bx^2)^{7/2}} - \frac{x^4(6B+7Cx)}{35b^2(a+bx^2)^{5/2}} - \frac{x^2(24B+35Cx)}{105b^3(a+bx^2)^{3/2}} - \frac{16B+35Cx}{35b^4\sqrt{a+bx^2}} + \frac{C\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

```
[Out] -1/7*x^6*(B*a-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)-1/35*x^4*(7*C*x+6*B)/b^2/(b*
x^2+a)^(5/2)-1/105*x^2*(35*C*x+24*B)/b^3/(b*x^2+a)^(3/2)+C*arctanh(x*b^(1/2
)/(b*x^2+a)^(1/2))/b^(9/2)+1/35*(-35*C*x-16*B)/b^4/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1818, 833, 792, 223, 212}

$$\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{x^6(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}} + \frac{C\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}} - \frac{16B+35Cx}{35b^4\sqrt{a+bx^2}} - \frac{x^2(24B+35Cx)}{105b^3(a+bx^2)^{3/2}} - \frac{x^4(6B+7Cx)}{35b^2(a+bx^2)^{5/2}}$$

```
[In] Int[(x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/7*(x^6*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) - (x^4*(6*B + 7*C*
x))/(35*b^2*(a + b*x^2)^(5/2)) - (x^2*(24*B + 35*C*x))/(105*b^3*(a + b*x^2)
```

```
^(3/2)) - (16*B + 35*C*x)/(35*b^4*Sqrt[a + b*x^2]) + (C*ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]])/b^(9/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{x^6(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{\int \frac{x^5(-6aB-7aCx)}{(a+bx^2)^{7/2}}\,dx}{7ab}$$

$$= -\frac{x^6(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^4(6B+7Cx)}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{\int \frac{x^3\left(-24a^2B-35a^2Cx\right)}{\left(a+bx^2\right)^{5/2}}\,dx}{35a^2b^2}$$

$$= -\frac{x^6(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^4(6B+7Cx)}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{x^2(24B+35Cx)}{105b^3\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{x\left(-48a^3B-105a^3Cx\right)}{\left(a+bx^2\right)^{3/2}}\,dx}{105a^3b^3}$$

$$= -\frac{x^6(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^4(6B+7Cx)}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{x^2(24B+35Cx)}{105b^3\left(a+bx^2\right)^{3/2}} - \frac{16B+35Cx}{35b^4\sqrt{a+bx^2}} + \frac{C\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b^4}$$

$$= -\frac{x^6(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^4(6B+7Cx)}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{x^2(24B+35Cx)}{105b^3\left(a+bx^2\right)^{3/2}} - \frac{16B+35Cx}{35b^4\sqrt{a+bx^2}} + \frac{C\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{b^4}$$

$$= -\frac{x^6(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^4(6B+7Cx)}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{x^2(24B+35Cx)}{105b^3\left(a+bx^2\right)^{3/2}} - \frac{16B+35Cx}{35b^4\sqrt{a+bx^2}} + \frac{C\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.85

$$\int \frac{x^6(A+Bx+Cx^2)}{\left(a+bx^2\right)^{9/2}}\,dx = \frac{15Ab^4x^7-14a^3bx^2(12B+25Cx)-14a^2b^2x^4(15B+29Cx)-3a^4(16B+35Cx)}{105ab^4\left(a+bx^2\right)^{7/2}} - \frac{C\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{9/2}}$$

```
[In] Integrate[(x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (15*A*b^4*x^7 - 14*a^3*b*x^2*(12*B + 25*C*x) - 14*a^2*b^2*x^4*(15*B + 29*C*
x) - 3*a^4*(16*B + 35*C*x) - a*b^3*x^6*(105*B + 176*C*x))/(105*a*b^4*(a + b
*x^2)^(7/2)) - (C*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(9/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(128) = 256$.

Time = 3.50 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.23

method	result
default	$C\left(-\frac{x^7}{7b(b\,x^2+a)^{\frac{7}{2}}}+\frac{-\frac{x^5}{5b\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{-\frac{x^3}{3b\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}}{b}}{b}}{b}\right)+B\left(-\frac{x^6}{b(b\,x^2+a)^{\frac{7}{2}}}+\frac{6a\left(-\frac{}{3b\left(b\,\right.}\right.}{}\right.$

```
[In] int(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] C*(-1/7*x^7/b/(b*x^2+a)^(7/2)+1/b*(-1/5*x^5/b/(b*x^2+a)^(5/2)+1/b*(-1/3*x^3
/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+
a)^(1/2))))))+B*(-x^6/b/(b*x^2+a)^(7/2)+6*a/b*(-1/3*x^4/b/(b*x^2+a)^(7/2)+4
/3*a/b*(-1/5*x^2/b/(b*x^2+a)^(7/2)-2/35*a/b^2/(b*x^2+a)^(7/2))))+A*(-1/2*x^
5/b/(b*x^2+a)^(7/2)+5/2*a/b*(-1/4*x^3/b/(b*x^2+a)^(7/2)+3/4*a/b*(-1/6*x/b/(
b*x^2+a)^(7/2)+1/6*a/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5
/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 467, normalized size of antiderivative = 3.11

$$\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \left[\frac{105\,(Cab^4x^8+4\,Ca^2b^3x^6+6\,Ca^3b^2x^4+4\,Ca^4bx^2+Ca^5)\sqrt{b}\log\left(-2\,bx^2-2\right.}{}\right.$$

$$-\frac{105\,(Cab^4x^8+4\,Ca^2b^3x^6+6\,Ca^3b^2x^4+4\,Ca^4bx^2+Ca^5)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(105\,Bab^4x^6+406\,Ca^2}{105\,(ab^9x^8+4\,a^2b^8x^6+}$$

```
[In] integrate(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(C*a*b^4*x^8 + 4*C*a^2*b^3*x^6 + 6*C*a^3*b^2*x^4 + 4*C*a^4*b*x^
2 + C*a^5)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(105
*B*a*b^4*x^6 + 406*C*a^2*b^3*x^5 + 210*B*a^2*b^3*x^4 + 350*C*a^3*b^2*x^3 +
168*B*a^3*b^2*x^2 + (176*C*a*b^4 - 15*A*b^5)*x^7 + 105*C*a^4*b*x + 48*B*a^4
*b)*sqrt(b*x^2 + a))/(a*b^9*x^8 + 4*a^2*b^8*x^6 + 6*a^3*b^7*x^4 + 4*a^4*b^6
*x^2 + a^5*b^5), -1/105*(105*(C*a*b^4*x^8 + 4*C*a^2*b^3*x^6 + 6*C*a^3*b^2*x
^4 + 4*C*a^4*b*x^2 + C*a^5)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (
105*B*a*b^4*x^6 + 406*C*a^2*b^3*x^5 + 210*B*a^2*b^3*x^4 + 350*C*a^3*b^2*x^3
 + 168*B*a^3*b^2*x^2 + (176*C*a*b^4 - 15*A*b^5)*x^7 + 105*C*a^4*b*x + 48*B*
a^4*b)*sqrt(b*x^2 + a))/(a*b^9*x^8 + 4*a^2*b^8*x^6 + 6*a^3*b^7*x^4 + 4*a^4*
b^6*x^2 + a^5*b^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(133) = 266.

Time = 44.84 (sec) , antiderivative size = 3448, normalized size of antiderivative = 22.99

$$\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x**6*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2/
a) + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt(1
 + b*x**2/a)) + B*Piecewise((-16*a**3/(35*a**3*b**4*sqrt(a + b*x**2) + 105*
a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**
7*x**6*sqrt(a + b*x**2)) - 56*a**2*b*x**2/(35*a**3*b**4*sqrt(a + b*x**2) +
105*a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35
*b**7*x**6*sqrt(a + b*x**2)) - 70*a*b**2*x**4/(35*a**3*b**4*sqrt(a + b*x**2
) + 105*a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2)
+ 35*b**7*x**6*sqrt(a + b*x**2)) - 35*b**3*x**6/(35*a**3*b**4*sqrt(a + b*x*
*2) + 105*a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2
) + 35*b**7*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**8/(8*a**(9/2)), True)) +
 C*(105*a**(205/2)*b**45*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a
**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqr
t(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100
*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)
*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/
a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 630*a**(203/2)*b
**46*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(9
9/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a)
 + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**
(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 +
b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(19
3/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 1575*a**(201/2)*b**47*x**4*sqrt
(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 +
b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(20
1/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*s
qrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 63
0*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2
)*x**12*sqrt(1 + b*x**2/a)) + 2100*a**(199/2)*b**48*x**6*sqrt(1 + b*x**2/a)
*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 63
0*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2
)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2
/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b
**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1
 + b*x**2/a)) + 1575*a**(197/2)*b**49*x**8*sqrt(1 + b*x**2/a)*asinh(sqrt(b)
```

```
*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b
**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1
+ b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**
(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**1
0*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a))
+ 630*a**(195/2)*b**50*x**10*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(1
05*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2
*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) +
2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(10
7/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x
**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 105*a**(193/
2)*b**51*x**12*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*
b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x*
*2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2
)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt
(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a
**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 105*a**102*b**(91/2)*x/(10
5*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*
sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2
100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107
/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x*
*2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 665*a**101*b*
*(93/2)*x**3/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*
b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1
 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a*
*(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**
10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a))
 - 1771*a**100*b**(95/2)*x**5/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a)
+ 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(1
03/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*
x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/
2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sq
rt(1 + b*x**2/a)) - 2549*a**99*b**(97/2)*x**7/(105*a**(205/2)*b**(99/2)*sqr
t(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*
a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*
x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a
) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**
(111/2)*x**12*sqrt(1 + b*x**2/a)) - 2096*a**98*b**(99/2)*x**9/(105*a**(205/
2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b
*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(19
9/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*s
qrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 10
5*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 934*a**97*b**(101/2)*x*
*11/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2
)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2
```

```
/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*
b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1
 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 176*a*
*96*b**(103/2)*x**13/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**
(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**
4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) +
 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(10
9/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*
x**2/a)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(127) = 254$.

Time = 0.22 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.98

$$\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{1}{35}\left(\frac{35\,x^6}{(bx^2+a)^{\frac{7}{2}}b}+\frac{70\,ax^4}{(bx^2+a)^{\frac{7}{2}}b^2}+\frac{56\,a^2x^2}{(bx^2+a)^{\frac{7}{2}}b^3}+\frac{16\,a^3}{(bx^2+a)^{\frac{7}{2}}b^4}\right)Cx - \frac{Bx^6}{(bx^2+a)^{\frac{7}{2}}b} - \frac{Cx\left(\frac{15\,x^4}{(bx^2+a)^{\frac{5}{2}}b}+\frac{20\,ax^2}{(bx^2+a)^{\frac{5}{2}}b^2}+\frac{8\,a^2}{(bx^2+a)^{\frac{5}{2}}b^3}\right)}{15\,b} - \frac{Ax^5}{2\,(bx^2+a)^{\frac{7}{2}}b} - \frac{Cx\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{3\,b^2} - \frac{2\,Bax^4}{(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Cax^3}{(bx^2+a)^{\frac{5}{2}}b^3} - \frac{5\,Aax^3}{8\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{8\,Ba^2x^2}{5\,(bx^2+a)^{\frac{7}{2}}b^3} + \frac{139\,Cx}{105\,\sqrt{bx^2+a}b^4} + \frac{17\,Cax}{105\,(bx^2+a)^{\frac{3}{2}}b^4} - \frac{29\,Ca^2x}{35\,(bx^2+a)^{\frac{5}{2}}b^4} + \frac{Ax}{14\,(bx^2+a)^{\frac{3}{2}}b^3} + \frac{Ax}{7\,\sqrt{bx^2+a}ab^3} + \frac{3\,Aax}{56\,(bx^2+a)^{\frac{5}{2}}b^3} - \frac{15\,Aa^2x}{56\,(bx^2+a)^{\frac{7}{2}}b^3} + \frac{C\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{9}{2}}} - \frac{16\,Ba^3}{35\,(bx^2+a)^{\frac{7}{2}}b^4}$$

```
[In] integrate(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56
*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*C*x - B*
x^6/((b*x^2 + a)^(7/2)*b) - 1/15*C*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x
^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*A*x^5/(
(b*x^2 + a)^(7/2)*b) - 1/3*C*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 +
 a)^(3/2)*b^2))/b^2 - 2*B*a*x^4/((b*x^2 + a)^(7/2)*b^2) - C*a*x^3/((b*x^2 +
 a)^(5/2)*b^3) - 5/8*A*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 8/5*B*a^2*x^2/((b*x^
```

```
2 + a)^(7/2)*b^3) + 139/105*C*x/(sqrt(b*x^2 + a)*b^4) + 17/105*C*a*x/((b*x^
2 + a)^(3/2)*b^4) - 29/35*C*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14*A*x/((b*x^
2 + a)^(3/2)*b^3) + 1/7*A*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*A*a*x/((b*x^2 +
a)^(5/2)*b^3) - 15/56*A*a^2*x/((b*x^2 + a)^(7/2)*b^3) + C*arcsinh(b*x/sqrt(
a*b))/b^(9/2) - 16/35*B*a^3/((b*x^2 + a)^(7/2)*b^4)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.92

$$\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{\left(\left(\left(\left(\left(x\left(\frac{105\,B}{b}+\frac{(176\,Ca^3b^7-15\,Aa^2b^8)x}{a^3b^8}\right)+\frac{406\,Ca}{b^2}\right)x+\frac{210\,Ba}{b^2}\right)x+\frac{350\,Ca^2}{b^3}\right)x+\frac{168\,Ba^2}{b^3}\right)x+\frac{105\,Ca^3}{b^4}\right)x+\frac{48\,Ba^3}{b^4}}{105\,(bx^2+a)^{\frac{7}{2}}} - \frac{C\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}$$

```
[In] integrate(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*((((((x*(105*B/b + (176*C*a^3*b^7 - 15*A*a^2*b^8)*x/(a^3*b^8)) + 406
*C*a/b^2)*x + 210*B*a/b^2)*x + 350*C*a^2/b^3)*x + 168*B*a^2/b^3)*x + 105*C*
a^3/b^4)*x + 48*B*a^3/b^4)/(b*x^2 + a)^(7/2) - C*log(abs(-sqrt(b)*x + sqrt(
b*x^2 + a)))/b^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \int \frac{x^6\,(C\,x^2+B\,x+A)}{(b\,x^2+a)^{9/2}}\,dx$$

```
[In] int((x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] int((x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2), x)
```

3.49 $\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

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Optimal result

Integrand size = 25, antiderivative size = 132

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{x^5(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{x^4(Ab+6aC-5bBx)}{35ab^2\,(a+bx^2)^{5/2}} + \frac{4(Ab+6aC)}{105b^4\,(a+bx^2)^{3/2}} - \frac{4(Ab+6aC)}{35ab^4\sqrt{a+bx^2}}$$

```
[Out] -1/7*x^5*(B*a-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)-1/35*x^4*(-5*B*b*x+A*b+6*C*a
)/a/b^2/(b*x^2+a)^(5/2)+4/105*(A*b+6*C*a)/b^4/(b*x^2+a)^(3/2)-4/35*(A*b+6*C
*a)/a/b^4/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1818, 819, 272, 45}

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{4(6aC+Ab)}{35ab^4\sqrt{a+bx^2}} + \frac{4(6aC+Ab)}{105b^4\,(a+bx^2)^{3/2}} - \frac{x^4(6aC+Ab-5bBx)}{35ab^2\,(a+bx^2)^{5/2}} - \frac{x^5(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

```
[In] Int[(x^5*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/7*(x^5*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) - (x^4*(A*b + 6*a*
C - 5*b*B*x))/(35*a*b^2*(a + b*x^2)^(5/2)) + (4*(A*b + 6*a*C))/(105*b^4*(a
+ b*x^2)^(3/2)) - (4*(A*b + 6*a*C))/(35*a*b^4*Sqrt[a + b*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[m*((c*d*f + a*e*g)/(2*a*c*(p + 1))), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^5(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{x^4(-5aB-(Ab+6aC)x)}{(a+bx^2)^{7/2}}\,dx}{7ab}\\
&= -\frac{x^5(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{x^4(Ab+6aC-5bBx)}{35ab^2\left(a+bx^2\right)^{5/2}}+\frac{(4(Ab+6aC))\int \frac{x^3}{(a+bx^2)^{5/2}}\,dx}{35ab^2}\\
&= -\frac{x^5(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{x^4(Ab+6aC-5bBx)}{35ab^2\left(a+bx^2\right)^{5/2}}\\
&\quad+\frac{(2(Ab+6aC))\text{Subst}\left(\int \frac{x}{(a+bx)^{5/2}}\,dx,x,x^2\right)}{35ab^2}
\end{aligned}
$$

$$= -\frac{x^5(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{x^4(Ab+6aC-5bBx)}{35ab^2\,(a+bx^2)^{5/2}} + \frac{(2(Ab+6aC))\mathrm{Subst}\left(\int\left(-\frac{a}{b(a+bx)^{5/2}}+\frac{1}{b(a+bx)^{3/2}}\right)dx,x,x^2\right)}{35ab^2}$$

$$= -\frac{x^5(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{x^4(Ab+6aC-5bBx)}{35ab^2\,(a+bx^2)^{5/2}} + \frac{4(Ab+6aC)}{105b^4\,(a+bx^2)^{3/2}} - \frac{4(Ab+6aC)}{35ab^4\sqrt{a+bx^2}}$$

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.67

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{-48a^4C+15b^4Bx^7-35ab^3x^4(A+3Cx^2)-14a^2b^2x^2(2A+15Cx^2)-8a^3b(A+}{105ab^4\,(a+bx^2)^{7/2}}$$

```
[In] Integrate[(x^5*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
[Out] (-48*a^4*C + 15*b^4*B*x^7 - 35*a*b^3*x^4*(A + 3*C*x^2) - 14*a^2*b^2*x^2*(2*
A + 15*C*x^2) - 8*a^3*b*(A + 21*C*x^2))/(105*a*b^4*(a + b*x^2)^(7/2))
```

Maple [A] (verified)

Time = 3.56 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.72

method	result
gosper	$-\frac{-15x^{7}B\,b^{4}+105C\,x^{6}a\,b^{3}+35Aa\,b^{3}x^{4}+210C\,a^{2}b^{2}x^{4}+28A\,a^{2}b^{2}x^{2}+168C\,a^{3}b\,x^{2}+8A\,a^{3}b+48C\,a^{4}}{105(b\,x^{2}+a)^{\frac{7}{2}}a\,b^{4}}$
trager	$-\frac{-15x^{7}B\,b^{4}+105C\,x^{6}a\,b^{3}+35Aa\,b^{3}x^{4}+210C\,a^{2}b^{2}x^{4}+28A\,a^{2}b^{2}x^{2}+168C\,a^{3}b\,x^{2}+8A\,a^{3}b+48C\,a^{4}}{105(b\,x^{2}+a)^{\frac{7}{2}}a\,b^{4}}$
default	$C\left(-\frac{x^{6}}{b(b\,x^{2}+a)^{\frac{7}{2}}}+\frac{6a\left(-\frac{x^{4}}{3b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{4a\left(-\frac{x^{2}}{5b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}-\frac{2a}{35b^{2}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}\right)}{3b}\right)}{b}\right)+B\left(-\frac{x^{5}}{2b(b\,x^{2}+a)^{\frac{7}{2}}}+\frac{5a\left(-\frac{x^{3}}{4b\left(b\,x^{2}+\cdots\right.}\cdots\right.}{\cdots}\right.$

```
[In] int(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/105*(-15*B*b^4*x^7+105*C*a*b^3*x^6+35*A*a*b^3*x^4+210*C*a^2*b^2*x^4+28*A
*a^2*b^2*x^2+168*C*a^3*b*x^2+8*A*a^3*b+48*C*a^4)/(b*x^2+a)^(7/2)/a/b^4
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.04

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{(15\,Bb^4x^7 - 105\,Cab^3x^6 - 48\,Ca^4 - 8\,Aa^3b - 35\,(6\,Ca^2b^2 + Aab^3)x^4 - 28\,(6\,Ca^3}{105\,(ab^8x^8 + 4\,a^2b^7x^6 + 6\,a^3b^6x^4 + 4\,a^4b^5x^2 + a^5b^4)}$$

```
[In] integrate(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*B*b^4*x^7 - 105*C*a*b^3*x^6 - 48*C*a^4 - 8*A*a^3*b - 35*(6*C*a^2*
b^2 + A*a*b^3)*x^4 - 28*(6*C*a^3*b + A*a^2*b^2)*x^2)*sqrt(b*x^2 + a)/(a*b^8
*x^8 + 4*a^2*b^7*x^6 + 6*a^3*b^6*x^4 + 4*a^4*b^5*x^2 + a^5*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(121) = 242.

Time = 20.66 (sec) , antiderivative size = 740, normalized size of antiderivative = 5.61

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = A\left(\begin{cases} -\frac{8a^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx^2}+105b^6x^6\sqrt{a+bx^2}} - \frac{}{105a^3b^3\sqrt{a+bx^2}+3} \\ \frac{x^6}{6a^{\frac{9}{2}}} \end{cases}\right.$$
$$+ \frac{Bx^7}{7a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}} + 21a^{\frac{7}{2}}bx^2\sqrt{1+\frac{bx^2}{a}} + 21a^{\frac{5}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}} + 7a^{\frac{3}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}$$
$$+C\left(\begin{cases} -\frac{16a^3}{35a^3b^4\sqrt{a+bx^2}+105a^2b^5x^2\sqrt{a+bx^2}+105ab^6x^4\sqrt{a+bx^2}+35b^7x^6\sqrt{a+bx^2}} - \frac{56a^2bx^2}{35a^3b^4\sqrt{a+bx^2}+105a^2b^5x^2\sqrt{a+bx^2}+105ab^6x^4\sqrt{a+bx^2}+3} \\ \frac{x^8}{8a^{\frac{9}{2}}} \end{cases}\right.$$

```
[In] integrate(x**5*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*Piecewise((-8*a**2/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*s
qrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a +
 b*x**2)) - 28*a*b*x**2/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**
2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(
a + b*x**2)) - 35*b**2*x**4/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4
*x**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*s
qrt(a + b*x**2)), Ne(b, 0)), (x**6/(6*a**(9/2)), True)) + B*x**7/(7*a**(9/2
```

```
)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 21*a**(5/2)*
b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + C
*Piecewise((-16*a**3/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*x**2*sq
rt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqrt(a + b
*x**2)) - 56*a**2*b*x**2/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*x**
2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqrt(a
 + b*x**2)) - 70*a*b**2*x**4/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5
*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sq
rt(a + b*x**2)) - 35*b**3*x**6/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b*
*5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*
sqrt(a + b*x**2)), Ne(b, 0)), (x**8/(8*a**(9/2)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(116) = 232$.

Time = 0.21 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.82

$$\begin{aligned}\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = &-\frac{Cx^6}{(bx^2+a)^{\frac{7}{2}}b} - \frac{Bx^5}{2\,(bx^2+a)^{\frac{7}{2}}b}\\ &- \frac{2\,Cax^4}{(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Ax^4}{3\,(bx^2+a)^{\frac{7}{2}}b} - \frac{5\,Bax^3}{8\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{8\,Ca^2x^2}{5\,(bx^2+a)^{\frac{7}{2}}b^3}\\ &- \frac{4\,Aax^2}{15\,(bx^2+a)^{\frac{7}{2}}b^2} + \frac{Bx}{14\,(bx^2+a)^{\frac{3}{2}}b^3} + \frac{Bx}{7\,\sqrt{bx^2+a}\,ab^3} + \frac{3\,Bax}{56\,(bx^2+a)^{\frac{5}{2}}b^3}\\ &- \frac{15\,Ba^2x}{56\,(bx^2+a)^{\frac{7}{2}}b^3} - \frac{16\,Ca^3}{35\,(bx^2+a)^{\frac{7}{2}}b^4} - \frac{8\,Aa^2}{105\,(bx^2+a)^{\frac{7}{2}}b^3}\end{aligned}$$

```
[In] integrate(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -C*x^6/((b*x^2 + a)^(7/2)*b) - 1/2*B*x^5/((b*x^2 + a)^(7/2)*b) - 2*C*a*x^4/
((b*x^2 + a)^(7/2)*b^2) - 1/3*A*x^4/((b*x^2 + a)^(7/2)*b) - 5/8*B*a*x^3/((b
*x^2 + a)^(7/2)*b^2) - 8/5*C*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) - 4/15*A*a*x^2
/((b*x^2 + a)^(7/2)*b^2) + 1/14*B*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*B*x/(sqrt
(b*x^2 + a)*a*b^3) + 3/56*B*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*B*a^2*x/((b
*x^2 + a)^(7/2)*b^3) - 16/35*C*a^3/((b*x^2 + a)^(7/2)*b^4) - 8/105*A*a^2/((
b*x^2 + a)^(7/2)*b^3)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(5\left(3\left(\frac{Bx}{a}-\frac{7C}{b}\right)x^2-\frac{7\left(6Ca^4b^2+Aa^3b^3\right)}{a^3b^4}\right)x^2-\frac{28\left(6Ca^5b+Aa^4b^2\right)}{a^3b^4}\right)x^2-\frac{8\left(6Ca^6+Aa^5b\right)}{a^3b^4}}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

```
[In] integrate(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((5*(3*(B*x/a - 7*C/b)*x^2 - 7*(6*C*a^4*b^2 + A*a^3*b^3)/(a^3*b^4))*x
^2 - 28*(6*C*a^5*b + A*a^4*b^2)/(a^3*b^4))*x^2 - 8*(6*C*a^6 + A*a^5*b)/(a^3
*b^4))/(b*x^2 + a)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 5.67 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.48

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{\frac{a\left(\frac{C}{3\,b^3}-\frac{7\,A\,b-14\,C\,a}{21\,a\,b^3}\right)}{b}-\frac{3\,B\,x}{7\,b^3}}{\left(b\,x^2+a\right)^{3/2}}-\frac{\frac{a^2\left(\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}\right)}{b^2}+\frac{B\,a^2\,x}{7\,b^3}}{\left(b\,x^2+a\right)^{7/2}}-\frac{\frac{C}{b^4}-\frac{B\,x}{7\,a\,b^3}}{\sqrt{b\,x^2+a}}-\frac{\frac{a\left(\frac{7\,C\,a^2-7\,A\,a\,b}{35\,a\,b^3}+\frac{a\left(\frac{C}{5\,b^2}-\frac{7\,A\,b^2-7\,C\,a\,b}{35\,a\,b^3}\right)}{b}\right)}{b}-\frac{3\,B\,a\,x}{7\,b^3}}{\left(b\,x^2+a\right)^{5/2}}$$

```
[In] int((x^5*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] ((a*(C/(3*b^3) - (7*A*b - 14*C*a)/(21*a*b^3)))/b - (3*B*x)/(7*b^3))/(a + b*
x^2)^(3/2) - ((a^2*(A/(7*b) - (C*a)/(7*b^2)))/b^2 + (B*a^2*x)/(7*b^3))/(a +
 b*x^2)^(7/2) - (C/b^4 - (B*x)/(7*a*b^3))/(a + b*x^2)^(1/2) - ((a*((7*C*a^2
 - 7*A*a*b)/(35*a*b^3) + (a*(C/(5*b^2) - (7*A*b^2 - 7*C*a*b)/(35*a*b^3)))/b
))/b - (3*B*a*x)/(7*b^3))/(a + b*x^2)^(5/2)
```

3.50 $\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

Optimal result . . . 345
Rubi [A] (verified) . . . 345
Mathematica [A] (verified) . . . 347
Maple [A] (verified) . . . 347
Fricas [A] (verification not implemented) . . . 349
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Maxima [A] (verification not implemented) . . . 350
Giac [A] (verification not implemented) . . . 350
Mupad [B] (verification not implemented) . . . 351

Optimal result

Integrand size = 25, antiderivative size = 149

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{x^4(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{x^2(4aB+(2Ab+5aC)x)}{35ab^2\,(a+bx^2)^{5/2}} - \frac{8aB+3(2Ab+5aC)x}{105ab^3\,(a+bx^2)^{3/2}} + \frac{(2Ab+5aC)x}{35a^2b^3\sqrt{a+bx^2}}$$

```
[Out] -1/7*x^4*(B*a-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)-1/35*x^2*(4*B*a+(2*A*b+5*C*a
)*x)/a/b^2/(b*x^2+a)^(5/2)+1/105*(-8*B*a-3*(2*A*b+5*C*a)*x)/a/b^3/(b*x^2+a)
^(3/2)+1/35*(2*A*b+5*C*a)*x/a^2/b^3/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1818, 833, 792, 197}

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{x(5aC+2Ab)}{35a^2b^3\sqrt{a+bx^2}} - \frac{3x(5aC+2Ab)+8aB}{105ab^3\,(a+bx^2)^{3/2}} - \frac{x^2(x(5aC+2Ab)+4aB)}{35ab^2\,(a+bx^2)^{5/2}} - \frac{x^4(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

```
[In] Int[(x^4*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/7*(x^4*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) - (x^2*(4*a*B + (2
*A*b + 5*a*C)*x))/(35*a*b^2*(a + b*x^2)^(5/2)) - (8*a*B + 3*(2*A*b + 5*a*C)
*x)/(105*a*b^3*(a + b*x^2)^(3/2)) + ((2*A*b + 5*a*C)*x)/(35*a^2*b^3*Sqrt[a
+ b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^4(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{x^3(-4aB-(2Ab+5aC)x)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab} \\
&= -\frac{x^4(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{x^2(4aB+(2Ab+5aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{\int \frac{x\left(-8a^2B-3a(2Ab+5aC)x\right)}{\left(a+bx^2\right)^{5/2}}\,dx}{35a^2b^2} \\
&= -\frac{x^4(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{x^2(4aB+(2Ab+5aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} \\
&\quad -\frac{8aB+3(2Ab+5aC)x}{105ab^3\left(a+bx^2\right)^{3/2}}+\frac{(2Ab+5aC)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{35ab^3}
\end{aligned}$$

$$= -\frac{x^4(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^2(4aB+(2Ab+5aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}} - \frac{8aB+3(2Ab+5aC)x}{105ab^3\left(a+bx^2\right)^{3/2}} + \frac{(2Ab+5aC)x}{35a^2b^3\sqrt{a+bx^2}}$$

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.53

$$\int \frac{x^4(A+Bx+Cx^2)}{\left(a+bx^2\right)^{9/2}}\,dx = \frac{-8a^4B-28a^3bBx^2-35a^2b^2Bx^4+21aAb^3x^5+6Ab^4x^7+15ab^3Cx^7}{105a^2b^3\left(a+bx^2\right)^{7/2}}$$

```
[In] Integrate[(x^4*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
[Out] (-8*a^4*B - 28*a^3*b*B*x^2 - 35*a^2*b^2*B*x^4 + 21*a*A*b^3*x^5 + 6*A*b^4*x^
7 + 15*a*b^3*C*x^7)/(105*a^2*b^3*(a + b*x^2)^(7/2))
```

Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.51

method	result
gosper	$\frac{6A\,b^4x^7+15Ca\,x^7b^3+21Aa\,b^3x^5-35a^2B\,b^2x^4-28B\,a^3b\,x^2-8B\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}a^2b^3}$
trager	$\frac{6A\,b^4x^7+15Ca\,x^7b^3+21Aa\,b^3x^5-35a^2B\,b^2x^4-28B\,a^3b\,x^2-8B\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}a^2b^3}$
default	$C\left(-\frac{x^5}{2b(b\,x^2+a)^{\frac{7}{2}}}+\frac{5a\left(-\frac{x^3}{4b\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{3a\left(-\frac{x}{6b\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)}{6b}\right)}{4b}\right)}{2b}\right.$

```
[In] int(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*(6*A*b^4*x^7+15*C*a*b^3*x^7+21*A*a*b^3*x^5-35*B*a^2*b^2*x^4-28*B*a^3*
b*x^2-8*B*a^4)/(b*x^2+a)^(7/2)/a^2/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.82

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{(21\,Aab^3x^5-35\,Ba^2b^2x^4+3\,(5\,Cab^3+2\,Ab^4)x^7-28\,Ba^3bx^2-8\,Ba^4)\sqrt{bx^2+}}{105\,(a^2b^7x^8+4\,a^3b^6x^6+6\,a^4b^5x^4+4\,a^5b^4x^2+a^6b^3)}$$

```
[In] integrate(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(21*A*a*b^3*x^5 - 35*B*a^2*b^2*x^4 + 3*(5*C*a*b^3 + 2*A*b^4)*x^7 - 28
*B*a^3*b*x^2 - 8*B*a^4)*sqrt(b*x^2 + a)/(a^2*b^7*x^8 + 4*a^3*b^6*x^6 + 6*a^
4*b^5*x^4 + 4*a^5*b^4*x^2 + a^6*b^3)
```

Sympy [A] (verification not implemented)

Time = 31.27 (sec) , antiderivative size = 575, normalized size of antiderivative = 3.86

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = A\left(\frac{7ax^5}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+35a^{\frac{5}{2}}b^3x^6\sqrt{1}}\right.$$
$$\left.+\frac{2bx^7}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+35a^{\frac{5}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}\right)$$
$$+B\left(\begin{cases}-\frac{8a^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx^2}+105b^6x^6\sqrt{a+bx^2}}-\frac{28abx^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+}} \\ \frac{x^6}{6a^{\frac{9}{2}}}\end{cases}\right.$$
$$+\frac{Cx^7}{7a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}}+21a^{\frac{7}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+21a^{\frac{5}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+7a^{\frac{3}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}$$

```
[In] integrate(x**4*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 +
 b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x
**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a
**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/
a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a))) + B*Piecewise((-8*a**2/(105
*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*sqrt(a + b*x**2) + 315*a*b
```

```
**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a + b*x**2)) - 28*a*b*x**2/(
105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*sqrt(a + b*x**2) + 315*
a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a + b*x**2)) - 35*b**2*x*
*4/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*sqrt(a + b*x**2) +
315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a + b*x**2)), Ne(b, 0
)), (x**6/(6*a**(9/2)), True)) + C*x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21
*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2
/a) + 7*a**(3/2)*b**3*x**6*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.70

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{Cx^5}{2\,(bx^2+a)^{\frac{7}{2}}b} - \frac{Bx^4}{3\,(bx^2+a)^{\frac{7}{2}}b} - \frac{5\,Cax^3}{8\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Ax^3}{4\,(bx^2+a)^{\frac{7}{2}}b} - \frac{4\,Bax^2}{15\,(bx^2+a)^{\frac{7}{2}}b^2} + \frac{Cx}{14\,(bx^2+a)^{\frac{3}{2}}b^3} + \frac{Cx}{7\,\sqrt{bx^2+a}ab^3} + \frac{3\,Cax}{56\,(bx^2+a)^{\frac{5}{2}}b^3} - \frac{15\,Ca^2x}{56\,(bx^2+a)^{\frac{7}{2}}b^3} + \frac{3\,Ax}{140\,(bx^2+a)^{\frac{5}{2}}b^2} + \frac{2\,Ax}{35\,\sqrt{bx^2+a}a^2b^2} + \frac{Ax}{35\,(bx^2+a)^{\frac{3}{2}}ab^2} - \frac{3\,Aax}{28\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{8\,Ba^2}{105\,(bx^2+a)^{\frac{7}{2}}b^3}$$

```
[In] integrate(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/2*C*x^5/((b*x^2 + a)^(7/2)*b) - 1/3*B*x^4/((b*x^2 + a)^(7/2)*b) - 5/8*C*
a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*A*x^3/((b*x^2 + a)^(7/2)*b) - 4/15*B*a*
x^2/((b*x^2 + a)^(7/2)*b^2) + 1/14*C*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*C*x/(s
qrt(b*x^2 + a)*a*b^3) + 3/56*C*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*C*a^2*x/
((b*x^2 + a)^(7/2)*b^3) + 3/140*A*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*A*x/(sqr
t(b*x^2 + a)*a^2*b^2) + 1/35*A*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*A*a*x/((b
*x^2 + a)^(7/2)*b^2) - 8/105*B*a^2/((b*x^2 + a)^(7/2)*b^3)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.54

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(3\,x\left(\frac{7\,A}{a}+\frac{(5\,Ca^2b^3+2\,Aab^4)x^2}{a^3b^3}\right)-\frac{35\,B}{b}\right)x^2-\frac{28\,Ba}{b^2}\right)x^2-\frac{8\,Ba^2}{b^3}}{105\,(bx^2+a)^{\frac{7}{2}}}$$

```
[In] integrate(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((3*x*(7*A/a + (5*C*a^2*b^3 + 2*A*a*b^4)*x^2/(a^3*b^3)) - 35*B/b)*x^
2 - 28*B*a/b^2)*x^2 - 8*B*a^2/b^3)/(b*x^2 + a)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 5.57 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.25

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{x\left(\frac{C\,a^2-A\,a\,b}{35\,a\,b^3}+\frac{a\left(\frac{C}{5\,b^2}-\frac{7\,A\,b^2-7\,C\,a\,b}{35\,a\,b^3}\right)}{b}\right)+\frac{2\,B\,a}{5\,b^3}}{\left(b\,x^2+a\right)^{5/2} - \frac{\frac{B}{3\,b^3}+x\left(\frac{C}{3\,b^3}-\frac{3\,A\,b-10\,C\,a}{105\,a\,b^3}\right)}{\left(b\,x^2+a\right)^{3/2}} - \frac{\frac{B\,a^2}{7\,b^3}-\frac{a\,x\left(\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}\right)}{b}}{\left(b\,x^2+a\right)^{7/2}} + \frac{x\,(2\,A\,b+5\,C\,a)}{35\,a^2\,b^3\,\sqrt{b\,x^2+a}}$$

```
[In] int((x^4*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] (x*((C*a^2 - A*a*b)/(35*a*b^3) + (a*(C/(5*b^2) - (7*A*b^2 - 7*C*a*b)/(35*a*
b^3)))/b) + (2*B*a)/(5*b^3))/(a + b*x^2)^(5/2) - (B/(3*b^3) + x*(C/(3*b^3)
- (3*A*b - 10*C*a)/(105*a*b^3)))/(a + b*x^2)^(3/2) - ((B*a^2)/(7*b^3) - (a*
x*(A/(7*b) - (C*a)/(7*b^2)))/b)/(a + b*x^2)^(7/2) + (x*(2*A*b + 5*C*a))/(35
*a^2*b^3*(a + b*x^2)^(1/2))
```

3.51 $\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}} dx$

Optimal result . . . 352
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Optimal result

Integrand size = 25, antiderivative size = 139

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}} dx = -\frac{x^3(aB-(Ab-aC)x)}{7ab(a+bx^2)^{7/2}} - \frac{x(3aB+(3Ab+4aC)x)}{35ab^2(a+bx^2)^{5/2}} - \frac{2(3Ab+4aC)-3bBx}{105ab^3(a+bx^2)^{3/2}} + \frac{2Bx}{35a^2b^2\sqrt{a+bx^2}}$$

```
[Out] -1/7*x^3*(B*a-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)-1/35*x*(3*B*a+(3*A*b+4*C*a)*
x)/a/b^2/(b*x^2+a)^(5/2)+1/105*(3*B*b*x-6*A*b-8*C*a)/a/b^3/(b*x^2+a)^(3/2)+
2/35*B*x/a^2/b^2/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1818, 833, 653, 197}

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}} dx = \frac{2Bx}{35a^2b^2\sqrt{a+bx^2}} - \frac{2(4aC+3Ab)-3bBx}{105ab^3(a+bx^2)^{3/2}} - \frac{x(x(4aC+3Ab)+3aB)}{35ab^2(a+bx^2)^{5/2}} - \frac{x^3(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}$$

```
[In] Int[(x^3*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/7*(x^3*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) - (x*(3*a*B + (3*A
*b + 4*a*C)*x))/(35*a*b^2*(a + b*x^2)^(5/2)) - (2*(3*A*b + 4*a*C) - 3*b*B*x
)/(105*a*b^3*(a + b*x^2)^(3/2)) + (2*B*x)/(35*a^2*b^2*Sqrt[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^3(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{x^2(-3aB-(3Ab+4aC)x)}{(a+bx^2)^{7/2}}\,dx}{7ab}\\
&= -\frac{x^3(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{x(3aB+(3Ab+4aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{\int \frac{-3a^2B-2a(3Ab+4aC)x}{(a+bx^2)^{5/2}}\,dx}{35a^2b^2}\\
&= -\frac{x^3(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{x(3aB+(3Ab+4aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}}\\
&\quad-\frac{2(3Ab+4aC)-3bBx}{105ab^3\left(a+bx^2\right)^{3/2}}+\frac{(2B)\int \frac{1}{(a+bx^2)^{3/2}}\,dx}{35ab^2}
\end{aligned}
$$

$$= -\frac{x^3(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{x(3aB+(3Ab+4aC)x)}{35ab^2\,(a+bx^2)^{5/2}} - \frac{2(3Ab+4aC)-3bBx}{105ab^3\,(a+bx^2)^{3/2}} + \frac{2Bx}{35a^2b^2\sqrt{a+bx^2}}$$

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.60

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{-8a^4C+21ab^3Bx^5+6b^4Bx^7-7a^2b^2x^2(3A+5Cx^2)-2a^3b(3A+14Cx^2)}{105a^2b^3\,(a+bx^2)^{7/2}}$$

```
[In] Integrate[(x^3*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-8*a^4*C + 21*a*b^3*B*x^5 + 6*b^4*B*x^7 - 7*a^2*b^2*x^2*(3*A + 5*C*x^2) -
2*a^3*b*(3*A + 14*C*x^2))/(105*a^2*b^3*(a + b*x^2)^(7/2))
```

Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.61

method	result
gosper	$-\frac{-6x^7 B\,b^4-21x^5 Ba\,b^3+35C\,a^2 b^2 x^4+21A\,a^2 b^2 x^2+28C\,a^3 b\,x^2+6A\,a^3 b+8C\,a^4}{105(b\,x^2+a)^{\frac{7}{2}} a^2 b^3}$
trager	$-\frac{-6x^7 B\,b^4-21x^5 Ba\,b^3+35C\,a^2 b^2 x^4+21A\,a^2 b^2 x^2+28C\,a^3 b\,x^2+6A\,a^3 b+8C\,a^4}{105(b\,x^2+a)^{\frac{7}{2}} a^2 b^3}$
default	$C\left(-\frac{x^4}{3b(b\,x^2+a)^{\frac{7}{2}}}+\frac{4a\left(-\frac{x^2}{5b\left(b\,x^2+a\right)^{\frac{7}{2}}}-\frac{2a}{35b^2\left(b\,x^2+a\right)^{\frac{7}{2}}}\right)}{3b}\right)+B\left(-\frac{x^3}{4b(b\,x^2+a)^{\frac{7}{2}}}+\frac{3a\left(-\frac{x}{6b\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^2+a\right)\ldots}\right.}{\ldots}\right.}{\ldots}\right.$

[In] int(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)

[Out] -1/105*(-6*B*b^4*x^7-21*B*a*b^3*x^5+35*C*a^2*b^2*x^4+21*A*a^2*b^2*x^2+28*C*a^3*b*x^2+6*A*a^3*b+8*C*a^4)/(b*x^2+a)^(7/2)/a^2/b^3

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{(6\,Bb^4x^7+21\,Bab^3x^5-35\,Ca^2b^2x^4-8\,Ca^4-6\,Aa^3b-7\,(4\,Ca^3b+3\,Aa^2b^2)x^2}{105\,(a^2b^7x^8+4\,a^3b^6x^6+6\,a^4b^5x^4+4\,a^5b^4x^2+a^6b^3)}$$

[In] integrate(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")

```
[Out] 1/105*(6*B*b^4*x^7 + 21*B*a*b^3*x^5 - 35*C*a^2*b^2*x^4 - 8*C*a^4 - 6*A*a^3*
b - 7*(4*C*a^3*b + 3*A*a^2*b^2)*x^2)*sqrt(b*x^2 + a)/(a^2*b^7*x^8 + 4*a^3*b
^6*x^6 + 6*a^4*b^5*x^4 + 4*a^5*b^4*x^2 + a^6*b^3)
```

Sympy [A] (verification not implemented)

Time = 20.05 (sec) , antiderivative size = 660, normalized size of antiderivative = 4.75

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = A\left(\left(\begin{cases} -\frac{2a}{35a^3b^2\sqrt{a+bx^2}+105a^2b^3x^2\sqrt{a+bx^2}+105ab^4x^4\sqrt{a+bx^2}+35b^5x^6\sqrt{a+bx^2}} - \frac{}{35a^3b^2\sqrt{a+bx^2}+105a\cdots} \\ \frac{x^4}{4a^{\frac{9}{2}}} \end{cases}\right.\right.$$

$$+B\left(\frac{7ax^5}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+35a^{\frac{5}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}\right.$$

$$\left.+\frac{2bx^7}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+35a^{\frac{5}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}\right)$$

$$+C\left(\left(\begin{cases} -\frac{8a^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx^2}+105b^6x^6\sqrt{a+bx^2}} - \frac{28abx^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx\cdots}} \\ \frac{x^6}{6a^{\frac{9}{2}}} \end{cases}\right.\right.$$

```
[In] integrate(x**3*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*Piecewise((-2*a/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(
a + b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x*
*2)) - 7*b*x**2/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(a
+ b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x**2
)), Ne(b, 0)), (x**4/(4*a**(9/2)), True)) + B*(7*a*x**5/(35*a**(11/2)*sqrt(
1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*
x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 2*b*x
**7/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/
a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt
(1 + b*x**2/a))) + C*Piecewise((-8*a**2/(105*a**3*b**3*sqrt(a + b*x**2) + 3
15*a**2*b**4*x**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105
*b**6*x**6*sqrt(a + b*x**2)) - 28*a*b*x**2/(105*a**3*b**3*sqrt(a + b*x**2)
+ 315*a**2*b**4*x**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) +
105*b**6*x**6*sqrt(a + b*x**2)) - 35*b**2*x**4/(105*a**3*b**3*sqrt(a + b*x*
*2) + 315*a**2*b**4*x**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2
) + 105*b**6*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**6/(6*a**(9/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.29

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{Cx^4}{3\,(bx^2+a)^{\frac{7}{2}}b} - \frac{Bx^3}{4\,(bx^2+a)^{\frac{7}{2}}b} - \frac{4\,Cax^2}{15\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Ax^2}{5\,(bx^2+a)^{\frac{7}{2}}b} + \frac{3\,Bx}{140\,(bx^2+a)^{\frac{5}{2}}b^2} + \frac{2\,Bx}{35\,\sqrt{bx^2+a}a^2b^2} + \frac{Bx}{35\,(bx^2+a)^{\frac{3}{2}}ab^2} - \frac{3\,Bax}{28\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{8\,Ca^2}{105\,(bx^2+a)^{\frac{7}{2}}b^3} - \frac{2\,Aa}{35\,(bx^2+a)^{\frac{7}{2}}b^2}$$

```
[In] integrate(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/3*C*x^4/((b*x^2 + a)^(7/2)*b) - 1/4*B*x^3/((b*x^2 + a)^(7/2)*b) - 4/15*C
*a*x^2/((b*x^2 + a)^(7/2)*b^2) - 1/5*A*x^2/((b*x^2 + a)^(7/2)*b) + 3/140*B*
x/((b*x^2 + a)^(5/2)*b^2) + 2/35*B*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*B*x/(
(b*x^2 + a)^(3/2)*a*b^2) - 3/28*B*a*x/((b*x^2 + a)^(7/2)*b^2) - 8/105*C*a^2
/((b*x^2 + a)^(7/2)*b^3) - 2/35*A*a/((b*x^2 + a)^(7/2)*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.68

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(3\left(\frac{2\,Bbx^2}{a^2}+\frac{7\,B}{a}\right)x-\frac{35\,C}{b}\right)x^2-\frac{7\,(4\,Ca^4b+3\,Aa^3b^2)}{a^3b^3}\right)x^2-\frac{2\,(4\,Ca^5+3\,Aa^4b)}{a^3b^3}}{105\,(bx^2+a)^{\frac{7}{2}}}$$

```
[In] integrate(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((3*(2*B*b*x^2/a^2 + 7*B/a)*x - 35*C/b)*x^2 - 7*(4*C*a^4*b + 3*A*a^3
*b^2)/(a^3*b^3))*x^2 - 2*(4*C*a^5 + 3*A*a^4*b)/(a^3*b^3))/(b*x^2 + a)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 5.50 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{\frac{a\left(\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}\right)}{b}+\frac{B\,a\,x}{7\,b^2}}{(b\,x^2+a)^{7/2}} - \frac{\frac{C}{3\,b^3}-\frac{B\,x}{35\,a\,b^2}}{(b\,x^2+a)^{3/2}} + \frac{\frac{a\left(\frac{C}{5\,b^2}-\frac{7\,A\,b-7\,C\,a}{35\,a\,b^2}\right)}{b}-\frac{8\,B\,x}{35\,b^2}}{(b\,x^2+a)^{5/2}} + \frac{2\,B\,x}{35\,a^2\,b^2\,\sqrt{b\,x^2+a}}$$

```
[In] int((x^3*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] ((a*(A/(7*b) - (C*a)/(7*b^2)))/b + (B*a*x)/(7*b^2))/(a + b*x^2)^(7/2) - (C/
(3*b^3) - (B*x)/(35*a*b^2))/(a + b*x^2)^(3/2) + ((a*(C/(5*b^2) - (7*A*b - 7
*C*a)/(35*a*b^2)))/b - (8*B*x)/(35*b^2))/(a + b*x^2)^(5/2) + (2*B*x)/(35*a^
2*b^2*(a + b*x^2)^(1/2))
```

3.52 $\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

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Optimal result

Integrand size = 25, antiderivative size = 139

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{x^2(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{2aB+(4Ab+3aC)x}{35ab^2\,(a+bx^2)^{5/2}} + \frac{(4Ab+3aC)x}{105a^2b^2\,(a+bx^2)^{3/2}} + \frac{2(4Ab+3aC)x}{105a^3b^2\sqrt{a+bx^2}}$$

```
[Out] -1/7*x^2*(B*a-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)+1/35*(-2*B*a-(4*A*b+3*C*a)*x
)/a/b^2/(b*x^2+a)^(5/2)+1/105*(4*A*b+3*C*a)*x/a^2/b^2/(b*x^2+a)^(3/2)+2/105
*(4*A*b+3*C*a)*x/a^3/b^2/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1818, 792, 198, 197}

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{2x(3aC+4Ab)}{105a^3b^2\sqrt{a+bx^2}} + \frac{x(3aC+4Ab)}{105a^2b^2\,(a+bx^2)^{3/2}} - \frac{x(3aC+4Ab)+2aB}{35ab^2\,(a+bx^2)^{5/2}} - \frac{x^2(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

```
[In] Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/7*(x^2*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) - (2*a*B + (4*A*b
+ 3*a*C)*x)/(35*a*b^2*(a + b*x^2)^(5/2)) + ((4*A*b + 3*a*C)*x)/(105*a^2*b^2
*(a + b*x^2)^(3/2)) + (2*(4*A*b + 3*a*C)*x)/(105*a^3*b^2*Sqrt[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{x(-2aB-(4Ab+3aC)x)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}\\
&= -\frac{x^2(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{2aB+(4Ab+3aC)x}{35ab^2\left(a+bx^2\right)^{5/2}}+\frac{(4Ab+3aC)\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{35ab^2}\\
&= -\frac{x^2(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{2aB+(4Ab+3aC)x}{35ab^2\left(a+bx^2\right)^{5/2}}\\
&\quad+\frac{(4Ab+3aC)x}{105a^2b^2\left(a+bx^2\right)^{3/2}}+\frac{(2(4Ab+3aC))\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{105a^2b^2}\\
&= -\frac{x^2(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{2aB+(4Ab+3aC)x}{35ab^2\left(a+bx^2\right)^{5/2}}+\frac{(4Ab+3aC)x}{105a^2b^2\left(a+bx^2\right)^{3/2}}+\frac{2(4Ab+3aC)x}{105a^3b^2\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.63

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{-6a^4B-21a^3bBx^2+8Ab^4x^7+7a^2b^2x^3(5A+3Cx^2)+2ab^3x^5(14A+3Cx^2)}{105a^3b^2\,(a+bx^2)^{7/2}}$$

```
[In] Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-6*a^4*B - 21*a^3*b*B*x^2 + 8*A*b^4*x^7 + 7*a^2*b^2*x^3*(5*A + 3*C*x^2) +
2*a*b^3*x^5*(14*A + 3*C*x^2))/(105*a^3*b^2*(a + b*x^2)^(7/2))
```

Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.63

method	result
gosper	$\frac{8A\,b^4x^7+6Ca\,x^7b^3+28Aa\,b^3x^5+21C\,a^2x^5b^2+35A\,a^2b^2x^3-21B\,a^3b\,x^2-6B\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}a^3b^2}$
trager	$\frac{8A\,b^4x^7+6Ca\,x^7b^3+28Aa\,b^3x^5+21C\,a^2x^5b^2+35A\,a^2b^2x^3-21B\,a^3b\,x^2-6B\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}a^3b^2}$
default	$C\left(-\frac{x^3}{4b(b\,x^2+a)^{\frac{7}{2}}}+\frac{3a\left(-\frac{x}{6b\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)}{6b}\right)}{4b}\right)+B\Big(-$

```
[In] int(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*(8*A*b^4*x^7+6*C*a*b^3*x^7+28*A*a*b^3*x^5+21*C*a^2*b^2*x^5+35*A*a^2*b
^2*x^3-21*B*a^3*b*x^2-6*B*a^4)/(b*x^2+a)^(7/2)/a^3/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.96

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{(35\,Aa^2b^2x^3+2\,(3\,Cab^3+4\,Ab^4)x^7-21\,Ba^3bx^2+7\,(3\,Ca^2b^2+4\,Aab^3)x^5-6\,B}{105\,(a^3b^6x^8+4\,a^4b^5x^6+6\,a^5b^4x^4+4\,a^6b^3x^2+a^7b^2)}$$

```
[In] integrate(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(35*A*a^2*b^2*x^3 + 2*(3*C*a*b^3 + 4*A*b^4)*x^7 - 21*B*a^3*b*x^2 + 7*
(3*C*a^2*b^2 + 4*A*a*b^3)*x^5 - 6*B*a^4)*sqrt(b*x^2 + a)/(a^3*b^6*x^8 + 4*a
^4*b^5*x^6 + 6*a^5*b^4*x^4 + 4*a^6*b^3*x^2 + a^7*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(129) = 258$.

Time = 30.65 (sec) , antiderivative size = 904, normalized size of antiderivative = 6.50

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = A\left(\frac{35a^5x^3}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x\right.$$

$$+\frac{63a^4bx^5}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1+}}$$

$$+\frac{36a^3b^2x^7}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1+}}$$

$$+\frac{8a^2b^3x^9}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1+}}$$

$$+B\left(\begin{cases}-\frac{2a}{35a^3b^2\sqrt{a+bx^2}+105a^2b^3x^2\sqrt{a+bx^2}+105ab^4x^4\sqrt{a+bx^2}+35b^5x^6\sqrt{a+bx^2}}-\frac{7bx^2}{35a^3b^2\sqrt{a+bx^2}+105a^2b^3x^2\sqrt{a+bx^2}+105ab^4x^4\sqrt{a+bx^2}}\\ \frac{x^4}{4a^{\frac{9}{2}}}\end{cases}\right.$$

$$+C\left(\frac{7ax^5}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+35a^{\frac{5}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}\right.$$

$$\left.+\frac{2bx^7}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+35a^{\frac{5}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}\right)$$

```
[In] integrate(x**2*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sq
rt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/
2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a
)) + 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**
2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**
(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x*
*2/a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2
)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) +
420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1
 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a*
```

```
*(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2
/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*
sqrt(1 + b*x**2/a))) + B*Piecewise((-2*a/(35*a**3*b**2*sqrt(a + b*x**2) + 1
05*a**2*b**3*x**2*sqrt(a + b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*
b**5*x**6*sqrt(a + b*x**2)) - 7*b*x**2/(35*a**3*b**2*sqrt(a + b*x**2) + 105
*a**2*b**3*x**2*sqrt(a + b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b*
*5*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**4/(4*a**(9/2)), True)) + C*(7*a*x
**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/
a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt
(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*
b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*
a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.42

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{Cx^3}{4\,(bx^2+a)^{\frac{7}{2}}b} - \frac{Bx^2}{5\,(bx^2+a)^{\frac{7}{2}}b} + \frac{3\,Cx}{140\,(bx^2+a)^{\frac{5}{2}}b^2} + \frac{2\,Cx}{35\,\sqrt{bx^2+a}a^2b^2} + \frac{Cx}{35\,(bx^2+a)^{\frac{3}{2}}ab^2} - \frac{3\,Cax}{28\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Ax}{7\,(bx^2+a)^{\frac{7}{2}}b} + \frac{8\,Ax}{105\,\sqrt{bx^2+a}a^3b} + \frac{4\,Ax}{105\,(bx^2+a)^{\frac{3}{2}}a^2b} + \frac{Ax}{35\,(bx^2+a)^{\frac{5}{2}}ab} - \frac{2\,Ba}{35\,(bx^2+a)^{\frac{7}{2}}b^2}$$

```
[In] integrate(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*C*x^3/((b*x^2 + a)^(7/2)*b) - 1/5*B*x^2/((b*x^2 + a)^(7/2)*b) + 3/140*
C*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*C*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*C*x
/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*C*a*x/((b*x^2 + a)^(7/2)*b^2) - 1/7*A*x/(
(b*x^2 + a)^(7/2)*b) + 8/105*A*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*A*x/((b*x^
2 + a)^(3/2)*a^2*b) + 1/35*A*x/((b*x^2 + a)^(5/2)*a*b) - 2/35*B*a/((b*x^2 +
 a)^(7/2)*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.68

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{2\,(3\,Cab^4+4\,Ab^5)x^2}{a^3b^3} + \frac{7\,(3\,Ca^2b^3+4\,Aab^4)}{a^3b^3}\right) + \frac{35\,A}{a}\right)x - \frac{21\,B}{b}\right)x^2 - \frac{6\,Ba}{b^2}}{105\,(bx^2+a)^{\frac{7}{2}}}$$

```
[In] integrate(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((x^2*(2*(3*C*a*b^4 + 4*A*b^5)*x^2/(a^3*b^3) + 7*(3*C*a^2*b^3 + 4*A*
a*b^4)/(a^3*b^3)) + 35*A/a)*x - 21*B/b)*x^2 - 6*B*a/b^2)/(b*x^2 + a)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 5.48 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{x\,(4\,A\,b+3\,C\,a)}{105\,a^2\,b^2\,(b\,x^2+a)^{3/2} } - \frac{\frac{B}{5\,b^2}+x\,\left(\frac{C}{5\,b^2}-\frac{A\,b-C\,a}{35\,a\,b^2}\right)}{(b\,x^2+a)^{5/2}} - \frac{x\,\left(\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}\right)-\frac{B\,a}{7\,b^2}}{(b\,x^2+a)^{7/2}} + \frac{x\,(8\,A\,b+6\,C\,a)}{105\,a^3\,b^2\,\sqrt{b\,x^2+a}}$$

```
[In] int((x^2*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] (x*(4*A*b + 3*C*a))/(105*a^2*b^2*(a + b*x^2)^(3/2)) - (B/(5*b^2) + x*(C/(5*
b^2) - (A*b - C*a)/(35*a*b^2)))/(a + b*x^2)^(5/2) - (x*(A/(7*b) - (C*a)/(7*
b^2)) - (B*a)/(7*b^2))/(a + b*x^2)^(7/2) + (x*(8*A*b + 6*C*a))/(105*a^3*b^2
*(a + b*x^2)^(1/2))
```

3.53 $\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}} dx$

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Optimal result

Integrand size = 23, antiderivative size = 119

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}} dx = -\frac{x(aB-(Ab-aC)x)}{7ab(a+bx^2)^{7/2}} - \frac{5Ab+2aC-bBx}{35ab^2(a+bx^2)^{5/2}} + \frac{4Bx}{105a^2b(a+bx^2)^{3/2}} + \frac{8Bx}{105a^3b\sqrt{a+bx^2}}$$

```
[Out] -1/7*x*(B*a-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)+1/35*(B*b*x-5*A*b-2*C*a)/a/b^2
/(b*x^2+a)^(5/2)+4/105*B*x/a^2/b/(b*x^2+a)^(3/2)+8/105*B*x/a^3/b/(b*x^2+a)^
(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1818, 653, 198, 197}

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}} dx = \frac{8Bx}{105a^3b\sqrt{a+bx^2}} + \frac{4Bx}{105a^2b(a+bx^2)^{3/2}} - \frac{2aC+5Ab-bBx}{35ab^2(a+bx^2)^{5/2}} - \frac{x(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}$$

```
[In] Int[(x*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/7*(x*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) - (5*A*b + 2*a*C - b
*B*x)/(35*a*b^2*(a + b*x^2)^(5/2)) + (4*B*x)/(105*a^2*b*(a + b*x^2)^(3/2))
+ (8*B*x)/(105*a^3*b*Sqrt[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{\int\frac{-aB-(5Ab+2aC)x}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}\\
&= -\frac{x(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{5Ab+2aC-bBx}{35ab^2\left(a+bx^2\right)^{5/2}}+\frac{(4B)\int\frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{35ab}\\
&= -\frac{x(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{5Ab+2aC-bBx}{35ab^2\left(a+bx^2\right)^{5/2}}+\frac{4Bx}{105a^2b\left(a+bx^2\right)^{3/2}}+\frac{(8B)\int\frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{105a^2b}\\
&= -\frac{x(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{5Ab+2aC-bBx}{35ab^2\left(a+bx^2\right)^{5/2}}+\frac{4Bx}{105a^2b\left(a+bx^2\right)^{3/2}}+\frac{8Bx}{105a^3b\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.64

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{-15a^3Ab-6a^4C-21a^3bCx^2+35a^2b^2Bx^3+28ab^3Bx^5+8b^4Bx^7}{105a^3b^2\left(a+bx^2\right)^{7/2}}$$

[In] Integrate[(x*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]

[Out] (-15*a^3*A*b - 6*a^4*C - 21*a^3*b*C*x^2 + 35*a^2*b^2*B*x^3 + 28*a*b^3*B*x^5 + 8*b^4*B*x^7)/(105*a^3*b^2*(a + b*x^2)^(7/2))

Maple [A] (verified)

Time = 3.59 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.61

method	result
gosper	$-\frac{-8x^7Bb^4-28x^5Bab^3-35Ba^2b^2x^3+21Ca^3bx^2+15Aa^3b+6Ca^4}{105(bx^2+a)^{\frac{7}{2}}a^3b^2}$
trager	$-\frac{-8x^7Bb^4-28x^5Bab^3-35Ba^2b^2x^3+21Ca^3bx^2+15Aa^3b+6Ca^4}{105(bx^2+a)^{\frac{7}{2}}a^3b^2}$
default	$C\left(-\frac{x^2}{5b(bx^2+a)^{\frac{7}{2}}}-\frac{2a}{35b^2(bx^2+a)^{\frac{7}{2}}}\right)+B\left(-\frac{x}{6b(bx^2+a)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(bx^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(bx^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b}}\right.}{7a}}{a}\right.}{6b}\right.$

[In] int(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)

[Out] -1/105*(-8*B*b^4*x^7-28*B*a*b^3*x^5-35*B*a^2*b^2*x^3+21*C*a^3*b*x^2+15*A*a^3*b+6*C*a^4)/(b*x^2+a)^(7/2)/a^3/b^2

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{(8\,Bb^4x^7+28\,Bab^3x^5+35\,Ba^2b^2x^3-21\,Ca^3bx^2-6\,Ca^4-15\,Aa^3b)\sqrt{bx^2+a}}{105\,(a^3b^6x^8+4\,a^4b^5x^6+6\,a^5b^4x^4+4\,a^6b^3x^2+a^7b^2)}$$

```
[In] integrate(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*B*b^4*x^7 + 28*B*a*b^3*x^5 + 35*B*a^2*b^2*x^3 - 21*C*a^3*b*x^2 - 6
*C*a^4 - 15*A*a^3*b)*sqrt(b*x^2 + a)/(a^3*b^6*x^8 + 4*a^4*b^5*x^6 + 6*a^5*b
^4*x^4 + 4*a^6*b^3*x^2 + a^7*b^2)
```

Sympy [A] (verification not implemented)

Time = 19.63 (sec) , antiderivative size = 796, normalized size of antiderivative = 6.69

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = A\left(\begin{cases} -\frac{1}{7a^3b\sqrt{a+bx^2}+21a^2b^2x^2\sqrt{a+bx^2}+21ab^3x^4\sqrt{a+bx^2}+7b^4x^6\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{9}{2}}} & \text{otherwise} \end{cases}\right)$$

$$+B\left(\frac{35a^5x^3}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{}}\right.$$

$$+\frac{63a^4bx^5}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1+}}$$

$$+\frac{36a^3b^2x^7}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1+}}$$

$$+\frac{8a^2b^3x^9}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1+}}$$

$$+C\left(\begin{cases} -\frac{2a}{35a^3b^2\sqrt{a+bx^2}+105a^2b^3x^2\sqrt{a+bx^2}+105ab^4x^4\sqrt{a+bx^2}+35b^5x^6\sqrt{a+bx^2}} - \frac{7bx^2}{35a^3b^2\sqrt{a+bx^2}+105a^2b^3x^2\sqrt{a+bx^2}+105ab^4x^4\sqrt{a+bx^2}} \\ \frac{x^4}{4a^{\frac{9}{2}}} \end{cases}\right.$$

```
[In] integrate(x*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*Piecewise((-1/(7*a**3*b*sqrt(a + b*x**2) + 21*a**2*b**2*x**2*sqrt(a + b*x
**2) + 21*a*b**3*x**4*sqrt(a + b*x**2) + 7*b**4*x**6*sqrt(a + b*x**2)), Ne(
b, 0)), (x**2/(2*a**(9/2)), True)) + B*(35*a**5*x**3/(105*a**(19/2)*sqrt(1
+ b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*
```

```
x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*
a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 63*a**4*b*x**5/(105*a**(19/2)*sqr
t(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b
**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) +
105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 36*a**3*b**2*x**7/(105*a**(19
/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(
15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**
2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*
a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 63
0*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 +
 b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a))) + C*Piecewise((-2
*a/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(a + b*x**2) + 1
05*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x**2)) - 7*b*x**2
/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(a + b*x**2) + 105
*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x**2)), Ne(b, 0)),
(x**4/(4*a**(9/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{Cx^2}{5\,(bx^2+a)^{\frac{7}{2}}b} - \frac{Bx}{7\,(bx^2+a)^{\frac{7}{2}}b} + \frac{8\,Bx}{105\,\sqrt{bx^2+a}a^3b} + \frac{4\,Bx}{105\,(bx^2+a)^{\frac{3}{2}}a^2b} + \frac{Bx}{35\,(bx^2+a)^{\frac{5}{2}}ab} - \frac{2\,Ca}{35\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{A}{7\,(bx^2+a)^{\frac{7}{2}}b}$$

```
[In] integrate(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/5*C*x^2/((b*x^2 + a)^(7/2)*b) - 1/7*B*x/((b*x^2 + a)^(7/2)*b) + 8/105*B*
x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*B*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*B*x/
((b*x^2 + a)^(5/2)*a*b) - 2/35*C*a/((b*x^2 + a)^(7/2)*b^2) - 1/7*A/((b*x^2
+ a)^(7/2)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.69

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(4\left(\frac{2\,Bb^2x^2}{a^3} + \frac{7\,Bb}{a^2}\right)x^2 + \frac{35\,B}{a}\right)x - \frac{21\,C}{b}\right)x^2 - \frac{3\,(2\,Ca^4b+5\,Aa^3b^2)}{a^3b^3}}{105\,(bx^2+a)^{\frac{7}{2}}}$$

```
[In] integrate(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((4*(2*B*b^2*x^2/a^3 + 7*B*b/a^2)*x^2 + 35*B/a)*x - 21*C/b)*x^2 - 3*
(2*C*a^4*b + 5*A*a^3*b^2)/(a^3*b^3))/(b*x^2 + a)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.83

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{8\,B\,x}{105\,a^3\,b\,\sqrt{b\,x^2+a}} - \frac{\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}+\frac{B\,x}{7\,b}}{(b\,x^2+a)^{7/2}} - \frac{\frac{C}{5\,b^2}-\frac{B\,x}{35\,a\,b}}{(b\,x^2+a)^{5/2}} + \frac{4\,B\,x}{105\,a^2\,b\,(b\,x^2+a)^{3/2}}$$

```
[In] int((x*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] (8*B*x)/(105*a^3*b*(a + b*x^2)^(1/2)) - (A/(7*b) - (C*a)/(7*b^2) + (B*x)/(7
*b))/(a + b*x^2)^(7/2) - (C/(5*b^2) - (B*x)/(35*a*b))/(a + b*x^2)^(5/2) + (
4*B*x)/(105*a^2*b*(a + b*x^2)^(3/2))
```

3.54 $\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx$

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Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = -\frac{aB-(Ab-aC)x}{7ab(a+bx^2)^{7/2}} + \frac{(6Ab+aC)x}{35a^2b(a+bx^2)^{5/2}} + \frac{4(6Ab+aC)x}{105a^3b(a+bx^2)^{3/2}} + \frac{8(6Ab+aC)x}{105a^4b\sqrt{a+bx^2}}$$

```
[Out] 1/7*(-B*a+(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)+1/35*(6*A*b+C*a)*x/a^2/b/(b*x^2+
a)^(5/2)+4/105*(6*A*b+C*a)*x/a^3/b/(b*x^2+a)^(3/2)+8/105*(6*A*b+C*a)*x/a^4/
b/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1828, 12, 198, 197}

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = \frac{8x(aC+6Ab)}{105a^4b\sqrt{a+bx^2}} + \frac{4x(aC+6Ab)}{105a^3b(a+bx^2)^{3/2}} + \frac{x(aC+6Ab)}{35a^2b(a+bx^2)^{5/2}} - \frac{aB-x(Ab-aC)}{7ab(a+bx^2)^{7/2}}$$

```
[In] Int[(A + B*x + C*x^2)/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/7*(a*B - (A*b - a*C)*x)/(a*b*(a + b*x^2)^(7/2)) + ((6*A*b + a*C)*x)/(35*
a^2*b*(a + b*x^2)^(5/2)) + (4*(6*A*b + a*C)*x)/(105*a^3*b*(a + b*x^2)^(3/2)
) + (8*(6*A*b + a*C)*x)/(105*a^4*b*Sqrt[a + b*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{aB-(Ab-aC)x}{7ab\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{-6A-\frac{aC}{b}}{\left(a+bx^2\right)^{7/2}}\,dx}{7a}\\
&= -\frac{aB-(Ab-aC)x}{7ab\left(a+bx^2\right)^{7/2}}+\frac{(6Ab+aC)\int \frac{1}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}\\
&= -\frac{aB-(Ab-aC)x}{7ab\left(a+bx^2\right)^{7/2}}+\frac{(6Ab+aC)x}{35a^2b\left(a+bx^2\right)^{5/2}}+\frac{(4(6Ab+aC))\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{35a^2b}\\
&= -\frac{aB-(Ab-aC)x}{7ab\left(a+bx^2\right)^{7/2}}+\frac{(6Ab+aC)x}{35a^2b\left(a+bx^2\right)^{5/2}}\\
&\quad+\frac{4(6Ab+aC)x}{105a^3b\left(a+bx^2\right)^{3/2}}+\frac{(8(6Ab+aC))\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{105a^3b}\\
&= -\frac{aB-(Ab-aC)x}{7ab\left(a+bx^2\right)^{7/2}}+\frac{(6Ab+aC)x}{35a^2b\left(a+bx^2\right)^{5/2}}+\frac{4(6Ab+aC)x}{105a^3b\left(a+bx^2\right)^{3/2}}+\frac{8(6Ab+aC)x}{105a^4b\sqrt{a+bx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = \frac{-15a^4B+48Ab^4x^7+35a^3bx(3A+Cx^2)+8ab^3x^5(21A+Cx^2)+14a^2b^2x^3(15A+2}{105a^4b\,(a+bx^2)^{7/2}}$$

[In] Integrate[(A + B*x + C*x^2)/(a + b*x^2)^(9/2),x]

[Out] (-15*a^4*B + 48*A*b^4*x^7 + 35*a^3*b*x*(3*A + C*x^2) + 8*a*b^3*x^5*(21*A + C*x^2) + 14*a^2*b^2*x^3*(15*A + 2*C*x^2))/(105*a^4*b*(a + b*x^2)^(7/2))

Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

method	result
gosper	$\frac{48A\,b^4x^7+8Ca\,x^7b^3+168Aa\,b^3x^5+28C\,a^2x^5b^2+210A\,a^2b^2x^3+35C\,a^3x^3b+105A\,a^3bx-15B\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}a^4b}$
trager	$\frac{48A\,b^4x^7+8Ca\,x^7b^3+168Aa\,b^3x^5+28C\,a^2x^5b^2+210A\,a^2b^2x^3+35C\,a^3x^3b+105A\,a^3bx-15B\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}a^4b}$
default	$A\left(\frac{x}{7a(b\,x^2+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)+C\left(-\frac{x}{6b(b\,x^2+a)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{6x}{35a\left(b\,x^2+\right.}\right.}{}\right.$

[In] int((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)

[Out] 1/105*(48*A*b^4*x^7+8*C*a*b^3*x^7+168*A*a*b^3*x^5+28*C*a^2*b^2*x^5+210*A*a^2*b^2*x^3+35*C*a^3*b*x^3+105*A*a^3*b*x-15*B*a^4)/(b*x^2+a)^(7/2)/a^4/b

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = \frac{(8\,(Cab^3+6\,Ab^4)x^7+105\,Aa^3bx+28\,(Ca^2b^2+6\,Aab^3)x^5-15\,Ba^4+35\,(Ca^3b+6}{105\,(a^4b^5x^8+4\,a^5b^4x^6+6\,a^6b^3x^4+4\,a^7b^2x^2+a^8b)}$$

```
[In] integrate((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(C*a*b^3 + 6*A*b^4)*x^7 + 105*A*a^3*b*x + 28*(C*a^2*b^2 + 6*A*a*b^
3)*x^5 - 15*B*a^4 + 35*(C*a^3*b + 6*A*a^2*b^2)*x^3)*sqrt(b*x^2 + a)/(a^4*b^
5*x^8 + 4*a^5*b^4*x^6 + 6*a^6*b^3*x^4 + 4*a^7*b^2*x^2 + a^8*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1266 vs. 2(117) = 234.

Time = 24.19 (sec) , antiderivative size = 1880, normalized size of antiderivative = 14.80

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(
1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*
b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) +
 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt
(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a
**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**
2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8
*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(
25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sq
rt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*
b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) +
 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt
(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b*
*3*x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*
x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x
**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a
**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b
*x**2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(
35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a
) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sq
```

```
rt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/
2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 104*a**9*b**5*x**11/(35*a**(37/2)*sqrt(
1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**
2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 52
5*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1
+ b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x*
*13/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2
/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*
sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(2
7/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**
2/a))) + B*Piecewise((-1/(7*a**3*b*sqrt(a + b*x**2) + 21*a**2*b**2*x**2*sqr
t(a + b*x**2) + 21*a*b**3*x**4*sqrt(a + b*x**2) + 7*b**4*x**6*sqrt(a + b*x*
*2)), Ne(b, 0)), (x**2/(2*a**(9/2)), True)) + C*(35*a**5*x**3/(105*a**(19/2
)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15
/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/
a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 63*a**4*b*x**5/(105*a**(
19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a*
*(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x
**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 36*a**3*b**2*x**7/(1
05*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) +
 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(
1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x
**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**
2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6
*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.20

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx &= \frac{16\,Ax}{35\,\sqrt{bx^2+a}a^4} + \frac{8\,Ax}{35\,(bx^2+a)^{\frac{3}{2}}a^3} \\ &\quad + \frac{6\,Ax}{35\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{Ax}{7\,(bx^2+a)^{\frac{7}{2}}a} - \frac{Cx}{7\,(bx^2+a)^{\frac{7}{2}}b} + \frac{8\,Cx}{105\,\sqrt{bx^2+a}a^3b} \\ &\quad + \frac{4\,Cx}{105\,(bx^2+a)^{\frac{3}{2}}a^2b} + \frac{Cx}{35\,(bx^2+a)^{\frac{5}{2}}ab} - \frac{B}{7\,(bx^2+a)^{\frac{7}{2}}b}\end{aligned}$$

```
[In] integrate((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*A*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*A
*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((b*x^2 + a)^(7/2)*a) - 1/7*C*x/((b*x^
2 + a)^(7/2)*b) + 8/105*C*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*C*x/((b*x^2 + a
)^(3/2)*a^2*b) + 1/35*C*x/((b*x^2 + a)^(5/2)*a*b) - 1/7*B/((b*x^2 + a)^(7/2
)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(4\,x^2\left(\frac{2\,(Cab^5+6\,Ab^6)x^2}{a^4b^3}+\frac{7\,(Ca^2b^4+6\,Aab^5)}{a^4b^3}\right)+\frac{35\,(Ca^3b^3+6\,Aa^2b^4)}{a^4b^3}\right)x^2+\frac{105\,A}{a}\right)x-\frac{15\,B}{b}}{105\,(bx^2+a)^{\frac{7}{2}}}$$

```
[In] integrate((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((4*x^2*(2*(C*a*b^5 + 6*A*b^6)*x^2/(a^4*b^3) + 7*(C*a^2*b^4 + 6*A*a*
b^5)/(a^4*b^3)) + 35*(C*a^3*b^3 + 6*A*a^2*b^4)/(a^4*b^3))*x^2 + 105*A/a)*x
- 15*B/b)/(b*x^2 + a)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = \frac{x\,(6\,A\,b+C\,a)}{35\,a^2\,b\,(b\,x^2+a)^{5/2}}-\frac{\frac{B}{7\,b}-x\left(\frac{A}{7\,a}-\frac{C}{7\,b}\right)}{(b\,x^2+a)^{7/2}}+\frac{x\,(24\,A\,b+4\,C\,a)}{105\,a^3\,b\,(b\,x^2+a)^{3/2}}+\frac{x\,(48\,A\,b+8\,C\,a)}{105\,a^4\,b\,\sqrt{b\,x^2+a}}$$

```
[In] int((A + B*x + C*x^2)/(a + b*x^2)^(9/2),x)
```

```
[Out] (x*(6*A*b + C*a))/(35*a^2*b*(a + b*x^2)^(5/2)) - (B/(7*b) - x*(A/(7*a) - C/
(7*b)))/(a + b*x^2)^(7/2) + (x*(24*A*b + 4*C*a))/(105*a^3*b*(a + b*x^2)^(3/
2)) + (x*(48*A*b + 8*C*a))/(105*a^4*b*(a + b*x^2)^(1/2))
```

3.55 $\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx$

Optimal result . . . 378
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Optimal result

Integrand size = 25, antiderivative size = 138

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx=\frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}}+\frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}}+\frac{35A+16Bx}{35a^4\sqrt{a+bx^2}}-\frac{A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}$$

```
[Out] 1/7*(B*b*x+A*b-C*a)/a/b/(b*x^2+a)^(7/2)+1/35*(6*B*x+7*A)/a^2/(b*x^2+a)^(5/2
)+1/105*(24*B*x+35*A)/a^3/(b*x^2+a)^(3/2)-A*arctanh((b*x^2+a)^(1/2)/a^(1/2)
)/a^(9/2)+1/35*(16*B*x+35*A)/a^4/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1819, 837, 12, 272, 65, 214}

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx=-\frac{A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{35A+16Bx}{35a^4\sqrt{a+bx^2}}+\frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}}+\frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

```
[In] Int[(A + B*x + C*x^2)/(x*(a + b*x^2)^(9/2)),x]
```

```
[Out] (A*b - a*C + b*B*x)/(7*a*b*(a + b*x^2)^(7/2)) + (7*A + 6*B*x)/(35*a^2*(a +
b*x^2)^(5/2)) + (35*A + 24*B*x)/(105*a^3*(a + b*x^2)^(3/2)) + (35*A + 16*B*
x)/(35*a^4*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(9/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{Ab - aC + bBx}{7ab\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{-7A-6Bx}{x(a+bx^2)^{7/2}}\,dx}{7a}$$

$$= \frac{Ab - aC + bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{35aAb+24abBx}{x(a+bx^2)^{5/2}}\,dx}{35a^3b}$$

$$= \frac{Ab - aC + bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-105a^2Ab^2-48a^2b^2Bx}{x(a+bx^2)^{3/2}}\,dx}{105a^5b^2}$$

$$= \frac{Ab - aC + bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35A+16Bx}{35a^4\sqrt{a+bx^2}} + \frac{\int \frac{105a^3Ab^3}{x\sqrt{a+bx^2}}\,dx}{105a^7b^3}$$

$$= \frac{Ab - aC + bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35A+16Bx}{35a^4\sqrt{a+bx^2}} + \frac{A\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{a^4}$$

$$= \frac{Ab - aC + bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35A+16Bx}{35a^4\sqrt{a+bx^2}} + \frac{A\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{2a^4}$$

$$= \frac{Ab - aC + bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35A+16Bx}{35a^4\sqrt{a+bx^2}} + \frac{A\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{a^4b}$$

$$= \frac{Ab - aC + bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35A+16Bx}{35a^4\sqrt{a+bx^2}} - \frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx = \frac{-15a^4C + 14ab^3x^4(25A+12Bx) + 14a^2b^2x^2(29A+15Bx) + 3b^4x^6(35A+16Bx) +}{105a^4b\left(a+bx^2\right)^{7/2}} + \frac{2A\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}$$

```
[In] Integrate[(A + B*x + C*x^2)/(x*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-15*a^4*C + 14*a*b^3*x^4*(25*A + 12*B*x) + 14*a^2*b^2*x^2*(29*A + 15*B*x)
+ 3*b^4*x^6*(35*A + 16*B*x) + a^3*b*(176*A + 105*B*x))/(105*a^4*b*(a + b*x^
2)^(7/2)) + (2*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(9/2)
```

Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.40

method	result
default	$B\left(\frac{x}{7a(bx^2+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a(bx^2+a)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a(bx^2+a)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{bx^2+a}}\right)}{7a}}{a}\right)-\frac{C}{7b(bx^2+a)^{\frac{7}{2}}}+A\left(\frac{1}{7a(bx^2+a)^{\frac{7}{2}}}+\frac{1}{5a(bx^2+a)}\right.$

```
[In] int((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b
*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))-1/7*C/b/(b*x^2+a)^(7/2)+A*(1/7/a
/(b*x^2+a)^(7/2)+1/a*(1/5/a/(b*x^2+a)^(5/2)+1/a*(1/3/a/(b*x^2+a)^(3/2)+1/a*
(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 465, normalized size of antiderivative = 3.37

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx = \left[\frac{105\left(Ab^5x^8+4\,Aab^4x^6+6\,Aa^2b^3x^4+4\,Aa^3b^2x^2+Aa^4b\right)\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+}{x^2}\right.}{}\right.$$

```
[In] integrate((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(A*b^5*x^8 + 4*A*a*b^4*x^6 + 6*A*a^2*b^3*x^4 + 4*A*a^3*b^2*x^2
+ A*a^4*b)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*
(48*B*a*b^4*x^7 + 105*A*a*b^4*x^6 + 168*B*a^2*b^3*x^5 + 350*A*a^2*b^3*x^4 +
 210*B*a^3*b^2*x^3 + 406*A*a^3*b^2*x^2 + 105*B*a^4*b*x - 15*C*a^5 + 176*A*a
^4*b)*sqrt(b*x^2 + a))/(a^5*b^5*x^8 + 4*a^6*b^4*x^6 + 6*a^7*b^3*x^4 + 4*a^8
*b^2*x^2 + a^9*b), 1/105*(105*(A*b^5*x^8 + 4*A*a*b^4*x^6 + 6*A*a^2*b^3*x^4
```

```
+ 4*A*a^3*b^2*x^2 + A*a^4*b)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (4
8*B*a*b^4*x^7 + 105*A*a*b^4*x^6 + 168*B*a^2*b^3*x^5 + 350*A*a^2*b^3*x^4 + 2
10*B*a^3*b^2*x^3 + 406*A*a^3*b^2*x^2 + 105*B*a^4*b*x - 15*C*a^5 + 176*A*a^4
*b)*sqrt(b*x^2 + a))/(a^5*b^5*x^8 + 4*a^6*b^4*x^6 + 6*a^7*b^3*x^4 + 4*a^8*b
^2*x^2 + a^9*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5251 vs. $2(122) = 244$.

Time = 32.01 (sec) , antiderivative size = 6613, normalized size of antiderivative = 47.92

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x**2+B*x+A)/x/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(352*a**32*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 94
50*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x
**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(5
9/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 2
10*a**(53/2)*b**10*x**20) + 105*a**32*log(b*x**2/a)/(210*a**(73/2) + 2100*a
**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44
100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6
*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(
55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 210*a**32*log(sqrt(1 + b*x*
*2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**
4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)
*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 945
0*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x*
*20) + 2924*a**31*b*x**2*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)
*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(
65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 +
 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b*
*9*x**18 + 210*a**(53/2)*b**10*x**20) + 1050*a**31*b*x**2*log(b*x**2/a)/(21
0*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(
67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 +
44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b*
*8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 2100*a*
*31*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x*
*2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)
*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 2520
0*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x*
*18 + 210*a**(53/2)*b**10*x**20) + 10852*a**30*b**2*x**4*sqrt(1 + b*x**2/a)
/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*
```

```
a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**1
0 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2
)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 472
5*a**30*b**2*x**4*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 94
50*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x
**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(5
9/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 2
10*a**(53/2)*b**10*x**20) - 9450*a**30*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1
)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200
*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**
10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/
2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 23
630*a**29*b**3*x**6*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x*
*2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)
*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 2520
0*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x*
*18 + 210*a**(53/2)*b**10*x**20) + 12600*a**29*b**3*x**6*log(b*x**2/a)/(210
*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(6
7/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 4
4100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**
8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 25200*a*
*29*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b
*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65
/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 2
5200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9
*x**18 + 210*a**(53/2)*b**10*x**20) + 33280*a**28*b**4*x**8*sqrt(1 + b*x**2
/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 252
00*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x
**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(5
7/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) +
22050*a**28*b**4*x**8*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2
+ 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b*
*4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a
**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18
 + 210*a**(53/2)*b**10*x**20) - 44100*a**28*b**4*x**8*log(sqrt(1 + b*x**2/a
) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 +
25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**
5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a*
*(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20)
 + 31442*a**27*b**5*x**10*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2
)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**
(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12
+ 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b
**9*x**18 + 210*a**(53/2)*b**10*x**20) + 26460*a**27*b**5*x**10*log(b*x**2/
a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 2520
```

0*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x*
*10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57
/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 5
2920*a**27*b**5*x**10*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**
(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 4410
0*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x
12 + 25200*a(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55
/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 19924*a**26*b**6*x**12*sqrt(1
 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x
4 + 25200*a(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/
2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9
450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*
x**20) + 22050*a**26*b**6*x**12*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/
2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a*
*(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12
 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*
b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 44100*a**26*b**6*x**12*log(sqrt(1
 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b
2*x4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a*
*(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**1
4 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b
10*x20) + 8162*a**25*b**7*x**14*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 210
0*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 +
 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b
6*x12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a
(55/2)*b9*x**18 + 210*a**(53/2)*b**10*x**20) + 12600*a**25*b**7*x**14*l
og(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x
4 + 25200*a(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/
2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9
450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*
x**20) - 25200*a**25*b**7*x**14*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2)
+ 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x
6 + 44100*a(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61
/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2
100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 1960*a**24*b**8*x**
16*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/
2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 5292
0*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*
x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/
2)*b**10*x**20) + 4725*a**24*b**8*x**16*log(b*x**2/a)/(210*a**(73/2) + 2100
*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 +
44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b*
*6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a*
*(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 9450*a**24*b**8*x**16*log
(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(

69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 5
2920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b*
*7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(
53/2)*b**10*x**20) + 210*a**23*b**9*x**18*sqrt(1 + b*x**2/a)/(210*a**(73/2)
 + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*
x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(6
1/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 +
2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 1050*a**23*b**9*x*
*18*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b
2*x4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a*
*(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**1
4 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b
10*x20) - 2100*a**23*b**9*x**18*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73
/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b*
*3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a*
*(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16
 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 105*a**22*b**10
*x**20*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2
)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920
*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x
14 + 9450*a(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2
)*b**10*x**20) - 210*a**22*b**10*x**20*log(sqrt(1 + b*x**2/a) + 1)/(210*a**
(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)
*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100
*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x*
*16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20)) + B*(35*a**14
*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/
a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*s
qrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27
/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2
/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*
x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*
a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b
*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*
x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sqrt(1 + b*x*
*2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*s
qrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29
/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2
/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b**3*x**7/(35
*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 5
25*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1
+ b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b*
*5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) +
 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**
2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**

```
(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x*
*2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**
12*sqrt(1 + b*x**2/a)) + 104*a**9*b**5*x**11/(35*a**(37/2)*sqrt(1 + b*x**2/
a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt
(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)
*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a)
 + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x**13/(35*a**
(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a
**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*
x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x
**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a))) + C*
Piecewise((-1/(7*a**3*b*sqrt(a + b*x**2) + 21*a**2*b**2*x**2*sqrt(a + b*x**
2) + 21*a*b**3*x**4*sqrt(a + b*x**2) + 7*b**4*x**6*sqrt(a + b*x**2)), Ne(b,
 0)), (x**2/(2*a**(9/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx = \frac{16\,Bx}{35\,\sqrt{bx^2+a}a^4} + \frac{8\,Bx}{35\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{6\,Bx}{35\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{Bx}{7\,(bx^2+a)^{\frac{7}{2}}a} - \frac{A\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{9}{2}}} + \frac{A}{\sqrt{bx^2+a}a^4} + \frac{A}{3\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{A}{5\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{A}{7\,(bx^2+a)^{\frac{7}{2}}a} - \frac{C}{7\,(bx^2+a)^{\frac{7}{2}}b}$$

```
[In] integrate((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*B*x/(sqrt(b*x^2 + a)*a^4) + 8/35*B*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*B
*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*B*x/((b*x^2 + a)^(7/2)*a) - A*arcsinh(a/(s
qrt(a*b)*abs(x)))/a^(9/2) + A/(sqrt(b*x^2 + a)*a^4) + 1/3*A/((b*x^2 + a)^(3
/2)*a^3) + 1/5*A/((b*x^2 + a)^(5/2)*a^2) + 1/7*A/((b*x^2 + a)^(7/2)*a) - 1/
7*C/((b*x^2 + a)^(7/2)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(\left(\left(\left(\left(3\left(\left(\frac{16\,Bb^3x}{a^4}+\frac{35\,Ab^3}{a^4}\right)x+\frac{56\,Bb^2}{a^3}\right)x+\frac{350\,Ab^2}{a^3}\right)x+\frac{210\,Bb}{a^2}\right)x+\frac{406\,Ab}{a^2}\right)x+\frac{105\,B}{a}\right)x}{105\left(bx^2+a\right)^{\frac{7}{2}}} + \frac{2\,A\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4}$$

```
[In] integrate((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((((3*((16*B*b^3*x/a^4 + 35*A*b^3/a^4)*x + 56*B*b^2/a^3)*x + 350*A*b
^2/a^3)*x + 210*B*b/a^2)*x + 406*A*b/a^2)*x + 105*B/a)*x - (15*C*a^14*b^2 -
 176*A*a^13*b^3)/(a^14*b^3))/(b*x^2 + a)^(7/2) + 2*A*arctan(-(sqrt(b)*x - s
qrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^4)
```

Mupad [B] (verification not implemented)

Time = 6.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx = \frac{\frac{A}{7\,a}+\frac{A\,\left(b\,x^2+a\right)^2}{3\,a^3}+\frac{A\,\left(b\,x^2+a\right)^3}{a^4}+\frac{A\,\left(b\,x^2+a\right)}{5\,a^2}}{\left(b\,x^2+a\right)^{7/2}} - \frac{C}{7\,b\,\left(b\,x^2+a\right)^{7/2}} - \frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{9/2}} + \frac{16\,B\,x}{35\,a^4\,\sqrt{b\,x^2+a}} + \frac{8\,B\,x}{35\,a^3\,\left(b\,x^2+a\right)^{3/2}} + \frac{6\,B\,x}{35\,a^2\,\left(b\,x^2+a\right)^{5/2}} + \frac{B\,x}{7\,a\,\left(b\,x^2+a\right)^{7/2}}$$

```
[In] int((A + B*x + C*x^2)/(x*(a + b*x^2)^(9/2)),x)
```

```
[Out] (A/(7*a) + (A*(a + b*x^2)^2)/(3*a^3) + (A*(a + b*x^2)^3)/a^4 + (A*(a + b*x^
2))/(5*a^2))/(a + b*x^2)^(7/2) - C/(7*b*(a + b*x^2)^(7/2)) - (A*atanh((a +
b*x^2)^(1/2)/a^(1/2)))/a^(9/2) + (16*B*x)/(35*a^4*(a + b*x^2)^(1/2)) + (8*B
*x)/(35*a^3*(a + b*x^2)^(3/2)) + (6*B*x)/(35*a^2*(a + b*x^2)^(5/2)) + (B*x)
/(7*a*(a + b*x^2)^(7/2))
```

3.56 $\int \frac{A+Bx+Cx^2}{x^2(a+bx^2)^{9/2}}\,dx$

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Optimal result

Integrand size = 25, antiderivative size = 188

$$\int \frac{A+Bx+Cx^2}{x^2(a+bx^2)^{9/2}}\,dx=\frac{B-\left(\frac{Ab}{a}-C\right)x}{7a(a+bx^2)^{7/2}}+\frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2(a+bx^2)^{5/2}}+\frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3(a+bx^2)^{3/2}}+\frac{35B-\left(\frac{93Ab}{a}-16C\right)x}{35a^4\sqrt{a+bx^2}}-\frac{A\sqrt{a+bx^2}}{a^5x}-\frac{B\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}$$

```
[Out] 1/7*(B-(A*b/a-C)*x)/a/(b*x^2+a)^(7/2)+1/35*(7*B-(13*A*b/a-6*C)*x)/a^2/(b*x^
2+a)^(5/2)+1/105*(35*B-3*(29*A*b/a-8*C)*x)/a^3/(b*x^2+a)^(3/2)-B*arctanh((b
*x^2+a)^(1/2)/a^(1/2))/a^(9/2)+1/35*(35*B-(93*A*b/a-16*C)*x)/a^4/(b*x^2+a)^
(1/2)-A*(b*x^2+a)^(1/2)/a^5/x
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1819, 821, 272, 65, 214}

$$\int \frac{A+Bx+Cx^2}{x^2(a+bx^2)^{9/2}}\,dx=-\frac{B\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}-\frac{A\sqrt{a+bx^2}}{a^5x}+\frac{35B-x\left(\frac{93Ab}{a}-16C\right)}{35a^4\sqrt{a+bx^2}}+\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{105a^3(a+bx^2)^{3/2}}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{35a^2(a+bx^2)^{5/2}}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a(a+bx^2)^{7/2}}$$

```
[In] Int[(A + B*x + C*x^2)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (B - ((A*b)/a - C)*x)/(7*a*(a + b*x^2)^(7/2)) + (7*B - ((13*A*b)/a - 6*C)*x
)/(35*a^2*(a + b*x^2)^(5/2)) + (35*B - 3*((29*A*b)/a - 8*C)*x)/(105*a^3*(a
```

```
+ b*x^2)^(3/2)) + (35*B - ((93*A*b)/a - 16*C)*x)/(35*a^4*Sqrt[a + b*x^2]) -
 (A*Sqrt[a + b*x^2])/(a^5*x) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(9/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{B - \left(\frac{Ab}{a} - C\right)x}{7a\left(a + bx^2\right)^{7/2}} - \frac{\int \frac{-7A - 7Bx + 6\left(\frac{Ab}{a} - C\right)x^2}{x^2\left(a + bx^2\right)^{7/2}}\,dx}{7a}$$

$$= \frac{B - \left(\frac{Ab}{a} - C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B - \left(\frac{13Ab}{a} - 6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{35A+35Bx-4\left(\frac{13Ab}{a}-6C\right)x^2}{x^2(a+bx^2)^{5/2}}\,dx}{35a^2}$$

$$= \frac{B - \left(\frac{Ab}{a} - C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B - \left(\frac{13Ab}{a} - 6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35B - 3\left(\frac{29Ab}{a} - 8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-105A-105Bx+6\left(\frac{29Ab}{a}-8C\right)x^2}{x^2(a+bx^2)^{3/2}}\,dx}{105a^3}$$

$$= \frac{B - \left(\frac{Ab}{a} - C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B - \left(\frac{13Ab}{a} - 6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35B - 3\left(\frac{29Ab}{a} - 8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35B - \left(\frac{93Ab}{a} - 16C\right)x}{35a^4\sqrt{a+bx^2}} + \frac{\int \frac{105A+105Bx}{x^2\sqrt{a+bx^2}}\,dx}{105a^4}$$

$$= \frac{B - \left(\frac{Ab}{a} - C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B - \left(\frac{13Ab}{a} - 6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35B - 3\left(\frac{29Ab}{a} - 8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35B - \left(\frac{93Ab}{a} - 16C\right)x}{35a^4\sqrt{a+bx^2}} - \frac{A\sqrt{a+bx^2}}{a^5x} + \frac{B\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{a^4}$$

$$= \frac{B - \left(\frac{Ab}{a} - C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B - \left(\frac{13Ab}{a} - 6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35B - 3\left(\frac{29Ab}{a} - 8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35B - \left(\frac{93Ab}{a} - 16C\right)x}{35a^4\sqrt{a+bx^2}} - \frac{A\sqrt{a+bx^2}}{a^5x} + \frac{B\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{2a^4}$$

$$= \frac{B - \left(\frac{Ab}{a} - C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B - \left(\frac{13Ab}{a} - 6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35B - 3\left(\frac{29Ab}{a} - 8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35B - \left(\frac{93Ab}{a} - 16C\right)x}{35a^4\sqrt{a+bx^2}} - \frac{A\sqrt{a+bx^2}}{a^5x} + \frac{B\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{a^4b}$$

$$= \frac{B - \left(\frac{Ab}{a} - C\right)x}{7a\left(a+bx^2\right)^{7/2}} + \frac{7B - \left(\frac{13Ab}{a} - 6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35B - 3\left(\frac{29Ab}{a} - 8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35B - \left(\frac{93Ab}{a} - 16C\right)x}{35a^4\sqrt{a+bx^2}} - \frac{A\sqrt{a+bx^2}}{a^5x} - \frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \frac{-384Ab^4x^8+14a^2b^2x^4(-120A+x(25B+12Cx))+14a^3bx^2(-60A+x(29B+15}{}$$

```
[In] Integrate[(A + B*x + C*x^2)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-384*A*b^4*x^8 + 14*a^2*b^2*x^4*(-120*A + x*(25*B + 12*C*x)) + 14*a^3*b*x^
2*(-60*A + x*(29*B + 15*C*x)) + 3*a*b^3*x^6*(-448*A + x*(35*B + 16*C*x)) +
a^4*(-105*A + x*(176*B + 105*C*x)) + 210*Sqrt[a]*B*x*(a + b*x^2)^(7/2)*ArcT
anh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/(105*a^5*x*(a + b*x^2)^(7/2))
```

Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.47

method	result
default	$C\left(\frac{x}{7a(bx^2+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(bx^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(bx^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{bx^2+a}}\right)}{7a}}{a}\right)+B\left(\frac{1}{7a(bx^2+a)^{\frac{7}{2}}}+\frac{\frac{1}{5a\left(bx^2+a\right)^{\frac{5}{2}}}+\frac{1}{3a\left(bx^2+a\right)^{\frac{3}{2}}}}{}\right.$
risch	Expression too large to display

```
[In] int((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] C*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b
*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))+B*(1/7/a/(b*x^2+a)^(7/2)+1/a*(1/
5/a/(b*x^2+a)^(5/2)+1/a*(1/3/a/(b*x^2+a)^(3/2)+1/a*(1/a/(b*x^2+a)^(1/2)-1/a
^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))+A*(-1/a/x/(b*x^2+a)^(7/2)-
8*b/a*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/
a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 525, normalized size of antiderivative = 2.79

$$\int \frac{A+Bx+Cx^2}{x^2 \left(a+bx^2\right)^{9/2}}\,dx = \left[\frac{105\left(Bb^4x^9+4\,Bab^3x^7+6\,Ba^2b^2x^5+4\,Ba^3bx^3+Ba^4x\right)\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2}{x^2}\right.}{}\right.$$

```
[In] integrate((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(B*b^4*x^9 + 4*B*a*b^3*x^7 + 6*B*a^2*b^2*x^5 + 4*B*a^3*b*x^3 +
B*a^4*x)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(1
05*B*a*b^3*x^7 + 350*B*a^2*b^2*x^5 + 48*(C*a*b^3 - 8*A*b^4)*x^8 + 406*B*a^3
*b*x^3 + 168*(C*a^2*b^2 - 8*A*a*b^3)*x^6 + 176*B*a^4*x - 105*A*a^4 + 210*(C
*a^3*b - 8*A*a^2*b^2)*x^4 + 105*(C*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(
a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x), 1/105*(
105*(B*b^4*x^9 + 4*B*a*b^3*x^7 + 6*B*a^2*b^2*x^5 + 4*B*a^3*b*x^3 + B*a^4*x)
*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (105*B*a*b^3*x^7 + 350*B*a^2*b
^2*x^5 + 48*(C*a*b^3 - 8*A*b^4)*x^8 + 406*B*a^3*b*x^3 + 168*(C*a^2*b^2 - 8*
A*a*b^3)*x^6 + 176*B*a^4*x - 105*A*a^4 + 210*(C*a^3*b - 8*A*a^2*b^2)*x^4 +
105*(C*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^5*b^4*x^9 + 4*a^6*b^3*x^7
+ 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6922 vs. 2(155) = 310.

Time = 42.51 (sec) , antiderivative size = 6922, normalized size of antiderivative = 36.82

$$\int \frac{A+Bx+Cx^2}{x^2 \left(a+bx^2\right)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x**2+B*x+A)/x**2/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(-35*a**4*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*
x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 28
0*a**3*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*
x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 56
0*a**2*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*
x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 44
8*a*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**
2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 128*b
**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 2
10*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)) + B*(352*a*
```

```
*32*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69
/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 529
20*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7
*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53
/2)*b**10*x**20) + 105*a**32*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*
b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(6
5/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 +
25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**
9*x**18 + 210*a**(53/2)*b**10*x**20) - 210*a**32*log(sqrt(1 + b*x**2/a) + 1
)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200
*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**
10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/
2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 29
24*a**31*b*x**2*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 +
 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**
4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a*
*(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18
+ 210*a**(53/2)*b**10*x**20) + 1050*a**31*b*x**2*log(b*x**2/a)/(210*a**(73/
2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**
3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**
(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16
+ 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 2100*a**31*b*x**
2*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450
*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**
8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/
2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210
*a**(53/2)*b**10*x**20) + 10852*a**30*b**2*x**4*sqrt(1 + b*x**2/a)/(210*a**
(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)
*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100
*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x*
*16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 4725*a**30*b
**2*x**4*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69
/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 529
20*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7
*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53
/2)*b**10*x**20) - 9450*a**30*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(210*a*
*(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2
)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 4410
0*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x
**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 23630*a**29
*b**3*x**6*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450
*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**
8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/
2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210
*a**(53/2)*b**10*x**20) + 12600*a**29*b**3*x**6*log(b*x**2/a)/(210*a**(73/2
```

) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3
*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(
61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 +
 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 25200*a**29*b**3*
x**6*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9
450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*
x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(
59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 +
210*a**(53/2)*b**10*x**20) + 33280*a**28*b**4*x**8*sqrt(1 + b*x**2/a)/(210*
a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67
/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44
100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8
*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 22050*a**
28*b**4*x**8*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a*
*(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 +
 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*
b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a*
*(53/2)*b**10*x**20) - 44100*a**28*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(2
10*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**
(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 +
 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b
8*x16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 31442*
a**27*b**5*x**10*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2
 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b*
*4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a
(59/2)*b7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18
 + 210*a**(53/2)*b**10*x**20) + 26460*a**27*b**5*x**10*log(b*x**2/a)/(210*a
(73/2) + 2100*a(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/
2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 441
00*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*
x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 52920*a**2
7*b**5*x**10*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*
x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/
2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25
200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*
x**18 + 210*a**(53/2)*b**10*x**20) + 19924*a**26*b**6*x**12*sqrt(1 + b*x**2
/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 252
00*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x
10 + 44100*a(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(5
7/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) +
22050*a**26*b**6*x**12*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2
 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b
4*x8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*
a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**1
8 + 210*a**(53/2)*b**10*x**20) - 44100*a**26*b**6*x**12*log(sqrt(1 + b*x**2

/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4
+ 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b
5*x10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*
a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**2
0) + 8162*a**25*b**7*x**14*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/
2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a*
*(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12
 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*
b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 12600*a**25*b**7*x**14*log(b*x**2
/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 252
00*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x
10 + 44100*a(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(5
7/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) -
25200*a**25*b**7*x**14*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a*
*(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 441
00*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*
x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(5
5/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 1960*a**24*b**8*x**16*sqrt(1
 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x
4 + 25200*a(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/
2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9
450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*
x**20) + 4725*a**24*b**8*x**16*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2
)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**
(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12
+ 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b
9*x18 + 210*a**(53/2)*b**10*x**20) - 9450*a**24*b**8*x**16*log(sqrt(1 +
 b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**
2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(
63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14
+ 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**
10*x**20) + 210*a**23*b**9*x**18*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a
(71/2)*b*x2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44
100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6
*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(
55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 1050*a**23*b**9*x**18*log(b
*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4
+ 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b
5*x10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*
a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**2
0) - 2100*a**23*b**9*x**18*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 210
0*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 +
 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b
6*x12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a
(55/2)*b9*x**18 + 210*a**(53/2)*b**10*x**20) + 105*a**22*b**10*x**20*lo

```
g(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x*
*4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2
)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 94
50*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x
**20) - 210*a**22*b**10*x**20*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) +
2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**
6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2
)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 210
0*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20)) + C*(35*a**14*x/(35*a*
*(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*
a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b
*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*
x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 17
5*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt
(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)
*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a)
+ 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqr
t(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 2
10*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b
*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*
x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*
a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b**3*x**7/(35*a**(37/2
)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33
/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/
a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*
sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 286*a**1
0*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1
+ b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b*
*3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 2
10*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1
 + b*x**2/a)) + 104*a**9*b**5*x**11/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*
a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x*
*2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**
8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**
(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x**13/(35*a**(37/2)*sq
rt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*
b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) +
 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt
(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \frac{16\,Cx}{35\,\sqrt{bx^2+a}a^4} + \frac{8\,Cx}{35\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{6\,Cx}{35\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{Cx}{7\,(bx^2+a)^{\frac{7}{2}}a} - \frac{128\,Abx}{35\,\sqrt{bx^2+a}a^5} - \frac{64\,Abx}{35\,(bx^2+a)^{\frac{3}{2}}a^4} - \frac{48\,Abx}{35\,(bx^2+a)^{\frac{5}{2}}a^3} - \frac{8\,Abx}{7\,(bx^2+a)^{\frac{7}{2}}a^2} - \frac{B\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{9}{2}}} + \frac{B}{\sqrt{bx^2+a}a^4} + \frac{B}{3\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{B}{5\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{B}{7\,(bx^2+a)^{\frac{7}{2}}a} - \frac{A}{(bx^2+a)^{\frac{7}{2}}ax}$$

```
[In] integrate((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*C*x/(sqrt(b*x^2 + a)*a^4) + 8/35*C*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*C
*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*C*x/((b*x^2 + a)^(7/2)*a) - 128/35*A*b*x/(
sqrt(b*x^2 + a)*a^5) - 64/35*A*b*x/((b*x^2 + a)^(3/2)*a^4) - 48/35*A*b*x/((
b*x^2 + a)^(5/2)*a^3) - 8/7*A*b*x/((b*x^2 + a)^(7/2)*a^2) - B*arcsinh(a/(sq
rt(a*b)*abs(x)))/a^(9/2) + B/(sqrt(b*x^2 + a)*a^4) + 1/3*B/((b*x^2 + a)^(3/
2)*a^3) + 1/5*B/((b*x^2 + a)^(5/2)*a^2) + 1/7*B/((b*x^2 + a)^(7/2)*a) - A/(
(b*x^2 + a)^(7/2)*a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.27

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(\left(\left(\left(3\left(x\left(\frac{35\,Bb^3}{a^4} + \frac{(16\,Ca^{20}b^6-93\,Aa^{19}b^7)x}{a^{24}b^3}\right) + \frac{28\,(2\,Ca^{21}b^5-11\,Aa^{20}b^6)}{a^{24}b^3}\right)x + \frac{350\,Bb^2}{a^3}\right)x + \frac{210}{}\right.\right.\right.}{105\,(bx^2+a)^{\frac{7}{2}}} + \frac{2\,B\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4} + \frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^4}$$

```
[In] integrate((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((((3*(x*(35*B*b^3/a^4 + (16*C*a^20*b^6 - 93*A*a^19*b^7)*x/(a^24*b^3
)) + 28*(2*C*a^21*b^5 - 11*A*a^20*b^6)/(a^24*b^3))*x + 350*B*b^2/a^3)*x + 2
10*(C*a^22*b^4 - 5*A*a^21*b^5)/(a^24*b^3))*x + 406*B*b/a^2)*x + 105*(C*a^23
*b^3 - 4*A*a^22*b^4)/(a^24*b^3))*x + 176*B/a)/(b*x^2 + a)^(7/2) + 2*B*arcta
n(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^4) + 2*A*sqrt(b)/(((
sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

Mupad [B] (verification not implemented)

Time = 6.75 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{9/2}}\,dx=\frac{\frac{B}{7\,a}+\frac{B\,{\left(b\,x^2+a\right)}^2}{3\,a^3}+\frac{B\,{\left(b\,x^2+a\right)}^3}{a^4}+\frac{B\,\left(b\,x^2+a\right)}{5\,a^2}}{{\left(b\,x^2+a\right)}^{7/2}}-\frac{\frac{A}{a^4}+\frac{128\,A\,b\,x^2}{35\,a^5}}{x\,\sqrt{b\,x^2+a}}-\frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{16\,C\,x}{35\,a^4\,\sqrt{b\,x^2+a}}+\frac{8\,C\,x}{35\,a^3\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{6\,C\,x}{35\,a^2\,{\left(b\,x^2+a\right)}^{5/2}}+\frac{C\,x}{7\,a\,{\left(b\,x^2+a\right)}^{7/2}}-\frac{29\,A\,b\,x}{35\,a^4\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{13\,A\,b\,x}{35\,a^3\,{\left(b\,x^2+a\right)}^{5/2}}-\frac{A\,b\,x}{7\,a^2\,{\left(b\,x^2+a\right)}^{7/2}}$$

```
[In] int((A + B*x + C*x^2)/(x^2*(a + b*x^2)^(9/2)),x)
```

```
[Out] (B/(7*a) + (B*(a + b*x^2)^2)/(3*a^3) + (B*(a + b*x^2)^3)/a^4 + (B*(a + b*x^
2))/(5*a^2))/(a + b*x^2)^(7/2) - (A/a^4 + (128*A*b*x^2)/(35*a^5))/(x*(a + b
*x^2)^(1/2)) - (B*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(9/2) + (16*C*x)/(35*
a^4*(a + b*x^2)^(1/2)) + (8*C*x)/(35*a^3*(a + b*x^2)^(3/2)) + (6*C*x)/(35*a
^2*(a + b*x^2)^(5/2)) + (C*x)/(7*a*(a + b*x^2)^(7/2)) - (29*A*b*x)/(35*a^4*
(a + b*x^2)^(3/2)) - (13*A*b*x)/(35*a^3*(a + b*x^2)^(5/2)) - (A*b*x)/(7*a^2
*(a + b*x^2)^(7/2))
```

3.57 $\int \frac{A+Bx+Cx^2}{x^3(a+bx^2)^{9/2}}\,dx$

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Optimal result

Integrand size = 25, antiderivative size = 219

$$\int \frac{A+Bx+Cx^2}{x^3(a+bx^2)^{9/2}}\,dx = -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2(a+bx^2)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3(a+bx^2)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4(a+bx^2)^{3/2}} - \frac{35(4Ab-aC)+93bBx}{35a^5\sqrt{a+bx^2}} - \frac{A\sqrt{a+bx^2}}{2a^5x^2} - \frac{B\sqrt{a+bx^2}}{a^5x} + \frac{(9Ab-2aC)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}}$$

```
[Out] 1/7*(-a*(A*b/a-C)-B*b*x)/a^2/(b*x^2+a)^(7/2)+1/35*(-13*B*b*x-14*A*b+7*C*a)/
a^3/(b*x^2+a)^(5/2)+1/105*(-87*B*b*x-105*A*b+35*C*a)/a^4/(b*x^2+a)^(3/2)+1/
2*(9*A*b-2*C*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(11/2)+1/35*(-93*B*b*x-1
40*A*b+35*C*a)/a^5/(b*x^2+a)^(1/2)-1/2*A*(b*x^2+a)^(1/2)/a^5/x^2-B*(b*x^2+a
)^(1/2)/a^5/x
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1819, 1821, 821, 272, 65, 214}

$$\int \frac{A+Bx+Cx^2}{x^3(a+bx^2)^{9/2}}\,dx = \frac{(9Ab-2aC)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}} - \frac{35(4Ab-aC)+93bBx}{35a^5\sqrt{a+bx^2}} - \frac{A\sqrt{a+bx^2}}{2a^5x^2} - \frac{B\sqrt{a+bx^2}}{a^5x} - \frac{35(3Ab-aC)+87bBx}{105a^4(a+bx^2)^{3/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3(a+bx^2)^{5/2}} - \frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2(a+bx^2)^{7/2}}$$

```
[In] Int[(A + B*x + C*x^2)/(x^3*(a + b*x^2)^(9/2)),x]
```

```
[Out] -1/7*(a*((A*b)/a - C) + b*B*x)/(a^2*(a + b*x^2)^(7/2)) - (7*(2*A*b - a*C) +
 13*b*B*x)/(35*a^3*(a + b*x^2)^(5/2)) - (35*(3*A*b - a*C) + 87*b*B*x)/(105*
a^4*(a + b*x^2)^(3/2)) - (35*(4*A*b - a*C) + 93*b*B*x)/(35*a^5*Sqrt[a + b*x
^2]) - (A*Sqrt[a + b*x^2])/(2*a^5*x^2) - (B*Sqrt[a + b*x^2])/(a^5*x) + ((9*
A*b - 2*a*C)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(11/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{-7A-7Bx+7\left(\frac{Ab}{a}-C\right)x^2+\frac{6bBx^3}{a}}{x^3\left(a+bx^2\right)^{7/2}}\,dx}{7a}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}-\frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}}+\frac{\int \frac{35A+35Bx-35\left(\frac{2Ab}{a}-C\right)x^2-\frac{52bBx^3}{a}}{x^3\left(a+bx^2\right)^{5/2}}\,dx}{35a^2}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}-\frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} -\frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}}-\frac{\int \frac{-105A-105Bx+105\left(\frac{3Ab}{a}-C\right)x^2+\frac{174bBx^3}{a}}{x^3\left(a+bx^2\right)^{3/2}}\,dx}{105a^3}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}-\frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}}-\frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} -\frac{35(4Ab-aC)+93bBx}{35a^5\sqrt{a+bx^2}}+\frac{\int \frac{105A+105Bx-105\left(\frac{4Ab}{a}-C\right)x^2}{x^3\sqrt{a+bx^2}}\,dx}{105a^4}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}-\frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}}-\frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} -\frac{35(4Ab-aC)+93bBx}{35a^5\sqrt{a+bx^2}}-\frac{A\sqrt{a+bx^2}}{2a^5x^2}-\frac{\int \frac{-210aB+105(9Ab-2aC)x}{x^2\sqrt{a+bx^2}}\,dx}{210a^5}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}-\frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}}-\frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} -\frac{35(4Ab-aC)+93bBx}{35a^5\sqrt{a+bx^2}}-\frac{A\sqrt{a+bx^2}}{2a^5x^2}-\frac{B\sqrt{a+bx^2}}{a^5x}-\frac{(9Ab-2aC)\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{2a^5}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4Ab-aC)+93bBx}{35a^5\sqrt{a+bx^2}} - \frac{A\sqrt{a+bx^2}}{2a^5x^2} - \frac{B\sqrt{a+bx^2}}{a^5x} - \frac{(9Ab-2aC)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{4a^5}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4Ab-aC)+93bBx}{35a^5\sqrt{a+bx^2}} - \frac{A\sqrt{a+bx^2}}{2a^5x^2} - \frac{B\sqrt{a+bx^2}}{a^5x} - \frac{(9Ab-2aC)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{2a^5b}$$

$$= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4Ab-aC)+93bBx}{35a^5\sqrt{a+bx^2}} - \frac{A\sqrt{a+bx^2}}{2a^5x^2} - \frac{B\sqrt{a+bx^2}}{a^5x} + \frac{(9Ab-2aC)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}}$$

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{9/2}}\,dx = \frac{-3b^4x^8(315A+256Bx)+a^4\left(-105A-210Bx+352Cx^2\right)-4a^3bx^2(396A+7x(60B}{210a^5x} + \frac{(-9Ab+2aC)\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{11/2}}$$

```
[In] Integrate[(A + B*x + C*x^2)/(x^3*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-3*b^4*x^8*(315*A + 256*B*x) + a^4*(-105*A - 210*B*x + 352*C*x^2) - 4*a^3*
b*x^2*(396*A + 7*x*(60*B - 29*C*x)) + 42*a*b^3*x^6*(-75*A + x*(-64*B + 5*C*
x)) + 14*a^2*b^2*x^4*(-261*A + 10*x*(-24*B + 5*C*x)))/(210*a^5*x^2*(a + b*x
^2)^(7/2)) + ((-9*A*b + 2*a*C)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a
]])/a^(11/2)
```

Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.49

method	result
default	$C\left(\frac{1}{7a(b\,x^2+a)^{\frac{7}{2}}}+\frac{\frac{1}{5a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{\frac{1}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}}{a}}{a}\right)+B\left(-\frac{1}{ax(b\,x^2+a)^{\frac{7}{2}}}-\frac{8b\left(\frac{\ldots}{7a\left(b\,x\ldots\right.}\right.}{\ldots}\right.$
risch	Expression too large to display

```
[In] int((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] C*(1/7/a/(b*x^2+a)^(7/2)+1/a*(1/5/a/(b*x^2+a)^(5/2)+1/a*(1/3/a/(b*x^2+a)^(3
/2)+1/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x
)))))+B*(-1/a/x/(b*x^2+a)^(7/2)-8*b/a*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x
/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)
))))+A*(-1/2/a/x^2/(b*x^2+a)^(7/2)-9/2*b/a*(1/7/a/(b*x^2+a)^(7/2)+1/a*(1/5/
a/(b*x^2+a)^(5/2)+1/a*(1/3/a/(b*x^2+a)^(3/2)+1/a*(1/a/(b*x^2+a)^(1/2)-1/a^(
3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 688, normalized size of antiderivative = 3.14

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{9/2}}\,dx=\left[-\frac{105\left((2\,Cab^4-9\,Ab^5)x^{10}+4\,(2\,Ca^2b^3-9\,Aab^4)x^8+6\,(2\,Ca^3b^2-9\,Aa^2b^3)x^6+\ldots\right.}{\ldots}\right.$$

```
[In] integrate((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [-1/420*(105*((2*C*a*b^4 - 9*A*b^5)*x^10 + 4*(2*C*a^2*b^3 - 9*A*a*b^4)*x^8
+ 6*(2*C*a^3*b^2 - 9*A*a^2*b^3)*x^6 + 4*(2*C*a^4*b - 9*A*a^3*b^2)*x^4 + (2*
C*a^5 - 9*A*a^4*b)*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2
*a)/x^2) + 2*(768*B*a*b^4*x^9 + 2688*B*a^2*b^3*x^7 + 3360*B*a^3*b^2*x^5 + 1
680*B*a^4*b*x^3 - 105*(2*C*a^2*b^3 - 9*A*a*b^4)*x^8 + 210*B*a^5*x - 350*(2*
C*a^3*b^2 - 9*A*a^2*b^3)*x^6 + 105*A*a^5 - 406*(2*C*a^4*b - 9*A*a^3*b^2)*x^
4 - 176*(2*C*a^5 - 9*A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(a^6*b^4*x^10 + 4*a^7*b
^3*x^8 + 6*a^8*b^2*x^6 + 4*a^9*b*x^4 + a^10*x^2), 1/210*(105*((2*C*a*b^4 -
9*A*b^5)*x^10 + 4*(2*C*a^2*b^3 - 9*A*a*b^4)*x^8 + 6*(2*C*a^3*b^2 - 9*A*a^2*
b^3)*x^6 + 4*(2*C*a^4*b - 9*A*a^3*b^2)*x^4 + (2*C*a^5 - 9*A*a^4*b)*x^2)*sqr
t(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (768*B*a*b^4*x^9 + 2688*B*a^2*b^3*
x^7 + 3360*B*a^3*b^2*x^5 + 1680*B*a^4*b*x^3 - 105*(2*C*a^2*b^3 - 9*A*a*b^4)
*x^8 + 210*B*a^5*x - 350*(2*C*a^3*b^2 - 9*A*a^2*b^3)*x^6 + 105*A*a^5 - 406*
(2*C*a^4*b - 9*A*a^3*b^2)*x^4 - 176*(2*C*a^5 - 9*A*a^4*b)*x^2)*sqrt(b*x^2 +
 a))/(a^6*b^4*x^10 + 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 + 4*a^9*b*x^4 + a^10*x^2
)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11198 vs. 2(196) = 392.

Time = 62.49 (sec) , antiderivative size = 11198, normalized size of antiderivative = 51.13

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((C*x**2+B*x+A)/x**3/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(-70*a**49*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x*
*4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/
2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 1
6800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9
*x**20 + 140*a**(87/2)*b**10*x**22) - 1476*a**48*b*x**2*sqrt(1 + b*x**2/a)/
(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 +
 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*
b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300
*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**
22) - 315*a**48*b*x**2*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)
*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a*
*(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**1
4 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)
*b**9*x**20 + 140*a**(87/2)*b**10*x**22) + 630*a**48*b*x**2*log(sqrt(1 + b*
x**2/a) + 1)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2
)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 352
80*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7
```

*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87
/2)*b**10*x**22) - 9822*a**47*b**2*x**4*sqrt(1 + b*x**2/a)/(140*a**(107/2)*
x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2
)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 294
00*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*
x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 3150*a**47
*b**2*x**4*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 63
00*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4
*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a*
*(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20
+ 140*a**(87/2)*b**10*x**22) + 6300*a**47*b**2*x**4*log(sqrt(1 + b*x**2/a)
+ 1)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x
6 + 16800*a(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(9
7/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 +
 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**1
0*x**22) - 33956*a**46*b**3*x**6*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 +
1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*
x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(
95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 +
 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 14175*a**46*b**3*
x**6*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**
(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10
 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2
)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*
a**(87/2)*b**10*x**22) + 28350*a**46*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/
(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 +
 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*
b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300
*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**
22) - 71940*a**45*b**4*x**8*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*
a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8
+ 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)
*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400
*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 37800*a**45*b**4*x**8*
log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/
2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35
280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**
7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(8
7/2)*b**10*x**22) + 75600*a**45*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(140*
a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 1680
0*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*
x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(
91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) -
 100260*a**44*b**5*x**10*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**
(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 2

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9400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b*
*6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a*
*(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 66150*a**44*b**5*x**10*lo
g(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)
*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 3528
0*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*
x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/
2)*b**10*x**22) + 132300*a**44*b**5*x**10*log(sqrt(1 + b*x**2/a) + 1)/(140*
a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 1680
0*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*
x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(
91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) -
 94396*a**43*b**6*x**12*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(
105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29
400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**
6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**
(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 79380*a**43*b**6*x**12*log
(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*
b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280
*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x
**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2
)*b**10*x**22) + 158760*a**43*b**6*x**12*log(sqrt(1 + b*x**2/a) + 1)/(140*a
**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800
*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x
**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(9
1/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) -
59772*a**42*b**7*x**14*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(1
05/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 294
00*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6
*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(
89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 66150*a**42*b**7*x**14*log(
b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b
**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*
a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x*
*16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)
*b**10*x**22) + 132300*a**42*b**7*x**14*log(sqrt(1 + b*x**2/a) + 1)/(140*a*
*(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*
a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x*
*12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91
/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 2
4486*a**41*b**8*x**16*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(10
5/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 2940
0*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*
x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(8
9/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 37800*a**41*b**8*x**16*log(b
```

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*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b*
*2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a
**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**
16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*
b**10*x**22) + 75600*a**41*b**8*x**16*log(sqrt(1 + b*x**2/a) + 1)/(140*a**(
107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a*
*(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**1
2 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2
)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 588
0*a**40*b**9*x**18*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2
)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a
**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**
14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2
)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 14175*a**40*b**9*x**18*log(b*x*
*2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*
x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(
97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16
 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**
10*x**22) + 28350*a**40*b**9*x**18*log(sqrt(1 + b*x**2/a) + 1)/(140*a**(107
/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(1
01/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 +
 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b
**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 630*a*
*39*b**10*x**20*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b
*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(
99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14
 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b
**9*x**20 + 140*a**(87/2)*b**10*x**22) - 3150*a**39*b**10*x**20*log(b*x**2/
a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**
6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/
2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6
300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*
x**22) + 6300*a**39*b**10*x**20*log(sqrt(1 + b*x**2/a) + 1)/(140*a**(107/2)
*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/
2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29
400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8
*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 315*a**38
*b**11*x**22*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 +
6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b*
*4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*
a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**2
0 + 140*a**(87/2)*b**10*x**22) + 630*a**38*b**11*x**22*log(sqrt(1 + b*x**2/
a) + 1)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**
2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a*
*(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**1
```

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6 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b
**10*x**22)) + B*(-35*a**4*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 +
140*a**8*b**17*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b
**20*x**8) - 280*a**3*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 +
140*a**8*b**17*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b
**20*x**8) - 560*a**2*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 +
140*a**8*b**17*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b
**20*x**8) - 448*a*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140
*a**8*b**17*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**2
0*x**8) - 128*b**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8
*b**17*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**
8)) + C*(352*a**32*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**
2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*
b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200
*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**
18 + 210*a**(53/2)*b**10*x**20) + 105*a**32*log(b*x**2/a)/(210*a**(73/2) +
2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**
6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2
)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 210
0*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 210*a**32*log(sqrt(1
+ b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b*
*2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**
(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14
 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b*
*10*x**20) + 2924*a**31*b*x**2*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**
(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 4410
0*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x
**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55
/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 1050*a**31*b*x**2*log(b*x**2/
a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 2520
0*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x*
*10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57
/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 2
100*a**31*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2
)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**
(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12
+ 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b
**9*x**18 + 210*a**(53/2)*b**10*x**20) + 10852*a**30*b**2*x**4*sqrt(1 + b*x
**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 +
25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**
5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a*
*(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20)
 + 4725*a**30*b**2*x**4*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**
2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*
b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200
```

```
*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**
18 + 210*a**(53/2)*b**10*x**20) - 9450*a**30*b**2*x**4*log(sqrt(1 + b*x**2/
a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 +
 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b*
*5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a
**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20
) + 23630*a**29*b**3*x**6*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2
)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**
(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12
+ 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b
**9*x**18 + 210*a**(53/2)*b**10*x**20) + 12600*a**29*b**3*x**6*log(b*x**2/a
)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200
*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**
10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/
2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 25
200*a**29*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(7
1/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*
a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**
12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2
)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 33280*a**28*b**4*x**8*sqrt(1 +
b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4
 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*
b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450
*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**
20) + 22050*a**28*b**4*x**8*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b
*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65
/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 2
5200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9
*x**18 + 210*a**(53/2)*b**10*x**20) - 44100*a**28*b**4*x**8*log(sqrt(1 + b*
x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x
**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/
2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9
450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*
x**20) + 31442*a**27*b**5*x**10*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a*
*(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 441
00*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*
x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(5
5/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 26460*a**27*b**5*x**10*log(b
*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4
+ 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b
**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*
a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**2
0) - 52920*a**27*b**5*x**10*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 21
00*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6
+ 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*
```

```
b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*
a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 19924*a**26*b**6*x**12*
sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*
b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a
**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**
14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*
b**10*x**20) + 22050*a**26*b**6*x**12*log(b*x**2/a)/(210*a**(73/2) + 2100*a
**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44
100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6
*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(
55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 44100*a**26*b**6*x**12*log(
sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(6
9/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52
920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**
7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(5
3/2)*b**10*x**20) + 8162*a**25*b**7*x**14*sqrt(1 + b*x**2/a)/(210*a**(73/2)
 + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*
x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(6
1/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 +
2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 12600*a**25*b**7*x
**14*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*
b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a
**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**
14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*
b**10*x**20) - 25200*a**25*b**7*x**14*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(
73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*
b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*
a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**
16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 1960*a**24*b*
*8*x**16*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a
**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8
+ 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)
*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a
**(53/2)*b**10*x**20) + 4725*a**24*b**8*x**16*log(b*x**2/a)/(210*a**(73/2)
+ 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x
**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61
/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2
100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 9450*a**24*b**8*x**
16*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 945
0*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x*
*8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59
/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 21
0*a**(53/2)*b**10*x**20) + 210*a**23*b**9*x**18*sqrt(1 + b*x**2/a)/(210*a**
(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)
*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100
```

```
*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x*
*16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 1050*a**23*b
**9*x**18*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(6
9/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52
920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**
7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(5
3/2)*b**10*x**20) - 2100*a**23*b**9*x**18*log(sqrt(1 + b*x**2/a) + 1)/(210*
a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67
/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44
100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8
*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 105*a**22
*b**10*x**20*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a*
*(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 +
 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*
b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a*
*(53/2)*b**10*x**20) - 210*a**22*b**10*x**20*log(sqrt(1 + b*x**2/a) + 1)/(2
10*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**
(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 +
 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b
**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx+Cx^2}{x^3(a+bx^2)^{9/2}}\,dx = -\frac{128\,Bbx}{35\,\sqrt{bx^2+a}a^5} - \frac{64\,Bbx}{35\,(bx^2+a)^{\frac{3}{2}}a^4} - \frac{48\,Bbx}{35\,(bx^2+a)^{\frac{5}{2}}a^3} - \frac{8\,Bbx}{7\,(bx^2+a)^{\frac{7}{2}}a^2} - \frac{C\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{9}{2}}} + \frac{9\,Ab\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{11}{2}}} + \frac{C}{\sqrt{bx^2+a}a^4} + \frac{C}{3\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{C}{5\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{C}{7\,(bx^2+a)^{\frac{7}{2}}a} - \frac{9\,Ab}{2\,\sqrt{bx^2+a}a^5} - \frac{3\,Ab}{2\,(bx^2+a)^{\frac{3}{2}}a^4} - \frac{9\,Ab}{10\,(bx^2+a)^{\frac{5}{2}}a^3} - \frac{9\,Ab}{14\,(bx^2+a)^{\frac{7}{2}}a^2} - \frac{B}{(bx^2+a)^{\frac{7}{2}}ax} - \frac{A}{2\,(bx^2+a)^{\frac{7}{2}}ax^2}$$

```
[In] integrate((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -128/35*B*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*B*b*x/((b*x^2 + a)^(3/2)*a^4) -
 48/35*B*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*B*b*x/((b*x^2 + a)^(7/2)*a^2) -
C*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(9/2) + 9/2*A*b*arcsinh(a/(sqrt(a*b)*abs(
x)))/a^(11/2) + C/(sqrt(b*x^2 + a)*a^4) + 1/3*C/((b*x^2 + a)^(3/2)*a^3) + 1
/5*C/((b*x^2 + a)^(5/2)*a^2) + 1/7*C/((b*x^2 + a)^(7/2)*a) - 9/2*A*b/(sqrt(
```

```
b*x^2 + a)*a^5) - 3/2*A*b/((b*x^2 + a)^(3/2)*a^4) - 9/10*A*b/((b*x^2 + a)^(
5/2)*a^3) - 9/14*A*b/((b*x^2 + a)^(7/2)*a^2) - B/((b*x^2 + a)^(7/2)*a*x) -
1/2*A/((b*x^2 + a)^(7/2)*a*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.48

$$\int \frac{A+Bx+Cx^2}{x^3 (a+bx^2)^{9/2}} \, dx = -\frac{\left(\left(\left(\left(3\left(\left(\frac{93\,Bb^4x}{a^5}-\frac{35\,(Ca^{24}b^6-4\,Aa^{23}b^7)}{a^{28}b^3}\right)x+\frac{308\,Bb^3}{a^4}\right)x-\frac{35\,(10\,Ca^{25}b^5-39\,Aa^{24}b^6)}{a^{28}b^3}\right)x+\frac{1050\,Bb^2}{a^3}\right)x-\frac{14\,(29\,Ca^{26}b^4-108}{a^{28}b^3}}{105\,(bx^2+a)^{\frac{7}{2}}} + \frac{(2\,Ca-9\,Ab)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^5} + \frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aab-2\,Ba^2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2a^5}$$

```
[In] integrate((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(((((3*((93*B*b^4*x/a^5 - 35*(C*a^24*b^6 - 4*A*a^23*b^7)/(a^28*b^3))
*x + 308*B*b^3/a^4)*x - 35*(10*C*a^25*b^5 - 39*A*a^24*b^6)/(a^28*b^3))*x +
1050*B*b^2/a^3)*x - 14*(29*C*a^26*b^4 - 108*A*a^25*b^5)/(a^28*b^3))*x + 420
*B*b/a^2)*x - 2*(88*C*a^27*b^3 - 291*A*a^26*b^4)/(a^28*b^3))/(b*x^2 + a)^(7
/2) + (2*C*a - 9*A*b)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt
(-a)*a^5) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b + 2*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b - 2*B*a^2*sqrt(
b))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a^5)
```

Mupad [B] (verification not implemented)

Time = 7.46 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.27

$$\int \frac{A+Bx+Cx^2}{x^3 (a+bx^2)^{9/2}}\,dx = \frac{\frac{C}{7a}+\frac{C\,(b\,x^2+a)^2}{3\,a^3}+\frac{C\,(b\,x^2+a)^3}{a^4}+\frac{C\,(b\,x^2+a)}{5\,a^2}}{(b\,x^2+a)^{7/2}} - \frac{\frac{A\,b}{7\,a}+\frac{9\,A\,b\,(b\,x^2+a)}{35\,a^2}+\frac{3\,A\,b\,(b\,x^2+a)^2}{5\,a^3}+\frac{3\,A\,b\,(b\,x^2+a)^3}{a^4}-\frac{9\,A\,b\,(b\,x^2+a)^4}{2\,a^5}}{a\,(b\,x^2+a)^{7/2}-(b\,x^2+a)^{9/2}} - \frac{\frac{B}{a^4}+\frac{128\,B\,b\,x^2}{35\,a^5}}{x\,\sqrt{b\,x^2+a}} - \frac{C\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{9/2}} + \frac{9\,A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{11/2}} - \frac{29\,B\,b\,x}{35\,a^4\,(b\,x^2+a)^{3/2}} - \frac{13\,B\,b\,x}{35\,a^3\,(b\,x^2+a)^{5/2}} - \frac{B\,b\,x}{7\,a^2\,(b\,x^2+a)^{7/2}}$$

```
[In] int((A + B*x + C*x^2)/(x^3*(a + b*x^2)^(9/2)),x)
```

```
[Out] (C/(7*a) + (C*(a + b*x^2)^2)/(3*a^3) + (C*(a + b*x^2)^3)/a^4 + (C*(a + b*x^
2))/(5*a^2))/(a + b*x^2)^(7/2) - ((A*b)/(7*a) + (9*A*b*(a + b*x^2))/(35*a^2
) + (3*A*b*(a + b*x^2)^2)/(5*a^3) + (3*A*b*(a + b*x^2)^3)/a^4 - (9*A*b*(a +
 b*x^2)^4)/(2*a^5))/(a*(a + b*x^2)^(7/2) - (a + b*x^2)^(9/2)) - (B/a^4 + (1
28*B*b*x^2)/(35*a^5))/(x*(a + b*x^2)^(1/2)) - (C*atanh((a + b*x^2)^(1/2)/a^
(1/2)))/a^(9/2) + (9*A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(11/2)) - (
29*B*b*x)/(35*a^4*(a + b*x^2)^(3/2)) - (13*B*b*x)/(35*a^3*(a + b*x^2)^(5/2)
) - (B*b*x)/(7*a^2*(a + b*x^2)^(7/2))
```

3.58 $\int \frac{A(cx)^m}{a+bx^2} \, dx$

Optimal result 414
Rubi [A] (verified) 414
Mathematica [A] (verified) 415
Maple [F] 415
Fricas [F] 415
Sympy [C] (verification not implemented) 416
Maxima [F] 416
Giac [F] 416
Mupad [F(-1)] 417

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int \frac{A(cx)^m}{a+bx^2} \, dx = \frac{A(cx)^{1+m} \text{Hypergeometric2F1}\left(1, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{bx^2}{a}\right)}{ac(1+m)}$$

```
[Out] A*(c*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/c/(1+m)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {12, 371}

$$\int \frac{A(cx)^m}{a+bx^2} \, dx = \frac{A(cx)^{m+1} \text{Hypergeometric2F1}\left(1, \frac{m+1}{2}, \frac{m+3}{2}, -\frac{bx^2}{a}\right)}{ac(m+1)}$$

```
[In] Int[(A*(c*x)^m)/(a + b*x^2),x]
```

```
[Out] (A*(c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/
(a*c*(1 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = A\int \frac{(cx)^m}{a+bx^2}\,dx$$
$$= \frac{A(cx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ac(1+m)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{A(cx)^m}{a+bx^2}\,dx = \frac{Ax(cx)^m\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},1+\frac{1+m}{2},-\frac{bx^2}{a}\right)}{a(1+m)}$$

```
[In] Integrate[(A*(c*x)^m)/(a + b*x^2),x]
[Out] (A*x*(c*x)^m*Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/
(a*(1 + m))
```

Maple [F]

$$\int \frac{A(cx)^m}{b\,x^2+a}dx$$

```
[In] int(A*(c*x)^m/(b*x^2+a),x)
[Out] int(A*(c*x)^m/(b*x^2+a),x)
```

Fricas [F]

$$\int \frac{A(cx)^m}{a+bx^2}\,dx = \int \frac{(cx)^m A}{bx^2+a}\,dx$$

```
[In] integrate(A*(c*x)^m/(b*x^2+a),x, algorithm="fricas")
[Out] integral((c*x)^m*A/(b*x^2 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.16

$$\int \frac{A(cx)^m}{a+bx^2}\,dx = A\left(\frac{c^m m x^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{c^m x^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}\right)$$

```
[In] integrate(A*(c*x)**m/(b*x**2+a),x)
```

```
[Out] A*(c**m*m*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma
(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + c**m*x**(m + 1)*lerchphi(b*x**2*exp_po
lar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)))
```

Maxima [F]

$$\int \frac{A(cx)^m}{a+bx^2}\,dx = \int \frac{(cx)^m A}{bx^2+a}\,dx$$

```
[In] integrate(A*(c*x)^m/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] A*integrate((c*x)^m/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{A(cx)^m}{a+bx^2}\,dx = \int \frac{(cx)^m A}{bx^2+a}\,dx$$

```
[In] integrate(A*(c*x)^m/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((c*x)^m*A/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A(cx)^m}{a+bx^2}\,dx = \int \frac{A\,(c\,x)^m}{b\,x^2+a}\,dx$$

```
[In] int((A*(c*x)^m)/(a + b*x^2),x)
[Out] int((A*(c*x)^m)/(a + b*x^2), x)
```

3.59 $\int \frac{(cx)^m(A+Bx)}{a+bx^2} dx$

Optimal result 418
Rubi [A] (verified) 418
Mathematica [A] (verified) 419
Maple [F] 420
Fricas [F] 420
Sympy [C] (verification not implemented) 420
Maxima [F] 421
Giac [F] 421
Mupad [F(-1)] 421

Optimal result

Integrand size = 20, antiderivative size = 91

$$\int \frac{(cx)^m(A+Bx)}{a+bx^2}\,dx = \frac{A(cx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{ac(1+m)} + \frac{B(cx)^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\frac{bx^2}{a}\right)}{ac^2(2+m)}$$

```
[Out] A*(c*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/c/(1+m)+B*(c
*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-b*x^2/a)/a/c^2/(2+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {822, 371}

$$\int \frac{(cx)^m(A+Bx)}{a+bx^2}\,dx = \frac{A(cx)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{ac(m+1)} + \frac{B(cx)^{m+2}\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{bx^2}{a}\right)}{ac^2(m+2)}$$

```
[In] Int[((c*x)^m*(A + B*x))/(a + b*x^2),x]
```

```
[Out] (A*(c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/
(a*c*(1 + m)) + (B*(c*x)^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2,
 -((b*x^2)/a)])/(a*c^2*(2 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = A\int \frac{(cx)^m}{a+bx^2}\,dx + \frac{B\int \frac{(cx)^{1+m}}{a+bx^2}\,dx}{c}$$

$$= \frac{A(cx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ac(1+m)} + \frac{B(cx)^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-\frac{bx^2}{a}\right)}{ac^2(2+m)}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.90

$$\int \frac{(cx)^m(A+Bx)}{a+bx^2}\,dx$$
$$= \frac{x(cx)^m\left(B(1+m)x\,\text{Hypergeometric2F1}\left(1,1+\frac{m}{2},2+\frac{m}{2},-\frac{bx^2}{a}\right)+A(2+m)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2}\right.\right.}{a(1+m)(2+m)}$$

```
[In] Integrate[((c*x)^m*(A + B*x))/(a + b*x^2),x]
```

```
[Out] (x*(c*x)^m*(B*(1 + m)*x*Hypergeometric2F1[1, 1 + m/2, 2 + m/2, -((b*x^2)/a)
] + A*(2 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)]))/(a
*(1 + m)*(2 + m))
```

Maple [F]

$$\int \frac{(cx)^m (Bx+A)}{b\,x^2+a}dx$$

```
[In] int((c*x)^m*(B*x+A)/(b*x^2+a),x)
```

```
[Out] int((c*x)^m*(B*x+A)/(b*x^2+a),x)
```

Fricas [F]

$$\int \frac{(cx)^m(A+Bx)}{a+bx^2}\,dx = \int \frac{(Bx+A)(cx)^m}{bx^2+a}\,dx$$

```
[In] integrate((c*x)^m*(B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.67 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.08

$$\begin{aligned}\int \frac{(cx)^m(A+Bx)}{a+bx^2}\,dx &= \frac{Ac^m m x^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} \\ &\quad + \frac{Ac^m x^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} \\ &\quad + \frac{Bc^m m x^{m+2}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{4a\Gamma\left(\frac{m}{2}+2\right)} \\ &\quad + \frac{Bc^m x^{m+2}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{2a\Gamma\left(\frac{m}{2}+2\right)}\end{aligned}$$

```
[In] integrate((c*x)**m*(B*x+A)/(b*x**2+a),x)
```

```
[Out] A*c**m*m*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(
m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + A*c**m*x**(m + 1)*lerchphi(b*x**2*exp_p
olar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + B*c**
m*m*x**(m + 2)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1)*gamma(m/2 + 1
)/(4*a*gamma(m/2 + 2)) + B*c**m*x**(m + 2)*lerchphi(b*x**2*exp_polar(I*pi)/
a, 1, m/2 + 1)*gamma(m/2 + 1)/(2*a*gamma(m/2 + 2))
```

Maxima [F]

$$\int \frac{(cx)^m(A+Bx)}{a+bx^2}\,dx = \int \frac{(Bx+A)(cx)^m}{bx^2+a}\,dx$$

```
[In] integrate((c*x)^m*(B*x+A)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate((B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(cx)^m(A+Bx)}{a+bx^2}\,dx = \int \frac{(Bx+A)(cx)^m}{bx^2+a}\,dx$$

```
[In] integrate((c*x)^m*(B*x+A)/(b*x^2+a),x, algorithm="giac")
[Out] integrate((B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m(A+Bx)}{a+bx^2}\,dx = \int \frac{(c\,x)^m\,(A+B\,x)}{b\,x^2+a}\,dx$$

```
[In] int(((c*x)^m*(A + B*x))/(a + b*x^2),x)
[Out] int(((c*x)^m*(A + B*x))/(a + b*x^2), x)
```

3.60 $\int \frac{(cx)^m(A+Cx^2)}{a+bx^2}\,dx$

Optimal result . . . 422
Rubi [A] (verified) . . . 422
Mathematica [A] (verified) . . . 423
Maple [F] . . . 423
Fricas [F] . . . 424
Sympy [C] (verification not implemented) . . . 424
Maxima [F] . . . 425
Giac [F] . . . 425
Mupad [F(-1)] . . . 425

Optimal result

Integrand size = 22, antiderivative size = 76

$$\int \frac{(cx)^m (A+Cx^2)}{a+bx^2}\,dx = \frac{C(cx)^{1+m}}{bc(1+m)} + \frac{(Ab-aC)(cx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{abc(1+m)}$$

```
[Out] C*(c*x)^(1+m)/b/c/(1+m)+(A*b-C*a)*(c*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2
+1/2*m],-b*x^2/a)/a/b/c/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {470, 371}

$$\int \frac{(cx)^m (A+Cx^2)}{a+bx^2}\,dx = \frac{(cx)^{m+1}(Ab-aC)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{abc(m+1)} + \frac{C(cx)^{m+1}}{bc(m+1)}$$

```
[In] Int[((c*x)^m*(A + C*x^2))/(a + b*x^2),x]
```

```
[Out] (C*(c*x)^(1 + m))/(b*c*(1 + m)) + ((A*b - a*C)*(c*x)^(1 + m)*Hypergeometric
2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*c*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{C(cx)^{1+m}}{bc(1+m)} - \frac{(-Ab(1+m)+aC(1+m))\int \frac{(cx)^m}{a+bx^2}\,dx}{b(1+m)}\\ &= \frac{C(cx)^{1+m}}{bc(1+m)} + \frac{(Ab-aC)(cx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{abc(1+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

$$\begin{aligned}&\int \frac{(cx)^m\,(A+Cx^2)}{a+bx^2}\,dx\\ &= \frac{x(cx)^m\left(aC+(Ab-aC)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)\right)}{ab(1+m)}\end{aligned}$$

```
[In] Integrate[((c*x)^m*(A + C*x^2))/(a + b*x^2),x]
[Out] (x*(c*x)^m*(a*C + (A*b - a*C)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -(
(b*x^2)/a)]))/(a*b*(1 + m))
```

Maple [F]

$$\int \frac{(cx)^m\,(C\,x^2+A)}{b\,x^2+a}dx$$

```
[In] int((c*x)^m*(C*x^2+A)/(b*x^2+a),x)
[Out] int((c*x)^m*(C*x^2+A)/(b*x^2+a),x)
```

Fricas [F]

$$\int \frac{(cx)^m (A + Cx^2)}{a + bx^2} \, dx = \int \frac{(Cx^2 + A)(cx)^m}{bx^2 + a} \, dx$$

```
[In] integrate((c*x)^m*(C*x^2+A)/(b*x^2+a),x, algorithm="fricas")
[Out] integral((C*x^2 + A)*(c*x)^m/(b*x^2 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.82 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.64

$$\int \frac{(cx)^m (A + Cx^2)}{a + bx^2} \, dx = \frac{Ac^m m x^{m+1} \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right) \Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{Ac^m x^{m+1} \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right) \Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{Cc^m m x^{m+3} \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{3}{2}\right) \Gamma\left(\frac{m}{2} + \frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{5}{2}\right)} + \frac{3Cc^m x^{m+3} \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{3}{2}\right) \Gamma\left(\frac{m}{2} + \frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{5}{2}\right)}$$

```
[In] integrate((c*x)**m*(C*x**2+A)/(b*x**2+a),x)
[Out] A*c**m*m*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(
m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + A*c**m*x**(m + 1)*lerchphi(b*x**2*exp_p
olar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + C*c**
m*m*x**(m + 3)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 +
 3/2)/(4*a*gamma(m/2 + 5/2)) + 3*C*c**m*x**(m + 3)*lerchphi(b*x**2*exp_pola
r(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2))
```

Maxima [F]

$$\int \frac{(cx)^m (A + Cx^2)}{a + bx^2} \, dx = \int \frac{(Cx^2 + A)(cx)^m}{bx^2 + a} \, dx$$

```
[In] integrate((c*x)^m*(C*x^2+A)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate((C*x^2 + A)*(c*x)^m/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(cx)^m (A + Cx^2)}{a + bx^2} \, dx = \int \frac{(Cx^2 + A)(cx)^m}{bx^2 + a} \, dx$$

```
[In] integrate((c*x)^m*(C*x^2+A)/(b*x^2+a),x, algorithm="giac")
[Out] integrate((C*x^2 + A)*(c*x)^m/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A + Cx^2)}{a + bx^2} \, dx = \int \frac{(C\,x^2 + A)\,(c\,x)^m}{b\,x^2 + a} \, dx$$

```
[In] int(((A + C*x^2)*(c*x)^m)/(a + b*x^2),x)
[Out] int(((A + C*x^2)*(c*x)^m)/(a + b*x^2), x)
```

3.61 $\int \frac{(cx)^m(A+Bx+Cx^2)}{a+bx^2}\,dx$

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Optimal result

Integrand size = 25, antiderivative size = 121

$$\int \frac{(cx)^m(A+Bx+Cx^2)}{a+bx^2}\,dx = \frac{C(cx)^{1+m}}{bc(1+m)}+\frac{(Ab-aC)(cx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{abc(1+m)}+\frac{B(cx)^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\frac{bx^2}{a}\right)}{ac^2(2+m)}$$

```
[Out] C*(c*x)^(1+m)/b/c/(1+m)+(A*b-C*a)*(c*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2
+1/2*m],-b*x^2/a)/a/b/c/(1+m)+B*(c*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m
],-b*x^2/a)/a/c^2/(2+m)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1816, 822, 371}

$$\int \frac{(cx)^m(A+Bx+Cx^2)}{a+bx^2}\,dx = \frac{(cx)^{m+1}(Ab-aC)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{abc(m+1)}+\frac{B(cx)^{m+2}\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{bx^2}{a}\right)}{ac^2(m+2)}+\frac{C(cx)^{m+1}}{bc(m+1)}$$

```
[In] Int[((c*x)^m*(A + B*x + C*x^2))/(a + b*x^2),x]
```

```
[Out] (C*(c*x)^(1 + m))/(b*c*(1 + m)) + ((A*b - a*C)*(c*x)^(1 + m)*Hypergeometric
2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*c*(1 + m)) + (B*(c*x)^(2 +
 m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/(a*c^2*(2 + m
))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{C(cx)^m}{b}+\frac{(cx)^m(Ab-aC+bBx)}{b\,(a+bx^2)}\right)\,dx\\
&= \frac{C(cx)^{1+m}}{bc(1+m)}+\frac{\int \frac{(cx)^m(Ab-aC+bBx)}{a+bx^2}\,dx}{b}\\
&= \frac{C(cx)^{1+m}}{bc(1+m)}+\frac{B\int \frac{(cx)^{1+m}}{a+bx^2}\,dx}{c}+\frac{(Ab-aC)\int \frac{(cx)^m}{a+bx^2}\,dx}{b}\\
&= \frac{C(cx)^{1+m}}{bc(1+m)}+\frac{(Ab-aC)(cx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{abc(1+m)}+\frac{B(cx)^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-\frac{bx^2}{a}\right)}{ac^2(2+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.82

$$\int \frac{(cx)^m \left(A+Bx+Cx^2\right)}{a+bx^2}\,dx$$
$$=\frac{x(cx)^m \left(aC(2+m)+bB(1+m)x\,\text{Hypergeometric2F1}\left(1,1+\frac{m}{2},2+\frac{m}{2},-\frac{bx^2}{a}\right)+(Ab-aC)(2+m)\,\text{Hyp}\right.}{ab(1+m)(2+m)}$$

```
[In] Integrate[((c*x)^m*(A + B*x + C*x^2))/(a + b*x^2),x]
```

```
[Out] (x*(c*x)^m*(a*C*(2 + m) + b*B*(1 + m)*x*Hypergeometric2F1[1, 1 + m/2, 2 + m
/2, -((b*x^2)/a)] + (A*b - a*C)*(2 + m)*Hypergeometric2F1[1, (1 + m)/2, (3
+ m)/2, -((b*x^2)/a)]))/(a*b*(1 + m)*(2 + m))
```

Maple [F]

$$\int \frac{(cx)^m \left(C\,x^2+Bx+A\right)}{b\,x^2+a}dx$$

```
[In] int((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x)
```

```
[Out] int((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x)
```

Fricas [F]

$$\int \frac{(cx)^m \left(A+Bx+Cx^2\right)}{a+bx^2}\,dx=\int \frac{(Cx^2+Bx+A)(cx)^m}{bx^2+a}\,dx$$

```
[In] integrate((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.31 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.40

$$\int \frac{(cx)^m (A+Bx+Cx^2)}{a+bx^2}\,dx = \frac{Ac^m m x^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Ac^m x^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Bc^m m x^{m+2}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{4a\Gamma\left(\frac{m}{2}+2\right)} + \frac{Bc^m x^{m+2}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{2a\Gamma\left(\frac{m}{2}+2\right)} + \frac{Cc^m m x^{m+3}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{3Cc^m x^{m+3}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}$$

```
[In] integrate((c*x)**m*(C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] A*c**m*m*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(
m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + A*c**m*x**(m + 1)*lerchphi(b*x**2*exp_p
olar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + B*c**
m*m*x**(m + 2)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1)*gamma(m/2 + 1
)/(4*a*gamma(m/2 + 2)) + B*c**m*x**(m + 2)*lerchphi(b*x**2*exp_polar(I*pi)/
a, 1, m/2 + 1)*gamma(m/2 + 1)/(2*a*gamma(m/2 + 2)) + C*c**m*m*x**(m + 3)*le
rchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(
m/2 + 5/2)) + 3*C*c**m*x**(m + 3)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2
 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2))
```

Maxima [F]

$$\int \frac{(cx)^m (A+Bx+Cx^2)}{a+bx^2}\,dx = \int \frac{(Cx^2+Bx+A)(cx)^m}{bx^2+a}\,dx$$

```
[In] integrate((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(cx)^m (A + Bx + Cx^2)}{a + bx^2} dx = \int \frac{(Cx^2 + Bx + A)(cx)^m}{bx^2 + a} dx$$

```
[In] integrate((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
[Out] integrate((C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A + Bx + Cx^2)}{a + bx^2} dx = \int \frac{(c\,x)^m\,(C\,x^2 + B\,x + A)}{b\,x^2 + a} dx$$

```
[In] int(((c*x)^m*(A + B*x + C*x^2))/(a + b*x^2),x)
[Out] int(((c*x)^m*(A + B*x + C*x^2))/(a + b*x^2), x)
```

3.62 $\int x^3(a+bx^2)(A+Bx+Cx^2+Dx^3)\,dx$

Optimal result 431
Rubi [A] (verified) 431
Mathematica [A] (verified) 432
Maple [A] (verified) 432
Fricas [A] (verification not implemented) 433
Sympy [A] (verification not implemented) 433
Maxima [A] (verification not implemented) 433
Giac [A] (verification not implemented) 434
Mupad [B] (verification not implemented) 434

Optimal result

Integrand size = 26, antiderivative size = 65

$$\int x^3(a+bx^2)(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{4}aAx^4+\frac{1}{5}aBx^5+\frac{1}{6}(Ab+aC)x^6 + \frac{1}{7}(bB+aD)x^7+\frac{1}{8}bCx^8+\frac{1}{9}bDx^9$$

```
[Out] 1/4*a*A*x^4+1/5*a*B*x^5+1/6*(A*b+C*a)*x^6+1/7*(B*b+D*a)*x^7+1/8*b*C*x^8+1/9
*b*D*x^9
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1816}

$$\int x^3(a+bx^2)(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{6}x^6(aC+Ab)+\frac{1}{4}aAx^4+\frac{1}{7}x^7(aD+bB) + \frac{1}{5}aBx^5+\frac{1}{8}bCx^8+\frac{1}{9}bDx^9$$

```
[In] Int[x^3*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^4)/4 + (a*B*x^5)/5 + ((A*b + a*C)*x^6)/6 + ((b*B + a*D)*x^7)/7 + (b*
C*x^8)/8 + (b*D*x^9)/9
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(aAx^3+aBx^4+(Ab+aC)x^5+(bB+aD)x^6+bCx^7+bDx^8\right)\,dx\\ &= \frac{1}{4}aAx^4+\frac{1}{5}aBx^5+\frac{1}{6}(Ab+aC)x^6+\frac{1}{7}(bB+aD)x^7+\frac{1}{8}bCx^8+\frac{1}{9}bDx^9\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int x^3\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{4}aAx^4+\frac{1}{5}aBx^5+\frac{1}{6}(Ab+aC)x^6+\frac{1}{7}(bB+aD)x^7+\frac{1}{8}bCx^8+\frac{1}{9}bDx^9$$

```
[In] Integrate[x^3*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
[Out] (a*A*x^4)/4 + (a*B*x^5)/5 + ((A*b + a*C)*x^6)/6 + ((b*B + a*D)*x^7)/7 + (b*
C*x^8)/8 + (b*D*x^9)/9
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{aA\,x^4}{4}+\frac{aB\,x^5}{5}+\frac{(Ab+Ca)x^6}{6}+\frac{(Bb+Da)x^7}{7}+\frac{bC\,x^8}{8}+\frac{bDx^9}{9}$	54
norman	$\frac{bDx^9}{9}+\frac{bC\,x^8}{8}+\left(\frac{Bb}{7}+\frac{Da}{7}\right)x^7+\left(\frac{Ab}{6}+\frac{Ca}{6}\right)x^6+\frac{aB\,x^5}{5}+\frac{aA\,x^4}{4}$	56
gosper	$\frac{1}{9}bDx^9+\frac{1}{8}bC\,x^8+\frac{1}{7}bB\,x^7+\frac{1}{7}x^7Da+\frac{1}{6}x^6Ab+\frac{1}{6}x^6Ca+\frac{1}{5}aB\,x^5+\frac{1}{4}aA\,x^4$	58
parallelrisch	$\frac{1}{9}bDx^9+\frac{1}{8}bC\,x^8+\frac{1}{7}bB\,x^7+\frac{1}{7}x^7Da+\frac{1}{6}x^6Ab+\frac{1}{6}x^6Ca+\frac{1}{5}aB\,x^5+\frac{1}{4}aA\,x^4$	58

```
[In] int(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
[Out] 1/4*a*A*x^4+1/5*a*B*x^5+1/6*(A*b+C*a)*x^6+1/7*(B*b+D*a)*x^7+1/8*b*C*x^8+1/9
*b*D*x^9
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int x^3(a+bx^2)\,(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{9}\,Dbx^9+\frac{1}{8}\,Cbx^8+\frac{1}{7}\,(Da+Bb)x^7 + \frac{1}{5}\,Bax^5+\frac{1}{6}\,(Ca+Ab)x^6+\frac{1}{4}\,Aax^4$$

[In] integrate(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")

[Out] 1/9*D*b*x^9 + 1/8*C*b*x^8 + 1/7*(D*a + B*b)*x^7 + 1/5*B*a*x^5 + 1/6*(C*a + A*b)*x^6 + 1/4*A*a*x^4

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int x^3(a+bx^2)\,(A+Bx+Cx^2+Dx^3)\,dx = \frac{Aax^4}{4}+\frac{Bax^5}{5}+\frac{Cbx^8}{8}+\frac{Dbx^9}{9} + x^7\left(\frac{Bb}{7}+\frac{Da}{7}\right)+x^6\left(\frac{Ab}{6}+\frac{Ca}{6}\right)$$

[In] integrate(x**3*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)

[Out] A*a*x**4/4 + B*a*x**5/5 + C*b*x**8/8 + D*b*x**9/9 + x**7*(B*b/7 + D*a/7) + x**6*(A*b/6 + C*a/6)

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int x^3(a+bx^2)\,(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{9}\,Dbx^9+\frac{1}{8}\,Cbx^8+\frac{1}{7}\,(Da+Bb)x^7 + \frac{1}{5}\,Bax^5+\frac{1}{6}\,(Ca+Ab)x^6+\frac{1}{4}\,Aax^4$$

[In] integrate(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")

[Out] 1/9*D*b*x^9 + 1/8*C*b*x^8 + 1/7*(D*a + B*b)*x^7 + 1/5*B*a*x^5 + 1/6*(C*a + A*b)*x^6 + 1/4*A*a*x^4

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int x^3(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{9}\,Dbx^9+\frac{1}{8}\,Cbx^8+\frac{1}{7}\,Dax^7+\frac{1}{7}\,Bbx^7+\frac{1}{6}\,Cax^6+\frac{1}{6}\,Abx^6+\frac{1}{5}\,Bax^5+\frac{1}{4}\,Aax^4$$

```
[In] integrate(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
[Out] 1/9*D*b*x^9 + 1/8*C*b*x^8 + 1/7*D*a*x^7 + 1/7*B*b*x^7 + 1/6*C*a*x^6 + 1/6*A
*b*x^6 + 1/5*B*a*x^5 + 1/4*A*a*x^4
```

Mupad [B] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int x^3(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{a\,x^7\,D}{7}+\frac{b\,x^9\,D}{9}+\frac{A\,a\,x^4}{4}+\frac{B\,a\,x^5}{5}+\frac{A\,b\,x^6}{6}+\frac{C\,a\,x^6}{6}+\frac{B\,b\,x^7}{7}+\frac{C\,b\,x^8}{8}$$

```
[In] int(x^3*(a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
[Out] (a*x^7*D)/7 + (b*x^9*D)/9 + (A*a*x^4)/4 + (B*a*x^5)/5 + (A*b*x^6)/6 + (C*a*
x^6)/6 + (B*b*x^7)/7 + (C*b*x^8)/8
```

3.63 $\int x^2(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal result 435
Rubi [A] (verified) 435
Mathematica [A] (verified) 436
Maple [A] (verified) 436
Fricas [A] (verification not implemented) 437
Sympy [A] (verification not implemented) 437
Maxima [A] (verification not implemented) 437
Giac [A] (verification not implemented) 438
Mupad [B] (verification not implemented) 438

Optimal result

Integrand size = 26, antiderivative size = 65

$$\int x^2\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{5}(Ab+aC)x^5+\frac{1}{6}(bB+aD)x^6+\frac{1}{7}bCx^7+\frac{1}{8}bDx^8$$

```
[Out] 1/3*a*A*x^3+1/4*a*B*x^4+1/5*(A*b+C*a)*x^5+1/6*(B*b+D*a)*x^6+1/7*b*C*x^7+1/8
*b*D*x^8
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1816}

$$\int x^2\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{6}x^6(aD+bB)+\frac{1}{4}aBx^4+\frac{1}{7}bCx^7+\frac{1}{8}bDx^8$$

```
[In] Int[x^2*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + ((b*B + a*D)*x^6)/6 + (b*
C*x^7)/7 + (b*D*x^8)/8
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(aAx^2 + aBx^3 + (Ab + aC)x^4 + (bB + aD)x^5 + bCx^6 + bDx^7\right) dx$$
$$= \frac{1}{3}aAx^3 + \frac{1}{4}aBx^4 + \frac{1}{5}(Ab + aC)x^5 + \frac{1}{6}(bB + aD)x^6 + \frac{1}{7}bCx^7 + \frac{1}{8}bDx^8$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int x^2(a + bx^2)\left(A + Bx + Cx^2 + Dx^3\right) dx = \frac{1}{3}aAx^3 + \frac{1}{4}aBx^4 + \frac{1}{5}(Ab + aC)x^5 + \frac{1}{6}(bB + aD)x^6 + \frac{1}{7}bCx^7 + \frac{1}{8}bDx^8$$

```
[In] Integrate[x^2*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + ((b*B + a*D)*x^6)/6 + (b*
C*x^7)/7 + (b*D*x^8)/8
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{aA\,x^3}{3}+\frac{Ba\,x^4}{4}+\frac{(Ab+Ca)x^5}{5}+\frac{(Bb+Da)x^6}{6}+\frac{bC\,x^7}{7}+\frac{bDx^8}{8}$	54
norman	$\frac{bDx^8}{8}+\frac{bC\,x^7}{7}+\left(\frac{Bb}{6}+\frac{Da}{6}\right)x^6+\left(\frac{Ab}{5}+\frac{Ca}{5}\right)x^5+\frac{Ba\,x^4}{4}+\frac{aA\,x^3}{3}$	56
gosper	$\frac{1}{8}bDx^8+\frac{1}{7}bC\,x^7+\frac{1}{6}bB\,x^6+\frac{1}{6}x^6Da+\frac{1}{5}x^5Ab+\frac{1}{5}x^5Ca+\frac{1}{4}Ba\,x^4+\frac{1}{3}aA\,x^3$	58
parallelrisch	$\frac{1}{8}bDx^8+\frac{1}{7}bC\,x^7+\frac{1}{6}bB\,x^6+\frac{1}{6}x^6Da+\frac{1}{5}x^5Ab+\frac{1}{5}x^5Ca+\frac{1}{4}Ba\,x^4+\frac{1}{3}aA\,x^3$	58

```
[In] int(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
[Out] 1/3*a*A*x^3+1/4*B*a*x^4+1/5*(A*b+C*a)*x^5+1/6*(B*b+D*a)*x^6+1/7*b*C*x^7+1/8
*b*D*x^8
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int x^2(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{8}\,Dbx^8+\frac{1}{7}\,Cbx^7+\frac{1}{6}\,(Da+Bb)x^6 +\frac{1}{4}\,Bax^4+\frac{1}{5}\,(Ca+Ab)x^5+\frac{1}{3}\,Aax^3$$

```
[In] integrate(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/8*D*b*x^8 + 1/7*C*b*x^7 + 1/6*(D*a + B*b)*x^6 + 1/4*B*a*x^4 + 1/5*(C*a +
A*b)*x^5 + 1/3*A*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int x^2(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{Aax^3}{3}+\frac{Bax^4}{4}+\frac{Cbx^7}{7}+\frac{Dbx^8}{8} +x^6\left(\frac{Bb}{6}+\frac{Da}{6}\right)+x^5\left(\frac{Ab}{5}+\frac{Ca}{5}\right)$$

```
[In] integrate(x**2*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a*x**3/3 + B*a*x**4/4 + C*b*x**7/7 + D*b*x**8/8 + x**6*(B*b/6 + D*a/6) +
x**5*(A*b/5 + C*a/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int x^2(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{8}\,Dbx^8+\frac{1}{7}\,Cbx^7+\frac{1}{6}\,(Da+Bb)x^6 +\frac{1}{4}\,Bax^4+\frac{1}{5}\,(Ca+Ab)x^5+\frac{1}{3}\,Aax^3$$

```
[In] integrate(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/8*D*b*x^8 + 1/7*C*b*x^7 + 1/6*(D*a + B*b)*x^6 + 1/4*B*a*x^4 + 1/5*(C*a +
A*b)*x^5 + 1/3*A*a*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int x^2(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{8}\,Dbx^8+\frac{1}{7}\,Cbx^7+\frac{1}{6}\,Dax^6+\frac{1}{6}\,Bbx^6+\frac{1}{5}\,Cax^5+\frac{1}{5}\,Abx^5+\frac{1}{4}\,Bax^4+\frac{1}{3}\,Aax^3$$

```
[In] integrate(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
[Out] 1/8*D*b*x^8 + 1/7*C*b*x^7 + 1/6*D*a*x^6 + 1/6*B*b*x^6 + 1/5*C*a*x^5 + 1/5*A
*b*x^5 + 1/4*B*a*x^4 + 1/3*A*a*x^3
```

Mupad [B] (verification not implemented)

Time = 6.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int x^2(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{a\,x^6\,D}{6}+\frac{b\,x^8\,D}{8}+\frac{A\,a\,x^3}{3}+\frac{B\,a\,x^4}{4}+\frac{A\,b\,x^5}{5}+\frac{C\,a\,x^5}{5}+\frac{B\,b\,x^6}{6}+\frac{C\,b\,x^7}{7}$$

```
[In] int(x^2*(a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
[Out] (a*x^6*D)/6 + (b*x^8*D)/8 + (A*a*x^3)/3 + (B*a*x^4)/4 + (A*b*x^5)/5 + (C*a*
x^5)/5 + (B*b*x^6)/6 + (C*b*x^7)/7
```

3.64 $\int x(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal result 439
Rubi [A] (verified) 439
Mathematica [A] (verified) 440
Maple [A] (verified) 440
Fricas [A] (verification not implemented) 441
Sympy [A] (verification not implemented) 441
Maxima [A] (verification not implemented) 441
Giac [A] (verification not implemented) 442
Mupad [B] (verification not implemented) 442

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int x(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{4}(Ab+aC)x^4+\frac{1}{5}(bB+aD)x^5+\frac{1}{6}bCx^6+\frac{1}{7}bDx^7$$

```
[Out] 1/2*a*A*x^2+1/3*a*B*x^3+1/4*(A*b+C*a)*x^4+1/5*(B*b+D*a)*x^5+1/6*b*C*x^6+1/7
*b*D*x^7
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1816}

$$\int x(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{5}x^5(aD+bB)+\frac{1}{3}aBx^3+\frac{1}{6}bCx^6+\frac{1}{7}bDx^7$$

```
[In] Int[x*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + ((b*B + a*D)*x^5)/5 + (b*
C*x^6)/6 + (b*D*x^7)/7
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(aAx + aBx^2 + (Ab + aC)x^3 + (bB + aD)x^4 + bCx^5 + bDx^6\right)\,dx$$
$$= \frac{1}{2}aAx^2 + \frac{1}{3}aBx^3 + \frac{1}{4}(Ab + aC)x^4 + \frac{1}{5}(bB + aD)x^5 + \frac{1}{6}bCx^6 + \frac{1}{7}bDx^7$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int x\left(a + bx^2\right)\left(A + Bx + Cx^2 + Dx^3\right)\,dx = \frac{1}{2}aAx^2 + \frac{1}{3}aBx^3 + \frac{1}{4}(Ab + aC)x^4 + \frac{1}{5}(bB + aD)x^5 + \frac{1}{6}bCx^6 + \frac{1}{7}bDx^7$$

```
[In] Integrate[x*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + ((b*B + a*D)*x^5)/5 + (b*
C*x^6)/6 + (b*D*x^7)/7
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{aA\,x^2}{2}+\frac{Ba\,x^3}{3}+\frac{(Ab+Ca)x^4}{4}+\frac{(Bb+Da)x^5}{5}+\frac{bC\,x^6}{6}+\frac{bDx^7}{7}$	54
norman	$\frac{bDx^7}{7}+\frac{bC\,x^6}{6}+\left(\frac{Bb}{5}+\frac{Da}{5}\right)x^5+\left(\frac{Ab}{4}+\frac{Ca}{4}\right)x^4+\frac{Ba\,x^3}{3}+\frac{aA\,x^2}{2}$	56
gosper	$\frac{1}{7}bDx^7+\frac{1}{6}bC\,x^6+\frac{1}{5}bB\,x^5+\frac{1}{5}x^5Da+\frac{1}{4}x^4Ab+\frac{1}{4}x^4Ca+\frac{1}{3}Ba\,x^3+\frac{1}{2}aA\,x^2$	58
parallelrisch	$\frac{1}{7}bDx^7+\frac{1}{6}bC\,x^6+\frac{1}{5}bB\,x^5+\frac{1}{5}x^5Da+\frac{1}{4}x^4Ab+\frac{1}{4}x^4Ca+\frac{1}{3}Ba\,x^3+\frac{1}{2}aA\,x^2$	58

```
[In] int(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
[Out] 1/2*a*A*x^2+1/3*B*a*x^3+1/4*(A*b+C*a)*x^4+1/5*(B*b+D*a)*x^5+1/6*b*C*x^6+1/7
*b*D*x^7
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int x(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{7}\,Dbx^7+\frac{1}{6}\,Cbx^6+\frac{1}{5}\,(Da+Bb)x^5+\frac{1}{3}\,Bax^3+\frac{1}{4}\,(Ca+Ab)x^4+\frac{1}{2}\,Aax^2$$

[In] integrate(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")

[Out] 1/7*D*b*x^7 + 1/6*C*b*x^6 + 1/5*(D*a + B*b)*x^5 + 1/3*B*a*x^3 + 1/4*(C*a + A*b)*x^4 + 1/2*A*a*x^2

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int x(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{Aax^2}{2}+\frac{Bax^3}{3}+\frac{Cbx^6}{6}+\frac{Dbx^7}{7}+x^5\left(\frac{Bb}{5}+\frac{Da}{5}\right)+x^4\left(\frac{Ab}{4}+\frac{Ca}{4}\right)$$

[In] integrate(x*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)

[Out] A*a*x**2/2 + B*a*x**3/3 + C*b*x**6/6 + D*b*x**7/7 + x**5*(B*b/5 + D*a/5) + x**4*(A*b/4 + C*a/4)

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int x(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{7}\,Dbx^7+\frac{1}{6}\,Cbx^6+\frac{1}{5}\,(Da+Bb)x^5+\frac{1}{3}\,Bax^3+\frac{1}{4}\,(Ca+Ab)x^4+\frac{1}{2}\,Aax^2$$

[In] integrate(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")

[Out] 1/7*D*b*x^7 + 1/6*C*b*x^6 + 1/5*(D*a + B*b)*x^5 + 1/3*B*a*x^3 + 1/4*(C*a + A*b)*x^4 + 1/2*A*a*x^2

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int x(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{7}\,Dbx^7+\frac{1}{6}\,Cbx^6+\frac{1}{5}\,Dax^5+\frac{1}{5}\,Bbx^5+\frac{1}{4}\,Cax^4+\frac{1}{4}\,Abx^4+\frac{1}{3}\,Bax^3+\frac{1}{2}\,Aax^2$$

```
[In] integrate(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
[Out] 1/7*D*b*x^7 + 1/6*C*b*x^6 + 1/5*D*a*x^5 + 1/5*B*b*x^5 + 1/4*C*a*x^4 + 1/4*A
*b*x^4 + 1/3*B*a*x^3 + 1/2*A*a*x^2
```

Mupad [B] (verification not implemented)

Time = 6.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int x(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{a\,x^5\,D}{5}+\frac{b\,x^7\,D}{7}+\frac{A\,a\,x^2}{2}+\frac{B\,a\,x^3}{3}+\frac{A\,b\,x^4}{4}+\frac{C\,a\,x^4}{4}+\frac{B\,b\,x^5}{5}+\frac{C\,b\,x^6}{6}$$

```
[In] int(x*(a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
[Out] (a*x^5*D)/5 + (b*x^7*D)/7 + (A*a*x^2)/2 + (B*a*x^3)/3 + (A*b*x^4)/4 + (C*a*
x^4)/4 + (B*b*x^5)/5 + (C*b*x^6)/6
```

3.65 $\int (a+bx^2)(A+Bx+Cx^2+Dx^3)\,dx$

Optimal result . . . 443
Rubi [A] (verified) . . . 443
Mathematica [A] (verified) . . . 444
Maple [A] (verified) . . . 444
Fricas [A] (verification not implemented) . . . 445
Sympy [A] (verification not implemented) . . . 445
Maxima [A] (verification not implemented) . . . 445
Giac [A] (verification not implemented) . . . 446
Mupad [B] (verification not implemented) . . . 446

Optimal result

Integrand size = 23, antiderivative size = 60

$$\int (a+bx^2)(A+Bx+Cx^2+Dx^3)\,dx = aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3 +\frac{1}{4}(bB+aD)x^4+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

```
[Out] a*A*x+1/2*a*B*x^2+1/3*(A*b+C*a)*x^3+1/4*(B*b+D*a)*x^4+1/5*b*C*x^5+1/6*b*D*x
^6
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1824}

$$\int (a+bx^2)(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{4}x^4(aD+bB) +\frac{1}{2}aBx^2+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

```
[In] Int[(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + ((b*B + a*D)*x^4)/4 + (b*C*x^5)
/5 + (b*D*x^6)/6
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(aA + aBx + (Ab + aC)x^2 + (bB + aD)x^3 + bCx^4 + bDx^5\right)\,dx \\
&= aAx + \frac{1}{2}aBx^2 + \frac{1}{3}(Ab + aC)x^3 + \frac{1}{4}(bB + aD)x^4 + \frac{1}{5}bCx^5 + \frac{1}{6}bDx^6
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$
\int \left(a + bx^2\right)\left(A + Bx + Cx^2 + Dx^3\right)\,dx = aAx + \frac{1}{2}aBx^2 + \frac{1}{3}(Ab + aC)x^3 + \frac{1}{4}(bB + aD)x^4 + \frac{1}{5}bCx^5 + \frac{1}{6}bDx^6
$$

```
[In] Integrate[(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + ((b*B + a*D)*x^4)/4 + (b*C*x^5)
/5 + (b*D*x^6)/6
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

method	result	size
default	$aAx + \frac{Ba\,x^2}{2} + \frac{(Ab+Ca)x^3}{3} + \frac{(Bb+Da)x^4}{4} + \frac{bC\,x^5}{5} + \frac{bDx^6}{6}$	51
norman	$\frac{bDx^6}{6} + \frac{bC\,x^5}{5} + \left(\frac{Bb}{4} + \frac{Da}{4}\right)x^4 + \left(\frac{Ab}{3} + \frac{Ca}{3}\right)x^3 + \frac{Ba\,x^2}{2} + aAx$	53
gosper	$\frac{1}{6}bDx^6 + \frac{1}{5}bC\,x^5 + \frac{1}{4}bB\,x^4 + \frac{1}{4}x^4Da + \frac{1}{3}Ab\,x^3 + \frac{1}{3}x^3Ca + \frac{1}{2}Ba\,x^2 + aAx$	55
parallelrisch	$\frac{1}{6}bDx^6 + \frac{1}{5}bC\,x^5 + \frac{1}{4}bB\,x^4 + \frac{1}{4}x^4Da + \frac{1}{3}Ab\,x^3 + \frac{1}{3}x^3Ca + \frac{1}{2}Ba\,x^2 + aAx$	55

```
[In] int((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
[Out] a*A*x+1/2*B*a*x^2+1/3*(A*b+C*a)*x^3+1/4*(B*b+D*a)*x^4+1/5*b*C*x^5+1/6*b*D*x
^6
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{6}\,Dbx^6+\frac{1}{5}\,Cbx^5+\frac{1}{4}\,(Da+Bb)x^4+\frac{1}{2}\,Bax^2+\frac{1}{3}\,(Ca+Ab)x^3+Aax$$

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
[Out] 1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*(D*a + B*b)*x^4 + 1/2*B*a*x^2 + 1/3*(C*a +
A*b)*x^3 + A*a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx = Aax+\frac{Bax^2}{2}+\frac{Cbx^5}{5}+\frac{Dbx^6}{6}+x^4\left(\frac{Bb}{4}+\frac{Da}{4}\right)+x^3\left(\frac{Ab}{3}+\frac{Ca}{3}\right)$$

```
[In] integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
[Out] A*a*x + B*a*x**2/2 + C*b*x**5/5 + D*b*x**6/6 + x**4*(B*b/4 + D*a/4) + x**3*
(A*b/3 + C*a/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{6}\,Dbx^6+\frac{1}{5}\,Cbx^5+\frac{1}{4}\,(Da+Bb)x^4+\frac{1}{2}\,Bax^2+\frac{1}{3}\,(Ca+Ab)x^3+Aax$$

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
[Out] 1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*(D*a + B*b)*x^4 + 1/2*B*a*x^2 + 1/3*(C*a +
A*b)*x^3 + A*a*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int (a+bx^2) (A+Bx+Cx^2+Dx^3) \, dx = \frac{1}{6} Dbx^6 + \frac{1}{5} Cbx^5 + \frac{1}{4} Dax^4 + \frac{1}{4} Bbx^4 + \frac{1}{3} Cax^3 + \frac{1}{3} Abx^3 + \frac{1}{2} Bax^2 + Aax$$

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
[Out] 1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*D*a*x^4 + 1/4*B*b*x^4 + 1/3*C*a*x^3 + 1/3*A
*b*x^3 + 1/2*B*a*x^2 + A*a*x
```

Mupad [B] (verification not implemented)

Time = 5.88 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int (a+bx^2) (A+Bx+Cx^2+Dx^3) \, dx = \frac{a\,x^4\,D}{4} + \frac{b\,x^6\,D}{6} + A\,a\,x + \frac{B\,a\,x^2}{2} + \frac{A\,b\,x^3}{3} + \frac{C\,a\,x^3}{3} + \frac{B\,b\,x^4}{4} + \frac{C\,b\,x^5}{5}$$

```
[In] int((a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
[Out] (a*x^4*D)/4 + (b*x^6*D)/6 + A*a*x + (B*a*x^2)/2 + (A*b*x^3)/3 + (C*a*x^3)/3
 + (B*b*x^4)/4 + (C*b*x^5)/5
```

3.66 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x} \, dx$

Optimal result 447
Rubi [A] (verified) 447
Mathematica [A] (verified) 448
Maple [A] (verified) 448
Fricas [A] (verification not implemented) 449
Sympy [A] (verification not implemented) 449
Maxima [A] (verification not implemented) 449
Giac [A] (verification not implemented) 450
Mupad [B] (verification not implemented) 450

Optimal result

Integrand size = 26, antiderivative size = 56

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x} \, dx = aBx + \frac{1}{2}(Ab+aC)x^2 + \frac{1}{3}(bB+aD)x^3 + \frac{1}{4}bCx^4 + \frac{1}{5}bDx^5 + aA\log(x)$$

```
[Out] a*B*x+1/2*(A*b+C*a)*x^2+1/3*(B*b+D*a)*x^3+1/4*b*C*x^4+1/5*b*D*x^5+a*A*ln(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1816}

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x} \, dx = \frac{1}{2}x^2(aC+Ab) + aA\log(x) + \frac{1}{3}x^3(aD+bB) + aBx + \frac{1}{4}bCx^4 + \frac{1}{5}bDx^5$$

```
[In] Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + ((b*B + a*D)*x^3)/3 + (b*C*x^4)/4 + (b*D*x^5)
/5 + a*A*Log[x]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(aB+\frac{aA}{x}+(Ab+aC)x+(bB+aD)x^2+bCx^3+bDx^4\right)dx\\
&= aBx+\frac{1}{2}(Ab+aC)x^2+\frac{1}{3}(bB+aD)x^3+\frac{1}{4}bCx^4+\frac{1}{5}bDx^5+aA\log(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$
\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x}\,dx = aBx+\frac{1}{2}(Ab+aC)x^2+\frac{1}{3}(bB+aD)x^3 +\frac{1}{4}bCx^4+\frac{1}{5}bDx^5+aA\log(x)
$$

```
[In] Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + ((b*B + a*D)*x^3)/3 + (b*C*x^4)/4 + (b*D*x^5)
/5 + a*A*Log[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

method	result	size
norman	$\left(\frac{Ab}{2}+\frac{Ca}{2}\right)x^2+\left(\frac{Bb}{3}+\frac{Da}{3}\right)x^3+Bax+\frac{bC\,x^4}{4}+\frac{bDx^5}{5}+aA\ln(x)$	51
default	$\frac{bDx^5}{5}+\frac{bC\,x^4}{4}+\frac{bB\,x^3}{3}+\frac{Da\,x^3}{3}+\frac{Ab\,x^2}{2}+\frac{Ca\,x^2}{2}+Bax+aA\ln(x)$	53
parallelrisch	$\frac{bDx^5}{5}+\frac{bC\,x^4}{4}+\frac{bB\,x^3}{3}+\frac{Da\,x^3}{3}+\frac{Ab\,x^2}{2}+\frac{Ca\,x^2}{2}+Bax+aA\ln(x)$	53

```
[In] int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*A*b+1/2*C*a)*x^2+(1/3*B*b+1/3*D*a)*x^3+B*a*x+1/4*b*C*x^4+1/5*b*D*x^5+a
*A*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{1}{5}\,Dbx^5 + \frac{1}{4}\,Cbx^4 + \frac{1}{3}\,(Da+Bb)x^3 + Bax + \frac{1}{2}\,(Ca+Ab)x^2 + Aa\log(x)$$

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="fricas")
[Out] 1/5*D*b*x^5 + 1/4*C*b*x^4 + 1/3*(D*a + B*b)*x^3 + B*a*x + 1/2*(C*a + A*b)*x
^2 + A*a*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x}\,dx = Aa\log(x) + Bax + \frac{Cbx^4}{4} + \frac{Dbx^5}{5} + x^3\left(\frac{Bb}{3}+\frac{Da}{3}\right) + x^2\left(\frac{Ab}{2}+\frac{Ca}{2}\right)$$

```
[In] integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x,x)
[Out] A*a*log(x) + B*a*x + C*b*x**4/4 + D*b*x**5/5 + x**3*(B*b/3 + D*a/3) + x**2*
(A*b/2 + C*a/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{1}{5}\,Dbx^5 + \frac{1}{4}\,Cbx^4 + \frac{1}{3}\,(Da+Bb)x^3 + Bax + \frac{1}{2}\,(Ca+Ab)x^2 + Aa\log(x)$$

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="maxima")
[Out] 1/5*D*b*x^5 + 1/4*C*b*x^4 + 1/3*(D*a + B*b)*x^3 + B*a*x + 1/2*(C*a + A*b)*x
^2 + A*a*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{1}{5}Dbx^5 + \frac{1}{4}Cbx^4 + \frac{1}{3}Dax^3 + \frac{1}{3}Bbx^3 + \frac{1}{2}Cax^2 + \frac{1}{2}Abx^2 + Bax + Aa\log(|x|)$$

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="giac")
```

```
[Out] 1/5*D*b*x^5 + 1/4*C*b*x^4 + 1/3*D*a*x^3 + 1/3*B*b*x^3 + 1/2*C*a*x^2 + 1/2*A
*b*x^2 + B*a*x + A*a*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 5.95 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{a\,x^3\,D}{3} + \frac{b\,x^5\,D}{5} + B\,a\,x + \frac{A\,b\,x^2}{2} + \frac{C\,a\,x^2}{2} + \frac{B\,b\,x^3}{3} + \frac{C\,b\,x^4}{4} + A\,a\,\ln(x)$$

```
[In] int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/x,x)
```

```
[Out] (a*x^3*D)/3 + (b*x^5*D)/5 + B*a*x + (A*b*x^2)/2 + (C*a*x^2)/2 + (B*b*x^3)/3
 + (C*b*x^4)/4 + A*a*log(x)
```

3.67 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2} dx$

Optimal result . . . 451
Rubi [A] (verified) . . . 451
Mathematica [A] (verified) . . . 452
Maple [A] (verified) . . . 452
Fricas [A] (verification not implemented) . . . 453
Sympy [A] (verification not implemented) . . . 453
Maxima [A] (verification not implemented) . . . 453
Giac [A] (verification not implemented) . . . 454
Mupad [B] (verification not implemented) . . . 454

Optimal result

Integrand size = 26, antiderivative size = 54

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2} dx = -\frac{aA}{x}+(Ab+aC)x+\frac{1}{2}(bB+aD)x^2+\frac{1}{3}bCx^3+\frac{1}{4}bDx^4+aB\log(x)$$

```
[Out] -a*A/x+(A*b+C*a)*x+1/2*(B*b+D*a)*x^2+1/3*b*C*x^3+1/4*b*D*x^4+a*B*ln(x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = $\{1816\}$

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2} dx = x(aC+Ab)-\frac{aA}{x}+\frac{1}{2}x^2(aD+bB)+aB\log(x)+\frac{1}{3}bCx^3+\frac{1}{4}bDx^4$$

```
[In] Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*C)*x + ((b*B + a*D)*x^2)/2 + (b*C*x^3)/3 + (b*D*x^4)/
4 + a*B*Log[x]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(Ab\left(1+\frac{aC}{Ab}\right)+\frac{aA}{x^2}+\frac{aB}{x}+(bB+aD)x+bCx^2+bDx^3\right)\,dx$$
$$= -\frac{aA}{x}+(Ab+aC)x+\frac{1}{2}(bB+aD)x^2+\frac{1}{3}bCx^3+\frac{1}{4}bDx^4+aB\log(x)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = -\frac{aA}{x}+(Ab+aC)x+\frac{1}{2}(bB+aD)x^2 + \frac{1}{3}bCx^3+\frac{1}{4}bDx^4+aB\log(x)$$

```
[In] Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^2,x]
[Out] -((a*A)/x) + (A*b + a*C)*x + ((b*B + a*D)*x^2)/2 + (b*C*x^3)/3 + (b*D*x^4)/
4 + a*B*Log[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{Db\,x^4}{4}+\frac{bC\,x^3}{3}+\frac{bB\,x^2}{2}+\frac{Da\,x^2}{2}+Abx+Cax+aB\ln\left(x\right)-\frac{aA}{x}$	50
norman	$\frac{\left(\frac{Bb}{2}+\frac{Da}{2}\right)x^3+(Ab+Ca)x^2-Aa+\frac{bC\,x^4}{3}+\frac{bD x^5}{4}}{x}+aB\ln\left(x\right)$	54
parallelrisch	$\frac{3bD x^5+4bC\,x^4+6bB\,x^3+6Da\,x^3+12Ab\,x^2+12Ba\ln(x)x+12Ca\,x^2-12Aa}{12x}$	60

```
[In] int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/4*D*b*x^4+1/3*b*C*x^3+1/2*b*B*x^2+1/2*D*a*x^2+A*b*x+C*a*x+a*B*ln(x)-a*A/x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{3\,Dbx^5+4\,Cbx^4+6\,(Da+Bb)x^3+12\,Bax\log(x)+12\,(Ca+Ab)x^2-12\,Aa}{12\,x}$$

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*D*b*x^5 + 4*C*b*x^4 + 6*(D*a + B*b)*x^3 + 12*B*a*x*log(x) + 12*(C*a
 + A*b)*x^2 - 12*A*a)/x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = -\frac{Aa}{x}+Ba\log(x)+\frac{Cbx^3}{3}+\frac{Dbx^4}{4}+x^2\left(\frac{Bb}{2}+\frac{Da}{2}\right)+x(Ab+Ca)$$

```
[In] integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

```
[Out] -A*a/x + B*a*log(x) + C*b*x**3/3 + D*b*x**4/4 + x**2*(B*b/2 + D*a/2) + x*(A
*b + C*a)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{1}{4}\,Dbx^4+\frac{1}{3}\,Cbx^3+\frac{1}{2}\,(Da+Bb)x^2+Ba\log(x)+(Ca+Ab)x-\frac{Aa}{x}$$

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*D*b*x^4 + 1/3*C*b*x^3 + 1/2*(D*a + B*b)*x^2 + B*a*log(x) + (C*a + A*b)*
x - A*a/x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{1}{4}Dbx^4 + \frac{1}{3}Cbx^3 + \frac{1}{2}Dax^2 + \frac{1}{2}Bbx^2 + Cax + Abx + Ba\log(|x|) - \frac{Aa}{x}$$

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*D*b*x^4 + 1/3*C*b*x^3 + 1/2*D*a*x^2 + 1/2*B*b*x^2 + C*a*x + A*b*x + B*a
*log(abs(x)) - A*a/x
```

Mupad [B] (verification not implemented)

Time = 5.96 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{a\,x^2\,D}{2} + \frac{b\,x^4\,D}{4} + A\,b\,x + C\,a\,x - \frac{A\,a}{x} + \frac{B\,b\,x^2}{2} + \frac{C\,b\,x^3}{3} + B\,a\,\ln(x)$$

```
[In] int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/x^2,x)
```

```
[Out] (a*x^2*D)/2 + (b*x^4*D)/4 + A*b*x + C*a*x - (A*a)/x + (B*b*x^2)/2 + (C*b*x^
3)/3 + B*a*log(x)
```

3.68 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3} dx$

Optimal result 455
Rubi [A] (verified) 455
Mathematica [A] (verified) 456
Maple [A] (verified) 456
Fricas [A] (verification not implemented) 457
Sympy [A] (verification not implemented) 457
Maxima [A] (verification not implemented) 457
Giac [A] (verification not implemented) 458
Mupad [B] (verification not implemented) 458

Optimal result

Integrand size = 26, antiderivative size = 54

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3} dx = -\frac{aA}{2x^2} - \frac{aB}{x} + (bB+aD)x + \frac{1}{2}bCx^2 + \frac{1}{3}bDx^3 + (Ab+aC)\log(x)$$

```
[Out] -1/2*a*A/x^2-a*B/x+(B*b+D*a)*x+1/2*b*C*x^2+1/3*b*D*x^3+(A*b+C*a)*ln(x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1816}

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3} dx = \log(x)(aC+Ab) - \frac{aA}{2x^2} + x(aD+bB) - \frac{aB}{x} + \frac{1}{2}bCx^2 + \frac{1}{3}bDx^3$$

```
[In] Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -1/2*(a*A)/x^2 - (a*B)/x + (b*B + a*D)*x + (b*C*x^2)/2 + (b*D*x^3)/3 + (A*b
 + a*C)*Log[x]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(bB\left(1+\frac{aD}{bB}\right)+\frac{aA}{x^3}+\frac{aB}{x^2}+\frac{Ab+aC}{x}+bCx+bDx^2\right)dx$$
$$= -\frac{aA}{2x^2}-\frac{aB}{x}+(bB+aD)x+\frac{1}{2}bCx^2+\frac{1}{3}bDx^3+(Ab+aC)\log(x)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}dx = \frac{1}{6}bx(6B+3Cx+2Dx^2) - \frac{a(A+2Bx-2Dx^3)}{2x^2}+(Ab+aC)\log(x)$$

```
[In] Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^3,x]
[Out] (b*x*(6*B + 3*C*x + 2*D*x^2))/6 - (a*(A + 2*B*x - 2*D*x^3))/(2*x^2) + (A*b
+ a*C)*Log[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{Db\,x^3}{3}+\frac{bC\,x^2}{2}+bBx+Dax+(Ab+Ca)\ln(x)-\frac{aB}{x}-\frac{aA}{2x^2}$	48
norman	$\frac{(Bb+Da)x^3-\frac{Aa}{2}-Bax+\frac{bC\,x^4}{2}+\frac{bDx^5}{3}}{x^2}+(Ab+Ca)\ln(x)$	51
parallelrisch	$\frac{2bDx^5+3bC\,x^4+6A\ln(x)x^2b+6bB\,x^3+6C\ln(x)x^2a+6Da\,x^3-6Bax-3Aa}{6x^2}$	62

```
[In] int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x,method=_RETURNVERBOSE)
[Out] 1/3*D*b*x^3+1/2*b*C*x^2+b*B*x+D*a*x+(A*b+C*a)*ln(x)-a*B/x-1/2*a*A/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{2\,Dbx^5+3\,Cbx^4+6\,(Da+Bb)x^3+6\,(Ca+Ab)x^2\log{(x)}-6\,Bax-3\,Aa}{6\,x^2}$$

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*D*b*x^5 + 3*C*b*x^4 + 6*(D*a + B*b)*x^3 + 6*(C*a + A*b)*x^2*log(x) -
 6*B*a*x - 3*A*a)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{Cbx^2}{2}+\frac{Dbx^3}{3}+x(Bb+Da)+(Ab+Ca)\log{(x)}+\frac{-Aa-2Bax}{2x^2}$$

```
[In] integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

```
[Out] C*b*x**2/2 + D*b*x**3/3 + x*(B*b + D*a) + (A*b + C*a)*log(x) + (-A*a - 2*B*
a*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{1}{3}\,Dbx^3+\frac{1}{2}\,Cbx^2+(Da+Bb)x+(Ca+Ab)\log{(x)}-\frac{2\,Bax+Aa}{2\,x^2}$$

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*D*b*x^3 + 1/2*C*b*x^2 + (D*a + B*b)*x + (C*a + A*b)*log(x) - 1/2*(2*B*a
*x + A*a)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{1}{3}\,Dbx^3 + \frac{1}{2}\,Cbx^2 + Dax + Bbx + (Ca+Ab)\log\left(|x|\right) - \frac{2\,Bax + Aa}{2\,x^2}$$

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/3*D*b*x^3 + 1/2*C*b*x^2 + D*a*x + B*b*x + (C*a + A*b)*log(abs(x)) - 1/2*(
2*B*a*x + A*a)/x^2
```

Mupad [B] (verification not implemented)

Time = 6.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{b\,x^3\,D}{3} + B\,b\,x - \frac{A\,a}{2\,x^2} - \frac{B\,a}{x} + \frac{C\,b\,x^2}{2} + A\,b\,\ln\left(x\right) + C\,a\,\ln\left(x\right) + a\,x\,D$$

```
[In] int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/x^3,x)
```

```
[Out] (b*x^3*D)/3 + B*b*x - (A*a)/(2*x^2) - (B*a)/x + (C*b*x^2)/2 + A*b*log(x) +
C*a*log(x) + a*x*D
```

3.69 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$

Optimal result 459
Rubi [A] (verified) 459
Mathematica [A] (verified) 460
Maple [A] (verified) 460
Fricas [A] (verification not implemented) 461
Sympy [A] (verification not implemented) 461
Maxima [A] (verification not implemented) 461
Giac [A] (verification not implemented) 462
Mupad [B] (verification not implemented) 462

Optimal result

Integrand size = 26, antiderivative size = 54

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = -\frac{aA}{3x^3}-\frac{aB}{2x^2}-\frac{Ab+aC}{x}+bCx+\frac{1}{2}bDx^2+(bB+aD)\log(x)$$

```
[Out] -1/3*a*A/x^3-1/2*a*B/x^2+(-A*b-C*a)/x+b*C*x+1/2*b*D*x^2+(B*b+D*a)*ln(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1816}

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = -\frac{aC+Ab}{x}-\frac{aA}{3x^3}+\log(x)(aD+bB)-\frac{aB}{2x^2}+bCx+\frac{1}{2}bDx^2$$

```
[In] Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -1/3*(a*A)/x^3 - (a*B)/(2*x^2) - (A*b + a*C)/x + b*C*x + (b*D*x^2)/2 + (b*B
 + a*D)*Log[x]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(bC+\frac{aA}{x^4}+\frac{aB}{x^3}+\frac{Ab+aC}{x^2}+\frac{bB+aD}{x}+bDx\right)\,dx\\
&= -\frac{aA}{3x^3}-\frac{aB}{2x^2}-\frac{Ab+aC}{x}+bCx+\frac{1}{2}bDx^2+(bB+aD)\log(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$
\int \frac{(a+bx^2)\,(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = -\frac{aA}{3x^3}-\frac{aB}{2x^2}+\frac{-Ab-aC}{x}+bCx+\frac{1}{2}bDx^2+(bB+aD)\log(x)
$$

```
[In] Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -1/3*(a*A)/x^3 - (a*B)/(2*x^2) + (-(A*b) - a*C)/x + b*C*x + (b*D*x^2)/2 + (
b*B + a*D)*Log[x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{Db\,x^2}{2}+bCx+(Bb+Da)\ln(x)-\frac{aA}{3x^3}-\frac{Ab+Ca}{x}-\frac{aB}{2x^2}$	49
norman	$\frac{(-Ab-Ca)x^2+bC\,x^4-\frac{Aa}{3}-\frac{Bax}{2}+\frac{bDx^5}{2}}{x^3}+(Bb+Da)\ln(x)$	52
parallelrisch	$-\frac{-3bDx^5-6B\ln(x)x^3b-6bC\,x^4-6D\ln(x)x^3a+6Ab\,x^2+6Ca\,x^2+3Bax+2Aa}{6x^3}$	62

```
[In] int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*D*b*x^2+b*C*x+(B*b+D*a)*ln(x)-1/3*a*A/x^3-(A*b+C*a)/x-1/2*a*B/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{3\,Dbx^5+6\,Cbx^4+6\,(Da+Bb)x^3\log(x)-3\,Bax-6\,(Ca+Ab)x^2-2\,Aa}{6\,x^3}$$

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="fricas")
[Out] 1/6*(3*D*b*x^5 + 6*C*b*x^4 + 6*(D*a + B*b)*x^3*log(x) - 3*B*a*x - 6*(C*a +
A*b)*x^2 - 2*A*a)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = Cbx+\frac{Dbx^2}{2}+(Bb+Da)\log(x)+\frac{-2Aa-3Bax+x^2(-6Ab-6Ca)}{6x^3}$$

```
[In] integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**4,x)
[Out] C*b*x + D*b*x**2/2 + (B*b + D*a)*log(x) + (-2*A*a - 3*B*a*x + x**2*(-6*A*b
- 6*C*a))/(6*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{1}{2}\,Dbx^2+Cbx+(Da+Bb)\log(x)-\frac{3\,Bax+6\,(Ca+Ab)x^2+2\,Aa}{6\,x^3}$$

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="maxima")
[Out] 1/2*D*b*x^2 + C*b*x + (D*a + B*b)*log(x) - 1/6*(3*B*a*x + 6*(C*a + A*b)*x^2
 + 2*A*a)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{1}{2}\,Dbx^2 + Cbx + (Da+Bb)\log(|x|) - \frac{3\,Bax + 6\,(Ca+Ab)x^2 + 2\,Aa}{6\,x^3}$$

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*D*b*x^2 + C*b*x + (D*a + B*b)*log(abs(x)) - 1/6*(3*B*a*x + 6*(C*a + A*b
)*x^2 + 2*A*a)/x^3
```

Mupad [B] (verification not implemented)

Time = 6.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{b\,x^2\,D}{2} + a\,\ln(x)\,D + C\,b\,x - \frac{A\,a}{3\,x^3} - \frac{A\,b}{x} - \frac{B\,a}{2\,x^2} - \frac{C\,a}{x} + B\,b\,\ln(x)$$

```
[In] int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/x^4,x)
```

```
[Out] (b*x^2*D)/2 + a*log(x)*D + C*b*x - (A*a)/(3*x^3) - (A*b)/x - (B*a)/(2*x^2)
- (C*a)/x + B*b*log(x)
```

3.70 $\int x^3(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)dx$

Optimal result 463
Rubi [A] (verified) 463
Mathematica [A] (verified) 464
Maple [A] (verified) 464
Fricas [A] (verification not implemented) 465
Sympy [A] (verification not implemented) 465
Maxima [A] (verification not implemented) 466
Giac [A] (verification not implemented) 466
Mupad [B] (verification not implemented) 467

Optimal result

Integrand size = 28, antiderivative size = 109

$$\int x^3\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{4}a^2Ax^4+\frac{1}{5}a^2Bx^5+\frac{1}{6}a(2Ab+aC)x^6+\frac{1}{7}a(2bB+aD)x^7+\frac{1}{8}b(Ab+2aC)x^8+\frac{1}{9}b(bB+2aD)x^9+\frac{1}{10}b^2Cx^{10}+\frac{1}{11}b^2Dx^{11}$$

```
[Out] 1/4*a^2*A*x^4+1/5*a^2*B*x^5+1/6*a*(2*A*b+C*a)*x^6+1/7*a*(2*B*b+D*a)*x^7+1/8
*b*(A*b+2*C*a)*x^8+1/9*b*(B*b+2*D*a)*x^9+1/10*b^2*C*x^10+1/11*b^2*D*x^11
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1816}

$$\int x^3\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{4}a^2Ax^4+\frac{1}{5}a^2Bx^5+\frac{1}{8}bx^8(2aC+Ab)+\frac{1}{6}ax^6(aC+2Ab)+\frac{1}{9}bx^9(2aD+bB)+\frac{1}{7}ax^7(aD+2bB)+\frac{1}{10}b^2Cx^{10}+\frac{1}{11}b^2Dx^{11}$$

```
[In] Int[x^3*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^2*A*x^4)/4 + (a^2*B*x^5)/5 + (a*(2*A*b + a*C)*x^6)/6 + (a*(2*b*B + a*D)*
x^7)/7 + (b*(A*b + 2*a*C)*x^8)/8 + (b*(b*B + 2*a*D)*x^9)/9 + (b^2*C*x^10)/1
0 + (b^2*D*x^11)/11
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^2Ax^3 + a^2Bx^4 + a(2Ab + aC)x^5 + a(2bB + aD)x^6 + b(Ab + 2aC)x^7 \right. \\ &\quad \left. + b(bB + 2aD)x^8 + b^2Cx^9 + b^2Dx^{10}\right) dx \\ &= \frac{1}{4}a^2Ax^4 + \frac{1}{5}a^2Bx^5 + \frac{1}{6}a(2Ab + aC)x^6 + \frac{1}{7}a(2bB + aD)x^7 \\ &\quad + \frac{1}{8}b(Ab + 2aC)x^8 + \frac{1}{9}b(bB + 2aD)x^9 + \frac{1}{10}b^2Cx^{10} + \frac{1}{11}b^2Dx^{11}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.90

$$\begin{aligned}\int x^3(a + bx^2)^2 (A + Bx + Cx^2 + Dx^3) \, dx &= a^2\left(\frac{Ax^4}{4} + \frac{Bx^5}{5} + \frac{1}{42}x^6(7C + 6Dx)\right) \\ &\quad + \frac{b^2x^8(495A + 4x(110B + 99Cx + 90Dx^2))}{3960} \\ &\quad + \frac{1}{252}abx^6(84A + x(72B + 7x(9C + 8Dx)))\end{aligned}$$

```
[In] Integrate[x^3*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
[Out] a^2*((A*x^4)/4 + (B*x^5)/5 + (x^6*(7*C + 6*D*x))/42) + (b^2*x^8*(495*A + 4*
x*(110*B + 99*C*x + 90*D*x^2)))/3960 + (a*b*x^6*(84*A + x*(72*B + 7*x*(9*C
+ 8*D*x))))/252
```

Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

method	result
default	$\frac{b^2Dx^{11}}{11} + \frac{b^2Cx^{10}}{10} + \frac{(Bb^2+2Dab)x^9}{9} + \frac{(b^2A+2Cab)x^8}{8} + \frac{(2abB+Da^2)x^7}{7} + \frac{(2abA+Ca^2)x^6}{6} + \frac{a^2Bx^5}{5} + \frac{a^2Ax^4}{4}$
norman	$\frac{b^2Dx^{11}}{11} + \frac{b^2Cx^{10}}{10} + \left(\frac{1}{9}Bb^2 + \frac{2}{9}Dab\right)x^9 + \left(\frac{1}{8}b^2A + \frac{1}{4}Cab\right)x^8 + \left(\frac{2}{7}abB + \frac{1}{7}Da^2\right)x^7 + \left(\frac{1}{3}abA + \right.$
gosper	$\frac{1}{11}b^2Dx^{11} + \frac{1}{10}b^2Cx^{10} + \frac{1}{9}b^2Bx^9 + \frac{2}{9}x^9Dab + \frac{1}{8}x^8b^2A + \frac{1}{4}x^8Cab + \frac{2}{7}x^7abB + \frac{1}{7}x^7Da^2 + \frac{1}{3}x^6$
parallelrisch	$\frac{1}{11}b^2Dx^{11} + \frac{1}{10}b^2Cx^{10} + \frac{1}{9}b^2Bx^9 + \frac{2}{9}x^9Dab + \frac{1}{8}x^8b^2A + \frac{1}{4}x^8Cab + \frac{2}{7}x^7abB + \frac{1}{7}x^7Da^2 + \frac{1}{3}x^6$

```
[In] int(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b^2*D*x^11+1/10*b^2*C*x^10+1/9*(B*b^2+2*D*a*b)*x^9+1/8*(A*b^2+2*C*a*b)
*x^8+1/7*(2*B*a*b+D*a^2)*x^7+1/6*(2*A*a*b+C*a^2)*x^6+1/5*a^2*B*x^5+1/4*a^2*
A*x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int x^3(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{11}\,Db^2x^{11}+\frac{1}{10}\,Cb^2x^{10}+\frac{1}{9}\,(2\,Dab+Bb^2)x^9+\frac{1}{8}\,(2\,Cab+Ab^2)x^8+\frac{1}{5}\,Ba^2x^5+\frac{1}{7}\,(Da^2+2\,Bab)x^7+\frac{1}{4}\,Aa^2x^4+\frac{1}{6}\,(Ca^2+2\,Aab)x^6$$

```
[In] integrate(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/11*D*b^2*x^11 + 1/10*C*b^2*x^10 + 1/9*(2*D*a*b + B*b^2)*x^9 + 1/8*(2*C*a*
b + A*b^2)*x^8 + 1/5*B*a^2*x^5 + 1/7*(D*a^2 + 2*B*a*b)*x^7 + 1/4*A*a^2*x^4
+ 1/6*(C*a^2 + 2*A*a*b)*x^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01

$$\int x^3(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{Aa^2x^4}{4}+\frac{Ba^2x^5}{5}+\frac{Cb^2x^{10}}{10}+\frac{Db^2x^{11}}{11}+x^9\left(\frac{Bb^2}{9}+\frac{2Dab}{9}\right)+x^8\left(\frac{Ab^2}{8}+\frac{Cab}{4}\right)+x^7\cdot\left(\frac{2Bab}{7}+\frac{Da^2}{7}\right)+x^6\left(\frac{Aab}{3}+\frac{Ca^2}{6}\right)$$

```
[In] integrate(x**3*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x**4/4 + B*a**2*x**5/5 + C*b**2*x**10/10 + D*b**2*x**11/11 + x**9*(B
*b**2/9 + 2*D*a*b/9) + x**8*(A*b**2/8 + C*a*b/4) + x**7*(2*B*a*b/7 + D*a**2
/7) + x**6*(A*a*b/3 + C*a**2/6)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int x^3(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{11}\,Db^2x^{11}+\frac{1}{10}\,Cb^2x^{10}+\frac{1}{9}\,(2\,Dab+Bb^2)x^9+\frac{1}{8}\,(2\,Cab+Ab^2)x^8+\frac{1}{5}\,Ba^2x^5+\frac{1}{7}\,(Da^2+2\,Bab)x^7+\frac{1}{4}\,Aa^2x^4+\frac{1}{6}\,(Ca^2+2\,Aab)x^6$$

```
[In] integrate(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/11*D*b^2*x^11 + 1/10*C*b^2*x^10 + 1/9*(2*D*a*b + B*b^2)*x^9 + 1/8*(2*C*a*
b + A*b^2)*x^8 + 1/5*B*a^2*x^5 + 1/7*(D*a^2 + 2*B*a*b)*x^7 + 1/4*A*a^2*x^4
+ 1/6*(C*a^2 + 2*A*a*b)*x^6
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96

$$\int x^3(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{11}\,Db^2x^{11}+\frac{1}{10}\,Cb^2x^{10}+\frac{2}{9}\,Dabx^9+\frac{1}{9}\,Bb^2x^9+\frac{1}{4}\,Cabx^8+\frac{1}{8}\,Ab^2x^8+\frac{1}{7}\,Da^2x^7+\frac{2}{7}\,Babx^7+\frac{1}{6}\,Ca^2x^6+\frac{1}{3}\,Aabx^6+\frac{1}{5}\,Ba^2x^5+\frac{1}{4}\,Aa^2x^4$$

```
[In] integrate(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/11*D*b^2*x^11 + 1/10*C*b^2*x^10 + 2/9*D*a*b*x^9 + 1/9*B*b^2*x^9 + 1/4*C*a
*b*x^8 + 1/8*A*b^2*x^8 + 1/7*D*a^2*x^7 + 2/7*B*a*b*x^7 + 1/6*C*a^2*x^6 + 1/
3*A*a*b*x^6 + 1/5*B*a^2*x^5 + 1/4*A*a^2*x^4
```

Mupad [B] (verification not implemented)

Time = 5.80 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

$$\int x^3\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{a^2\,x^7\,D}{7}+\frac{b^2\,x^{11}\,D}{11}+\frac{A\,x^4\,\left(6\,a^2+8\,a\,b\,x^2+3\,b^2\,x^4\right)}{24}+\frac{B\,x^5\,\left(63\,a^2+90\,a\,b\,x^2+35\,b^2\,x^4\right)}{315}+\frac{C\,x^6\,\left(10\,a^2+15\,a\,b\,x^2+6\,b^2\,x^4\right)}{60}+\frac{2\,a\,b\,x^9\,D}{9}$$

```
[In] int(x^3*(a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (a^2*x^7*D)/7 + (b^2*x^11*D)/11 + (A*x^4*(6*a^2 + 3*b^2*x^4 + 8*a*b*x^2))/2
4 + (B*x^5*(63*a^2 + 35*b^2*x^4 + 90*a*b*x^2))/315 + (C*x^6*(10*a^2 + 6*b^2
*x^4 + 15*a*b*x^2))/60 + (2*a*b*x^9*D)/9
```

3.71 $\int x^2(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal result 468
Rubi [A] (verified) 468
Mathematica [A] (verified) 469
Maple [A] (verified) 469
Fricas [A] (verification not implemented) 470
Sympy [A] (verification not implemented) 470
Maxima [A] (verification not implemented) 471
Giac [A] (verification not implemented) 471
Mupad [B] (verification not implemented) 472

Optimal result

Integrand size = 28, antiderivative size = 109

$$\int x^2\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{5}a(2Ab+aC)x^5+\frac{1}{6}a(2bB+aD)x^6+\frac{1}{7}b(Ab+2aC)x^7+\frac{1}{8}b(bB+2aD)x^8+\frac{1}{9}b^2Cx^9+\frac{1}{10}b^2Dx^{10}$$

```
[Out] 1/3*a^2*A*x^3+1/4*a^2*B*x^4+1/5*a*(2*A*b+C*a)*x^5+1/6*a*(2*B*b+D*a)*x^6+1/7
*b*(A*b+2*C*a)*x^7+1/8*b*(B*b+2*D*a)*x^8+1/9*b^2*C*x^9+1/10*b^2*D*x^10
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1816}

$$\int x^2\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{7}bx^7(2aC+Ab)+\frac{1}{5}ax^5(aC+2Ab)+\frac{1}{8}bx^8(2aD+bB)+\frac{1}{6}ax^6(aD+2bB)+\frac{1}{9}b^2Cx^9+\frac{1}{10}b^2Dx^{10}$$

```
[In] Int[x^2*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*(2*b*B + a*D)*
x^6)/6 + (b*(A*b + 2*a*C)*x^7)/7 + (b*(b*B + 2*a*D)*x^8)/8 + (b^2*C*x^9)/9
+ (b^2*D*x^10)/10
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(a^2Ax^2 + a^2Bx^3 + a(2Ab + aC)x^4 + a(2bB + aD)x^5 + b(Ab + 2aC)x^6 \right. \\ &\qquad \left. + b(bB + 2aD)x^7 + b^2Cx^8 + b^2Dx^9\right) dx \\ &= \frac{1}{3}a^2Ax^3 + \frac{1}{4}a^2Bx^4 + \frac{1}{5}a(2Ab + aC)x^5 + \frac{1}{6}a(2bB + aD)x^6 \\ &\quad + \frac{1}{7}b(Ab + 2aC)x^7 + \frac{1}{8}b(bB + 2aD)x^8 + \frac{1}{9}b^2Cx^9 + \frac{1}{10}b^2Dx^{10} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int x^2(a + bx^2)^2 (A + Bx + Cx^2 + Dx^3) dx = \frac{42a^2x^3(20A + x(15B + 2x(6C + 5Dx))) + 6abx^5(168A + 5x(28B + 3x(8C + 7Dx))) + b^2x^7(360A + 7x}{2520}$$

```
[In] Integrate[x^2*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (42*a^2*x^3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))) + 6*a*b*x^5*(168*A + 5*x*
(28*B + 3*x*(8*C + 7*D*x))) + b^2*x^7*(360*A + 7*x*(45*B + 4*x*(10*C + 9*D*
x))))/2520
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

method	result
default	$\frac{b^2Dx^{10}}{10} + \frac{b^2Cx^9}{9} + \frac{(Bb^2+2Dab)x^8}{8} + \frac{(b^2A+2Cab)x^7}{7} + \frac{(2abB+Da^2)x^6}{6} + \frac{(2abA+Ca^2)x^5}{5} + \frac{a^2Bx^4}{4} + \frac{a^2Ax^3}{3}$
norman	$\frac{b^2Dx^{10}}{10} + \frac{b^2Cx^9}{9} + (\frac{1}{8}Bb^2 + \frac{1}{4}Dab) x^8 + (\frac{1}{7}b^2A + \frac{2}{7}Cab) x^7 + (\frac{1}{3}abB + \frac{1}{6}Da^2) x^6 + (\frac{2}{5}abA +$
gosper	$\frac{1}{10}b^2Dx^{10} + \frac{1}{9}b^2Cx^9 + \frac{1}{8}b^2Bx^8 + \frac{1}{4}x^8Dab + \frac{1}{7}x^7b^2A + \frac{2}{7}x^7Cab + \frac{1}{3}x^6abB + \frac{1}{6}x^6Da^2 + \frac{2}{5}x^5$
parallelrisch	$\frac{1}{10}b^2Dx^{10} + \frac{1}{9}b^2Cx^9 + \frac{1}{8}b^2Bx^8 + \frac{1}{4}x^8Dab + \frac{1}{7}x^7b^2A + \frac{2}{7}x^7Cab + \frac{1}{3}x^6abB + \frac{1}{6}x^6Da^2 + \frac{2}{5}x^5$

```
[In] int(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*b^2*D*x^10+1/9*b^2*C*x^9+1/8*(B*b^2+2*D*a*b)*x^8+1/7*(A*b^2+2*C*a*b)*x
^7+1/6*(2*B*a*b+D*a^2)*x^6+1/5*(2*A*a*b+C*a^2)*x^5+1/4*a^2*B*x^4+1/3*a^2*A*
x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int x^2(a+bx^2)^2(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{10}Db^2x^{10} + \frac{1}{9}Cb^2x^9 + \frac{1}{8}(2Dab+Bb^2)x^8 + \frac{1}{7}(2Cab+Ab^2)x^7 + \frac{1}{4}Ba^2x^4 + \frac{1}{6}(Da^2+2Bab)x^6 + \frac{1}{3}Aa^2x^3 + \frac{1}{5}(Ca^2+2Aab)x^5$$

```
[In] integrate(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/10*D*b^2*x^10 + 1/9*C*b^2*x^9 + 1/8*(2*D*a*b + B*b^2)*x^8 + 1/7*(2*C*a*b
+ A*b^2)*x^7 + 1/4*B*a^2*x^4 + 1/6*(D*a^2 + 2*B*a*b)*x^6 + 1/3*A*a^2*x^3 +
1/5*(C*a^2 + 2*A*a*b)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01

$$\int x^2(a+bx^2)^2(A+Bx+Cx^2+Dx^3)\,dx = \frac{Aa^2x^3}{3} + \frac{Ba^2x^4}{4} + \frac{Cb^2x^9}{9} + \frac{Db^2x^{10}}{10} + x^8\left(\frac{Bb^2}{8} + \frac{Dab}{4}\right) + x^7\left(\frac{Ab^2}{7} + \frac{2Cab}{7}\right) + x^6\left(\frac{Bab}{3} + \frac{Da^2}{6}\right) + x^5\cdot\left(\frac{2Aab}{5} + \frac{Ca^2}{5}\right)$$

```
[In] integrate(x**2*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x**3/3 + B*a**2*x**4/4 + C*b**2*x**9/9 + D*b**2*x**10/10 + x**8*(B*b
**2/8 + D*a*b/4) + x**7*(A*b**2/7 + 2*C*a*b/7) + x**6*(B*a*b/3 + D*a**2/6)
+ x**5*(2*A*a*b/5 + C*a**2/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int x^2(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{10}\,Db^2x^{10}+\frac{1}{9}\,Cb^2x^9+\frac{1}{8}\,(2\,Dab+Bb^2)x^8+\frac{1}{7}\,(2\,Cab+Ab^2)x^7+\frac{1}{4}\,Ba^2x^4+\frac{1}{6}\,(Da^2+2\,Bab)x^6+\frac{1}{3}\,Aa^2x^3+\frac{1}{5}\,(Ca^2+2\,Aab)x^5$$

```
[In] integrate(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/10*D*b^2*x^10 + 1/9*C*b^2*x^9 + 1/8*(2*D*a*b + B*b^2)*x^8 + 1/7*(2*C*a*b
+ A*b^2)*x^7 + 1/4*B*a^2*x^4 + 1/6*(D*a^2 + 2*B*a*b)*x^6 + 1/3*A*a^2*x^3 +
1/5*(C*a^2 + 2*A*a*b)*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96

$$\int x^2(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{10}\,Db^2x^{10}+\frac{1}{9}\,Cb^2x^9+\frac{1}{4}\,Dabx^8+\frac{1}{8}\,Bb^2x^8+\frac{2}{7}\,Cabx^7+\frac{1}{7}\,Ab^2x^7+\frac{1}{6}\,Da^2x^6+\frac{1}{3}\,Babx^6+\frac{1}{5}\,Ca^2x^5+\frac{2}{5}\,Aabx^5+\frac{1}{4}\,Ba^2x^4+\frac{1}{3}\,Aa^2x^3$$

```
[In] integrate(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/10*D*b^2*x^10 + 1/9*C*b^2*x^9 + 1/4*D*a*b*x^8 + 1/8*B*b^2*x^8 + 2/7*C*a*b
*x^7 + 1/7*A*b^2*x^7 + 1/6*D*a^2*x^6 + 1/3*B*a*b*x^6 + 1/5*C*a^2*x^5 + 2/5*
A*a*b*x^5 + 1/4*B*a^2*x^4 + 1/3*A*a^2*x^3
```

Mupad [B] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

$$\int x^2(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{a^2\,x^6\,D}{6}+\frac{b^2\,x^{10}\,D}{10}+\frac{A\,x^3\,(35\,a^2+42\,a\,b\,x^2+15\,b^2\,x^4)}{105}+\frac{B\,x^4\,(6\,a^2+8\,a\,b\,x^2+3\,b^2\,x^4)}{24}+\frac{C\,x^5\,(63\,a^2+90\,a\,b\,x^2+35\,b^2\,x^4)}{315}+\frac{a\,b\,x^8\,D}{4}$$

```
[In] int(x^2*(a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (a^2*x^6*D)/6 + (b^2*x^10*D)/10 + (A*x^3*(35*a^2 + 15*b^2*x^4 + 42*a*b*x^2)
)/105 + (B*x^4*(6*a^2 + 3*b^2*x^4 + 8*a*b*x^2))/24 + (C*x^5*(63*a^2 + 35*b^
2*x^4 + 90*a*b*x^2))/315 + (a*b*x^8*D)/4
```

3.72 $\int x(a+bx^2)^2 (A+Bx+Cx^2+Dx^3) \, dx$

Optimal result 473
Rubi [A] (verified) 473
Mathematica [A] (verified) 474
Maple [A] (verified) 475
Fricas [A] (verification not implemented) 475
Sympy [A] (verification not implemented) 476
Maxima [A] (verification not implemented) 476
Giac [A] (verification not implemented) 477
Mupad [B] (verification not implemented) 477

Optimal result

Integrand size = 26, antiderivative size = 104

$$\int x(a+bx^2)^2 (A+Bx+Cx^2+Dx^3) \, dx = \frac{1}{3}a^2Bx^3 + \frac{1}{4}a^2Cx^4 + \frac{1}{5}a(2bB+aD)x^5 + \frac{1}{3}abCx^6 + \frac{1}{7}b(bB+2aD)x^7 + \frac{1}{8}b^2Cx^8 + \frac{1}{9}b^2Dx^9 + \frac{A(a+bx^2)^3}{6b}$$

```
[Out] 1/3*a^2*B*x^3+1/4*a^2*C*x^4+1/5*a*(2*B*b+D*a)*x^5+1/3*a*b*C*x^6+1/7*b*(B*b+
2*D*a)*x^7+1/8*b^2*C*x^8+1/9*b^2*D*x^9+1/6*A*(b*x^2+a)^3/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1596, 1824}

$$\int x(a+bx^2)^2 (A+Bx+Cx^2+Dx^3) \, dx = \frac{1}{3}a^2Bx^3 + \frac{1}{4}a^2Cx^4 + \frac{A(a+bx^2)^3}{6b} + \frac{1}{7}bx^7(2aD+bB) + \frac{1}{5}ax^5(aD+2bB) + \frac{1}{3}abCx^6 + \frac{1}{8}b^2Cx^8 + \frac{1}{9}b^2Dx^9$$

```
[In] Int[x*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^2*B*x^3)/3 + (a^2*C*x^4)/4 + (a*(2*b*B + a*D)*x^5)/5 + (a*b*C*x^6)/3 + (
b*(b*B + 2*a*D)*x^7)/7 + (b^2*C*x^8)/8 + (b^2*D*x^9)/9 + (A*(a + b*x^2)^3)/
(6*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{A(a+bx^2)^3}{6b} + \int (a+bx^2)^2 \left(-Ax + x\left(A+Bx+Cx^2+Dx^3\right)\right) dx \\
&= \frac{A(a+bx^2)^3}{6b} + \int \left(a^2Bx^2 + a^2Cx^3 + a(2bB+aD)x^4 + 2abCx^5 + b(bB+2aD)x^6 \right. \\
&\qquad \left. + b^2Cx^7 + b^2Dx^8\right) dx \\
&= \frac{1}{3}a^2Bx^3 + \frac{1}{4}a^2Cx^4 + \frac{1}{5}a(2bB+aD)x^5 + \frac{1}{3}abCx^6 \\
&\quad + \frac{1}{7}b(bB+2aD)x^7 + \frac{1}{8}b^2Cx^8 + \frac{1}{9}b^2Dx^9 + \frac{A(a+bx^2)^3}{6b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int x\left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) dx$$
$$= \frac{42a^2x^2(30A+x(20B+3x(5C+4Dx))) + 12abx^4(105A+2x(42B+5x(7C+6Dx))) + 5b^2x^6(84A+x(7}{2520}$$

```
[In] Integrate[x*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (42*a^2*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))) + 12*a*b*x^4*(105*A + 2*x
*(42*B + 5*x*(7*C + 6*D*x))) + 5*b^2*x^6*(84*A + x*(72*B + 7*x*(9*C + 8*D*x
))))/2520
```

Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

method	result
default	$\frac{b^2 D x^9}{9}+\frac{b^2 C x^8}{8}+\frac{(B b^2+2Dab) x^7}{7}+\frac{(b^2 A+2Cab) x^6}{6}+\frac{(2abB+D a^2) x^5}{5}+\frac{(2abA+C a^2) x^4}{4}+\frac{a^2 B x^3}{3}+\frac{a^2 A x^2}{2}$
norman	$\frac{b^2 D x^9}{9}+\frac{b^2 C x^8}{8}+\left(\frac{1}{7}B b^2+\frac{2}{7}Dab\right) x^7+\left(\frac{1}{6}b^2 A+\frac{1}{3}Cab\right) x^6+\left(\frac{2}{5}abB+\frac{1}{5}D a^2\right) x^5+\left(\frac{1}{2}abA+\right.$
gosper	$\frac{1}{9}b^2 D x^9+\frac{1}{8}b^2 C x^8+\frac{1}{7}b^2 B x^7+\frac{2}{7}x^7 Dab+\frac{1}{6}x^6 b^2 A+\frac{1}{3}abC x^6+\frac{2}{5}x^5 abB+\frac{1}{5}x^5 D a^2+\frac{1}{2}x^4 ab$
parallelrisch	$\frac{1}{9}b^2 D x^9+\frac{1}{8}b^2 C x^8+\frac{1}{7}b^2 B x^7+\frac{2}{7}x^7 Dab+\frac{1}{6}x^6 b^2 A+\frac{1}{3}abC x^6+\frac{2}{5}x^5 abB+\frac{1}{5}x^5 D a^2+\frac{1}{2}x^4 ab$

```
[In] int(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b^2*D*x^9+1/8*b^2*C*x^8+1/7*(B*b^2+2*D*a*b)*x^7+1/6*(A*b^2+2*C*a*b)*x^6
+1/5*(2*B*a*b+D*a^2)*x^5+1/4*(2*A*a*b+C*a^2)*x^4+1/3*a^2*B*x^3+1/2*a^2*A*x^
2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97

$$\int x(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{9}\,Db^2x^9+\frac{1}{8}\,Cb^2x^8+\frac{1}{7}\,\left(2\,Dab+Bb^2\right)x^7+\frac{1}{6}\,\left(2\,Cab+Ab^2\right)x^6+\frac{1}{3}\,Ba^2x^3+\frac{1}{5}\,\left(Da^2+2\,Bab\right)x^5+\frac{1}{2}\,Aa^2x^2+\frac{1}{4}\,\left(Ca^2+2\,Aab\right)x^4$$

```
[In] integrate(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/9*D*b^2*x^9 + 1/8*C*b^2*x^8 + 1/7*(2*D*a*b + B*b^2)*x^7 + 1/6*(2*C*a*b +
A*b^2)*x^6 + 1/3*B*a^2*x^3 + 1/5*(D*a^2 + 2*B*a*b)*x^5 + 1/2*A*a^2*x^2 + 1/
4*(C*a^2 + 2*A*a*b)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.06

$$\int x(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{Aa^2x^2}{2}+\frac{Ba^2x^3}{3}+\frac{Cb^2x^8}{8}+\frac{Db^2x^9}{9}+x^7\left(\frac{Bb^2}{7}+\frac{2Dab}{7}\right)+x^6\left(\frac{Ab^2}{6}+\frac{Cab}{3}\right)+x^5\cdot\left(\frac{2Bab}{5}+\frac{Da^2}{5}\right)+x^4\left(\frac{Aab}{2}+\frac{Ca^2}{4}\right)$$

```
[In] integrate(x*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x**2/2 + B*a**2*x**3/3 + C*b**2*x**8/8 + D*b**2*x**9/9 + x**7*(B*b**
2/7 + 2*D*a*b/7) + x**6*(A*b**2/6 + C*a*b/3) + x**5*(2*B*a*b/5 + D*a**2/5)
+ x**4*(A*a*b/2 + C*a**2/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97

$$\int x(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{9}\,Db^2x^9+\frac{1}{8}\,Cb^2x^8+\frac{1}{7}\,(2\,Dab+Bb^2)x^7+\frac{1}{6}\,(2\,Cab+Ab^2)x^6+\frac{1}{3}\,Ba^2x^3+\frac{1}{5}\,(Da^2+2\,Bab)x^5+\frac{1}{2}\,Aa^2x^2+\frac{1}{4}\,(Ca^2+2\,Aab)x^4$$

```
[In] integrate(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/9*D*b^2*x^9 + 1/8*C*b^2*x^8 + 1/7*(2*D*a*b + B*b^2)*x^7 + 1/6*(2*C*a*b +
A*b^2)*x^6 + 1/3*B*a^2*x^3 + 1/5*(D*a^2 + 2*B*a*b)*x^5 + 1/2*A*a^2*x^2 + 1/
4*(C*a^2 + 2*A*a*b)*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int x(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{9}Db^2x^9+\frac{1}{8}Cb^2x^8+\frac{2}{7}Dabx^7+\frac{1}{7}Bb^2x^7+\frac{1}{3}Cabx^6+\frac{1}{6}Ab^2x^6+\frac{1}{5}Da^2x^5+\frac{2}{5}Babx^5+\frac{1}{4}Ca^2x^4+\frac{1}{2}Aabx^4+\frac{1}{3}Ba^2x^3+\frac{1}{2}Aa^2x^2$$

```
[In] integrate(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/9*D*b^2*x^9 + 1/8*C*b^2*x^8 + 2/7*D*a*b*x^7 + 1/7*B*b^2*x^7 + 1/3*C*a*b*x
^6 + 1/6*A*b^2*x^6 + 1/5*D*a^2*x^5 + 2/5*B*a*b*x^5 + 1/4*C*a^2*x^4 + 1/2*A*
a*b*x^4 + 1/3*B*a^2*x^3 + 1/2*A*a^2*x^2
```

Mupad [B] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

$$\int x(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{a^2\,x^5\,D}{5}+\frac{b^2\,x^9\,D}{9}+\frac{A\,x^2\,\left(3\,a^2+3\,a\,b\,x^2+b^2\,x^4\right)}{6}+\frac{B\,x^3\,\left(35\,a^2+42\,a\,b\,x^2+15\,b^2\,x^4\right)}{105}+\frac{C\,x^4\,\left(6\,a^2+8\,a\,b\,x^2+3\,b^2\,x^4\right)}{24}+\frac{2\,a\,b\,x^7\,D}{7}$$

```
[In] int(x*(a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (a^2*x^5*D)/5 + (b^2*x^9*D)/9 + (A*x^2*(3*a^2 + b^2*x^4 + 3*a*b*x^2))/6 + (
B*x^3*(35*a^2 + 15*b^2*x^4 + 42*a*b*x^2))/105 + (C*x^4*(6*a^2 + 3*b^2*x^4 +
 8*a*b*x^2))/24 + (2*a*b*x^7*D)/7
```

3.73 $\int (a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\, dx$

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Optimal result

Integrand size = 25, antiderivative size = 99

$$\int (a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\, dx = a^2Ax+\frac{1}{3}a(2Ab+aC)x^3+\frac{1}{4}a^2Dx^4+\frac{1}{5}b(Ab+2aC)x^5 + \frac{1}{3}abDx^6+\frac{1}{7}b^2Cx^7+\frac{1}{8}b^2Dx^8+\frac{B(a+bx^2)^3}{6b}$$

```
[Out] a^2*A*x+1/3*a*(2*A*b+C*a)*x^3+1/4*a^2*D*x^4+1/5*b*(A*b+2*C*a)*x^5+1/3*a*b*D
*x^6+1/7*b^2*C*x^7+1/8*b^2*D*x^8+1/6*B*(b*x^2+a)^3/b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1596, 1824}

$$\int (a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\, dx = a^2Ax+\frac{1}{4}a^2Dx^4+\frac{1}{5}bx^5(2aC+Ab)+\frac{1}{3}ax^3(aC+2Ab) + \frac{B(a+bx^2)^3}{6b}+\frac{1}{3}abDx^6+\frac{1}{7}b^2Cx^7+\frac{1}{8}b^2Dx^8$$

```
[In] Int[(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a^2*A*x + (a*(2*A*b + a*C)*x^3)/3 + (a^2*D*x^4)/4 + (b*(A*b + 2*a*C)*x^5)/5
 + (a*b*D*x^6)/3 + (b^2*C*x^7)/7 + (b^2*D*x^8)/8 + (B*(a + b*x^2)^3)/(6*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
```

```
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{B(a+bx^2)^3}{6b} + \int (a+bx^2)^2 \left(A+Cx^2+Dx^3\right) dx \\ &= \frac{B(a+bx^2)^3}{6b} \\ &\quad + \int \left(a^2A+a(2Ab+aC)x^2+a^2Dx^3+b(Ab+2aC)x^4+2abDx^5+b^2Cx^6+b^2Dx^7\right) dx \\ &= a^2Ax + \frac{1}{3}a(2Ab+aC)x^3 + \frac{1}{4}a^2Dx^4 + \frac{1}{5}b(Ab+2aC)x^5 \\ &\quad + \frac{1}{3}abDx^6 + \frac{1}{7}b^2Cx^7 + \frac{1}{8}b^2Dx^8 + \frac{B(a+bx^2)^3}{6b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

$$\begin{aligned}\int (a+bx^2)^2 \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{1}{840}&\left(70a^2x(12A+x(6B+x(4C+3Dx)))\right. \\ &+ 28abx^3(20A+x(15B+2x(6C+5Dx))) \\ &\left.+ b^2x^5(168A+5x(28B+3x(8C+7Dx)))\right)\end{aligned}$$

```
[In] Integrate[(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]

[Out] (70*a^2*x*(12*A + x*(6*B + x*(4*C + 3*D*x))) + 28*a*b*x^3*(20*A + x*(15*B +
 2*x*(6*C + 5*D*x))) + b^2*x^5*(168*A + 5*x*(28*B + 3*x*(8*C + 7*D*x))))/84
0
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

method	result
default	$\frac{b^2Dx^8}{8}+\frac{b^2C\,x^7}{7}+\frac{(B\,b^2+2Dab)x^6}{6}+\frac{(b^2A+2Cab)x^5}{5}+\frac{(2abB+Da^2)x^4}{4}+\frac{(2abA+C\,a^2)x^3}{3}+\frac{a^2B\,x^2}{2}+a^2Ax$
norman	$\frac{b^2Dx^8}{8}+\frac{b^2C\,x^7}{7}+\left(\frac{1}{6}B\,b^2+\frac{1}{3}Dab\right)x^6+\left(\frac{1}{5}b^2A+\frac{2}{5}Cab\right)x^5+\left(\frac{1}{2}abB+\frac{1}{4}Da^2\right)x^4+\left(\frac{2}{3}abA+\frac{1}{3}C\right.$
gosper	$\frac{1}{8}b^2Dx^8+\frac{1}{7}b^2C\,x^7+\frac{1}{6}b^2B\,x^6+\frac{1}{3}abDx^6+\frac{1}{5}x^5b^2A+\frac{2}{5}x^5Cab+\frac{1}{2}Bab\,x^4+\frac{1}{4}a^2Dx^4+\frac{2}{3}x^3abA$
parallelrisch	$\frac{1}{8}b^2Dx^8+\frac{1}{7}b^2C\,x^7+\frac{1}{6}b^2B\,x^6+\frac{1}{3}abDx^6+\frac{1}{5}x^5b^2A+\frac{2}{5}x^5Cab+\frac{1}{2}Bab\,x^4+\frac{1}{4}a^2Dx^4+\frac{2}{3}x^3abA$

```
[In] int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*b^2*D*x^8+1/7*b^2*C*x^7+1/6*(B*b^2+2*D*a*b)*x^6+1/5*(A*b^2+2*C*a*b)*x^5
+1/4*(2*B*a*b+D*a^2)*x^4+1/3*(2*A*a*b+C*a^2)*x^3+1/2*a^2*B*x^2+a^2*A*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \frac{1}{8}\,Db^2x^8+\frac{1}{7}\,Cb^2x^7+\frac{1}{6}\,(2\,Dab+Bb^2)x^6\\ &\quad+\frac{1}{5}\,(2\,Cab+Ab^2)x^5+\frac{1}{2}\,Ba^2x^2\\ &\quad+\frac{1}{4}\,(Da^2+2\,Bab)x^4\\ &\quad+Aa^2x+\frac{1}{3}\,(Ca^2+2\,Aab)x^3\end{aligned}$$

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/6*(2*D*a*b + B*b^2)*x^6 + 1/5*(2*C*a*b +
A*b^2)*x^5 + 1/2*B*a^2*x^2 + 1/4*(D*a^2 + 2*B*a*b)*x^4 + A*a^2*x + 1/3*(C*a
^2 + 2*A*a*b)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08

$$\int (a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx = Aa^2x+\frac{Ba^2x^2}{2}+\frac{Cb^2x^7}{7}+\frac{Db^2x^8}{8}+x^6\left(\frac{Bb^2}{6}+\frac{Dab}{3}\right)+x^5\left(\frac{Ab^2}{5}+\frac{2Cab}{5}\right)+x^4\left(\frac{Bab}{2}+\frac{Da^2}{4}\right)+x^3\cdot\left(\frac{2Aab}{3}+\frac{Ca^2}{3}\right)$$

```
[In] integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x + B*a**2*x**2/2 + C*b**2*x**7/7 + D*b**2*x**8/8 + x**6*(B*b**2/6 +
 D*a*b/3) + x**5*(A*b**2/5 + 2*C*a*b/5) + x**4*(B*a*b/2 + D*a**2/4) + x**3*
(2*A*a*b/3 + C*a**2/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99

$$\int (a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{8}Db^2x^8+\frac{1}{7}Cb^2x^7+\frac{1}{6}(2Dab+Bb^2)x^6+\frac{1}{5}(2Cab+Ab^2)x^5+\frac{1}{2}Ba^2x^2+\frac{1}{4}(Da^2+2Bab)x^4+Aa^2x+\frac{1}{3}(Ca^2+2Aab)x^3$$

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/6*(2*D*a*b + B*b^2)*x^6 + 1/5*(2*C*a*b +
A*b^2)*x^5 + 1/2*B*a^2*x^2 + 1/4*(D*a^2 + 2*B*a*b)*x^4 + A*a^2*x + 1/3*(C*a
^2 + 2*A*a*b)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.03

$$\int (a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\, dx = \frac{1}{8}Db^2x^8 + \frac{1}{7}Cb^2x^7 + \frac{1}{3}Dabx^6 + \frac{1}{6}Bb^2x^6 + \frac{2}{5}Cabx^5 + \frac{1}{5}Ab^2x^5 + \frac{1}{4}Da^2x^4 + \frac{1}{2}Babx^4 + \frac{1}{3}Ca^2x^3 + \frac{2}{3}Aabx^3 + \frac{1}{2}Ba^2x^2 + Aa^2x$$

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/3*D*a*b*x^6 + 1/6*B*b^2*x^6 + 2/5*C*a*b*x
^5 + 1/5*A*b^2*x^5 + 1/4*D*a^2*x^4 + 1/2*B*a*b*x^4 + 1/3*C*a^2*x^3 + 2/3*A*
a*b*x^3 + 1/2*B*a^2*x^2 + A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 5.66 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06

$$\int (a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\, dx = \frac{A\,x\,\left(15\,a^2+10\,a\,b\,x^2+3\,b^2\,x^4\right)}{15} + \frac{a^2\,x^4\,D}{4} + \frac{b^2\,x^8\,D}{8} + \frac{B\,x^2\,\left(3\,a^2+3\,a\,b\,x^2+b^2\,x^4\right)}{6} + \frac{C\,x^3\,\left(35\,a^2+42\,a\,b\,x^2+15\,b^2\,x^4\right)}{105} + \frac{a\,b\,x^6\,D}{3}$$

```
[In] int((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (A*x*(15*a^2 + 3*b^2*x^4 + 10*a*b*x^2))/15 + (a^2*x^4*D)/4 + (b^2*x^8*D)/8
+ (B*x^2*(3*a^2 + b^2*x^4 + 3*a*b*x^2))/6 + (C*x^3*(35*a^2 + 15*b^2*x^4 + 4
2*a*b*x^2))/105 + (a*b*x^6*D)/3
```

3.74 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x} dx$

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Maxima [A] (verification not implemented) 486
Giac [A] (verification not implemented) 486
Mupad [B] (verification not implemented) 487

Optimal result

Integrand size = 28, antiderivative size = 92

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x} dx = a^2Bx+aAbx^2+\frac{1}{3}a(2bB+aD)x^3+\frac{1}{4}Ab^2x^4+\frac{1}{5}b(bB+2aD)x^5+\frac{1}{7}b^2Dx^7+\frac{C(a+bx^2)^3}{6b}+a^2A\log(x)$$

```
[Out] a^2*B*x+a*A*b*x^2+1/3*a*(2*B*b+D*a)*x^3+1/4*A*b^2*x^4+1/5*b*(B*b+2*D*a)*x^5
+1/7*b^2*D*x^7+1/6*C*(b*x^2+a)^3/b+a^2*A*ln(x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1597, 1816}

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x} dx = a^2A\log(x)+a^2Bx+aAbx^2+\frac{1}{5}bx^5(2aD+bB)+\frac{1}{3}ax^3(aD+2bB)+\frac{C(a+bx^2)^3}{6b}+\frac{1}{4}Ab^2x^4+\frac{1}{7}b^2Dx^7$$

```
[In] Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] a^2*B*x + a*A*b*x^2 + (a*(2*b*B + a*D)*x^3)/3 + (A*b^2*x^4)/4 + (b*(b*B + 2
*a*D)*x^5)/5 + (b^2*D*x^7)/7 + (C*(a + b*x^2)^3)/(6*b) + a^2*A*Log[x]
```

Rule 1597

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{C(a+bx^2)^3}{6b} + \int \frac{(a+bx^2)^2(A+Bx+Dx^3)}{x}\,dx \\
&= \frac{C(a+bx^2)^3}{6b} \\
&\quad + \int \left(a^2B + \frac{a^2A}{x} + 2aAbx + a(2bB+aD)x^2 + Ab^2x^3 + b(bB+2aD)x^4 + b^2Dx^6\right)dx \\
&= a^2Bx + aAbx^2 + \frac{1}{3}a(2bB+aD)x^3 + \frac{1}{4}Ab^2x^4 \\
&\quad + \frac{1}{5}b(bB+2aD)x^5 + \frac{1}{7}b^2Dx^7 + \frac{C(a+bx^2)^3}{6b} + a^2A\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.96

$$\begin{aligned}
\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x}\,dx = &\frac{1}{420}x\big(70a^2(6B+x(3C+2Dx)) \\
&+ 14abx(30A+x(20B+3x(5C+4Dx))) \\
&+ b^2x^3(105A+2x(42B+5x(7C+6Dx)))\big) \\
&+ a^2A\log(x)
\end{aligned}$$

```
[In] Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] (x*(70*a^2*(6*B + x*(3*C + 2*D*x)) + 14*a*b*x*(30*A + x*(20*B + 3*x*(5*C +
4*D*x))) + b^2*x^3*(105*A + 2*x*(42*B + 5*x*(7*C + 6*D*x)))))/420 + a^2*A*L
og[x]
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04

method	result
norman	$\left(\frac{1}{5}B\,b^{2}+\frac{2}{5}Dab\right)x^{5}+\left(\frac{1}{4}b^{2}A+\frac{1}{2}Cab\right)x^{4}+\left(abA+\frac{1}{2}C\,a^{2}\right)x^{2}+\left(\frac{2}{3}abB+\frac{1}{3}Da^{2}\right)x^{3}+a^{2}Bx$
default	$\frac{b^{2}Dx^{7}}{7}+\frac{C\,b^{2}x^{6}}{6}+\frac{b^{2}B\,x^{5}}{5}+\frac{2Dab\,x^{5}}{5}+\frac{A\,b^{2}x^{4}}{4}+\frac{Cab\,x^{4}}{2}+\frac{2Bab\,x^{3}}{3}+\frac{Da^{2}x^{3}}{3}+aAb\,x^{2}+\frac{C\,a^{2}x^{2}}{2}+a^{2}B$
parallelrisch	$\frac{b^{2}Dx^{7}}{7}+\frac{C\,b^{2}x^{6}}{6}+\frac{b^{2}B\,x^{5}}{5}+\frac{2Dab\,x^{5}}{5}+\frac{A\,b^{2}x^{4}}{4}+\frac{Cab\,x^{4}}{2}+\frac{2Bab\,x^{3}}{3}+\frac{Da^{2}x^{3}}{3}+aAb\,x^{2}+\frac{C\,a^{2}x^{2}}{2}+a^{2}B$

```
[In] int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (1/5*B*b^2+2/5*D*a*b)*x^5+(1/4*b^2*A+1/2*C*a*b)*x^4+(a*b*A+1/2*C*a^2)*x^2+(
2/3*a*b*B+1/3*D*a^2)*x^3+a^2*B*x+1/6*C*b^2*x^6+1/7*b^2*D*x^7+a^2*A*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2\,(A+Bx+Cx^2+Dx^3)}{x}\,dx=\frac{1}{7}\,Db^2x^7+\frac{1}{6}\,Cb^2x^6+\frac{1}{5}\,\big(2\,Dab+Bb^2\big)x^5+\frac{1}{4}\,\big(2\,Cab+Ab^2\big)x^4+Ba^2x+\frac{1}{3}\,\big(Da^2+2\,Bab\big)x^3+Aa^2\log\left(x\right)+\frac{1}{2}\,\big(Ca^2+2\,Aab\big)x^2$$

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="fricas")
```

```
[Out] 1/7*D*b^2*x^7 + 1/6*C*b^2*x^6 + 1/5*(2*D*a*b + B*b^2)*x^5 + 1/4*(2*C*a*b +
A*b^2)*x^4 + B*a^2*x + 1/3*(D*a^2 + 2*B*a*b)*x^3 + A*a^2*log(x) + 1/2*(C*a^
2 + 2*A*a*b)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^2)^2\,(A+Bx+Cx^2+Dx^3)}{x}\,dx=Aa^2\log\left(x\right)+Ba^2x+\frac{Cb^2x^6}{6}+\frac{Db^2x^7}{7}+x^5\left(\frac{Bb^2}{5}+\frac{2Dab}{5}\right)+x^4\left(\frac{Ab^2}{4}+\frac{Cab}{2}\right)+x^3\cdot\left(\frac{2Bab}{3}+\frac{Da^2}{3}\right)+x^2\left(Aab+\frac{Ca^2}{2}\right)$$

```
[In] integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x,x)
```

```
[Out] A*a**2*log(x) + B*a**2*x + C*b**2*x**6/6 + D*b**2*x**7/7 + x**5*(B*b**2/5 +
 2*D*a*b/5) + x**4*(A*b**2/4 + C*a*b/2) + x**3*(2*B*a*b/3 + D*a**2/3) + x**
2*(A*a*b + C*a**2/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{1}{7}Db^2x^7 + \frac{1}{6}Cb^2x^6 + \frac{1}{5}\left(2Dab + Bb^2\right)x^5 + \frac{1}{4}\left(2Cab + Ab^2\right)x^4 + Ba^2x + \frac{1}{3}\left(Da^2 + 2Bab\right)x^3 + Aa^2\log(x) + \frac{1}{2}\left(Ca^2 + 2Aab\right)x^2$$

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="maxima")
```

```
[Out] 1/7*D*b^2*x^7 + 1/6*C*b^2*x^6 + 1/5*(2*D*a*b + B*b^2)*x^5 + 1/4*(2*C*a*b +
A*b^2)*x^4 + B*a^2*x + 1/3*(D*a^2 + 2*B*a*b)*x^3 + A*a^2*log(x) + 1/2*(C*a^
2 + 2*A*a*b)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{1}{7}Db^2x^7 + \frac{1}{6}Cb^2x^6 + \frac{2}{5}Dabx^5 + \frac{1}{5}Bb^2x^5 + \frac{1}{2}Cabx^4 + \frac{1}{4}Ab^2x^4 + \frac{1}{3}Da^2x^3 + \frac{2}{3}Babx^3 + \frac{1}{2}Ca^2x^2 + Aabx^2 + Ba^2x + Aa^2\log(|x|)$$

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="giac")
```

```
[Out] 1/7*D*b^2*x^7 + 1/6*C*b^2*x^6 + 2/5*D*a*b*x^5 + 1/5*B*b^2*x^5 + 1/2*C*a*b*x
^4 + 1/4*A*b^2*x^4 + 1/3*D*a^2*x^3 + 2/3*B*a*b*x^3 + 1/2*C*a^2*x^2 + A*a*b*
x^2 + B*a^2*x + A*a^2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{A\,(4\,a^2\,\ln(x)+b^2\,x^4+4\,a\,b\,x^2)}{4} + \frac{B\,x\,(15\,a^2+10\,a\,b\,x^2+3\,b^2\,x^4)}{15} + \frac{a^2\,x^3\,D}{3} + \frac{b^2\,x^7\,D}{7} + \frac{C\,x^2\,(3\,a^2+3\,a\,b\,x^2+b^2\,x^4)}{6} + \frac{2\,a\,b\,x^5\,D}{5}$$

```
[In] int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/x,x)
```

```
[Out] (A*(4*a^2*log(x) + b^2*x^4 + 4*a*b*x^2))/4 + (B*x*(15*a^2 + 3*b^2*x^4 + 10*
a*b*x^2))/15 + (a^2*x^3*D)/3 + (b^2*x^7*D)/7 + (C*x^2*(3*a^2 + b^2*x^4 + 3*
a*b*x^2))/6 + (2*a*b*x^5*D)/5
```

3.75 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^2}\,dx$

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Optimal result

Integrand size = 28, antiderivative size = 90

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = -\frac{a^2A}{x}+a(2Ab+aC)x+abBx^2+\frac{1}{3}b(Ab+2aC)x^3+\frac{1}{4}b^2Bx^4+\frac{1}{5}b^2Cx^5+\frac{D(a+bx^2)^3}{6b}+a^2B\log(x)$$

```
[Out] -a^2*A/x+a*(2*A*b+C*a)*x+a*b*B*x^2+1/3*b*(A*b+2*C*a)*x^3+1/4*b^2*B*x^4+1/5*
b^2*C*x^5+1/6*D*(b*x^2+a)^3/b+a^2*B*ln(x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1597, 1642}

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = -\frac{a^2A}{x}+a^2B\log(x)+\frac{1}{3}bx^3(2aC+Ab)+ax(aC+2Ab)+abBx^2+\frac{D(a+bx^2)^3}{6b}+\frac{1}{4}b^2Bx^4+\frac{1}{5}b^2Cx^5$$

```
[In] Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + (b*(A*b + 2*a*C)*x^3)/3 + (b
^2*B*x^4)/4 + (b^2*C*x^5)/5 + (D*(a + b*x^2)^3)/(6*b) + a^2*B*Log[x]
```

Rule 1597

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{D(a+bx^2)^3}{6b} + \int \frac{(a+bx^2)^2\,(A+Bx+Cx^2)}{x^2}\,dx \\
&= \frac{D(a+bx^2)^3}{6b} \\
&\quad + \int \left(a(2Ab+aC)+\frac{a^2A}{x^2}+\frac{a^2B}{x}+2abBx+b(Ab+2aC)x^2+b^2Bx^3+b^2Cx^4\right)dx \\
&= -\frac{a^2A}{x}+a(2Ab+aC)x+abBx^2+\frac{1}{3}b(Ab+2aC)x^3 \\
&\quad + \frac{1}{4}b^2Bx^4+\frac{1}{5}b^2Cx^5+\frac{D(a+bx^2)^3}{6b}+a^2B\log(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$
\begin{aligned}
\int \frac{(a+bx^2)^2\,(A+Bx+Cx^2+Dx^3)}{x^2}\,dx &= a^2\left(-\frac{A}{x}+Cx+\frac{Dx^2}{2}\right) \\
&\quad + \frac{1}{6}abx(12A+x(6B+x(4C+3Dx))) \\
&\quad + \frac{1}{60}b^2x^3(20A+x(15B+2x(6C+5Dx))) \\
&\quad + a^2B\log(x)
\end{aligned}
$$

```
[In] Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] a^2*(-(A/x) + C*x + (D*x^2)/2) + (a*b*x*(12*A + x*(6*B + x*(4*C + 3*D*x))))
/6 + (b^2*x^3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))))/60 + a^2*B*Log[x]
```

Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.09

method	result
default	$\frac{b^{2} D x^{6}}{6}+\frac{b^{2} C \,x^{5}}{5}+\frac{b^{2} B \,x^{4}}{4}+\frac{D a b \,x^{4}}{2}+\frac{A \,b^{2} x^{3}}{3}+\frac{2 C a b \,x^{3}}{3}+B a b \,x^{2}+\frac{D a^{2} x^{2}}{2}+2 a A b x +C \,a^{2} x +a^{2} B \ln$
norman	$\frac{\left(\frac{1}{4} B \,b^{2}+\frac{1}{2} D a b \right) x^{5}+\left(\frac{1}{3} b^{2} A +\frac{2}{3} C a b \right) x^{4}+\left(a b B +\frac{1}{2} D \,a^{2}\right) x^{3}+\left(2 a b A +C \,a^{2}\right) x^{2}-a^{2} A +\frac{C \,b^{2} x^{6}}{5}+\frac{b^{2} D \,x^{7}}{6}}{x}+a^{2} B \ln \left(x \right)$
parallelrisch	$\frac{10 b^{2} D \,x^{7}+12 C \,b^{2} x^{6}+15 b^{2} B \,x^{5}+30 D a b \,x^{5}+20 A \,b^{2} x^{4}+40 C a b \,x^{4}+60 B a b \,x^{3}+30 D \,a^{2} x^{3}+120 a A b \,x^{2}+60 a^{2} B \ln \left(x \right) x +60 C \,a^{2} x^{2}-}{60 x}$

```
[In] int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b^2*D*x^6+1/5*b^2*C*x^5+1/4*b^2*B*x^4+1/2*D*a*b*x^4+1/3*A*b^2*x^3+2/3*C
*a*b*x^3+B*a*b*x^2+1/2*D*a^2*x^2+2*a*A*b*x+C*a^2*x+a^2*B*ln(x)-a^2*A/x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^2} \, dx = \frac{10 \, D b^{2} x^{7} + 12 \, C b^{2} x^{6} + 15 \, {\left(2 \, D a b + B b^{2}\right)} x^{5} + 20 \, {\left(2 \, C a b + A b^{2}\right)} x^{4} + 60 \, B a^{2} x \log\left(x\right) + 30 \, {\left(D a^{2} + 2 \, B a b\right)} x^{3}}{60 \, x}$$

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*D*b^2*x^7 + 12*C*b^2*x^6 + 15*(2*D*a*b + B*b^2)*x^5 + 20*(2*C*a*b
+ A*b^2)*x^4 + 60*B*a^2*x*log(x) + 30*(D*a^2 + 2*B*a*b)*x^3 - 60*A*a^2 + 60
*(C*a^2 + 2*A*a*b)*x^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^2} \, dx = - \frac{A a^{2}}{x} + B a^{2} \log{\left(x \right)} + \frac{C b^{2} x^{5}}{5} + \frac{D b^{2} x^{6}}{6} + x^{4} \left(\frac{B b^{2}}{4} + \frac{D a b}{2}\right) + x^{3} \left(\frac{A b^{2}}{3} + \frac{2 C a b}{3}\right) + x^{2} \left(B a b + \frac{D a^{2}}{2}\right) + x \left(2 A a b + C a^{2}\right)$$

```
[In] integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

```
[Out] -A*a**2/x + B*a**2*log(x) + C*b**2*x**5/5 + D*b**2*x**6/6 + x**4*(B*b**2/4
+ D*a*b/2) + x**3*(A*b**2/3 + 2*C*a*b/3) + x**2*(B*a*b + D*a**2/2) + x*(2*A
*a*b + C*a**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{1}{6}\,Db^2x^6 + \frac{1}{5}\,Cb^2x^5 + \frac{1}{4}\,(2\,Dab+Bb^2)x^4 + \frac{1}{3}\,(2\,Cab+Ab^2)x^3 + Ba^2\log(x) + \frac{1}{2}\,(Da^2+2\,Bab)x^2 - \frac{Aa^2}{x} + (Ca^2+2\,Aab)x$$

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/6*D*b^2*x^6 + 1/5*C*b^2*x^5 + 1/4*(2*D*a*b + B*b^2)*x^4 + 1/3*(2*C*a*b +
A*b^2)*x^3 + B*a^2*log(x) + 1/2*(D*a^2 + 2*B*a*b)*x^2 - A*a^2/x + (C*a^2 +
2*A*a*b)*x
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{1}{6}\,Db^2x^6 + \frac{1}{5}\,Cb^2x^5 + \frac{1}{2}\,Dabx^4 + \frac{1}{4}\,Bb^2x^4 + \frac{2}{3}\,Cabx^3 + \frac{1}{3}\,Ab^2x^3 + \frac{1}{2}\,Da^2x^2 + Babx^2 + Ca^2x + 2\,Aabx + Ba^2\log(|x|) - \frac{Aa^2}{x}$$

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/6*D*b^2*x^6 + 1/5*C*b^2*x^5 + 1/2*D*a*b*x^4 + 1/4*B*b^2*x^4 + 2/3*C*a*b*x
^3 + 1/3*A*b^2*x^3 + 1/2*D*a^2*x^2 + B*a*b*x^2 + C*a^2*x + 2*A*a*b*x + B*a^
2*log(abs(x)) - A*a^2/x
```

Mupad [B] (verification not implemented)

Time = 5.56 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^2} \, dx = \frac{B\,(4\,a^2\,\ln(x)+b^2\,x^4+4\,a\,b\,x^2)}{4}+\frac{(b\,x^2+a)^3\,D}{6\,b}+\frac{C\,x\,(15\,a^2+10\,a\,b\,x^2+3\,b^2\,x^4)}{15}+\frac{A\,(-3\,a^2+6\,a\,b\,x^2+b^2\,x^4)}{3\,x}$$

```
[In] int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/x^2,x)
```

```
[Out] (B*(4*a^2*log(x) + b^2*x^4 + 4*a*b*x^2))/4 + ((a + b*x^2)^3*D)/(6*b) + (C*x
*(15*a^2 + 3*b^2*x^4 + 10*a*b*x^2))/15 + (A*(b^2*x^4 - 3*a^2 + 6*a*b*x^2))/
(3*x)
```

3.76 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^3} dx$

Optimal result 493
Rubi [A] (verified) 493
Mathematica [A] (verified) 494
Maple [A] (verified) 494
Fricas [A] (verification not implemented) 495
Sympy [A] (verification not implemented) 495
Maxima [A] (verification not implemented) 496
Giac [A] (verification not implemented) 496
Mupad [B] (verification not implemented) 497

Optimal result

Integrand size = 28, antiderivative size = 98

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^3} dx = -\frac{a^2A}{2x^2} - \frac{a^2B}{x} + a(2bB+aD)x + \frac{1}{2}b(Ab+2aC)x^2 + \frac{1}{3}b(bB+2aD)x^3 + \frac{1}{4}b^2Cx^4 + \frac{1}{5}b^2Dx^5 + a(2Ab+aC)\log(x)$$

```
[Out] -1/2*a^2*A/x^2-a^2*B/x+a*(2*B*b+D*a)*x+1/2*b*(A*b+2*C*a)*x^2+1/3*b*(B*b+2*D
*a)*x^3+1/4*b^2*C*x^4+1/5*b^2*D*x^5+a*(2*A*b+C*a)*ln(x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1816}

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^3} dx = -\frac{a^2A}{2x^2} - \frac{a^2B}{x} + \frac{1}{2}bx^2(2aC+Ab) + a\log(x)(aC+2Ab) + \frac{1}{3}bx^3(2aD+bB) + ax(aD+2bB) + \frac{1}{4}b^2Cx^4 + \frac{1}{5}b^2Dx^5$$

```
[In] Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -1/2*(a^2*A)/x^2 - (a^2*B)/x + a*(2*b*B + a*D)*x + (b*(A*b + 2*a*C)*x^2)/2
+ (b*(b*B + 2*a*D)*x^3)/3 + (b^2*C*x^4)/4 + (b^2*D*x^5)/5 + a*(2*A*b + a*C)
*Log[x]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a(2bB+aD)+\frac{a^2A}{x^3}+\frac{a^2B}{x^2}+\frac{a(2Ab+aC)}{x}+b(Ab+2aC)x \right. \\ &\qquad \left. +b(bB+2aD)x^2+b^2Cx^3+b^2Dx^4\right)\,dx \\ &= -\frac{a^2A}{2x^2}-\frac{a^2B}{x}+a(2bB+aD)x+\frac{1}{2}b(Ab+2aC)x^2 \\ &\quad +\frac{1}{3}b(bB+2aD)x^3+\frac{1}{4}b^2Cx^4+\frac{1}{5}b^2Dx^5+a(2Ab+aC)\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.89

$$\begin{aligned}\int \frac{(a+bx^2)^2\,(A+Bx+Cx^2+Dx^3)}{x^3}\,dx &= -\frac{a^2(A+2Bx-2Dx^3)}{2x^2} \\ &\quad +\frac{1}{3}abx(6B+x(3C+2Dx)) \\ &\quad +\frac{1}{60}b^2x^2(30A+x(20B+3x(5C+4Dx))) \\ &\quad +a(2Ab+aC)\log(x)\end{aligned}$$

```
[In] Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^3,x]
[Out] -1/2*(a^2*(A + 2*B*x - 2*D*x^3))/x^2 + (a*b*x*(6*B + x*(3*C + 2*D*x)))/3 +
(b^2*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))))/60 + a*(2*A*b + a*C)*Log[x]
```

Maple [A] (verified)

Time = 3.45 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

method	result
default	$\frac{b^2Dx^5}{5}+\frac{b^2C\,x^4}{4}+\frac{b^2B\,x^3}{3}+\frac{2Dab\,x^3}{3}+\frac{A\,b^2x^2}{2}+Cab\,x^2+2Babx+Da^2x+a(2Ab+Ca)\ln(x)-$
norman	$\frac{(\frac{1}{3}B\,b^2+\frac{2}{3}Dab)x^5+(\frac{1}{2}b^2A+Cab)x^4+(2abB+Da^2)x^3-\frac{a^2A}{2}+\frac{C\,b^2x^6}{4}-a^2Bx+\frac{b^2Dx^7}{5}}{x^2}+(2abA+C\,a^2)\ln(x)$
parallelrisch	$\frac{12b^2Dx^7+15C\,b^2x^6+20b^2B\,x^5+40Dab\,x^5+30A\,b^2x^4+60Cab\,x^4+120A\ln(x)x^2ab+120Bab\,x^3+60C\ln(x)x^2a^2+60Da^2x^3-60a}{60x^2}$

```
[In] int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*b^2*D*x^5+1/4*b^2*C*x^4+1/3*b^2*B*x^3+2/3*D*a*b*x^3+1/2*A*b^2*x^2+C*a*b
*x^2+2*B*a*b*x+D*a^2*x+a*(2*A*b+C*a)*ln(x)-a^2*B/x-1/2*a^2*A/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{12\,Db^2x^7+15\,Cb^2x^6+20\,(2\,Dab+Bb^2)x^5+30\,(2\,Cab+Ab^2)x^4-60\,Ba^2x+60\,(Da^2+2\,Bab)x^3+60}{60\,x^2}$$

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/60*(12*D*b^2*x^7 + 15*C*b^2*x^6 + 20*(2*D*a*b + B*b^2)*x^5 + 30*(2*C*a*b
+ A*b^2)*x^4 - 60*B*a^2*x + 60*(D*a^2 + 2*B*a*b)*x^3 + 60*(C*a^2 + 2*A*a*b)
*x^2*log(x) - 30*A*a^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{Cb^2x^4}{4}+\frac{Db^2x^5}{5}+a(2Ab+Ca)\log{(x)} + x^3\left(\frac{Bb^2}{3}+\frac{2Dab}{3}\right)+x^2\left(\frac{Ab^2}{2}+Cab\right) + x\left(2Bab+Da^2\right)+\frac{-Aa^2-2Ba^2x}{2x^2}$$

```
[In] integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

```
[Out] C*b**2*x**4/4 + D*b**2*x**5/5 + a*(2*A*b + C*a)*log(x) + x**3*(B*b**2/3 + 2
*D*a*b/3) + x**2*(A*b**2/2 + C*a*b) + x*(2*B*a*b + D*a**2) + (-A*a**2 - 2*B
*a**2*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^3} dx = \frac{1}{5} Db^2x^5 + \frac{1}{4} Cb^2x^4 + \frac{1}{3} (2 Dab + Bb^2)x^3 + \frac{1}{2} (2 Cab + Ab^2)x^2 + (Da^2 + 2 Bab)x + (Ca^2 + 2 Aab) \log(x) - \frac{2 Ba^2x + Aa^2}{2 x^2}$$

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/5*D*b^2*x^5 + 1/4*C*b^2*x^4 + 1/3*(2*D*a*b + B*b^2)*x^3 + 1/2*(2*C*a*b +
A*b^2)*x^2 + (D*a^2 + 2*B*a*b)*x + (C*a^2 + 2*A*a*b)*log(x) - 1/2*(2*B*a^2*
x + A*a^2)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^3} dx = \frac{1}{5} Db^2x^5 + \frac{1}{4} Cb^2x^4 + \frac{2}{3} Dabx^3 + \frac{1}{3} Bb^2x^3 + Cabx^2 + \frac{1}{2} Ab^2x^2 + Da^2x + 2 Babx + (Ca^2 + 2 Aab) \log(|x|) - \frac{2 Ba^2x + Aa^2}{2 x^2}$$

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/5*D*b^2*x^5 + 1/4*C*b^2*x^4 + 2/3*D*a*b*x^3 + 1/3*B*b^2*x^3 + C*a*b*x^2 +
 1/2*A*b^2*x^2 + D*a^2*x + 2*B*a*b*x + (C*a^2 + 2*A*a*b)*log(abs(x)) - 1/2*
(2*B*a^2*x + A*a^2)/x^2
```

Mupad [B] (verification not implemented)

Time = 5.67 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{C\,(4\,a^2\,\ln(x)+b^2\,x^4+4\,a\,b\,x^2)}{4}+a^2\,x\,D + \frac{b^2\,x^5\,D}{5}+\frac{A\,(b^2\,x^4-a^2+4\,a\,b\,x^2\,\ln(x))}{2\,x^2} + \frac{B\,(-3\,a^2+6\,a\,b\,x^2+b^2\,x^4)}{3\,x}+\frac{2\,a\,b\,x^3\,D}{3}$$

```
[In] int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/x^3,x)
```

```
[Out] (C*(4*a^2*log(x) + b^2*x^4 + 4*a*b*x^2))/4 + a^2*x*D + (b^2*x^5*D)/5 + (A*(
b^2*x^4 - a^2 + 4*a*b*x^2*log(x)))/(2*x^2) + (B*(b^2*x^4 - 3*a^2 + 6*a*b*x^
2))/(3*x) + (2*a*b*x^3*D)/3
```

3.77 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$

Optimal result 498
Rubi [A] (verified) 498
Mathematica [A] (verified) 499
Maple [A] (verified) 499
Fricas [A] (verification not implemented) 500
Sympy [A] (verification not implemented) 500
Maxima [A] (verification not implemented) 501
Giac [A] (verification not implemented) 501
Mupad [B] (verification not implemented) 501

Optimal result

Integrand size = 28, antiderivative size = 98

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = -\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}-\frac{a(2Ab+aC)}{x}+b(Ab+2aC)x+\frac{1}{2}b(bB+2aD)x^2+\frac{1}{3}b^2Cx^3+\frac{1}{4}b^2Dx^4+a(2bB+aD)\log(x)$$

```
[Out] -1/3*a^2*A/x^3-1/2*a^2*B/x^2-a*(2*A*b+C*a)/x+b*(A*b+2*C*a)*x+1/2*b*(B*b+2*D
*a)*x^2+1/3*b^2*C*x^3+1/4*b^2*D*x^4+a*(2*B*b+D*a)*ln(x)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1816}

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = -\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+bx(2aC+Ab)-\frac{a(aC+2Ab)}{x}+\frac{1}{2}bx^2(2aD+bB)+a\log(x)(aD+2bB)+\frac{1}{3}b^2Cx^3+\frac{1}{4}b^2Dx^4$$

```
[In] Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -1/3*(a^2*A)/x^3 - (a^2*B)/(2*x^2) - (a*(2*A*b + a*C))/x + b*(A*b + 2*a*C)*
x + (b*(b*B + 2*a*D)*x^2)/2 + (b^2*C*x^3)/3 + (b^2*D*x^4)/4 + a*(2*b*B + a*
D)*Log[x]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(b(Ab+2aC) + \frac{a^2A}{x^4} + \frac{a^2B}{x^3} + \frac{a(2Ab+aC)}{x^2} + \frac{a(2bB+aD)}{x} + b(bB+2aD)x + b^2Cx^2 + b^2Dx^3\right) dx$$

$$= -\frac{a^2A}{3x^3} - \frac{a^2B}{2x^2} - \frac{a(2Ab+aC)}{x} + b(Ab+2aC)x + \frac{1}{2}b(bB+2aD)x^2 + \frac{1}{3}b^2Cx^3 + \frac{1}{4}b^2Dx^4 + a(2bB+aD)\log(x)$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^4}\,dx = -\frac{2aAb}{x} + abx(2C+Dx) - \frac{a^2(2A+3x(B+2Cx))}{6x^3} + \frac{1}{12}b^2x\left(12A+x\left(6B+4Cx+3Dx^2\right)\right) + a(2bB+aD)\log(x)$$

```
[In] Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^4,x]
[Out] (-2*a*A*b)/x + a*b*x*(2*C + D*x) - (a^2*(2*A + 3*x*(B + 2*C*x)))/(6*x^3) +
(b^2*x*(12*A + x*(6*B + 4*C*x + 3*D*x^2)))/12 + a*(2*b*B + a*D)*Log[x]
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

method	result
default	$\frac{b^2Dx^4}{4}+\frac{C b^2x^3}{3}+\frac{b^2B x^2}{2}+Dab\,x^2+A\,b^2x+2Cabx+a(2Bb+Da)\ln(x)-\frac{a^2A}{3x^3}-\frac{a(2Ab+Ca)}{x}$
norman	$\frac{\left(\frac{1}{2}B\,b^2+Dab\right)x^5+\left(b^2A+2Cab\right)x^4+\left(-2abA-C\,a^2\right)x^2-\frac{a^2A}{3}+\frac{C\,b^2x^6}{3}-\frac{a^2Bx}{2}+\frac{b^2Dx^7}{4}}{x^3}+\left(2abB+Da^2\right)\ln(x)$
parallelrisch	$\frac{3b^2Dx^7+4C\,b^2x^6+6b^2B\,x^5+12Dab\,x^5+12A\,b^2x^4+24B\ln(x)x^3ab+24Cab\,x^4+12D\ln(x)x^3a^2-24aAb\,x^2-12C\,a^2x^2-6a^2Bx-}{12x^3}$

```
[In] int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b^2*D*x^4+1/3*C*b^2*x^3+1/2*b^2*B*x^2+D*a*b*x^2+A*b^2*x+2*C*a*b*x+a*(2*
B*b+D*a)*ln(x)-1/3*a^2*A/x^3-a*(2*A*b+C*a)/x-1/2*a^2*B/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^4} dx$$
$$= \frac{3 Db^2x^7 + 4 Cb^2x^6 + 6 (2 Dab + Bb^2)x^5 + 12 (2 Cab + Ab^2)x^4 + 12 (Da^2 + 2 Bab)x^3 \log(x) - 6 Ba^2x - 4 A}{12 x^3}$$

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*D*b^2*x^7 + 4*C*b^2*x^6 + 6*(2*D*a*b + B*b^2)*x^5 + 12*(2*C*a*b + A
*b^2)*x^4 + 12*(D*a^2 + 2*B*a*b)*x^3*log(x) - 6*B*a^2*x - 4*A*a^2 - 12*(C*a
^2 + 2*A*a*b)*x^2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^4} dx = \frac{Cb^2x^3}{3} + \frac{Db^2x^4}{4} + a(2Bb+Da)\log(x) + x^2\left(\frac{Bb^2}{2} + Dab\right) + x\left(Ab^2 + 2Cab\right) + \frac{-2Aa^2 - 3Ba^2x + x^2(-12Aab - 6Ca^2)}{6x^3}$$

```
[In] integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

```
[Out] C*b**2*x**3/3 + D*b**2*x**4/4 + a*(2*B*b + D*a)*log(x) + x**2*(B*b**2/2 + D
*a*b) + x*(A*b**2 + 2*C*a*b) + (-2*A*a**2 - 3*B*a**2*x + x**2*(-12*A*a*b -
6*C*a**2))/(6*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{1}{4}Db^2x^4 + \frac{1}{3}Cb^2x^3 + \frac{1}{2}(2\,Dab+Bb^2)x^2 + (2\,Cab+Ab^2)x + (Da^2+2\,Bab)\log(x) - \frac{3\,Ba^2x + 2\,Aa^2 + 6\,(Ca^2+2\,Aab)x^2}{6\,x^3}$$

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/4*D*b^2*x^4 + 1/3*C*b^2*x^3 + 1/2*(2*D*a*b + B*b^2)*x^2 + (2*C*a*b + A*b^
2)*x + (D*a^2 + 2*B*a*b)*log(x) - 1/6*(3*B*a^2*x + 2*A*a^2 + 6*(C*a^2 + 2*A
*a*b)*x^2)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{1}{4}Db^2x^4 + \frac{1}{3}Cb^2x^3 + Dabx^2 + \frac{1}{2}Bb^2x^2 + 2\,Cabx + Ab^2x + (Da^2+2\,Bab)\log(|x|) - \frac{3\,Ba^2x + 2\,Aa^2 + 6\,(Ca^2+2\,Aab)x^2}{6\,x^3}$$

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/4*D*b^2*x^4 + 1/3*C*b^2*x^3 + D*a*b*x^2 + 1/2*B*b^2*x^2 + 2*C*a*b*x + A*b
^2*x + (D*a^2 + 2*B*a*b)*log(abs(x)) - 1/6*(3*B*a^2*x + 2*A*a^2 + 6*(C*a^2
+ 2*A*a*b)*x^2)/x^3
```

Mupad [B] (verification not implemented)

Time = 5.83 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{b^2\,x^4\,D}{4} + \frac{a^2\,\ln(x^2)\,D}{2} - \frac{A\,(a^2+6\,a\,b\,x^2-3\,b^2\,x^4)}{3\,x^3} + \frac{B\,(b^2\,x^4-a^2+4\,a\,b\,x^2\,\ln(x))}{2\,x^2} + \frac{C\,(-3\,a^2+6\,a\,b\,x^2+b^2\,x^4)}{3\,x} + a\,b\,x^2\,D$$

```
[In] int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/x^4,x)
```

```
[Out] (b^2*x^4*D)/4 + (a^2*log(x^2)*D)/2 - (A*(a^2 - 3*b^2*x^4 + 6*a*b*x^2))/(3*x
^3) + (B*(b^2*x^4 - a^2 + 4*a*b*x^2*log(x)))/(2*x^2) + (C*(b^2*x^4 - 3*a^2
+ 6*a*b*x^2))/(3*x) + a*b*x^2*D
```

3.78 $\int x^3(a+bx^2)^3\left(A+Bx+Cx^2+Dx^3\right)dx$

Optimal result . . . 503
Rubi [A] (verified) . . . 503
Mathematica [A] (verified) . . . 504
Maple [A] (verified) . . . 505
Fricas [A] (verification not implemented) . . . 505
Sympy [A] (verification not implemented) . . . 506
Maxima [A] (verification not implemented) . . . 506
Giac [A] (verification not implemented) . . . 507
Mupad [B] (verification not implemented) . . . 507

Optimal result

Integrand size = 28, antiderivative size = 149

$$\int x^3\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{4}a^3Ax^4+\frac{1}{5}a^3Bx^5+\frac{1}{6}a^2(3Ab+aC)x^6+\frac{1}{7}a^2(3bB+aD)x^7+\frac{3}{8}ab(Ab+aC)x^8+\frac{1}{3}ab(bB+aD)x^9+\frac{1}{10}b^2(Ab+3aC)x^{10}+\frac{1}{11}b^2(bB+3aD)x^{11}+\frac{1}{12}b^3Cx^{12}+\frac{1}{13}b^3Dx^{13}$$

```
[Out] 1/4*a^3*A*x^4+1/5*a^3*B*x^5+1/6*a^2*(3*A*b+C*a)*x^6+1/7*a^2*(3*B*b+D*a)*x^7
+3/8*a*b*(A*b+C*a)*x^8+1/3*a*b*(B*b+D*a)*x^9+1/10*b^2*(A*b+3*C*a)*x^10+1/11
*b^2*(B*b+3*D*a)*x^11+1/12*b^3*C*x^12+1/13*b^3*D*x^13
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1816}

$$\int x^3\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{4}a^3Ax^4+\frac{1}{5}a^3Bx^5+\frac{1}{6}a^2x^6(aC+3Ab)+\frac{1}{7}a^2x^7(aD+3bB)+\frac{1}{10}b^2x^{10}(3aC+Ab)+\frac{3}{8}abx^8(aC+Ab)+\frac{1}{11}b^2x^{11}(3aD+bB)+\frac{1}{3}abx^9(aD+bB)+\frac{1}{12}b^3Cx^{12}+\frac{1}{13}b^3Dx^{13}$$

```
[In] Int[x^3*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^3*A*x^4)/4 + (a^3*B*x^5)/5 + (a^2*(3*A*b + a*C)*x^6)/6 + (a^2*(3*b*B + a
*D)*x^7)/7 + (3*a*b*(A*b + a*C)*x^8)/8 + (a*b*(b*B + a*D)*x^9)/3 + (b^2*(A*
b + 3*a*C)*x^10)/10 + (b^2*(b*B + 3*a*D)*x^11)/11 + (b^3*C*x^12)/12 + (b^3*
D*x^13)/13
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^3Ax^3 + a^3Bx^4 + a^2(3Ab + aC)x^5 + a^2(3bB + aD)x^6 + 3ab(Ab + aC)x^7 \right.\\ &\quad \left. + 3ab(bB + aD)x^8 + b^2(Ab + 3aC)x^9 + b^2(bB + 3aD)x^{10} + b^3Cx^{11} + b^3Dx^{12}\right) dx \\ &= \frac{1}{4}a^3Ax^4 + \frac{1}{5}a^3Bx^5 + \frac{1}{6}a^2(3Ab + aC)x^6 + \frac{1}{7}a^2(3bB + aD)x^7 + \frac{3}{8}ab(Ab + aC)x^8 \\ &\quad + \frac{1}{3}ab(bB + aD)x^9 + \frac{1}{10}b^2(Ab + 3aC)x^{10} + \frac{1}{11}b^2(bB + 3aD)x^{11} + \frac{1}{12}b^3Cx^{12} \\ &\quad + \frac{1}{13}b^3Dx^{13}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^3(a + bx^2)^3 \left(A + Bx + Cx^2 + Dx^3\right) dx &= \frac{1}{4}a^3Ax^4 + \frac{1}{5}a^3Bx^5 + \frac{1}{6}a^2(3Ab + aC)x^6 \\ &\quad + \frac{1}{7}a^2(3bB + aD)x^7 + \frac{3}{8}ab(Ab + aC)x^8 \\ &\quad + \frac{1}{3}ab(bB + aD)x^9 + \frac{1}{10}b^2(Ab + 3aC)x^{10} \\ &\quad + \frac{1}{11}b^2(bB + 3aD)x^{11} + \frac{1}{12}b^3Cx^{12} + \frac{1}{13}b^3Dx^{13}\end{aligned}$$

```
[In] Integrate[x^3*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^3*A*x^4)/4 + (a^3*B*x^5)/5 + (a^2*(3*A*b + a*C)*x^6)/6 + (a^2*(3*b*B + a
*D)*x^7)/7 + (3*a*b*(A*b + a*C)*x^8)/8 + (a*b*(b*B + a*D)*x^9)/3 + (b^2*(A*
b + 3*a*C)*x^10)/10 + (b^2*(b*B + 3*a*D)*x^11)/11 + (b^3*C*x^12)/12 + (b^3*
D*x^13)/13
```

Maple [A] (verified)

Time = 3.58 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99

method	result
norman	$\frac{b^3 D x^{13}}{13}+\frac{b^3 C x^{12}}{12}+\left(\frac{1}{11} B b^3+\frac{3}{11} a b^2 D\right) x^{11}+\left(\frac{1}{10} b^3 A+\frac{3}{10} C b^2 a\right) x^{10}+\left(\frac{1}{3} a b^2 B+\frac{1}{3} D a^2 b\right) x^9$
default	$\frac{b^3 D x^{13}}{13}+\frac{b^3 C x^{12}}{12}+\frac{\left(B b^3+3 a b^2 D\right) x^{11}}{11}+\frac{\left(b^3 A+3 C b^2 a\right) x^{10}}{10}+\frac{\left(3 a b^2 B+3 D a^2 b\right) x^9}{9}+\frac{\left(3 a b^2 A+3 C a^2 b\right) x^8}{8}+$
gosper	$\frac{1}{13} b^3 D x^{13}+\frac{1}{12} b^3 C x^{12}+\frac{1}{11} x^{11} B b^3+\frac{3}{11} x^{11} a b^2 D+\frac{1}{10} x^{10} b^3 A+\frac{3}{10} x^{10} C b^2 a+\frac{1}{3} x^9 a b^2 B+\frac{1}{3} x$
parallelrisch	$\frac{1}{13} b^3 D x^{13}+\frac{1}{12} b^3 C x^{12}+\frac{1}{11} x^{11} B b^3+\frac{3}{11} x^{11} a b^2 D+\frac{1}{10} x^{10} b^3 A+\frac{3}{10} x^{10} C b^2 a+\frac{1}{3} x^9 a b^2 B+\frac{1}{3} x$

```
[In] int(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b^3*D*x^13+1/12*b^3*C*x^12+(1/11*B*b^3+3/11*a*b^2*D)*x^11+(1/10*b^3*A+
3/10*C*b^2*a)*x^10+(1/3*a*b^2*B+1/3*D*a^2*b)*x^9+(3/8*a*b^2*A+3/8*C*a^2*b)*
x^8+(3/7*a^2*b*B+1/7*D*a^3)*x^7+(1/2*a^2*b*A+1/6*C*a^3)*x^6+1/5*a^3*B*x^5+1
/4*a^3*A*x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int x^3\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{13}Db^3x^{13}+\frac{1}{12}Cb^3x^{12}+\frac{1}{11}\left(3\,Dab^2+Bb^3\right)x^{11}+\frac{1}{10}\left(3\,Cab^2+Ab^3\right)x^{10}+\frac{1}{3}\left(Da^2b+Bab^2\right)x^9+\frac{1}{5}Ba^3x^5+\frac{3}{8}\left(Ca^2b+Aab^2\right)x^8+\frac{1}{4}Aa^3x^4+\frac{1}{7}\left(Da^3+3\,Ba^2b\right)x^7+\frac{1}{6}\left(Ca^3+3\,Aa^2b\right)x^6$$

```
[In] integrate(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/13*D*b^3*x^13 + 1/12*C*b^3*x^12 + 1/11*(3*D*a*b^2 + B*b^3)*x^11 + 1/10*(3
*C*a*b^2 + A*b^3)*x^10 + 1/3*(D*a^2*b + B*a*b^2)*x^9 + 1/5*B*a^3*x^5 + 3/8*
(C*a^2*b + A*a*b^2)*x^8 + 1/4*A*a^3*x^4 + 1/7*(D*a^3 + 3*B*a^2*b)*x^7 + 1/6
*(C*a^3 + 3*A*a^2*b)*x^6
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.09

$$\int x^3(a+bx^2)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{Aa^3x^4}{4}+\frac{Ba^3x^5}{5}+\frac{Cb^3x^{12}}{12}+\frac{Db^3x^{13}}{13}+x^{11}\left(\frac{Bb^3}{11}+\frac{3Dab^2}{11}\right)+x^{10}\left(\frac{Ab^3}{10}+\frac{3Cab^2}{10}\right)+x^9\left(\frac{Bab^2}{3}+\frac{Da^2b}{3}\right)+x^8\cdot\left(\frac{3Aab^2}{8}+\frac{3Ca^2b}{8}\right)+x^7\cdot\left(\frac{3Ba^2b}{7}+\frac{Da^3}{7}\right)+x^6\left(\frac{Aa^2b}{2}+\frac{Ca^3}{6}\right)$$

```
[In] integrate(x**3*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x**4/4 + B*a**3*x**5/5 + C*b**3*x**12/12 + D*b**3*x**13/13 + x**11*(
B*b**3/11 + 3*D*a*b**2/11) + x**10*(A*b**3/10 + 3*C*a*b**2/10) + x**9*(B*a*
b**2/3 + D*a**2*b/3) + x**8*(3*A*a*b**2/8 + 3*C*a**2*b/8) + x**7*(3*B*a**2*
b/7 + D*a**3/7) + x**6*(A*a**2*b/2 + C*a**3/6)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int x^3(a+bx^2)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{13}\,Db^3x^{13}+\frac{1}{12}\,Cb^3x^{12}+\frac{1}{11}\,(3\,Dab^2+Bb^3)x^{11}+\frac{1}{10}\,(3\,Cab^2+Ab^3)x^{10}+\frac{1}{3}\,(Da^2b+Bab^2)x^9+\frac{1}{5}\,Ba^3x^5+\frac{3}{8}\,(Ca^2b+Aab^2)x^8+\frac{1}{4}\,Aa^3x^4+\frac{1}{7}\,(Da^3+3\,Ba^2b)x^7+\frac{1}{6}\,(Ca^3+3\,Aa^2b)x^6$$

```
[In] integrate(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/13*D*b^3*x^13 + 1/12*C*b^3*x^12 + 1/11*(3*D*a*b^2 + B*b^3)*x^11 + 1/10*(3
*C*a*b^2 + A*b^3)*x^10 + 1/3*(D*a^2*b + B*a*b^2)*x^9 + 1/5*B*a^3*x^5 + 3/8*
(C*a^2*b + A*a*b^2)*x^8 + 1/4*A*a^3*x^4 + 1/7*(D*a^3 + 3*B*a^2*b)*x^7 + 1/6
*(C*a^3 + 3*A*a^2*b)*x^6
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\int x^3(a+bx^2)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{13}\,Db^3x^{13}+\frac{1}{12}\,Cb^3x^{12}+\frac{3}{11}\,Dab^2x^{11}+\frac{1}{11}\,Bb^3x^{11}+\frac{3}{10}\,Cab^2x^{10}+\frac{1}{10}\,Ab^3x^{10}+\frac{1}{3}\,Da^2bx^9+\frac{1}{3}\,Bab^2x^9+\frac{3}{8}\,Ca^2bx^8+\frac{3}{8}\,Aab^2x^8+\frac{1}{7}\,Da^3x^7+\frac{3}{7}\,Ba^2bx^7+\frac{1}{6}\,Ca^3x^6+\frac{1}{2}\,Aa^2bx^6+\frac{1}{5}\,Ba^3x^5+\frac{1}{4}\,Aa^3x^4$$

```
[In] integrate(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/13*D*b^3*x^13 + 1/12*C*b^3*x^12 + 3/11*D*a*b^2*x^11 + 1/11*B*b^3*x^11 + 3
/10*C*a*b^2*x^10 + 1/10*A*b^3*x^10 + 1/3*D*a^2*b*x^9 + 1/3*B*a*b^2*x^9 + 3/
8*C*a^2*b*x^8 + 3/8*A*a*b^2*x^8 + 1/7*D*a^3*x^7 + 3/7*B*a^2*b*x^7 + 1/6*C*a
^3*x^6 + 1/2*A*a^2*b*x^6 + 1/5*B*a^3*x^5 + 1/4*A*a^3*x^4
```

Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\int x^3(a+bx^2)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{A\,a^3\,x^4}{4}+\frac{B\,a^3\,x^5}{5}+\frac{A\,b^3\,x^{10}}{10}+\frac{C\,a^3\,x^6}{6}+\frac{B\,b^3\,x^{11}}{11}+\frac{C\,b^3\,x^{12}}{12}+\frac{a^3\,x^7\,D}{7}+\frac{b^3\,x^{13}\,D}{13}+\frac{a^2\,b\,x^9\,D}{3}+\frac{3\,a\,b^2\,x^{11}\,D}{11}+\frac{A\,a^2\,b\,x^6}{2}+\frac{3\,A\,a\,b^2\,x^8}{8}+\frac{3\,B\,a^2\,b\,x^7}{7}+\frac{B\,a\,b^2\,x^9}{3}+\frac{3\,C\,a^2\,b\,x^8}{8}+\frac{3\,C\,a\,b^2\,x^{10}}{10}$$

```
[In] int(x^3*(a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (A*a^3*x^4)/4 + (B*a^3*x^5)/5 + (A*b^3*x^10)/10 + (C*a^3*x^6)/6 + (B*b^3*x^
11)/11 + (C*b^3*x^12)/12 + (a^3*x^7*D)/7 + (b^3*x^13*D)/13 + (a^2*b*x^9*D)/
3 + (3*a*b^2*x^11*D)/11 + (A*a^2*b*x^6)/2 + (3*A*a*b^2*x^8)/8 + (3*B*a^2*b*
x^7)/7 + (B*a*b^2*x^9)/3 + (3*C*a^2*b*x^8)/8 + (3*C*a*b^2*x^10)/10
```

3.79 $\int x^2(a+bx^2)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal result 508
Rubi [A] (verified) 508
Mathematica [A] (verified) 509
Maple [A] (verified) 510
Fricas [A] (verification not implemented) 510
Sympy [A] (verification not implemented) 511
Maxima [A] (verification not implemented) 511
Giac [A] (verification not implemented) 512
Mupad [B] (verification not implemented) 512

Optimal result

Integrand size = 28, antiderivative size = 149

$$\int x^2\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{3}a^3Ax^3+\frac{1}{4}a^3Bx^4+\frac{1}{5}a^2(3Ab+aC)x^5+\frac{1}{6}a^2(3bB+aD)x^6+\frac{3}{7}ab(Ab+aC)x^7+\frac{3}{8}ab(bB+aD)x^8+\frac{1}{9}b^2(Ab+3aC)x^9+\frac{1}{10}b^2(bB+3aD)x^{10}+\frac{1}{11}b^3Cx^{11}+\frac{1}{12}b^3Dx^{12}$$

```
[Out] 1/3*a^3*A*x^3+1/4*a^3*B*x^4+1/5*a^2*(3*A*b+C*a)*x^5+1/6*a^2*(3*B*b+D*a)*x^6
+3/7*a*b*(A*b+C*a)*x^7+3/8*a*b*(B*b+D*a)*x^8+1/9*b^2*(A*b+3*C*a)*x^9+1/10*b
^2*(B*b+3*D*a)*x^10+1/11*b^3*C*x^11+1/12*b^3*D*x^12
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1816}

$$\int x^2\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{3}a^3Ax^3+\frac{1}{4}a^3Bx^4+\frac{1}{5}a^2x^5(aC+3Ab)+\frac{1}{6}a^2x^6(aD+3bB)+\frac{1}{9}b^2x^9(3aC+Ab)+\frac{3}{7}abx^7(aC+Ab)+\frac{1}{10}b^2x^{10}(3aD+bB)+\frac{3}{8}abx^8(aD+bB)+\frac{1}{11}b^3Cx^{11}+\frac{1}{12}b^3Dx^{12}$$

```
[In] Int[x^2*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^3*A*x^3)/3 + (a^3*B*x^4)/4 + (a^2*(3*A*b + a*C)*x^5)/5 + (a^2*(3*b*B + a
*D)*x^6)/6 + (3*a*b*(A*b + a*C)*x^7)/7 + (3*a*b*(b*B + a*D)*x^8)/8 + (b^2*(
A*b + 3*a*C)*x^9)/9 + (b^2*(b*B + 3*a*D)*x^10)/10 + (b^3*C*x^11)/11 + (b^3*
D*x^12)/12
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^3Ax^2+a^3Bx^3+a^2(3Ab+aC)x^4+a^2(3bB+aD)x^5+3ab(Ab+aC)x^6\right.\\&\quad\left.+3ab(bB+aD)x^7+b^2(Ab+3aC)x^8+b^2(bB+3aD)x^9+b^3Cx^{10}+b^3Dx^{11}\right)\,dx\\&=\frac{1}{3}a^3Ax^3+\frac{1}{4}a^3Bx^4+\frac{1}{5}a^2(3Ab+aC)x^5+\frac{1}{6}a^2(3bB+aD)x^6+\frac{3}{7}ab(Ab+aC)x^7\\&\quad+\frac{3}{8}ab(bB+aD)x^8+\frac{1}{9}b^2(Ab+3aC)x^9+\frac{1}{10}b^2(bB+3aD)x^{10}+\frac{1}{11}b^3Cx^{11}+\frac{1}{12}b^3Dx^{12}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int x^2\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{14b^3x^9(220A+3x(66B+60Cx+55Dx^2))+462a^3x^3(20A+x(15B+2x(6C+5Dx)))+99a^2bx^5(168A}{27720}$$

```
[In] Integrate[x^2*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (14*b^3*x^9*(220*A + 3*x*(66*B + 60*C*x + 55*D*x^2)) + 462*a^3*x^3*(20*A +
x*(15*B + 2*x*(6*C + 5*D*x))) + 99*a^2*b*x^5*(168*A + 5*x*(28*B + 3*x*(8*C
+ 7*D*x))) + 33*a*b^2*x^7*(360*A + 7*x*(45*B + 4*x*(10*C + 9*D*x))))/27720
```

Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99

method	result
norman	$\frac{b^3Dx^{12}}{12}+\frac{b^3C\,x^{11}}{11}+\left(\frac{1}{10}B\,b^3+\frac{3}{10}a\,b^2D\right)x^{10}+\left(\frac{1}{9}b^3A+\frac{1}{3}C\,b^2a\right)x^9+\left(\frac{3}{8}a\,b^2B+\frac{3}{8}Da^2b\right)x^8+\left(\frac{3}{7}\right.$
default	$\frac{b^3Dx^{12}}{12}+\frac{b^3C\,x^{11}}{11}+\frac{(B\,b^3+3a\,b^2D)x^{10}}{10}+\frac{(b^3A+3C\,b^2a)x^9}{9}+\frac{(3a\,b^2B+3Da^2b)x^8}{8}+\frac{(3a\,b^2A+3C\,a^2b)x^7}{7}+\frac{(3a^2bB}{}$
gosper	$\frac{1}{12}b^3Dx^{12}+\frac{1}{11}b^3C\,x^{11}+\frac{1}{10}x^{10}B\,b^3+\frac{3}{10}x^{10}a\,b^2D+\frac{1}{9}x^9b^3A+\frac{1}{3}x^9C\,b^2a+\frac{3}{8}x^8a\,b^2B+\frac{3}{8}x^8Da^2$
parallelrisch	$\frac{1}{12}b^3Dx^{12}+\frac{1}{11}b^3C\,x^{11}+\frac{1}{10}x^{10}B\,b^3+\frac{3}{10}x^{10}a\,b^2D+\frac{1}{9}x^9b^3A+\frac{1}{3}x^9C\,b^2a+\frac{3}{8}x^8a\,b^2B+\frac{3}{8}x^8Da^2$

```
[In] int(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b^3*D*x^12+1/11*b^3*C*x^11+(1/10*B*b^3+3/10*a*b^2*D)*x^10+(1/9*b^3*A+1
/3*C*b^2*a)*x^9+(3/8*a*b^2*B+3/8*D*a^2*b)*x^8+(3/7*a*b^2*A+3/7*C*a^2*b)*x^7
+(1/2*a^2*b*B+1/6*D*a^3)*x^6+(3/5*a^2*b*A+1/5*C*a^3)*x^5+1/4*a^3*B*x^4+1/3*
a^3*A*x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int x^2\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{12}\,Db^3x^{12}+\frac{1}{11}\,Cb^3x^{11}+\frac{1}{10}\,\left(3\,Dab^2+Bb^3\right)x^{10}+\frac{1}{9}\,\left(3\,Cab^2+Ab^3\right)x^9+\frac{3}{8}\,\left(Da^2b+Bab^2\right)x^8+\frac{1}{4}\,Ba^3x^4+\frac{3}{7}\,\left(Ca^2b+Aab^2\right)x^7+\frac{1}{3}\,Aa^3x^3+\frac{1}{6}\,\left(Da^3+3\,Ba^2b\right)x^6+\frac{1}{5}\,\left(Ca^3+3\,Aa^2b\right)x^5$$

```
[In] integrate(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/12*D*b^3*x^12 + 1/11*C*b^3*x^11 + 1/10*(3*D*a*b^2 + B*b^3)*x^10 + 1/9*(3*
C*a*b^2 + A*b^3)*x^9 + 3/8*(D*a^2*b + B*a*b^2)*x^8 + 1/4*B*a^3*x^4 + 3/7*(C
*a^2*b + A*a*b^2)*x^7 + 1/3*A*a^3*x^3 + 1/6*(D*a^3 + 3*B*a^2*b)*x^6 + 1/5*(
C*a^3 + 3*A*a^2*b)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11

$$\int x^2(a+bx^2)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{Aa^3x^3}{3}+\frac{Ba^3x^4}{4}+\frac{Cb^3x^{11}}{11}+\frac{Db^3x^{12}}{12}+x^{10}\left(\frac{Bb^3}{10}+\frac{3Dab^2}{10}\right)+x^9\left(\frac{Ab^3}{9}+\frac{Cab^2}{3}\right)+x^8\cdot\left(\frac{3Bab^2}{8}+\frac{3Da^2b}{8}\right)+x^7\cdot\left(\frac{3Aab^2}{7}+\frac{3Ca^2b}{7}\right)+x^6\left(\frac{Ba^2b}{2}+\frac{Da^3}{6}\right)+x^5\cdot\left(\frac{3Aa^2b}{5}+\frac{Ca^3}{5}\right)$$

```
[In] integrate(x**2*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x**3/3 + B*a**3*x**4/4 + C*b**3*x**11/11 + D*b**3*x**12/12 + x**10*(
B*b**3/10 + 3*D*a*b**2/10) + x**9*(A*b**3/9 + C*a*b**2/3) + x**8*(3*B*a*b**
2/8 + 3*D*a**2*b/8) + x**7*(3*A*a*b**2/7 + 3*C*a**2*b/7) + x**6*(B*a**2*b/2
 + D*a**3/6) + x**5*(3*A*a**2*b/5 + C*a**3/5)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int x^2(a+bx^2)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{12}\,Db^3x^{12}+\frac{1}{11}\,Cb^3x^{11}+\frac{1}{10}\,(3\,Dab^2+Bb^3)x^{10}+\frac{1}{9}\,(3\,Cab^2+Ab^3)x^9+\frac{3}{8}\,(Da^2b+Bab^2)x^8+\frac{1}{4}\,Ba^3x^4+\frac{3}{7}\,(Ca^2b+Aab^2)x^7+\frac{1}{3}\,Aa^3x^3+\frac{1}{6}\,(Da^3+3\,Ba^2b)x^6+\frac{1}{5}\,(Ca^3+3\,Aa^2b)x^5$$

```
[In] integrate(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/12*D*b^3*x^12 + 1/11*C*b^3*x^11 + 1/10*(3*D*a*b^2 + B*b^3)*x^10 + 1/9*(3*
C*a*b^2 + A*b^3)*x^9 + 3/8*(D*a^2*b + B*a*b^2)*x^8 + 1/4*B*a^3*x^4 + 3/7*(C
*a^2*b + A*a*b^2)*x^7 + 1/3*A*a^3*x^3 + 1/6*(D*a^3 + 3*B*a^2*b)*x^6 + 1/5*(
C*a^3 + 3*A*a^2*b)*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\int x^2\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{12}Db^3x^{12}+\frac{1}{11}Cb^3x^{11}+\frac{3}{10}Dab^2x^{10}+\frac{1}{10}Bb^3x^{10}+\frac{1}{3}Cab^2x^9+\frac{1}{9}Ab^3x^9+\frac{3}{8}Da^2bx^8+\frac{3}{8}Bab^2x^8+\frac{3}{7}Ca^2bx^7+\frac{3}{7}Aab^2x^7+\frac{1}{6}Da^3x^6+\frac{1}{2}Ba^2bx^6+\frac{1}{5}Ca^3x^5+\frac{3}{5}Aa^2bx^5+\frac{1}{4}Ba^3x^4+\frac{1}{3}Aa^3x^3$$

```
[In] integrate(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/12*D*b^3*x^12 + 1/11*C*b^3*x^11 + 3/10*D*a*b^2*x^10 + 1/10*B*b^3*x^10 + 1
/3*C*a*b^2*x^9 + 1/9*A*b^3*x^9 + 3/8*D*a^2*b*x^8 + 3/8*B*a*b^2*x^8 + 3/7*C*
a^2*b*x^7 + 3/7*A*a*b^2*x^7 + 1/6*D*a^3*x^6 + 1/2*B*a^2*b*x^6 + 1/5*C*a^3*x
^5 + 3/5*A*a^2*b*x^5 + 1/4*B*a^3*x^4 + 1/3*A*a^3*x^3
```

Mupad [B] (verification not implemented)

Time = 5.85 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\int x^2\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{A\,a^3\,x^3}{3}+\frac{B\,a^3\,x^4}{4}+\frac{A\,b^3\,x^9}{9}+\frac{C\,a^3\,x^5}{5}+\frac{B\,b^3\,x^{10}}{10}+\frac{C\,b^3\,x^{11}}{11}+\frac{a^3\,x^6\,D}{6}+\frac{b^3\,x^{12}\,D}{12}+\frac{3\,a^2\,b\,x^8\,D}{8}+\frac{3\,a\,b^2\,x^{10}\,D}{10}+\frac{3\,A\,a^2\,b\,x^5}{5}+\frac{3\,A\,a\,b^2\,x^7}{7}+\frac{B\,a^2\,b\,x^6}{2}+\frac{3\,B\,a\,b^2\,x^8}{8}+\frac{3\,C\,a^2\,b\,x^7}{7}+\frac{C\,a\,b^2\,x^9}{3}$$

```
[In] int(x^2*(a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (A*a^3*x^3)/3 + (B*a^3*x^4)/4 + (A*b^3*x^9)/9 + (C*a^3*x^5)/5 + (B*b^3*x^10
)/10 + (C*b^3*x^11)/11 + (a^3*x^6*D)/6 + (b^3*x^12*D)/12 + (3*a^2*b*x^8*D)/
8 + (3*a*b^2*x^10*D)/10 + (3*A*a^2*b*x^5)/5 + (3*A*a*b^2*x^7)/7 + (B*a^2*b*
x^6)/2 + (3*B*a*b^2*x^8)/8 + (3*C*a^2*b*x^7)/7 + (C*a*b^2*x^9)/3
```

3.80 $\int x(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\, dx$

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Optimal result

Integrand size = 26, antiderivative size = 138

$$\int x(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\, dx = \frac{1}{3}a^3Bx^3 + \frac{1}{4}a^3Cx^4 + \frac{1}{5}a^2(3bB+aD)x^5 + \frac{1}{2}a^2bCx^6 + \frac{3}{7}ab(bB+aD)x^7 + \frac{3}{8}ab^2Cx^8 + \frac{1}{9}b^2(bB+3aD)x^9 + \frac{1}{10}b^3Cx^{10} + \frac{1}{11}b^3Dx^{11} + \frac{A(a+bx^2)^4}{8b}$$

```
[Out] 1/3*a^3*B*x^3+1/4*a^3*C*x^4+1/5*a^2*(3*B*b+D*a)*x^5+1/2*a^2*b*C*x^6+3/7*a*b
*(B*b+D*a)*x^7+3/8*a*b^2*C*x^8+1/9*b^2*(B*b+3*D*a)*x^9+1/10*b^3*C*x^10+1/11
*b^3*D*x^11+1/8*A*(b*x^2+a)^4/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1596, 1824}

$$\int x(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\, dx = \frac{1}{3}a^3Bx^3 + \frac{1}{4}a^3Cx^4 + \frac{1}{5}a^2x^5(aD+3bB) + \frac{1}{2}a^2bCx^6 + \frac{A(a+bx^2)^4}{8b} + \frac{1}{9}b^2x^9(3aD+bB) + \frac{3}{8}ab^2Cx^8 + \frac{3}{7}abx^7(aD+bB) + \frac{1}{10}b^3Cx^{10} + \frac{1}{11}b^3Dx^{11}$$

```
[In] Int[x*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^3*B*x^3)/3 + (a^3*C*x^4)/4 + (a^2*(3*b*B + a*D)*x^5)/5 + (a^2*b*C*x^6)/2
 + (3*a*b*(b*B + a*D)*x^7)/7 + (3*a*b^2*C*x^8)/8 + (b^2*(b*B + 3*a*D)*x^9)/
9 + (b^3*C*x^10)/10 + (b^3*D*x^11)/11 + (A*(a + b*x^2)^4)/(8*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{A(a+bx^2)^4}{8b}+\int (a+bx^2)^3\left(-Ax+x\left(A+Bx+Cx^2+Dx^3\right)\right)\,dx\\&=\frac{A(a+bx^2)^4}{8b}+\int \left(a^3Bx^2+a^3Cx^3+a^2(3bB+aD)x^4+3a^2bCx^5+3ab(bB+aD)x^6\right.\\&\qquad\left.+3ab^2Cx^7+b^2(bB+3aD)x^8+b^3Cx^9+b^3Dx^{10}\right)\,dx\\&=\frac{1}{3}a^3Bx^3+\frac{1}{4}a^3Cx^4+\frac{1}{5}a^2(3bB+aD)x^5+\frac{1}{2}a^2bCx^6+\frac{3}{7}ab(bB+aD)x^7\\&\quad+\frac{3}{8}ab^2Cx^8+\frac{1}{9}b^2(bB+3aD)x^9+\frac{1}{10}b^3Cx^{10}+\frac{1}{11}b^3Dx^{11}+\frac{A(a+bx^2)^4}{8b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.90

$$\int x(a+bx^2)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{7b^3x^8(495A+4x(110B+99Cx+90Dx^2))+462a^3x^2(30A+x(20B+3x(5C+4Dx)))+198a^2bx^4(105A}{27720}$$

```
[In] Integrate[x*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (7*b^3*x^8*(495*A + 4*x*(110*B + 99*C*x + 90*D*x^2)) + 462*a^3*x^2*(30*A +
x*(20*B + 3*x*(5*C + 4*D*x))) + 198*a^2*b*x^4*(105*A + 2*x*(42*B + 5*x*(7*C
 + 6*D*x))) + 165*a*b^2*x^6*(84*A + x*(72*B + 7*x*(9*C + 8*D*x))))/27720
```

Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.07

method	result
norman	$\frac{b^3 D x^{11}}{11}+\frac{b^3 C x^{10}}{10}+\left(\frac{1}{9} B b^3+\frac{1}{3} a b^2 D\right) x^9+\left(\frac{1}{8} b^3 A+\frac{3}{8} C b^2 a\right) x^8+\left(\frac{3}{7} a b^2 B+\frac{3}{7} D a^2 b\right) x^7+\left(\frac{1}{2} a\right.$
default	$\frac{b^3 D x^{11}}{11}+\frac{b^3 C x^{10}}{10}+\frac{\left(B b^3+3 a b^2 D\right) x^9}{9}+\frac{\left(b^3 A+3 C b^2 a\right) x^8}{8}+\frac{\left(3 a b^2 B+3 D a^2 b\right) x^7}{7}+\frac{\left(3 a b^2 A+3 C a^2 b\right) x^6}{6}+\frac{(3 a^2 b B}{}$
gosper	$\frac{1}{11} b^3 D x^{11}+\frac{1}{10} b^3 C x^{10}+\frac{1}{9} b^3 B x^9+\frac{1}{3} x^9 a b^2 D+\frac{1}{8} x^8 b^3 A+\frac{3}{8} a b^2 C x^8+\frac{3}{7} x^7 a b^2 B+\frac{3}{7} x^7 D a^2 b$
parallelrisch	$\frac{1}{11} b^3 D x^{11}+\frac{1}{10} b^3 C x^{10}+\frac{1}{9} b^3 B x^9+\frac{1}{3} x^9 a b^2 D+\frac{1}{8} x^8 b^3 A+\frac{3}{8} a b^2 C x^8+\frac{3}{7} x^7 a b^2 B+\frac{3}{7} x^7 D a^2 b$

```
[In] int(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b^3*D*x^11+1/10*b^3*C*x^10+(1/9*B*b^3+1/3*a*b^2*D)*x^9+(1/8*b^3*A+3/8*
C*b^2*a)*x^8+(3/7*a*b^2*B+3/7*D*a^2*b)*x^7+(1/2*a*b^2*A+1/2*C*a^2*b)*x^6+(3
/5*a^2*b*B+1/5*D*a^3)*x^5+(3/4*a^2*b*A+1/4*C*a^3)*x^4+1/3*a^3*B*x^3+1/2*a^3
*A*x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.05

$$\begin{aligned}\int x\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx =& \frac{1}{11}\,Db^3x^{11}+\frac{1}{10}\,Cb^3x^{10}+\frac{1}{9}\,\left(3\,Dab^2+Bb^3\right)x^9\\&+\frac{1}{8}\,\left(3\,Cab^2+Ab^3\right)x^8+\frac{3}{7}\,\left(Da^2b+Bab^2\right)x^7\\&+\frac{1}{3}\,Ba^3x^3+\frac{1}{2}\,\left(Ca^2b+Aab^2\right)x^6+\frac{1}{2}\,Aa^3x^2\\&+\frac{1}{5}\,\left(Da^3+3\,Ba^2b\right)x^5+\frac{1}{4}\,\left(Ca^3+3\,Aa^2b\right)x^4\end{aligned}$$

```
[In] integrate(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/11*D*b^3*x^11 + 1/10*C*b^3*x^10 + 1/9*(3*D*a*b^2 + B*b^3)*x^9 + 1/8*(3*C*
a*b^2 + A*b^3)*x^8 + 3/7*(D*a^2*b + B*a*b^2)*x^7 + 1/3*B*a^3*x^3 + 1/2*(C*a
^2*b + A*a*b^2)*x^6 + 1/2*A*a^3*x^2 + 1/5*(D*a^3 + 3*B*a^2*b)*x^5 + 1/4*(C*
a^3 + 3*A*a^2*b)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.18

$$\int x(a+bx^2)^3(A+Bx+Cx^2+Dx^3)\,dx = \frac{Aa^3x^2}{2}+\frac{Ba^3x^3}{3}+\frac{Cb^3x^{10}}{10}+\frac{Db^3x^{11}}{11}+x^9\left(\frac{Bb^3}{9}+\frac{Dab^2}{3}\right)+x^8\left(\frac{Ab^3}{8}+\frac{3Cab^2}{8}\right)+x^7\cdot\left(\frac{3Bab^2}{7}+\frac{3Da^2b}{7}\right)+x^6\left(\frac{Aab^2}{2}+\frac{Ca^2b}{2}\right)+x^5\cdot\left(\frac{3Ba^2b}{5}+\frac{Da^3}{5}\right)+x^4\cdot\left(\frac{3Aa^2b}{4}+\frac{Ca^3}{4}\right)$$

```
[In] integrate(x*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x**2/2 + B*a**3*x**3/3 + C*b**3*x**10/10 + D*b**3*x**11/11 + x**9*(B
*b**3/9 + D*a*b**2/3) + x**8*(A*b**3/8 + 3*C*a*b**2/8) + x**7*(3*B*a*b**2/7
 + 3*D*a**2*b/7) + x**6*(A*a*b**2/2 + C*a**2*b/2) + x**5*(3*B*a**2*b/5 + D*
a**3/5) + x**4*(3*A*a**2*b/4 + C*a**3/4)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.05

$$\int x(a+bx^2)^3(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{11}\,Db^3x^{11}+\frac{1}{10}\,Cb^3x^{10}+\frac{1}{9}\,(3\,Dab^2+Bb^3)x^9+\frac{1}{8}\,(3\,Cab^2+Ab^3)x^8+\frac{3}{7}\,(Da^2b+Bab^2)x^7+\frac{1}{3}\,Ba^3x^3+\frac{1}{2}\,(Ca^2b+Aab^2)x^6+\frac{1}{2}\,Aa^3x^2+\frac{1}{5}\,(Da^3+3\,Ba^2b)x^5+\frac{1}{4}\,(Ca^3+3\,Aa^2b)x^4$$

```
[In] integrate(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/11*D*b^3*x^11 + 1/10*C*b^3*x^10 + 1/9*(3*D*a*b^2 + B*b^3)*x^9 + 1/8*(3*C*
a*b^2 + A*b^3)*x^8 + 3/7*(D*a^2*b + B*a*b^2)*x^7 + 1/3*B*a^3*x^3 + 1/2*(C*a
^2*b + A*a*b^2)*x^6 + 1/2*A*a^3*x^2 + 1/5*(D*a^3 + 3*B*a^2*b)*x^5 + 1/4*(C*
a^3 + 3*A*a^2*b)*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11

$$\int x\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{11}\,Db^3x^{11}+\frac{1}{10}\,Cb^3x^{10}+\frac{1}{3}\,Dab^2x^9+\frac{1}{9}\,Bb^3x^9+\frac{3}{8}\,Cab^2x^8+\frac{1}{8}\,Ab^3x^8+\frac{3}{7}\,Da^2bx^7+\frac{3}{7}\,Bab^2x^7+\frac{1}{2}\,Ca^2bx^6+\frac{1}{2}\,Aab^2x^6+\frac{1}{5}\,Da^3x^5+\frac{3}{5}\,Ba^2bx^5+\frac{1}{4}\,Ca^3x^4+\frac{3}{4}\,Aa^2bx^4+\frac{1}{3}\,Ba^3x^3+\frac{1}{2}\,Aa^3x^2$$

```
[In] integrate(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/11*D*b^3*x^11 + 1/10*C*b^3*x^10 + 1/3*D*a*b^2*x^9 + 1/9*B*b^3*x^9 + 3/8*C
*a*b^2*x^8 + 1/8*A*b^3*x^8 + 3/7*D*a^2*b*x^7 + 3/7*B*a*b^2*x^7 + 1/2*C*a^2*
b*x^6 + 1/2*A*a*b^2*x^6 + 1/5*D*a^3*x^5 + 3/5*B*a^2*b*x^5 + 1/4*C*a^3*x^4 +
 3/4*A*a^2*b*x^4 + 1/3*B*a^3*x^3 + 1/2*A*a^3*x^2
```

Mupad [B] (verification not implemented)

Time = 5.79 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11

$$\int x\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{A\,a^3\,x^2}{2}+\frac{B\,a^3\,x^3}{3}+\frac{A\,b^3\,x^8}{8}+\frac{C\,a^3\,x^4}{4}+\frac{B\,b^3\,x^9}{9}+\frac{C\,b^3\,x^{10}}{10}+\frac{a^3\,x^5\,D}{5}+\frac{b^3\,x^{11}\,D}{11}+\frac{3\,a^2\,b\,x^7\,D}{7}+\frac{a\,b^2\,x^9\,D}{3}+\frac{3\,A\,a^2\,b\,x^4}{4}+\frac{A\,a\,b^2\,x^6}{2}+\frac{3\,B\,a^2\,b\,x^5}{5}+\frac{3\,B\,a\,b^2\,x^7}{7}+\frac{C\,a^2\,b\,x^6}{2}+\frac{3\,C\,a\,b^2\,x^8}{8}$$

```
[In] int(x*(a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (A*a^3*x^2)/2 + (B*a^3*x^3)/3 + (A*b^3*x^8)/8 + (C*a^3*x^4)/4 + (B*b^3*x^9)
/9 + (C*b^3*x^10)/10 + (a^3*x^5*D)/5 + (b^3*x^11*D)/11 + (3*a^2*b*x^7*D)/7
+ (a*b^2*x^9*D)/3 + (3*A*a^2*b*x^4)/4 + (A*a*b^2*x^6)/2 + (3*B*a^2*b*x^5)/5
 + (3*B*a*b^2*x^7)/7 + (C*a^2*b*x^6)/2 + (3*C*a*b^2*x^8)/8
```

3.81 $\int (a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\,dx$

Optimal result 518
Rubi [A] (verified) 518
Mathematica [A] (verified) 519
Maple [A] (verified) 520
Fricas [A] (verification not implemented) 520
Sympy [A] (verification not implemented) 521
Maxima [A] (verification not implemented) 521
Giac [A] (verification not implemented) 522
Mupad [B] (verification not implemented) 522

Optimal result

Integrand size = 25, antiderivative size = 133

$$\int (a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\,dx = a^3Ax+\frac{1}{3}a^2(3Ab+aC)x^3+\frac{1}{4}a^3Dx^4+\frac{3}{5}ab(Ab+aC)x^5+\frac{1}{2}a^2bDx^6+\frac{1}{7}b^2(Ab+3aC)x^7+\frac{3}{8}ab^2Dx^8+\frac{1}{9}b^3Cx^9+\frac{1}{10}b^3Dx^{10}+\frac{B(a+bx^2)^4}{8b}$$

```
[Out] a^3*A*x+1/3*a^2*(3*A*b+C*a)*x^3+1/4*a^3*D*x^4+3/5*a*b*(A*b+C*a)*x^5+1/2*a^2
*b*D*x^6+1/7*b^2*(A*b+3*C*a)*x^7+3/8*a*b^2*D*x^8+1/9*b^3*C*x^9+1/10*b^3*D*x
^10+1/8*B*(b*x^2+a)^4/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1596, 1824}

$$\int (a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\,dx = a^3Ax+\frac{1}{4}a^3Dx^4+\frac{1}{3}a^2x^3(aC+3Ab)+\frac{1}{2}a^2bDx^6+\frac{1}{7}b^2x^7(3aC+Ab)+\frac{3}{5}abx^5(aC+Ab)+\frac{3}{8}ab^2Dx^8+\frac{B(a+bx^2)^4}{8b}+\frac{1}{9}b^3Cx^9+\frac{1}{10}b^3Dx^{10}$$

```
[In] Int[(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a^3*A*x + (a^2*(3*A*b + a*C)*x^3)/3 + (a^3*D*x^4)/4 + (3*a*b*(A*b + a*C)*x^
5)/5 + (a^2*b*D*x^6)/2 + (b^2*(A*b + 3*a*C)*x^7)/7 + (3*a*b^2*D*x^8)/8 + (b
^3*C*x^9)/9 + (b^3*D*x^10)/10 + (B*(a + b*x^2)^4)/(8*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{B(a+bx^2)^4}{8b} + \int (a+bx^2)^3 \left(A+Cx^2+Dx^3\right) dx \\
&= \frac{B(a+bx^2)^4}{8b} + \int \left(a^3A + a^2(3Ab+aC)x^2 + a^3Dx^3 + 3ab(Ab+aC)x^4 + 3a^2bDx^5 \right. \\
&\qquad \left. + b^2(Ab+3aC)x^6 + 3ab^2Dx^7 + b^3Cx^8 + b^3Dx^9\right) dx \\
&= a^3Ax + \frac{1}{3}a^2(3Ab+aC)x^3 + \frac{1}{4}a^3Dx^4 + \frac{3}{5}ab(Ab+aC)x^5 + \frac{1}{2}a^2bDx^6 \\
&\quad + \frac{1}{7}b^2(Ab+3aC)x^7 + \frac{3}{8}ab^2Dx^8 + \frac{1}{9}b^3Cx^9 + \frac{1}{10}b^3Dx^{10} + \frac{B(a+bx^2)^4}{8b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.91

$$\int (a+bx^2)^3 \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{210a^3x(12A+x(6B+x(4C+3Dx))) + 126a^2bx^3(20A+x(15B+2x(6C+5Dx))) + 9ab^2x^5(168A+5}{2520}$$

```
[In] Integrate[(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (210*a^3*x*(12*A + x*(6*B + x*(4*C + 3*D*x))) + 126*a^2*b*x^3*(20*A + x*(15
*B + 2*x*(6*C + 5*D*x))) + 9*a*b^2*x^5*(168*A + 5*x*(28*B + 3*x*(8*C + 7*D*
x))) + b^3*x^7*(360*A + 7*x*(45*B + 4*x*(10*C + 9*D*x))))/2520
```

Maple [A] (verified)

Time = 3.45 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.08

method	result
norman	$\frac{b^3 D x^{10}}{10}+\frac{b^3 C x^9}{9}+\left(\frac{1}{8}B b^3+\frac{3}{8}a b^2 D\right)x^8+\left(\frac{1}{7}b^3 A+\frac{3}{7}C b^2 a\right)x^7+\left(\frac{1}{2}a b^2 B+\frac{1}{2}D a^2 b\right)x^6+\left(\frac{3}{5}a b^2\right.$
default	$\frac{b^3 D x^{10}}{10}+\frac{b^3 C x^9}{9}+\frac{(B b^3+3a b^2 D)x^8}{8}+\frac{(b^3 A+3C b^2 a)x^7}{7}+\frac{(3a b^2 B+3D a^2 b)x^6}{6}+\frac{(3a b^2 A+3C a^2 b)x^5}{5}+\frac{(3a^2 b B+}{4}$
gosper	$\frac{1}{10}b^3 D x^{10}+\frac{1}{9}b^3 C x^9+\frac{1}{8}b^3 B x^8+\frac{3}{8}a b^2 D x^8+\frac{1}{7}x^7 b^3 A+\frac{3}{7}x^7 C b^2 a+\frac{1}{2}x^6 a b^2 B+\frac{1}{2}a^2 b D x^6+$
parallelrisch	$\frac{1}{10}b^3 D x^{10}+\frac{1}{9}b^3 C x^9+\frac{1}{8}b^3 B x^8+\frac{3}{8}a b^2 D x^8+\frac{1}{7}x^7 b^3 A+\frac{3}{7}x^7 C b^2 a+\frac{1}{2}x^6 a b^2 B+\frac{1}{2}a^2 b D x^6+$

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*b^3*D*x^10+1/9*b^3*C*x^9+(1/8*B*b^3+3/8*a*b^2*D)*x^8+(1/7*b^3*A+3/7*C*
b^2*a)*x^7+(1/2*a*b^2*B+1/2*D*a^2*b)*x^6+(3/5*a*b^2*A+3/5*C*a^2*b)*x^5+(3/4
*a^2*b*B+1/4*D*a^3)*x^4+(a^2*b*A+1/3*C*a^3)*x^3+1/2*a^3*B*x^2+a^3*A*x
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \frac{1}{10}Db^3x^{10}+\frac{1}{9}Cb^3x^9+\frac{1}{8}\left(3\,Dab^2+Bb^3\right)x^8\\&\quad+\frac{1}{7}\left(3\,Cab^2+Ab^3\right)x^7+\frac{1}{2}\left(Da^2b+Bab^2\right)x^6\\&\quad+\frac{1}{2}Ba^3x^2+\frac{3}{5}\left(Ca^2b+Aab^2\right)x^5+Aa^3x\\&\quad+\frac{1}{4}\left(Da^3+3\,Ba^2b\right)x^4+\frac{1}{3}\left(Ca^3+3\,Aa^2b\right)x^3\end{aligned}$$

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 1/8*(3*D*a*b^2 + B*b^3)*x^8 + 1/7*(3*C*a*
b^2 + A*b^3)*x^7 + 1/2*(D*a^2*b + B*a*b^2)*x^6 + 1/2*B*a^3*x^2 + 3/5*(C*a^2
*b + A*a*b^2)*x^5 + A*a^3*x + 1/4*(D*a^3 + 3*B*a^2*b)*x^4 + 1/3*(C*a^3 + 3*
A*a^2*b)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.19

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right)\,dx = Aa^3x+\frac{Ba^3x^2}{2}+\frac{Cb^3x^9}{9}+\frac{Db^3x^{10}}{10} + x^8\left(\frac{Bb^3}{8}+\frac{3Dab^2}{8}\right)+x^7\left(\frac{Ab^3}{7}+\frac{3Cab^2}{7}\right) + x^6\left(\frac{Bab^2}{2}+\frac{Da^2b}{2}\right)+x^5\cdot\left(\frac{3Aab^2}{5}+\frac{3Ca^2b}{5}\right) + x^4\cdot\left(\frac{3Ba^2b}{4}+\frac{Da^3}{4}\right)+x^3\left(Aa^2b+\frac{Ca^3}{3}\right)$$

```
[In] integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x + B*a**3*x**2/2 + C*b**3*x**9/9 + D*b**3*x**10/10 + x**8*(B*b**3/8
 + 3*D*a*b**2/8) + x**7*(A*b**3/7 + 3*C*a*b**2/7) + x**6*(B*a*b**2/2 + D*a*
*2*b/2) + x**5*(3*A*a*b**2/5 + 3*C*a**2*b/5) + x**4*(3*B*a**2*b/4 + D*a**3/
4) + x**3*(A*a**2*b + C*a**3/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.07

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{10}\,Db^3x^{10}+\frac{1}{9}\,Cb^3x^9+\frac{1}{8}\,(3\,Dab^2+Bb^3)x^8 + \frac{1}{7}\,(3\,Cab^2+Ab^3)x^7+\frac{1}{2}\,(Da^2b+Bab^2)x^6 + \frac{1}{2}\,Ba^3x^2+\frac{3}{5}\,(Ca^2b+Aab^2)x^5+Aa^3x + \frac{1}{4}\,(Da^3+3\,Ba^2b)x^4+\frac{1}{3}\,(Ca^3+3\,Aa^2b)x^3$$

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 1/8*(3*D*a*b^2 + B*b^3)*x^8 + 1/7*(3*C*a*
b^2 + A*b^3)*x^7 + 1/2*(D*a^2*b + B*a*b^2)*x^6 + 1/2*B*a^3*x^2 + 3/5*(C*a^2
*b + A*a*b^2)*x^5 + A*a^3*x + 1/4*(D*a^3 + 3*B*a^2*b)*x^4 + 1/3*(C*a^3 + 3*
A*a^2*b)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{10}\,Db^3x^{10}+\frac{1}{9}\,Cb^3x^9+\frac{3}{8}\,Dab^2x^8+\frac{1}{8}\,Bb^3x^8+\frac{3}{7}\,Cab^2x^7+\frac{1}{7}\,Ab^3x^7+\frac{1}{2}\,Da^2bx^6+\frac{1}{2}\,Bab^2x^6+\frac{3}{5}\,Ca^2bx^5+\frac{3}{5}\,Aab^2x^5+\frac{1}{4}\,Da^3x^4+\frac{3}{4}\,Ba^2bx^4+\frac{1}{3}\,Ca^3x^3+Aa^2bx^3+\frac{1}{2}\,Ba^3x^2+Aa^3x$$

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 3/8*D*a*b^2*x^8 + 1/8*B*b^3*x^8 + 3/7*C*a
*b^2*x^7 + 1/7*A*b^3*x^7 + 1/2*D*a^2*b*x^6 + 1/2*B*a*b^2*x^6 + 3/5*C*a^2*b*
x^5 + 3/5*A*a*b^2*x^5 + 1/4*D*a^3*x^4 + 3/4*B*a^2*b*x^4 + 1/3*C*a^3*x^3 + A
*a^2*b*x^3 + 1/2*B*a^3*x^2 + A*a^3*x
```

Mupad [B] (verification not implemented)

Time = 5.77 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{B\,a^3\,x^2}{2}+\frac{A\,b^3\,x^7}{7}+\frac{C\,a^3\,x^3}{3}+\frac{B\,b^3\,x^8}{8}+\frac{C\,b^3\,x^9}{9}+\frac{a^3\,x^4\,D}{4}+\frac{b^3\,x^{10}\,D}{10}+A\,a^3\,x+\frac{a^2\,b\,x^6\,D}{2}+\frac{3\,a\,b^2\,x^8\,D}{8}+A\,a^2\,b\,x^3+\frac{3\,A\,a\,b^2\,x^5}{5}+\frac{3\,B\,a^2\,b\,x^4}{4}+\frac{B\,a\,b^2\,x^6}{2}+\frac{3\,C\,a^2\,b\,x^5}{5}+\frac{3\,C\,a\,b^2\,x^7}{7}$$

```
[In] int((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (B*a^3*x^2)/2 + (A*b^3*x^7)/7 + (C*a^3*x^3)/3 + (B*b^3*x^8)/8 + (C*b^3*x^9)
/9 + (a^3*x^4*D)/4 + (b^3*x^10*D)/10 + A*a^3*x + (a^2*b*x^6*D)/2 + (3*a*b^2
*x^8*D)/8 + A*a^2*b*x^3 + (3*A*a*b^2*x^5)/5 + (3*B*a^2*b*x^4)/4 + (B*a*b^2*
x^6)/2 + (3*C*a^2*b*x^5)/5 + (3*C*a*b^2*x^7)/7
```

3.82 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x} \, dx$

Optimal result 523
Rubi [A] (verified) 523
Mathematica [A] (verified) 524
Maple [A] (verified) 525
Fricas [A] (verification not implemented) 525
Sympy [A] (verification not implemented) 526
Maxima [A] (verification not implemented) 526
Giac [A] (verification not implemented) 527
Mupad [B] (verification not implemented) 527

Optimal result

Integrand size = 28, antiderivative size = 129

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x} \, dx = a^3Bx+\frac{3}{2}a^2Abx^2+\frac{1}{3}a^2(3bB+aD)x^3+\frac{3}{4}aAb^2x^4+\frac{3}{5}ab(bB+aD)x^5+\frac{1}{6}Ab^3x^6+\frac{1}{7}b^2(bB+3aD)x^7+\frac{1}{9}b^3Dx^9+\frac{C(a+bx^2)^4}{8b}+a^3A\log(x)$$

```
[Out] a^3*B*x+3/2*a^2*A*b*x^2+1/3*a^2*(3*B*b+D*a)*x^3+3/4*a*A*b^2*x^4+3/5*a*b*(B*
b+D*a)*x^5+1/6*A*b^3*x^6+1/7*b^2*(B*b+3*D*a)*x^7+1/9*b^3*D*x^9+1/8*C*(b*x^2
+a)^4/b+a^3*A*ln(x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1597, 1816}

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x} \, dx = a^3A\log(x)+a^3Bx+\frac{3}{2}a^2Abx^2+\frac{1}{3}a^2x^3(aD+3bB)+\frac{3}{4}aAb^2x^4+\frac{1}{7}b^2x^7(3aD+bB)+\frac{3}{5}abx^5(aD+bB)+\frac{C(a+bx^2)^4}{8b}+\frac{1}{6}Ab^3x^6+\frac{1}{9}b^3Dx^9$$

```
[In] Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] a^3*B*x + (3*a^2*A*b*x^2)/2 + (a^2*(3*b*B + a*D)*x^3)/3 + (3*a*A*b^2*x^4)/4
 + (3*a*b*(b*B + a*D)*x^5)/5 + (A*b^3*x^6)/6 + (b^2*(b*B + 3*a*D)*x^7)/7 +
(b^3*D*x^9)/9 + (C*(a + b*x^2)^4)/(8*b) + a^3*A*Log[x]
```

Rule 1597

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{C(a+bx^2)^4}{8b} + \int \frac{(a+bx^2)^3\,(A+Bx+Dx^3)}{x}\,dx \\
&= \frac{C(a+bx^2)^4}{8b} + \int \Bigg(a^3B + \frac{a^3A}{x} + 3a^2Abx + a^2(3bB+aD)x^2 + 3aAb^2x^3 \\
&\qquad + 3ab(bB+aD)x^4 + Ab^3x^5 + b^2(bB+3aD)x^6 + b^3Dx^8\Bigg)\,dx \\
&= a^3Bx + \frac{3}{2}a^2Abx^2 + \frac{1}{3}a^2(3bB+aD)x^3 + \frac{3}{4}aAb^2x^4 + \frac{3}{5}ab(bB+aD)x^5 \\
&\quad + \frac{1}{6}Ab^3x^6 + \frac{1}{7}b^2(bB+3aD)x^7 + \frac{1}{9}b^3Dx^9 + \frac{C(a+bx^2)^4}{8b} + a^3A\log(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.94

$$
\begin{aligned}
&\int \frac{(a+bx^2)^3\,(A+Bx+Cx^2+Dx^3)}{x}\,dx \\
&= \frac{x(420a^3(6B+x(3C+2Dx)) + 126a^2bx(30A+x(20B+3x(5C+4Dx))) + 18ab^2x^3(105A+2x(42B+5}{2520} \\
&\quad + a^3A\log(x)
\end{aligned}
$$

```
[In] Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] (x*(420*a^3*(6*B + x*(3*C + 2*D*x)) + 126*a^2*b*x*(30*A + x*(20*B + 3*x*(5*
C + 4*D*x))) + 18*a*b^2*x^3*(105*A + 2*x*(42*B + 5*x*(7*C + 6*D*x))) + 5*b^
3*x^5*(84*A + x*(72*B + 7*x*(9*C + 8*D*x)))))/2520 + a^3*A*Log[x]
```

Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.10

method	result
norman	$\left(\frac{1}{7}B\,b^3+\frac{3}{7}a\,b^2D\right)x^7+\left(\frac{1}{6}b^3A+\frac{1}{2}C\,b^2a\right)x^6+\left(\frac{3}{4}a\,b^2A+\frac{3}{4}C\,a^2b\right)x^4+\left(\frac{3}{5}a\,b^2B+\frac{3}{5}Da^2b\right)x^5$
default	$\frac{b^3Dx^9}{9}+\frac{b^3C\,x^8}{8}+\frac{b^3B\,x^7}{7}+\frac{3Da\,b^2x^7}{7}+\frac{x^6b^3A}{6}+\frac{Ca\,b^2x^6}{2}+\frac{3Ba\,b^2x^5}{5}+\frac{3Da^2b\,x^5}{5}+\frac{3aA\,b^2x^4}{4}+\frac{3C\,a^2b\,x^4}{4}$
parallelrisch	$\frac{b^3Dx^9}{9}+\frac{b^3C\,x^8}{8}+\frac{b^3B\,x^7}{7}+\frac{3Da\,b^2x^7}{7}+\frac{x^6b^3A}{6}+\frac{Ca\,b^2x^6}{2}+\frac{3Ba\,b^2x^5}{5}+\frac{3Da^2b\,x^5}{5}+\frac{3aA\,b^2x^4}{4}+\frac{3C\,a^2b\,x^4}{4}$

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (1/7*B*b^3+3/7*a*b^2*D)*x^7+(1/6*b^3*A+1/2*C*b^2*a)*x^6+(3/4*a*b^2*A+3/4*C*
a^2*b)*x^4+(3/5*a*b^2*B+3/5*D*a^2*b)*x^5+(3/2*a^2*b*A+1/2*C*a^3)*x^2+(a^2*b
*B+1/3*D*a^3)*x^3+a^3*B*x+1/8*b^3*C*x^8+1/9*b^3*D*x^9+a^3*A*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{1}{9}Db^3x^9+\frac{1}{8}Cb^3x^8+\frac{1}{7}\left(3\,Dab^2+Bb^3\right)x^7 + \frac{1}{6}\left(3\,Cab^2+Ab^3\right)x^6+\frac{3}{5}\left(Da^2b+Bab^2\right)x^5 + Ba^3x+\frac{3}{4}\left(Ca^2b+Aab^2\right)x^4+Aa^3\log(x) + \frac{1}{3}\left(Da^3+3\,Ba^2b\right)x^3+\frac{1}{2}\left(Ca^3+3\,Aa^2b\right)x^2$$

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="fricas")
```

```
[Out] 1/9*D*b^3*x^9 + 1/8*C*b^3*x^8 + 1/7*(3*D*a*b^2 + B*b^3)*x^7 + 1/6*(3*C*a*b^
2 + A*b^3)*x^6 + 3/5*(D*a^2*b + B*a*b^2)*x^5 + B*a^3*x + 3/4*(C*a^2*b + A*a
*b^2)*x^4 + A*a^3*log(x) + 1/3*(D*a^3 + 3*B*a^2*b)*x^3 + 1/2*(C*a^3 + 3*A*a
^2*b)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x}\,dx = Aa^3\log(x) + Ba^3x + \frac{Cb^3x^8}{8} + \frac{Db^3x^9}{9} + x^7\left(\frac{Bb^3}{7}+\frac{3Dab^2}{7}\right) + x^6\left(\frac{Ab^3}{6}+\frac{Cab^2}{2}\right) + x^5 \cdot \left(\frac{3Bab^2}{5}+\frac{3Da^2b}{5}\right) + x^4\cdot\left(\frac{3Aab^2}{4}+\frac{3Ca^2b}{4}\right) + x^3\left(Ba^2b+\frac{Da^3}{3}\right) + x^2\cdot\left(\frac{3Aa^2b}{2}+\frac{Ca^3}{2}\right)$$

```
[In] integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x,x)
```

```
[Out] A*a**3*log(x) + B*a**3*x + C*b**3*x**8/8 + D*b**3*x**9/9 + x**7*(B*b**3/7 +
 3*D*a*b**2/7) + x**6*(A*b**3/6 + C*a*b**2/2) + x**5*(3*B*a*b**2/5 + 3*D*a*
*2*b/5) + x**4*(3*A*a*b**2/4 + 3*C*a**2*b/4) + x**3*(B*a**2*b + D*a**3/3) +
 x**2*(3*A*a**2*b/2 + C*a**3/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{1}{9}Db^3x^9 + \frac{1}{8}Cb^3x^8 + \frac{1}{7}(3Dab^2+Bb^3)x^7 + \frac{1}{6}(3Cab^2+Ab^3)x^6 + \frac{3}{5}(Da^2b+Bab^2)x^5 + Ba^3x + \frac{3}{4}(Ca^2b+Aab^2)x^4 + Aa^3\log(x) + \frac{1}{3}(Da^3+3Ba^2b)x^3 + \frac{1}{2}(Ca^3+3Aa^2b)x^2$$

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="maxima")
```

```
[Out] 1/9*D*b^3*x^9 + 1/8*C*b^3*x^8 + 1/7*(3*D*a*b^2 + B*b^3)*x^7 + 1/6*(3*C*a*b^
2 + A*b^3)*x^6 + 3/5*(D*a^2*b + B*a*b^2)*x^5 + B*a^3*x + 3/4*(C*a^2*b + A*a
*b^2)*x^4 + A*a^3*log(x) + 1/3*(D*a^3 + 3*B*a^2*b)*x^3 + 1/2*(C*a^3 + 3*A*a
^2*b)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{1}{9}\,Db^3x^9 + \frac{1}{8}\,Cb^3x^8 + \frac{3}{7}\,Dab^2x^7 + \frac{1}{7}\,Bb^3x^7 + \frac{1}{2}\,Cab^2x^6 + \frac{1}{6}\,Ab^3x^6 + \frac{3}{5}\,Da^2bx^5 + \frac{3}{5}\,Bab^2x^5 + \frac{3}{4}\,Ca^2bx^4 + \frac{3}{4}\,Aab^2x^4 + \frac{1}{3}\,Da^3x^3 + Ba^2bx^3 + \frac{1}{2}\,Ca^3x^2 + \frac{3}{2}\,Aa^2bx^2 + Ba^3x + Aa^3\log\left(|x|\right)$$

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="giac")
```

```
[Out] 1/9*D*b^3*x^9 + 1/8*C*b^3*x^8 + 3/7*D*a*b^2*x^7 + 1/7*B*b^3*x^7 + 1/2*C*a*b
^2*x^6 + 1/6*A*b^3*x^6 + 3/5*D*a^2*b*x^5 + 3/5*B*a*b^2*x^5 + 3/4*C*a^2*b*x^
4 + 3/4*A*a*b^2*x^4 + 1/3*D*a^3*x^3 + B*a^2*b*x^3 + 1/2*C*a^3*x^2 + 3/2*A*a
^2*b*x^2 + B*a^3*x + A*a^3*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 5.85 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{A\,b^3\,x^6}{6} + \frac{C\,a^3\,x^2}{2} + \frac{B\,b^3\,x^7}{7} + \frac{C\,b^3\,x^8}{8} + A\,a^3\,\ln\left(x\right) + \frac{a^3\,x^3\,D}{3} + \frac{b^3\,x^9\,D}{9} + B\,a^3\,x + \frac{3\,a^2\,b\,x^5\,D}{5} + \frac{3\,a\,b^2\,x^7\,D}{7} + \frac{3\,A\,a^2\,b\,x^2}{2} + \frac{3\,A\,a\,b^2\,x^4}{4} + B\,a^2\,b\,x^3 + \frac{3\,B\,a\,b^2\,x^5}{5} + \frac{3\,C\,a^2\,b\,x^4}{4} + \frac{C\,a\,b^2\,x^6}{2}$$

```
[In] int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/x,x)
```

```
[Out] (A*b^3*x^6)/6 + (C*a^3*x^2)/2 + (B*b^3*x^7)/7 + (C*b^3*x^8)/8 + A*a^3*log(x
) + (a^3*x^3*D)/3 + (b^3*x^9*D)/9 + B*a^3*x + (3*a^2*b*x^5*D)/5 + (3*a*b^2*
x^7*D)/7 + (3*A*a^2*b*x^2)/2 + (3*A*a*b^2*x^4)/4 + B*a^2*b*x^3 + (3*B*a*b^2
*x^5)/5 + (3*C*a^2*b*x^4)/4 + (C*a*b^2*x^6)/2
```

3.83 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^2}\,dx$

Optimal result 528
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Optimal result

Integrand size = 28, antiderivative size = 124

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = -\frac{a^3A}{x}+a^2(3Ab+aC)x+\frac{3}{2}a^2bBx^2 +ab(Ab+aC)x^3+\frac{3}{4}ab^2Bx^4+\frac{1}{5}b^2(Ab+3aC)x^5 +\frac{1}{6}b^3Bx^6+\frac{1}{7}b^3Cx^7+\frac{D(a+bx^2)^4}{8b}+a^3B\log(x)$$

```
[Out] -a^3*A/x+a^2*(3*A*b+C*a)*x+3/2*a^2*b*B*x^2+a*b*(A*b+C*a)*x^3+3/4*a*b^2*B*x^
4+1/5*b^2*(A*b+3*C*a)*x^5+1/6*b^3*B*x^6+1/7*b^3*C*x^7+1/8*D*(b*x^2+a)^4/b+a
^3*B*ln(x)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1597, 1642}

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = -\frac{a^3A}{x}+a^3B\log(x)+a^2x(aC+3Ab) +\frac{3}{2}a^2bBx^2+\frac{1}{5}b^2x^5(3aC+Ab)+abx^3(aC+Ab) +\frac{3}{4}ab^2Bx^4+\frac{D(a+bx^2)^4}{8b}+\frac{1}{6}b^3Bx^6+\frac{1}{7}b^3Cx^7$$

```
[In] Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] -((a^3*A)/x) + a^2*(3*A*b + a*C)*x + (3*a^2*b*B*x^2)/2 + a*b*(A*b + a*C)*x^
3 + (3*a*b^2*B*x^4)/4 + (b^2*(A*b + 3*a*C)*x^5)/5 + (b^3*B*x^6)/6 + (b^3*C*
x^7)/7 + (D*(a + b*x^2)^4)/(8*b) + a^3*B*Log[x]
```

Rule 1597

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{D(a+bx^2)^4}{8b} + \int \frac{(a+bx^2)^3\,(A+Bx+Cx^2)}{x^2}\,dx \\
&= \frac{D(a+bx^2)^4}{8b} + \int \bigg(a^2(3Ab+aC) + \frac{a^3A}{x^2} + \frac{a^3B}{x} + 3a^2bBx + 3ab(Ab+aC)x^2 \\
&\qquad + 3ab^2Bx^3 + b^2(Ab+3aC)x^4 + b^3Bx^5 + b^3Cx^6\bigg)\,dx \\
&= -\frac{a^3A}{x} + a^2(3Ab+aC)x + \frac{3}{2}a^2bBx^2 + ab(Ab+aC)x^3 + \frac{3}{4}ab^2Bx^4 \\
&\quad + \frac{1}{5}b^2(Ab+3aC)x^5 + \frac{1}{6}b^3Bx^6 + \frac{1}{7}b^3Cx^7 + \frac{D(a+bx^2)^4}{8b} + a^3B\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.99

$$\begin{aligned}
\int \frac{(a+bx^2)^3\,(A+Bx+Cx^2+Dx^3)}{x^2}\,dx &= a^3\left(-\frac{A}{x} + Cx + \frac{Dx^2}{2}\right) \\
&\quad + \frac{1}{4}a^2bx(12A + x(6B + x(4C + 3Dx))) \\
&\quad + \frac{1}{20}ab^2x^3(20A + x(15B + 2x(6C + 5Dx))) \\
&\quad + \frac{1}{840}b^3x^5(168A + 5x(28B + 3x(8C + 7Dx))) \\
&\quad + a^3B\log(x)
\end{aligned}$$

```
[In] Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] a^3*(-(A/x) + C*x + (D*x^2)/2) + (a^2*b*x*(12*A + x*(6*B + x*(4*C + 3*D*x))
))/4 + (a*b^2*x^3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))))/20 + (b^3*x^5*(168
*A + 5*x*(28*B + 3*x*(8*C + 7*D*x))))/840 + a^3*B*Log[x]
```

Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.17

method	result
default	$\frac{b^3 D x^8}{8}+\frac{b^3 C x^7}{7}+\frac{b^3 B x^6}{6}+\frac{D a b^2 x^6}{2}+\frac{A b^3 x^5}{5}+\frac{3 C a b^2 x^5}{5}+\frac{3 B a b^2 x^4}{4}+\frac{3 D a^2 b x^4}{4}+a A b^2 x^3+C a^2 b x^3$
norman	$\frac{\left(\frac{1}{6} B b^3+\frac{1}{2} a b^2 D\right) x^7+\left(\frac{1}{5} b^3 A+\frac{3}{5} C b^2 a\right) x^6+\left(\frac{3}{4} a b^2 B+\frac{3}{4} D a^2 b\right) x^5+\left(\frac{3}{2} a^2 b B+\frac{1}{2} D a^3\right) x^3+\left(a b^2 A+C a^2 b\right) x^4+\left(3 a^2 b A+C a^3\right) x^2-a}{x}$
parallelrisch	$\frac{105 b^3 D x^9+120 b^3 C x^8+140 b^3 B x^7+420 D a b^2 x^7+168 x^6 b^3 A+504 C a b^2 x^6+630 B a b^2 x^5+630 D a^2 b x^5+840 a A b^2 x^4+840 C a^2 b x^4}{840 x}$

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*b^3*D*x^8+1/7*b^3*C*x^7+1/6*b^3*B*x^6+1/2*D*a*b^2*x^6+1/5*A*b^3*x^5+3/5
*C*a*b^2*x^5+3/4*B*a*b^2*x^4+3/4*D*a^2*b*x^4+a*A*b^2*x^3+C*a^2*b*x^3+3/2*B*
a^2*b*x^2+1/2*D*a^3*x^2+3*a^2*A*b*x+C*a^3*x+a^3*B*ln(x)-a^3*A/x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{105\,Db^3x^9+120\,Cb^3x^8+140\,(3\,Dab^2+Bb^3)x^7+168\,(3\,Cab^2+Ab^3)x^6+630\,(Da^2b+Bab^2)x^5+840\,Ba}{840\,x}$$

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/840*(105*D*b^3*x^9 + 120*C*b^3*x^8 + 140*(3*D*a*b^2 + B*b^3)*x^7 + 168*(3
*C*a*b^2 + A*b^3)*x^6 + 630*(D*a^2*b + B*a*b^2)*x^5 + 840*B*a^3*x*log(x) +
840*(C*a^2*b + A*a*b^2)*x^4 - 840*A*a^3 + 420*(D*a^3 + 3*B*a^2*b)*x^3 + 840
*(C*a^3 + 3*A*a^2*b)*x^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^2} \, dx = -\frac{Aa^3}{x} + Ba^3 \log(x) + \frac{Cb^3x^7}{7} + \frac{Db^3x^8}{8} + x^6\left(\frac{Bb^3}{6} + \frac{Dab^2}{2}\right) + x^5\left(\frac{Ab^3}{5} + \frac{3Cab^2}{5}\right) + x^4\cdot\left(\frac{3Bab^2}{4} + \frac{3Da^2b}{4}\right) + x^3(Aab^2 + Ca^2b) + x^2\cdot\left(\frac{3Ba^2b}{2} + \frac{Da^3}{2}\right) + x(3Aa^2b + Ca^3)$$

```
[In] integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

```
[Out] -A*a**3/x + B*a**3*log(x) + C*b**3*x**7/7 + D*b**3*x**8/8 + x**6*(B*b**3/6
+ D*a*b**2/2) + x**5*(A*b**3/5 + 3*C*a*b**2/5) + x**4*(3*B*a*b**2/4 + 3*D*a
**2*b/4) + x**3*(A*a*b**2 + C*a**2*b) + x**2*(3*B*a**2*b/2 + D*a**3/2) + x*
(3*A*a**2*b + C*a**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^2} \, dx = \frac{1}{8} Db^3x^8 + \frac{1}{7} Cb^3x^7 + \frac{1}{6} (3 Dab^2 + Bb^3)x^6 + \frac{1}{5} (3 Cab^2 + Ab^3)x^5 + \frac{3}{4} (Da^2b + Bab^2)x^4 + Ba^3 \log(x) + (Ca^2b + Aab^2)x^3 - \frac{Aa^3}{x} + \frac{1}{2} (Da^3 + 3 Ba^2b)x^2 + (Ca^3 + 3 Aa^2b)x$$

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/8*D*b^3*x^8 + 1/7*C*b^3*x^7 + 1/6*(3*D*a*b^2 + B*b^3)*x^6 + 1/5*(3*C*a*b^
2 + A*b^3)*x^5 + 3/4*(D*a^2*b + B*a*b^2)*x^4 + B*a^3*log(x) + (C*a^2*b + A*
a*b^2)*x^3 - A*a^3/x + 1/2*(D*a^3 + 3*B*a^2*b)*x^2 + (C*a^3 + 3*A*a^2*b)*x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^2} dx = \frac{1}{8} Db^3x^8 + \frac{1}{7} Cb^3x^7 + \frac{1}{2} Dab^2x^6 + \frac{1}{6} Bb^3x^6 + \frac{3}{5} Cab^2x^5 + \frac{1}{5} Ab^3x^5 + \frac{3}{4} Da^2bx^4 + \frac{3}{4} Bab^2x^4 + Ca^2bx^3 + Aab^2x^3 + \frac{1}{2} Da^3x^2 + \frac{3}{2} Ba^2bx^2 + Ca^3x + 3 Aa^2bx + Ba^3 \log(|x|) - \frac{Aa^3}{x}$$

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*D*b^3*x^8 + 1/7*C*b^3*x^7 + 1/2*D*a*b^2*x^6 + 1/6*B*b^3*x^6 + 3/5*C*a*b
^2*x^5 + 1/5*A*b^3*x^5 + 3/4*D*a^2*b*x^4 + 3/4*B*a*b^2*x^4 + C*a^2*b*x^3 +
A*a*b^2*x^3 + 1/2*D*a^3*x^2 + 3/2*B*a^2*b*x^2 + C*a^3*x + 3*A*a^2*b*x + B*a
^3*log(abs(x)) - A*a^3/x
```

Mupad [B] (verification not implemented)

Time = 6.02 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^2} dx = \frac{(b\,x^2+a)^4\,D}{8\,b} - \frac{A\,a^3}{x} + \frac{A\,b^3\,x^5}{5} + \frac{B\,b^3\,x^6}{6} + \frac{C\,b^3\,x^7}{7} + B\,a^3\,\ln(x) + C\,a^3\,x + 3\,A\,a^2\,b\,x + A\,a\,b^2\,x^3 + \frac{3\,B\,a^2\,b\,x^2}{2} + \frac{3\,B\,a\,b^2\,x^4}{4} + C\,a^2\,b\,x^3 + \frac{3\,C\,a\,b^2\,x^5}{5}$$

```
[In] int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/x^2,x)
```

```
[Out] ((a + b*x^2)^4*D)/(8*b) - (A*a^3)/x + (A*b^3*x^5)/5 + (B*b^3*x^6)/6 + (C*b^
3*x^7)/7 + B*a^3*log(x) + C*a^3*x + 3*A*a^2*b*x + A*a*b^2*x^3 + (3*B*a^2*b*
x^2)/2 + (3*B*a*b^2*x^4)/4 + C*a^2*b*x^3 + (3*C*a*b^2*x^5)/5
```

3.84 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^3} dx$

Optimal result 533
Rubi [A] (verified) 533
Mathematica [A] (verified) 534
Maple [A] (verified) 535
Fricas [A] (verification not implemented) 535
Sympy [A] (verification not implemented) 535
Maxima [A] (verification not implemented) 536
Giac [A] (verification not implemented) 536
Mupad [B] (verification not implemented) 537

Optimal result

Integrand size = 28, antiderivative size = 135

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^3} dx = -\frac{a^3A}{2x^2} - \frac{a^3B}{x} + a^2(3bB+aD)x + \frac{3}{2}ab(Ab+aC)x^2 + ab(bB+aD)x^3 + \frac{1}{4}b^2(Ab+3aC)x^4 + \frac{1}{5}b^2(bB+3aD)x^5 + \frac{1}{6}b^3Cx^6 + \frac{1}{7}b^3Dx^7 + a^2(3Ab+aC)\log(x)$$

```
[Out] -1/2*a^3*A/x^2-a^3*B/x+a^2*(3*B*b+D*a)*x+3/2*a*b*(A*b+C*a)*x^2+a*b*(B*b+D*a
)*x^3+1/4*b^2*(A*b+3*C*a)*x^4+1/5*b^2*(B*b+3*D*a)*x^5+1/6*b^3*C*x^6+1/7*b^3
*D*x^7+a^2*(3*A*b+C*a)*ln(x)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1816}

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^3} dx = -\frac{a^3A}{2x^2} - \frac{a^3B}{x} + a^2\log(x)(aC+3Ab) + a^2x(aD+3bB) + \frac{1}{4}b^2x^4(3aC+Ab) + \frac{3}{2}abx^2(aC+Ab) + \frac{1}{5}b^2x^5(3aD+bB) + abx^3(aD+bB) + \frac{1}{6}b^3Cx^6 + \frac{1}{7}b^3Dx^7$$

```
[In] Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -1/2*(a^3*A)/x^2 - (a^3*B)/x + a^2*(3*b*B + a*D)*x + (3*a*b*(A*b + a*C)*x^2
)/2 + a*b*(b*B + a*D)*x^3 + (b^2*(A*b + 3*a*C)*x^4)/4 + (b^2*(b*B + 3*a*D)*
x^5)/5 + (b^3*C*x^6)/6 + (b^3*D*x^7)/7 + a^2*(3*A*b + a*C)*Log[x]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(a^2(3bB+aD)+\frac{a^3A}{x^3}+\frac{a^3B}{x^2}+\frac{a^2(3Ab+aC)}{x}+3ab(Ab+aC)x+3ab(bB+aD)x^2 + b^2(Ab+3aC)x^3+b^2(bB+3aD)x^4+b^3Cx^5+b^3Dx^6\right)dx$$

$$= -\frac{a^3A}{2x^2}-\frac{a^3B}{x}+a^2(3bB+aD)x+\frac{3}{2}ab(Ab+aC)x^2+ab(bB+aD)x^3 + \frac{1}{4}b^2(Ab+3aC)x^4+\frac{1}{5}b^2(bB+3aD)x^5+\frac{1}{6}b^3Cx^6+\frac{1}{7}b^3Dx^7+a^2(3Ab+aC)\log(x)$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = -\frac{a^3(A+2Bx-2Dx^3)}{2x^2} + \frac{1}{2}a^2bx(6B+x(3C+2Dx)) + \frac{1}{20}ab^2x^2(30A+x(20B+3x(5C+4Dx))) + \frac{1}{420}b^3x^4(105A+2x(42B+5x(7C+6Dx))) + a^2(3Ab+aC)\log(x)$$

```
[In] Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -1/2*(a^3*(A + 2*B*x - 2*D*x^3))/x^2 + (a^2*b*x*(6*B + x*(3*C + 2*D*x)))/2
+ (a*b^2*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))))/20 + (b^3*x^4*(105*A +
2*x*(42*B + 5*x*(7*C + 6*D*x))))/420 + a^2*(3*A*b + a*C)*Log[x]
```

Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.05

method	result
default	$\frac{b^3Dx^7}{7}+\frac{b^3C\,x^6}{6}+\frac{b^3B\,x^5}{5}+\frac{3Da\,b^2x^5}{5}+\frac{A\,b^3x^4}{4}+\frac{3Ca\,b^2x^4}{4}+Ba\,b^2x^3+Da^2b\,x^3+\frac{3aA\,b^2x^2}{2}+\frac{3C\,a^2b\,x}{2}$
norman	$\frac{\left(\frac{1}{5}B\,b^3+\frac{3}{5}a\,b^2D\right)x^7+\left(\frac{1}{4}b^3A+\frac{3}{4}C\,b^2a\right)x^6+\left(\frac{3}{2}a\,b^2A+\frac{3}{2}C\,a^2b\right)x^4+\left(a\,b^2B+Da^2b\right)x^5+\left(3a^2bB+Da^3\right)x^3-\frac{a^3A}{2}-a^3Bx+\frac{b^3C\,x^8}{6}}{x^2}$
parallelrisch	$\frac{60b^3Dx^9+70b^3C\,x^8+84b^3B\,x^7+252Da\,b^2x^7+105x^6b^3A+315Ca\,b^2x^6+420Ba\,b^2x^5+420Da^2b\,x^5+630aA\,b^2x^4+630C\,a^2b\,x^4+}{420x^2}$

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*b^3*D*x^7+1/6*b^3*C*x^6+1/5*b^3*B*x^5+3/5*D*a*b^2*x^5+1/4*A*b^3*x^4+3/4
*C*a*b^2*x^4+B*a*b^2*x^3+D*a^2*b*x^3+3/2*a*A*b^2*x^2+3/2*C*a^2*b*x^2+3*a^2*
b*B*x+D*a^3*x+a^2*(3*A*b+C*a)*ln(x)-a^3*B/x-1/2*a^3*A/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{60\,Db^3x^9+70\,Cb^3x^8+84\,(3\,Dab^2+Bb^3)x^7+105\,(3\,Cab^2+Ab^3)x^6+420\,(Da^2b+Bab^2)x^5-420\,Ba^3x}{420\,x^2}$$

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/420*(60*D*b^3*x^9 + 70*C*b^3*x^8 + 84*(3*D*a*b^2 + B*b^3)*x^7 + 105*(3*C*
a*b^2 + A*b^3)*x^6 + 420*(D*a^2*b + B*a*b^2)*x^5 - 420*B*a^3*x + 630*(C*a^2
*b + A*a*b^2)*x^4 - 210*A*a^3 + 420*(D*a^3 + 3*B*a^2*b)*x^3 + 420*(C*a^3 +
3*A*a^2*b)*x^2*log(x))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.12

$$\begin{aligned}\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^3}\,dx &= \frac{Cb^3x^6}{6}+\frac{Db^3x^7}{7}+a^2\cdot(3Ab+Ca)\log{(x)} \\ &\quad + x^5\left(\frac{Bb^3}{5}+\frac{3Dab^2}{5}\right)+x^4\left(\frac{Ab^3}{4}+\frac{3Cab^2}{4}\right) \\ &\quad + x^3\left(Bab^2+Da^2b\right)+x^2\cdot\left(\frac{3Aab^2}{2}+\frac{3Ca^2b}{2}\right) \\ &\quad + x\left(3Ba^2b+Da^3\right)+\frac{-Aa^3-2Ba^3x}{2x^2}\end{aligned}$$

```
[In] integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

```
[Out] C*b**3*x**6/6 + D*b**3*x**7/7 + a**2*(3*A*b + C*a)*log(x) + x**5*(B*b**3/5
+ 3*D*a*b**2/5) + x**4*(A*b**3/4 + 3*C*a*b**2/4) + x**3*(B*a*b**2 + D*a**2*
b) + x**2*(3*A*a*b**2/2 + 3*C*a**2*b/2) + x*(3*B*a**2*b + D*a**3) + (-A*a**
3 - 2*B*a**3*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^3} dx = \frac{1}{7} Db^3x^7 + \frac{1}{6} Cb^3x^6 + \frac{1}{5} (3 Dab^2 + Bb^3)x^5 + \frac{1}{4} (3 Cab^2 + Ab^3)x^4 + (Da^2b + Bab^2)x^3 + \frac{3}{2} (Ca^2b + Aab^2)x^2 + (Da^3 + 3 Ba^2b)x + (Ca^3 + 3 Aa^2b) \log(x) - \frac{2 Ba^3x + Aa^3}{2 x^2}$$

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/7*D*b^3*x^7 + 1/6*C*b^3*x^6 + 1/5*(3*D*a*b^2 + B*b^3)*x^5 + 1/4*(3*C*a*b^
2 + A*b^3)*x^4 + (D*a^2*b + B*a*b^2)*x^3 + 3/2*(C*a^2*b + A*a*b^2)*x^2 + (D
*a^3 + 3*B*a^2*b)*x + (C*a^3 + 3*A*a^2*b)*log(x) - 1/2*(2*B*a^3*x + A*a^3)/
x^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^3} dx = \frac{1}{7} Db^3x^7 + \frac{1}{6} Cb^3x^6 + \frac{3}{5} Dab^2x^5 + \frac{1}{5} Bb^3x^5 + \frac{3}{4} Cab^2x^4 + \frac{1}{4} Ab^3x^4 + Da^2bx^3 + Bab^2x^3 + \frac{3}{2} Ca^2bx^2 + \frac{3}{2} Aab^2x^2 + Da^3x + 3 Ba^2bx + (Ca^3 + 3 Aa^2b) \log(|x|) - \frac{2 Ba^3x + Aa^3}{2 x^2}$$

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/7*D*b^3*x^7 + 1/6*C*b^3*x^6 + 3/5*D*a*b^2*x^5 + 1/5*B*b^3*x^5 + 3/4*C*a*b
^2*x^4 + 1/4*A*b^3*x^4 + D*a^2*b*x^3 + B*a*b^2*x^3 + 3/2*C*a^2*b*x^2 + 3/2*
A*a*b^2*x^2 + D*a^3*x + 3*B*a^2*b*x + (C*a^3 + 3*A*a^2*b)*log(abs(x)) - 1/2
*(2*B*a^3*x + A*a^3)/x^2
```

Mupad [B] (verification not implemented)

Time = 6.01 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^3} dx = \frac{A b^3 x^4}{4} - \frac{B a^3}{x} - \frac{A a^3}{2 x^2} + \frac{B b^3 x^5}{5} + \frac{C b^3 x^6}{6} + C a^3 \ln(x) + a^3 x D + \frac{b^3 x^7 D}{7} + a^2 b x^3 D + \frac{3 a b^2 x^5 D}{5} + 3 B a^2 b x + \frac{3 A a b^2 x^2}{2} + B a b^2 x^3 + \frac{3 C a^2 b x^2}{2} + \frac{3 C a b^2 x^4}{4} + 3 A a^2 b \ln(x)$$

```
[In] int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/x^3,x)
```

```
[Out] (A*b^3*x^4)/4 - (B*a^3)/x - (A*a^3)/(2*x^2) + (B*b^3*x^5)/5 + (C*b^3*x^6)/6
 + C*a^3*log(x) + a^3*x*D + (b^3*x^7*D)/7 + a^2*b*x^3*D + (3*a*b^2*x^5*D)/5
 + 3*B*a^2*b*x + (3*A*a*b^2*x^2)/2 + B*a*b^2*x^3 + (3*C*a^2*b*x^2)/2 + (3*C
*a*b^2*x^4)/4 + 3*A*a^2*b*log(x)
```

3.85 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$

Optimal result 538
Rubi [A] (verified) 538
Mathematica [A] (verified) 539
Maple [A] (verified) 540
Fricas [A] (verification not implemented) 540
Sympy [A] (verification not implemented) 540
Maxima [A] (verification not implemented) 541
Giac [A] (verification not implemented) 541
Mupad [B] (verification not implemented) 542

Optimal result

Integrand size = 28, antiderivative size = 139

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = -\frac{a^3A}{3x^3}-\frac{a^3B}{2x^2}-\frac{a^2(3Ab+aC)}{x} + 3ab(Ab+aC)x+\frac{3}{2}ab(bB+aD)x^2 + \frac{1}{3}b^2(Ab+3aC)x^3+\frac{1}{4}b^2(bB+3aD)x^4 + \frac{1}{5}b^3Cx^5+\frac{1}{6}b^3Dx^6+a^2(3bB+aD)\log(x)$$

```
[Out] -1/3*a^3*A/x^3-1/2*a^3*B/x^2-a^2*(3*A*b+C*a)/x+3*a*b*(A*b+C*a)*x+3/2*a*b*(B
*b+D*a)*x^2+1/3*b^2*(A*b+3*C*a)*x^3+1/4*b^2*(B*b+3*D*a)*x^4+1/5*b^3*C*x^5+1
/6*b^3*D*x^6+a^2*(3*B*b+D*a)*ln(x)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1816}

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = -\frac{a^3A}{3x^3}-\frac{a^3B}{2x^2}-\frac{a^2(aC+3Ab)}{x} + a^2\log(x)(aD+3bB)+\frac{1}{3}b^2x^3(3aC+Ab) + 3abx(aC+Ab)+\frac{1}{4}b^2x^4(3aD+bB) + \frac{3}{2}abx^2(aD+bB)+\frac{1}{5}b^3Cx^5+\frac{1}{6}b^3Dx^6$$

```
[In] Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -1/3*(a^3*A)/x^3 - (a^3*B)/(2*x^2) - (a^2*(3*A*b + a*C))/x + 3*a*b*(A*b + a
*C)*x + (3*a*b*(b*B + a*D)*x^2)/2 + (b^2*(A*b + 3*a*C)*x^3)/3 + (b^2*(b*B +
 3*a*D)*x^4)/4 + (b^3*C*x^5)/5 + (b^3*D*x^6)/6 + a^2*(3*b*B + a*D)*Log[x]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(3ab(Ab+aC)+\frac{a^3A}{x^4}+\frac{a^3B}{x^3}+\frac{a^2(3Ab+aC)}{x^2}+\frac{a^2(3bB+aD)}{x}+3ab(bB+aD)x\right.\\
&\qquad \left.+b^2(Ab+3aC)x^2+b^2(bB+3aD)x^3+b^3Cx^4+b^3Dx^5\right)\,dx\\
&= -\frac{a^3A}{3x^3}-\frac{a^3B}{2x^2}-\frac{a^2(3Ab+aC)}{x}+3ab(Ab+aC)x+\frac{3}{2}ab(bB+aD)x^2\\
&\quad +\frac{1}{3}b^2(Ab+3aC)x^3+\frac{1}{4}b^2(bB+3aD)x^4+\frac{1}{5}b^3Cx^5+\frac{1}{6}b^3Dx^6+a^2(3bB+aD)\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.89

$$\begin{aligned}
\int \frac{(a+bx^2)^3\,(A+Bx+Cx^2+Dx^3)}{x^4}\,dx &= -\frac{a^3(2A+3x(B+2Cx))}{6x^3}\\
&\quad +\frac{3a^2b(-2A+x^2(2C+Dx))}{2x}\\
&\quad +\frac{1}{4}ab^2x(12A+x(6B+x(4C+3Dx)))\\
&\quad +\frac{1}{60}b^3x^3(20A+x(15B+2x(6C+5Dx)))\\
&\quad +a^2(3bB+aD)\log(x)
\end{aligned}$$

```
[In] Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -1/6*(a^3*(2*A + 3*x*(B + 2*C*x)))/x^3 + (3*a^2*b*(-2*A + x^2*(2*C + D*x)))
/(2*x) + (a*b^2*x*(12*A + x*(6*B + x*(4*C + 3*D*x))))/4 + (b^3*x^3*(20*A +
x*(15*B + 2*x*(6*C + 5*D*x))))/60 + a^2*(3*b*B + a*D)*Log[x]
```

Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01

method	result
default	$\frac{b^{3} D x^{6}}{6}+\frac{b^{3} C x^{5}}{5}+\frac{b^{3} B x^{4}}{4}+\frac{3 D a b^{2} x^{4}}{4}+\frac{A b^{3} x^{3}}{3}+C a b^{2} x^{3}+\frac{3 B a b^{2} x^{2}}{2}+\frac{3 D a^{2} b x^{2}}{2}+3 a b^{2} A x+3 C a^{2} b x$
norman	$\frac{\left(\frac{1}{4} B b^{3}+\frac{3}{4} a b^{2} D\right) x^{7}+\left(\frac{1}{3} b^{3} A+C b^{2} a\right) x^{6}+\left(\frac{3}{2} a b^{2} B+\frac{3}{2} D a^{2} b\right) x^{5}+\left(3 a b^{2} A+3 C a^{2} b\right) x^{4}+\left(-3 a^{2} b A-C a^{3}\right) x^{2}-\frac{a^{3} A}{3}-\frac{a^{3} B x}{2}+\frac{b^{3} C x^{8}}{5}}{x^{3}}$
parallelrisch	$\frac{10 b^{3} D x^{9}+12 b^{3} C x^{8}+15 b^{3} B x^{7}+45 D a b^{2} x^{7}+20 x^{6} b^{3} A+60 C a b^{2} x^{6}+90 B a b^{2} x^{5}+90 D a^{2} b x^{5}+180 a A b^{2} x^{4}+180 B \ln (x) x^{3} a^{2} b+18}{60 x^{3}}$

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b^3*D*x^6+1/5*b^3*C*x^5+1/4*b^3*B*x^4+3/4*D*a*b^2*x^4+1/3*A*b^3*x^3+C*a
*b^2*x^3+3/2*B*a*b^2*x^2+3/2*D*a^2*b*x^2+3*a*b^2*A*x+3*C*a^2*b*x+a^2*(3*B*b
+D*a)*ln(x)-1/3*a^3*A/x^3-a^2*(3*A*b+C*a)/x-1/2*a^3*B/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^4}\,dx$$
$$= \frac{10\,Db^3x^9 + 12\,Cb^3x^8 + 15\,(3\,Dab^2 + Bb^3)x^7 + 20\,(3\,Cab^2 + Ab^3)x^6 + 90\,(Da^2b + Bab^2)x^5 - 30\,Ba^3x + 18}{60\,x^3}$$

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/60*(10*D*b^3*x^9 + 12*C*b^3*x^8 + 15*(3*D*a*b^2 + B*b^3)*x^7 + 20*(3*C*a*
b^2 + A*b^3)*x^6 + 90*(D*a^2*b + B*a*b^2)*x^5 - 30*B*a^3*x + 180*(C*a^2*b +
 A*a*b^2)*x^4 + 60*(D*a^3 + 3*B*a^2*b)*x^3*log(x) - 20*A*a^3 - 60*(C*a^3 +
3*A*a^2*b)*x^2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{Cb^3x^5}{5} + \frac{Db^3x^6}{6} + a^2\cdot(3Bb+Da)\log(x) + x^4\left(\frac{Bb^3}{4} + \frac{3Dab^2}{4}\right) + x^3\left(\frac{Ab^3}{3} + Cab^2\right) + x^2\cdot\left(\frac{3Bab^2}{2} + \frac{3Da^2b}{2}\right) + x(3Aab^2 + 3Ca^2b) + \frac{-2Aa^3 - 3Ba^3x + x^2(-18Aa^2b - 6Ca^3)}{6x^3}$$

```
[In] integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

```
[Out] C*b**3*x**5/5 + D*b**3*x**6/6 + a**2*(3*B*b + D*a)*log(x) + x**4*(B*b**3/4
+ 3*D*a*b**2/4) + x**3*(A*b**3/3 + C*a*b**2) + x**2*(3*B*a*b**2/2 + 3*D*a**
2*b/2) + x*(3*A*a*b**2 + 3*C*a**2*b) + (-2*A*a**3 - 3*B*a**3*x + x**2*(-18*
A*a**2*b - 6*C*a**3))/(6*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{1}{6}\,Db^3x^6 + \frac{1}{5}\,Cb^3x^5 + \frac{1}{4}\,\left(3\,Dab^2 + Bb^3\right)x^4 + \frac{1}{3}\,\left(3\,Cab^2 + Ab^3\right)x^3 + \frac{3}{2}\,\left(Da^2b + Bab^2\right)x^2 + 3\,\left(Ca^2b + Aab^2\right)x + \left(Da^3 + 3\,Ba^2b\right)\log\left(x\right) - \frac{3\,Ba^3x + 2\,Aa^3 + 6\,(Ca^3 + 3\,Aa^2b)x^2}{6\,x^3}$$

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*D*b^3*x^6 + 1/5*C*b^3*x^5 + 1/4*(3*D*a*b^2 + B*b^3)*x^4 + 1/3*(3*C*a*b^
2 + A*b^3)*x^3 + 3/2*(D*a^2*b + B*a*b^2)*x^2 + 3*(C*a^2*b + A*a*b^2)*x + (D
*a^3 + 3*B*a^2*b)*log(x) - 1/6*(3*B*a^3*x + 2*A*a^3 + 6*(C*a^3 + 3*A*a^2*b)
*x^2)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{1}{6}\,Db^3x^6 + \frac{1}{5}\,Cb^3x^5 + \frac{3}{4}\,Dab^2x^4 + \frac{1}{4}\,Bb^3x^4 + Cab^2x^3 + \frac{1}{3}\,Ab^3x^3 + \frac{3}{2}\,Da^2bx^2 + \frac{3}{2}\,Bab^2x^2 + 3\,Ca^2bx + 3\,Aab^2x + \left(Da^3 + 3\,Ba^2b\right)\log\left(|x|\right) - \frac{3\,Ba^3x + 2\,Aa^3 + 6\,(Ca^3 + 3\,Aa^2b)x^2}{6\,x^3}$$

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*D*b^3*x^6 + 1/5*C*b^3*x^5 + 3/4*D*a*b^2*x^4 + 1/4*B*b^3*x^4 + C*a*b^2*x
^3 + 1/3*A*b^3*x^3 + 3/2*D*a^2*b*x^2 + 3/2*B*a*b^2*x^2 + 3*C*a^2*b*x + 3*A*
a*b^2*x + (D*a^3 + 3*B*a^2*b)*log(abs(x)) - 1/6*(3*B*a^3*x + 2*A*a^3 + 6*(C
*a^3 + 3*A*a^2*b)*x^2)/x^3
```

Mupad [B] (verification not implemented)

Time = 6.22 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^4} \, dx = \frac{B\,b^3\,x^4}{4} - \frac{C\,a^3}{x} - \frac{B\,a^3}{2\,x^2} + \frac{C\,b^3\,x^5}{5} + \frac{b^3\,x^6\,D}{6} - \frac{A\,(a^3+9\,a^2\,b\,x^2-9\,a\,b^2\,x^4-b^3\,x^6)}{3\,x^3} + \frac{a^3\,\ln(x^2)\,D}{2} + \frac{3\,a^2\,b\,x^2\,D}{2} + 3\,C\,a^2\,b\,x + \frac{3\,a\,b^2\,x^4\,D}{4} + \frac{3\,B\,a\,b^2\,x^2}{2} + C\,a\,b^2\,x^3 + 3\,B\,a^2\,b\,\ln(x)$$

```
[In] int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/x^4,x)
[Out] (B*b^3*x^4)/4 - (C*a^3)/x - (B*a^3)/(2*x^2) + (C*b^3*x^5)/5 + (b^3*x^6*D)/6
 - (A*(a^3 - b^3*x^6 + 9*a^2*b*x^2 - 9*a*b^2*x^4))/(3*x^3) + (a^3*log(x^2)*
D)/2 + (3*a^2*b*x^2*D)/2 + 3*C*a^2*b*x + (3*a*b^2*x^4*D)/4 + (3*B*a*b^2*x^2
)/2 + C*a*b^2*x^3 + 3*B*a^2*b*log(x)
```

3.86 $\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$

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Optimal result

Integrand size = 28, antiderivative size = 151

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = -\frac{a(Ab-aC)x}{b^3}-\frac{a(bB-aD)x^2}{2b^3}+\frac{(Ab-aC)x^3}{3b^2}+\frac{(bB-aD)x^4}{4b^2}+\frac{Cx^5}{5b}+\frac{Dx^6}{6b}+\frac{a^{3/2}(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{a^2(bB-aD)\log(a+bx^2)}{2b^4}$$

```
[Out] -a*(A*b-C*a)*x/b^3-1/2*a*(B*b-D*a)*x^2/b^3+1/3*(A*b-C*a)*x^3/b^2+1/4*(B*b-D
*a)*x^4/b^2+1/5*C*x^5/b+1/6*D*x^6/b+a^(3/2)*(A*b-C*a)*arctan(x*b^(1/2)/a^(1
/2))/b^(7/2)+1/2*a^2*(B*b-D*a)*ln(b*x^2+a)/b^4
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1816, 649, 211, 266}

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{a^{3/2}(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{a^2(bB-aD)\log(a+bx^2)}{2b^4}-\frac{ax(Ab-aC)}{b^3}+\frac{x^3(Ab-aC)}{3b^2}-\frac{ax^2(bB-aD)}{2b^3}+\frac{x^4(bB-aD)}{4b^2}+\frac{Cx^5}{5b}+\frac{Dx^6}{6b}$$

```
[In] Int[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] -((a*(A*b - a*C)*x)/b^3) - (a*(b*B - a*D)*x^2)/(2*b^3) + ((A*b - a*C)*x^3)/
(3*b^2) + ((b*B - a*D)*x^4)/(4*b^2) + (C*x^5)/(5*b) + (D*x^6)/(6*b) + (a^(3
/2)*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2) + (a^2*(b*B - a*D)*Log
[a + b*x^2])/(2*b^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{a(Ab-aC)}{b^3}-\frac{a(bB-aD)x}{b^3}+\frac{(Ab-aC)x^2}{b^2}+\frac{(bB-aD)x^3}{b^2}+\frac{Cx^4}{b}\right.\\
&\qquad \left.+\frac{Dx^5}{b}+\frac{a^2(Ab-aC)+a^2(bB-aD)x}{b^3\,(a+bx^2)}\right)dx\\
&= -\frac{a(Ab-aC)x}{b^3}-\frac{a(bB-aD)x^2}{2b^3}+\frac{(Ab-aC)x^3}{3b^2}\\
&\quad +\frac{(bB-aD)x^4}{4b^2}+\frac{Cx^5}{5b}+\frac{Dx^6}{6b}+\frac{\int \frac{a^2(Ab-aC)+a^2(bB-aD)x}{a+bx^2}\,dx}{b^3}\\
&= -\frac{a(Ab-aC)x}{b^3}-\frac{a(bB-aD)x^2}{2b^3}+\frac{(Ab-aC)x^3}{3b^2}+\frac{(bB-aD)x^4}{4b^2}\\
&\quad +\frac{Cx^5}{5b}+\frac{Dx^6}{6b}+\frac{\left(a^2(Ab-aC)\right)\int \frac{1}{a+bx^2}\,dx}{b^3}+\frac{\left(a^2(bB-aD)\right)\int \frac{x}{a+bx^2}\,dx}{b^3}
\end{aligned}$$

$$= -\frac{a(Ab-aC)x}{b^3} - \frac{a(bB-aD)x^2}{2b^3} + \frac{(Ab-aC)x^3}{3b^2} + \frac{(bB-aD)x^4}{4b^2} + \frac{Cx^5}{5b} + \frac{Dx^6}{6b} + \frac{a^{3/2}(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}} + \frac{a^2(bB-aD)\log(a+bx^2)}{2b^4}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.86

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{bx(30a^2(2C+Dx)-5ab(12A+x(6B+x(4C+3Dx)))+b^2x^2(20A+x(15B+2x(6C+5Dx))))-60a}{60b^4}$$

```
[In] Integrate[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] (b*x*(30*a^2*(2*C + D*x) - 5*a*b*(12*A + x*(6*B + x*(4*C + 3*D*x))) + b^2*x
^2*(20*A + x*(15*B + 2*x*(6*C + 5*D*x)))) - 60*a^(3/2)*Sqrt[b]*(-(A*b) + a*
C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] - 30*a^2*(-(b*B) + a*D)*Log[a + b*x^2])/(60*
b^4)
```

Maple [A] (verified)

Time = 3.44 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.93

method	result
default	$-\frac{-\frac{1}{6}b^2Dx^6-\frac{1}{5}b^2Cx^5-\frac{1}{4}b^2Bx^4+\frac{1}{4}Dabx^4-\frac{1}{3}Ab^2x^3+\frac{1}{3}Cabx^3+\frac{1}{2}Babx^2-\frac{1}{2}Da^2x^2+aAbx-Ca^2x}{b^3}+\frac{a^2\left(\frac{(Bb-Da)\ln(bx^2+a)}{2b}+\right.}{b^3}$

```
[In] int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^3*(-1/6*b^2*D*x^6-1/5*b^2*C*x^5-1/4*b^2*B*x^4+1/4*D*a*b*x^4-1/3*A*b^2*
x^3+1/3*C*a*b*x^3+1/2*B*a*b*x^2-1/2*D*a^2*x^2+a*A*b*x-C*a^2*x)+a^2/b^3*(1/2
*(B*b-D*a)/b*ln(b*x^2+a)+(A*b-C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.20

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$=\left[\frac{10\,Db^3x^6+12\,Cb^3x^5-15\,(Dab^2-Bb^3)x^4-20\,(Cab^2-Ab^3)x^3+30\,(Da^2b-Bab^2)x^2-30\,(Ca^2b-Aa}{60\,b^4}\right.$$

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/60*(10*D*b^3*x^6 + 12*C*b^3*x^5 - 15*(D*a*b^2 - B*b^3)*x^4 - 20*(C*a*b^2
 - A*b^3)*x^3 + 30*(D*a^2*b - B*a*b^2)*x^2 - 30*(C*a^2*b - A*a*b^2)*sqrt(-a
/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 60*(C*a^2*b - A*a*b^2
)*x - 30*(D*a^3 - B*a^2*b)*log(b*x^2 + a))/b^4, 1/60*(10*D*b^3*x^6 + 12*C*b
^3*x^5 - 15*(D*a*b^2 - B*b^3)*x^4 - 20*(C*a*b^2 - A*b^3)*x^3 + 30*(D*a^2*b
- B*a*b^2)*x^2 - 60*(C*a^2*b - A*a*b^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) +
 60*(C*a^2*b - A*a*b^2)*x - 30*(D*a^3 - B*a^2*b)*log(b*x^2 + a))/b^4]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(134) = 268.

Time = 0.55 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.09

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$=\frac{Cx^5}{5b}+\frac{Dx^6}{6b}+x^4\left(\frac{B}{4b}-\frac{Da}{4b^2}\right)+x^3\left(\frac{A}{3b}-\frac{Ca}{3b^2}\right)$$
$$+x^2\left(-\frac{Ba}{2b^2}+\frac{Da^2}{2b^3}\right)+x\left(-\frac{Aa}{b^2}+\frac{Ca^2}{b^3}\right)+\left(-\frac{a^2(-Bb+Da)}{2b^4}\right.$$
$$\left.-\frac{\sqrt{-a^3b^9}(-Ab+Ca)}{2b^8}\right)\log\left(x+\frac{Ba^2b-Da^3-2b^4\left(-\frac{a^2(-Bb+Da)}{2b^4}-\frac{\sqrt{-a^3b^9}(-Ab+Ca)}{2b^8}\right)}{-Aab^2+Ca^2b}\right)$$
$$+\left(-\frac{a^2(-Bb+Da)}{2b^4}\right.$$
$$\left.+\frac{\sqrt{-a^3b^9}(-Ab+Ca)}{2b^8}\right)\log\left(x+\frac{Ba^2b-Da^3-2b^4\left(-\frac{a^2(-Bb+Da)}{2b^4}+\frac{\sqrt{-a^3b^9}(-Ab+Ca)}{2b^8}\right)}{-Aab^2+Ca^2b}\right)$$

```
[In] integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**5/(5*b) + D*x**6/(6*b) + x**4*(B/(4*b) - D*a/(4*b**2)) + x**3*(A/(3*b)
 - C*a/(3*b**2)) + x**2*(-B*a/(2*b**2) + D*a**2/(2*b**3)) + x*(-A*a/b**2 +
C*a**2/b**3) + (-a**2*(-B*b + D*a)/(2*b**4) - sqrt(-a**3*b**9)*(-A*b + C*a)
/(2*b**8))*log(x + (B*a**2*b - D*a**3 - 2*b**4*(-a**2*(-B*b + D*a)/(2*b**4)
 - sqrt(-a**3*b**9)*(-A*b + C*a)/(2*b**8)))/(-A*a*b**2 + C*a**2*b)) + (-a**
2*(-B*b + D*a)/(2*b**4) + sqrt(-a**3*b**9)*(-A*b + C*a)/(2*b**8))*log(x + (
B*a**2*b - D*a**3 - 2*b**4*(-a**2*(-B*b + D*a)/(2*b**4) + sqrt(-a**3*b**9)*
(-A*b + C*a)/(2*b**8)))/(-A*a*b**2 + C*a**2*b))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.96

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = -\frac{(Ca^3-Aa^2b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{10\,Db^2x^6+12\,Cb^2x^5-15\,(Dab-Bb^2)x^4-20\,(Cab-Ab^2)x^3+30\,(Da^2-Bab)x^2+60\,(Ca^2-Aab)x}{60\,b^3} - \frac{(Da^3-Ba^2b)\log\left(bx^2+a\right)}{2\,b^4}$$

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(C*a^3 - A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/60*(10*D*b^2*x
^6 + 12*C*b^2*x^5 - 15*(D*a*b - B*b^2)*x^4 - 20*(C*a*b - A*b^2)*x^3 + 30*(D
*a^2 - B*a*b)*x^2 + 60*(C*a^2 - A*a*b)*x)/b^3 - 1/2*(D*a^3 - B*a^2*b)*log(b
*x^2 + a)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.07

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = -\frac{(Ca^3-Aa^2b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} - \frac{(Da^3-Ba^2b)\log\left(bx^2+a\right)}{2\,b^4} + \frac{10\,Db^5x^6+12\,Cb^5x^5-15\,Dab^4x^4+15\,Bb^5x^4-20\,Cab^4x^3+20\,Ab^5x^3+30\,Da^2b^3x^2-30\,Bab^4x^2+60}{60\,b^6}$$

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(C*a^3 - A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(D*a^3 - B*a
^2*b)*log(b*x^2 + a)/b^4 + 1/60*(10*D*b^5*x^6 + 12*C*b^5*x^5 - 15*D*a*b^4*x
^4 + 15*B*b^5*x^4 - 20*C*a*b^4*x^3 + 20*A*b^5*x^3 + 30*D*a^2*b^3*x^2 - 30*B
*a*b^4*x^2 + 60*C*a^2*b^3*x - 60*A*a*b^4*x)/b^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx=\int \frac{x^4\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{b\,x^2+a}\,dx$$

```
[In] int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
[Out] int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

3.87 $\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2} dx$

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Optimal result

Integrand size = 28, antiderivative size = 130

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2} dx = -\frac{a(bB-aD)x}{b^3}+\frac{(Ab-aC)x^2}{2b^2}+\frac{(bB-aD)x^3}{3b^2}+\frac{Cx^4}{4b}+\frac{Dx^5}{5b}+\frac{a^{3/2}(bB-aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}-\frac{a(Ab-aC)\log(a+bx^2)}{2b^3}$$

```
[Out] -a*(B*b-D*a)*x/b^3+1/2*(A*b-C*a)*x^2/b^2+1/3*(B*b-D*a)*x^3/b^2+1/4*C*x^4/b+
1/5*D*x^5/b+a^(3/2)*(B*b-D*a)*arctan(x*b^(1/2)/a^(1/2))/b^(7/2)-1/2*a*(A*b-
C*a)*ln(b*x^2+a)/b^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1816, 649, 211, 266}

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2} dx = \frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bB-aD)}{b^{7/2}}-\frac{a(Ab-aC)\log(a+bx^2)}{2b^3}+\frac{x^2(Ab-aC)}{2b^2}-\frac{ax(bB-aD)}{b^3}+\frac{x^3(bB-aD)}{3b^2}+\frac{Cx^4}{4b}+\frac{Dx^5}{5b}$$

```
[In] Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] -((a*(b*B - a*D)*x)/b^3) + ((A*b - a*C)*x^2)/(2*b^2) + ((b*B - a*D)*x^3)/(3
*b^2) + (C*x^4)/(4*b) + (D*x^5)/(5*b) + (a^(3/2)*(b*B - a*D)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/b^(7/2) - (a*(A*b - a*C)*Log[a + b*x^2])/(2*b^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{a(bB-aD)}{b^3}+\frac{(Ab-aC)x}{b^2}+\frac{(bB-aD)x^2}{b^2}+\frac{Cx^3}{b}+\frac{Dx^4}{b}\right.\\
&\qquad\left.+\frac{a^2(bB-aD)-ab(Ab-aC)x}{b^3\left(a+bx^2\right)}\right)dx\\
&= -\frac{a(bB-aD)x}{b^3}+\frac{(Ab-aC)x^2}{2b^2}+\frac{(bB-aD)x^3}{3b^2}+\frac{Cx^4}{4b}+\frac{Dx^5}{5b}+\frac{\int \frac{a^2(bB-aD)-ab(Ab-aC)x}{a+bx^2}\,dx}{b^3}\\
&= -\frac{a(bB-aD)x}{b^3}+\frac{(Ab-aC)x^2}{2b^2}+\frac{(bB-aD)x^3}{3b^2}+\frac{Cx^4}{4b}+\frac{Dx^5}{5b}\\
&\quad -\frac{(a(Ab-aC))\int \frac{x}{a+bx^2}\,dx}{b^2}+\frac{\left(a^2(bB-aD)\right)\int \frac{1}{a+bx^2}\,dx}{b^3}\\
&= -\frac{a(bB-aD)x}{b^3}+\frac{(Ab-aC)x^2}{2b^2}+\frac{(bB-aD)x^3}{3b^2}+\frac{Cx^4}{4b}+\frac{Dx^5}{5b}\\
&\quad +\frac{a^{3/2}(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}-\frac{a(Ab-aC)\log\left(a+bx^2\right)}{2b^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.88

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = -\frac{a^{3/2}(-bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}} + \frac{x(60a^2D-10ab(6B+x(3C+2Dx))+b^2x(30A+x(20B+3x(5C+4Dx))))+30a(-Ab+aC)\log(}{60b^3}$$

```
[In] Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] -((a^(3/2)*(-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)) + (x*(60*a^
2*D - 10*a*b*(6*B + x*(3*C + 2*D*x)) + b^2*x*(30*A + x*(20*B + 3*x*(5*C + 4
*D*x)))) + 30*a*(-(A*b) + a*C)*Log[a + b*x^2])/(60*b^3)
```

Maple [A] (verified)

Time = 3.44 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

method	result
default	$\frac{\frac{1}{5}b^2Dx^5+\frac{1}{4}b^2Cx^4+\frac{1}{3}b^2Bx^3-\frac{1}{3}Dabx^3+\frac{1}{2}Ab^2x^2-\frac{1}{2}Cabx^2-Babx+Da^2x}{b^3}-\frac{a\left(\frac{(b^2A-Cab)\ln(bx^2+a)}{2b}+\frac{(-abB+Da^2)\arctan\left(\frac{bx}{\sqrt{a}}\right.}{\sqrt{ab}}\right.}{b^3}$

```
[In] int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/5*b^2*D*x^5+1/4*b^2*C*x^4+1/3*b^2*B*x^3-1/3*D*a*b*x^3+1/2*A*b^2*x^
2-1/2*C*a*b*x^2-B*a*b*x+D*a^2*x)-a/b^3*(1/2*(A*b^2-C*a*b)/b*ln(b*x^2+a)+(-B
*a*b+D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.08

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$

$$= \left[\frac{12\,Db^2x^5+15\,Cb^2x^4-20\,(Dab-Bb^2)x^3-30\,(Cab-Ab^2)x^2+30\,(Da^2-Bab)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}}{bx^2+a}\right.}{60\,b^3}\right.$$

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/60*(12*D*b^2*x^5 + 15*C*b^2*x^4 - 20*(D*a*b - B*b^2)*x^3 - 30*(C*a*b - A
*b^2)*x^2 + 30*(D*a^2 - B*a*b)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a
)/(b*x^2 + a)) + 60*(D*a^2 - B*a*b)*x + 30*(C*a^2 - A*a*b)*log(b*x^2 + a))/
b^3, 1/60*(12*D*b^2*x^5 + 15*C*b^2*x^4 - 20*(D*a*b - B*b^2)*x^3 - 30*(C*a*b
 - A*b^2)*x^2 - 60*(D*a^2 - B*a*b)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 60*(
D*a^2 - B*a*b)*x + 30*(C*a^2 - A*a*b)*log(b*x^2 + a))/b^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(116) = 232$.

Time = 0.53 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.11

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{Cx^4}{4b}+\frac{Dx^5}{5b}+x^3\left(\frac{B}{3b}-\frac{Da}{3b^2}\right)+x^2\left(\frac{A}{2b}-\frac{Ca}{2b^2}\right)+x\left(-\frac{Ba}{b^2}+\frac{Da^2}{b^3}\right)+\left(\frac{a(-Ab+Ca)}{2b^3}-\frac{\sqrt{-a^3b^7}(-Bb+Da)}{2b^7}\right)\log\left(x+\frac{-Aab+Ca^2-2b^3\left(\frac{a(-Ab+Ca)}{2b^3}-\frac{\sqrt{-a^3b^7}(-Bb+Da)}{2b^7}\right)}{-Bab+Da^2}\right)+\left(\frac{a(-Ab+Ca)}{2b^3}+\frac{\sqrt{-a^3b^7}(-Bb+Da)}{2b^7}\right)\log\left(x+\frac{-Aab+Ca^2-2b^3\left(\frac{a(-Ab+Ca)}{2b^3}+\frac{\sqrt{-a^3b^7}(-Bb+Da)}{2b^7}\right)}{-Bab+Da^2}\right)$$

```
[In] integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**4/(4*b) + D*x**5/(5*b) + x**3*(B/(3*b) - D*a/(3*b**2)) + x**2*(A/(2*b)
 - C*a/(2*b**2)) + x*(-B*a/b**2 + D*a**2/b**3) + (a*(-A*b + C*a)/(2*b**3) -
 sqrt(-a**3*b**7)*(-B*b + D*a)/(2*b**7))*log(x + (-A*a*b + C*a**2 - 2*b**3*
(a*(-A*b + C*a)/(2*b**3) - sqrt(-a**3*b**7)*(-B*b + D*a)/(2*b**7)))/(-B*a*b
 + D*a**2)) + (a*(-A*b + C*a)/(2*b**3) + sqrt(-a**3*b**7)*(-B*b + D*a)/(2*b
**7))*log(x + (-A*a*b + C*a**2 - 2*b**3*(a*(-A*b + C*a)/(2*b**3) + sqrt(-a*
*3*b**7)*(-B*b + D*a)/(2*b**7)))/(-B*a*b + D*a**2))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{(Ca^2-Aab)\log(bx^2+a)}{2\,b^3} - \frac{(Da^3-Ba^2b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{12\,Db^2x^5+15\,Cb^2x^4-20\,(Dab-Bb^2)x^3-30\,(Cab-Ab^2)x^2+60\,(Da^2-Bab)x}{60\,b^3}$$

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(C*a^2 - A*a*b)*log(b*x^2 + a)/b^3 - (D*a^3 - B*a^2*b)*arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*b^3) + 1/60*(12*D*b^2*x^5 + 15*C*b^2*x^4 - 20*(D*a*b - B*b
^2)*x^3 - 30*(C*a*b - A*b^2)*x^2 + 60*(D*a^2 - B*a*b)*x)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{(Ca^2-Aab)\log(bx^2+a)}{2\,b^3} - \frac{(Da^3-Ba^2b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{12\,Db^4x^5+15\,Cb^4x^4-20\,Dab^3x^3+20\,Bb^4x^3-30\,Cab^3x^2+30\,Ab^4x^2+60\,Da^2b^2x-60\,Bab^3x}{60\,b^5}$$

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(C*a^2 - A*a*b)*log(b*x^2 + a)/b^3 - (D*a^3 - B*a^2*b)*arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*b^3) + 1/60*(12*D*b^4*x^5 + 15*C*b^4*x^4 - 20*D*a*b^3*x^3
+ 20*B*b^4*x^3 - 30*C*a*b^3*x^2 + 30*A*b^4*x^2 + 60*D*a^2*b^2*x - 60*B*a*b^
3*x)/b^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \int \frac{x^3\,(A+B\,x+C\,x^2+x^3\,D)}{b\,x^2+a}\,dx$$

```
[In] int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
[Out] int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

3.88 $\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2} dx$

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Mupad [F(-1)] 559

Optimal result

Integrand size = 28, antiderivative size = 111

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2} dx = \frac{(Ab-aC)x}{b^2} + \frac{(bB-aD)x^2}{2b^2} + \frac{Cx^3}{3b} + \frac{Dx^4}{4b} - \frac{\sqrt{a}(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}} - \frac{a(bB-aD)\log(a+bx^2)}{2b^3}$$

```
[Out] (A*b-C*a)*x/b^2+1/2*(B*b-D*a)*x^2/b^2+1/3*C*x^3/b+1/4*D*x^4/b-1/2*a*(B*b-D*
a)*ln(b*x^2+a)/b^3-(A*b-C*a)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(5/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1816, 649, 211, 266}

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2} dx = -\frac{\sqrt{a}(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}} + \frac{x(Ab-aC)}{b^2} - \frac{a(bB-aD)\log(a+bx^2)}{2b^3} + \frac{x^2(bB-aD)}{2b^2} + \frac{Cx^3}{3b} + \frac{Dx^4}{4b}$$

```
[In] Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] ((A*b - a*C)*x)/b^2 + ((b*B - a*D)*x^2)/(2*b^2) + (C*x^3)/(3*b) + (D*x^4)/(
4*b) - (Sqrt[a]*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) - (a*(b*B
- a*D)*Log[a + b*x^2])/(2*b^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{Ab-aC}{b^2}+\frac{(bB-aD)x}{b^2}+\frac{Cx^2}{b}+\frac{Dx^3}{b}-\frac{a(Ab-aC)+a(bB-aD)x}{b^2\left(a+bx^2\right)}\right)\,dx\\
&= \frac{(Ab-aC)x}{b^2}+\frac{(bB-aD)x^2}{2b^2}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}-\frac{\int \frac{a(Ab-aC)+a(bB-aD)x}{a+bx^2}\,dx}{b^2}\\
&= \frac{(Ab-aC)x}{b^2}+\frac{(bB-aD)x^2}{2b^2}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}\\
&\quad -\frac{(a(Ab-aC))\int \frac{1}{a+bx^2}\,dx}{b^2}-\frac{(a(bB-aD))\int \frac{x}{a+bx^2}\,dx}{b^2}\\
&= \frac{(Ab-aC)x}{b^2}+\frac{(bB-aD)x^2}{2b^2}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}\\
&\quad -\frac{\sqrt{a}(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{a(bB-aD)\log\left(a+bx^2\right)}{2b^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$= \frac{bx(12Ab-6a(2C+Dx)+bx(6B+4Cx+3Dx^2))+12\sqrt{a}\sqrt{b}(-Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+6a(-bB+a}{12b^3}$$

```
[In] Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] (b*x*(12*A*b - 6*a*(2*C + D*x) + b*x*(6*B + 4*C*x + 3*D*x^2)) + 12*Sqrt[a]*
Sqrt[b]*(-(A*b) + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 6*a*(-(b*B) + a*D)*Log
[a + b*x^2])/(12*b^3)
```

Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\frac{1}{4}Db\,x^4+\frac{1}{3}bC\,x^3+\frac{1}{2}bB\,x^2-\frac{1}{2}Da\,x^2+Abx-Cax}{b^2}-\frac{a\left(\frac{(Bb-Da)\ln\left(b\,x^2+a\right)}{2b}+\frac{(Ab-Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{b^2}$	95

```
[In] int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/4*D*b*x^4+1/3*b*C*x^3+1/2*b*B*x^2-1/2*D*a*x^2+A*b*x-C*a*x)-a/b^2*(
1/2*(B*b-D*a)/b*ln(b*x^2+a)+(A*b-C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.14

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$= \left[\frac{3\,Db^2x^4+4\,Cb^2x^3-6\,(Dab-Bb^2)x^2-6\,(Cab-Ab^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-12\,(Cab-Ab^2)x+}{12\,b^3}\right.$$

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*D*b^2*x^4 + 4*C*b^2*x^3 - 6*(D*a*b - B*b^2)*x^2 - 6*(C*a*b - A*b^2
)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 12*(C*a*b -
A*b^2)*x + 6*(D*a^2 - B*a*b)*log(b*x^2 + a))/b^3, 1/12*(3*D*b^2*x^4 + 4*C*b
^2*x^3 - 6*(D*a*b - B*b^2)*x^2 + 12*(C*a*b - A*b^2)*sqrt(a/b)*arctan(b*x*sq
rt(a/b)/a) - 12*(C*a*b - A*b^2)*x + 6*(D*a^2 - B*a*b)*log(b*x^2 + a))/b^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(97) = 194$.

Time = 0.48 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.21

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{Cx^3}{3b} + \frac{Dx^4}{4b} + x^2\left(\frac{B}{2b}-\frac{Da}{2b^2}\right) + x\left(\frac{A}{b}-\frac{Ca}{b^2}\right) + \left(\frac{a(-Bb+Da)}{2b^3} - \frac{\sqrt{-ab^7}(-Ab+Ca)}{2b^6}\right)\log\left(x+\frac{Bab-Da^2+2b^3\left(\frac{a(-Bb+Da)}{2b^3}-\frac{\sqrt{-ab^7}(-Ab+Ca)}{2b^6}\right)}{-Ab^2+Cab}\right) + \left(\frac{a(-Bb+Da)}{2b^3} + \frac{\sqrt{-ab^7}(-Ab+Ca)}{2b^6}\right)\log\left(x+\frac{Bab-Da^2+2b^3\left(\frac{a(-Bb+Da)}{2b^3}+\frac{\sqrt{-ab^7}(-Ab+Ca)}{2b^6}\right)}{-Ab^2+Cab}\right)$$

```
[In] integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**3/(3*b) + D*x**4/(4*b) + x**2*(B/(2*b) - D*a/(2*b**2)) + x*(A/b - C*a/
b**2) + (a*(-B*b + D*a)/(2*b**3) - sqrt(-a*b**7)*(-A*b + C*a)/(2*b**6))*log
(x + (B*a*b - D*a**2 + 2*b**3*(a*(-B*b + D*a)/(2*b**3) - sqrt(-a*b**7)*(-A*
b + C*a)/(2*b**6)))/(-A*b**2 + C*a*b)) + (a*(-B*b + D*a)/(2*b**3) + sqrt(-a
*b**7)*(-A*b + C*a)/(2*b**6))*log(x + (B*a*b - D*a**2 + 2*b**3*(a*(-B*b + D
*a)/(2*b**3) + sqrt(-a*b**7)*(-A*b + C*a)/(2*b**6)))/(-A*b**2 + C*a*b))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.88

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{(Ca^2-Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{3\,Dbx^4+4\,Cbx^3-6\,(Da-Bb)x^2-12\,(Ca-Ab)x}{12\,b^2} + \frac{(Da^2-Bab)\log\left(bx^2+a\right)}{2\,b^3}$$

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (C*a^2 - A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/12*(3*D*b*x^4 + 4
*C*b*x^3 - 6*(D*a - B*b)*x^2 - 12*(C*a - A*b)*x)/b^2 + 1/2*(D*a^2 - B*a*b)*
log(b*x^2 + a)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.01

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{(Ca^2-Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{(Da^2-Bab)\log\left(bx^2+a\right)}{2\,b^3} + \frac{3\,Db^3x^4+4\,Cb^3x^3-6\,Dab^2x^2+6\,Bb^3x^2-12\,Cab^2x+12\,Ab^3x}{12\,b^4}$$

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (C*a^2 - A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*(D*a^2 - B*a*b)
*log(b*x^2 + a)/b^3 + 1/12*(3*D*b^3*x^4 + 4*C*b^3*x^3 - 6*D*a*b^2*x^2 + 6*B
*b^3*x^2 - 12*C*a*b^2*x + 12*A*b^3*x)/b^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \int \frac{x^2\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{b\,x^2+a}\,dx$$

```
[In] int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

```
[Out] int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

3.89 $\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2} dx$

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Optimal result

Integrand size = 26, antiderivative size = 92

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2} dx = \frac{(bB-aD)x}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b} - \frac{\sqrt{a}(bB-aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{(Ab-aC)\log(a+bx^2)}{2b^2}$$

```
[Out] (B*b-D*a)*x/b^2+1/2*C*x^2/b+1/3*D*x^3/b+1/2*(A*b-C*a)*ln(b*x^2+a)/b^2-(B*b-
D*a)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(5/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1816, 649, 211, 266}

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2} dx = \frac{(Ab-aC)\log(a+bx^2)}{2b^2} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bB-aD)}{b^{5/2}} + \frac{x(bB-aD)}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}$$

```
[In] Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] ((b*B - a*D)*x)/b^2 + (C*x^2)/(2*b) + (D*x^3)/(3*b) - (Sqrt[a]*(b*B - a*D)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) + ((A*b - a*C)*Log[a + b*x^2])/(2*b^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{bB-aD}{b^2}+\frac{Cx}{b}+\frac{Dx^2}{b}-\frac{a(bB-aD)-b(Ab-aC)x}{b^2\left(a+bx^2\right)}\right)dx \\
&= \frac{(bB-aD)x}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}-\frac{\int \frac{a(bB-aD)-b(Ab-aC)x}{a+bx^2}\,dx}{b^2} \\
&= \frac{(bB-aD)x}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}+\frac{(Ab-aC)\int \frac{x}{a+bx^2}\,dx}{b}-\frac{(a(bB-aD))\int \frac{1}{a+bx^2}\,dx}{b^2} \\
&= \frac{(bB-aD)x}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}-\frac{\sqrt{a}(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{(Ab-aC)\log\left(a+bx^2\right)}{2b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\begin{aligned}
&\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx \\
&= \frac{\sqrt{a}(-bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}} \\
&\quad + \frac{x(6bB-6aD+bx(3C+2Dx))+3(Ab-aC)\log\left(a+bx^2\right)}{6b^2}
\end{aligned}$$

```
[In] Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
[Out] (Sqrt[a]*(-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) + (x*(6*b*B -
6*a*D + b*x*(3*C + 2*D*x)) + 3*(A*b - a*C)*Log[a + b*x^2])/(6*b^2)
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\frac{1}{3}Db\,x^{3}+\frac{1}{2}bC\,x^{2}+bBx-Dax}{b^{2}}+\frac{\frac{\left(b^{2}A-Cab\right)\ln\left(b\,x^{2}+a\right)}{2b}+\frac{\left(-abB+Da^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{b^{2}}$	85

```
[In] int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] 1/b^2*(1/3*D*b*x^3+1/2*b*C*x^2+b*B*x-D*a*x)+1/b^2*(1/2*(A*b^2-C*a*b)/b*ln(b
*x^2+a)+(-B*a*b+D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.96

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \left[\frac{2\,Dbx^{3}+3\,Cbx^{2}+3\,(Da-Bb)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^{2}+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^{2}+a}\right)-6\,(Da-Bb)x-3\,(Ca-Ab)\log\left(bx^{2}+a\right)}{6\,b^{2}},\right.$$

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
[Out] [1/6*(2*D*b*x^3 + 3*C*b*x^2 + 3*(D*a - B*b)*sqrt(-a/b)*log((b*x^2 + 2*b*x*s
qrt(-a/b) - a)/(b*x^2 + a)) - 6*(D*a - B*b)*x - 3*(C*a - A*b)*log(b*x^2 + a
))/b^2, 1/6*(2*D*b*x^3 + 3*C*b*x^2 + 6*(D*a - B*b)*sqrt(a/b)*arctan(b*x*sqr
t(a/b)/a) - 6*(D*a - B*b)*x - 3*(C*a - A*b)*log(b*x^2 + a))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(80) = 160$.

Time = 0.45 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.29

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$= \frac{Cx^2}{2b} + \frac{Dx^3}{3b} + x\left(\frac{B}{b} - \frac{Da}{b^2}\right) + \left(-\frac{-Ab+Ca}{2b^2} - \frac{\sqrt{-ab^5}(-Bb+Da)}{2b^5}\right)\log\left(x + \frac{-Ab+Ca+2b^2\left(-\frac{-Ab+Ca}{2b^2} - \frac{\sqrt{-ab^5}(-Bb+Da)}{2b^5}\right)}{-Bb+Da}\right)$$
$$+ \left(-\frac{-Ab+Ca}{2b^2} + \frac{\sqrt{-ab^5}(-Bb+Da)}{2b^5}\right)\log\left(x + \frac{-Ab+Ca+2b^2\left(-\frac{-Ab+Ca}{2b^2} + \frac{\sqrt{-ab^5}(-Bb+Da)}{2b^5}\right)}{-Bb+Da}\right)$$

```
[In] integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**2/(2*b) + D*x**3/(3*b) + x*(B/b - D*a/b**2) + (-(-A*b + C*a)/(2*b**2)
- sqrt(-a*b**5)*(-B*b + D*a)/(2*b**5))*log(x + (-A*b + C*a + 2*b**2*(-(-A*b
 + C*a)/(2*b**2) - sqrt(-a*b**5)*(-B*b + D*a)/(2*b**5)))/(-B*b + D*a)) + (-
(-A*b + C*a)/(2*b**2) + sqrt(-a*b**5)*(-B*b + D*a)/(2*b**5))*log(x + (-A*b
+ C*a + 2*b**2*(-(-A*b + C*a)/(2*b**2) + sqrt(-a*b**5)*(-B*b + D*a)/(2*b**5
)))/(-B*b + D*a))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = -\frac{(Ca-Ab)\log(bx^2+a)}{2\,b^2} + \frac{(Da^2-Bab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{2\,Dbx^3+3\,Cbx^2-6\,(Da-Bb)x}{6\,b^2}$$

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*(C*a - A*b)*log(b*x^2 + a)/b^2 + (D*a^2 - B*a*b)*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*b^2) + 1/6*(2*D*b*x^3 + 3*C*b*x^2 - 6*(D*a - B*b)*x)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.96

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = -\frac{(Ca-Ab)\log(bx^2+a)}{2\,b^2} + \frac{(Da^2-Bab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{2\,Db^2x^3+3\,Cb^2x^2-6\,Dabx+6\,Bb^2x}{6\,b^3}$$

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(C*a - A*b)*log(b*x^2 + a)/b^2 + (D*a^2 - B*a*b)*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*b^2) + 1/6*(2*D*b^2*x^3 + 3*C*b^2*x^2 - 6*D*a*b*x + 6*B*b^2*x)/
b^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \int \frac{x\,(A+B\,x+C\,x^2+x^3\,D)}{b\,x^2+a}\,dx$$

```
[In] int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

```
[Out] int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

3.90 $\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2} dx$

Optimal result 565
Rubi [A] (verified) 565
Mathematica [A] (verified) 566
Maple [A] (verified) 567
Fricas [A] (verification not implemented) 567
Sympy [B] (verification not implemented) 568
Maxima [A] (verification not implemented) 568
Giac [A] (verification not implemented) 569
Mupad [B] (verification not implemented) 569

Optimal result

Integrand size = 25, antiderivative size = 73

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2} dx = \frac{Cx}{b}+\frac{Dx^2}{2b}+\frac{(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}} + \frac{(bB-aD)\log(a+bx^2)}{2b^2}$$

```
[Out] C*x/b+1/2*D*x^2/b+1/2*(B*b-D*a)*ln(b*x^2+a)/b^2+(A*b-C*a)*arctan(x*b^(1/2)/
a^(1/2))/b^(3/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1824, 649, 211, 266}

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2} dx = \frac{(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}} + \frac{(bB-aD)\log(a+bx^2)}{2b^2}+\frac{Cx}{b}+\frac{Dx^2}{2b}$$

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2),x]
```

```
[Out] (C*x)/b + (D*x^2)/(2*b) + ((A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a
]*b^(3/2)) + ((b*B - a*D)*Log[a + b*x^2])/(2*b^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{C}{b}+\frac{Dx}{b}+\frac{Ab-aC+(bB-aD)x}{b\left(a+bx^2\right)}\right)dx\\
&= \frac{Cx}{b}+\frac{Dx^2}{2b}+\frac{\int \frac{Ab-aC+(bB-aD)x}{a+bx^2}\,dx}{b}\\
&= \frac{Cx}{b}+\frac{Dx^2}{2b}+\frac{(Ab-aC)\int \frac{1}{a+bx^2}\,dx}{b}+\frac{(bB-aD)\int \frac{x}{a+bx^2}\,dx}{b}\\
&= \frac{Cx}{b}+\frac{Dx^2}{2b}+\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{(bB-aD)\log\left(a+bx^2\right)}{2b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$
\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx = \frac{bx(2C+Dx)+\frac{2\sqrt{b}(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}+(bB-aD)\log\left(a+bx^2\right)}{2b^2}
$$

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2),x]
[Out] (b*x*(2*C + D*x) + (2*Sqrt[b]*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt
[a] + (b*B - a*D)*Log[a + b*x^2])/(2*b^2)
```

Maple [A] (verified)

Time = 3.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\frac{1}{2}Dx^2+Cx}{b}+\frac{\frac{(Bb-Da)\ln\left(b\,x^2+a\right)}{2b}+\frac{(Ab-Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{b}$	65

```
[In] int((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*D*x^2+C*x)+1/b*(1/2*(B*b-D*a)/b*ln(b*x^2+a)+(A*b-C*a)/(a*b)^(1/2)*
arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.15

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx = \left[\frac{Dabx^2+2\,Cabx+(Ca-Ab)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)-(Da^2-Bab)\log\left(bx^2+a\right)}{2\,ab^2}, \frac{Dabx^2+2\,Cabx}{}\right.$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(D*a*b*x^2 + 2*C*a*b*x + (C*a - A*b)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a
*b)*x - a)/(b*x^2 + a)) - (D*a^2 - B*a*b)*log(b*x^2 + a))/(a*b^2), 1/2*(D*a
*b*x^2 + 2*C*a*b*x - 2*(C*a - A*b)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (D*a^2
 - B*a*b)*log(b*x^2 + a))/(a*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(65) = 130.

Time = 0.42 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.00

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx$$
$$= \frac{Cx}{b} + \frac{Dx^2}{2b} + \left(-\frac{-Bb+Da}{2b^2} - \frac{\sqrt{-ab^5}(-Ab+Ca)}{2ab^4}\right)\log\left(x + \frac{Bab - Da^2 - 2ab^2\left(-\frac{-Bb+Da}{2b^2} - \frac{\sqrt{-ab^5}(-Ab+Ca)}{2ab^4}\right)}{-Ab^2+Cab}\right)$$
$$+ \left(-\frac{-Bb+Da}{2b^2} + \frac{\sqrt{-ab^5}(-Ab+Ca)}{2ab^4}\right)\log\left(x + \frac{Bab - Da^2 - 2ab^2\left(-\frac{-Bb+Da}{2b^2} + \frac{\sqrt{-ab^5}(-Ab+Ca)}{2ab^4}\right)}{-Ab^2+Cab}\right)$$

```
[In] integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x/b + D*x**2/(2*b) + (-(-B*b + D*a)/(2*b**2) - sqrt(-a*b**5)*(-A*b + C*a)
/(2*a*b**4))*log(x + (B*a*b - D*a**2 - 2*a*b**2*(-(-B*b + D*a)/(2*b**2) - s
qrt(-a*b**5)*(-A*b + C*a)/(2*a*b**4)))/(-A*b**2 + C*a*b)) + (-(-B*b + D*a)/
(2*b**2) + sqrt(-a*b**5)*(-A*b + C*a)/(2*a*b**4))*log(x + (B*a*b - D*a**2 -
 2*a*b**2*(-(-B*b + D*a)/(2*b**2) + sqrt(-a*b**5)*(-A*b + C*a)/(2*a*b**4)))
/(-A*b**2 + C*a*b))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx = -\frac{(Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{Dx^2+2\,Cx}{2\,b} - \frac{(Da-Bb)\log\left(bx^2+a\right)}{2\,b^2}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + 1/2*(D*x^2 + 2*C*x)/b -
1/2*(D*a - B*b)*log(b*x^2 + a)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx = -\frac{(Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} - \frac{(Da-Bb)\log\left(bx^2+a\right)}{2\,b^2} + \frac{Dbx^2+2\,Cbx}{2\,b^2}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*(D*a - B*b)*log(b*x^
2 + a)/b^2 + 1/2*(D*b*x^2 + 2*C*b*x)/b^2
```

Mupad [B] (verification not implemented)

Time = 5.57 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx = \frac{B\,\ln\left(b\,x^2+a\right)}{2\,b} - \frac{\left(a\,\ln\left(b\,x^2+a\right)-b\,x^2\right)\,D}{2\,b^2} + \frac{C\,x}{b} + \frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}} - \frac{C\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[In] int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2),x)
```

```
[Out] (B*log(a + b*x^2))/(2*b) - ((a*log(a + b*x^2) - b*x^2)*D)/(2*b^2) + (C*x)/b
 + (A*atan((b^(1/2)*x)/a^(1/2)))/(a^(1/2)*b^(1/2)) - (C*a^(1/2)*atan((b^(1/
2)*x)/a^(1/2)))/b^(3/2)
```

3.91 $\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)}\,dx$

Optimal result . . . 570
Rubi [A] (verified) . . . 570
Mathematica [A] (verified) . . . 571
Maple [A] (verified) . . . 572
Fricas [A] (verification not implemented) . . . 572
Sympy [F(-1)] . . . 572
Maxima [A] (verification not implemented) . . . 573
Giac [A] (verification not implemented) . . . 573
Mupad [F(-1)] . . . 573

Optimal result

Integrand size = 28, antiderivative size = 72

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)}\,dx=\frac{Dx}{b}+\frac{(bB-aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{A\log(x)}{a}-\frac{(Ab-aC)\log(a+bx^2)}{2ab}$$

```
[Out] D*x/b+A*ln(x)/a-1/2*(A*b-C*a)*ln(b*x^2+a)/a/b+(B*b-D*a)*arctan(x*b^(1/2)/a^
(1/2))/b^(3/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1816, 649, 211, 266}

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)}\,dx=-\frac{(Ab-aC)\log(a+bx^2)}{2ab}+\frac{A\log(x)}{a}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bB-aD)}{\sqrt{a}b^{3/2}}+\frac{Dx}{b}$$

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)),x]
```

```
[Out] (D*x)/b + ((b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)) + (A*
Log[x])/a - ((A*b - a*C)*Log[a + b*x^2])/(2*a*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{D}{b}+\frac{A}{ax}+\frac{a(bB-aD)-b(Ab-aC)x}{ab\,(a+bx^2)}\right)dx\\
&= \frac{Dx}{b}+\frac{A\log(x)}{a}+\frac{\int \frac{a(bB-aD)-b(Ab-aC)x}{a+bx^2}\,dx}{ab}\\
&= \frac{Dx}{b}+\frac{A\log(x)}{a}-\frac{(Ab-aC)\int \frac{x}{a+bx^2}\,dx}{a}+\frac{(bB-aD)\int \frac{1}{a+bx^2}\,dx}{b}\\
&= \frac{Dx}{b}+\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{A\log(x)}{a}-\frac{(Ab-aC)\log(a+bx^2)}{2ab}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\,(a+bx^2)}\,dx = \frac{Dx}{b}-\frac{(-bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{A\log(x)}{a}+\frac{(-Ab+aC)\log(a+bx^2)}{2ab}$$

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)),x]
[Out] (D*x)/b - ((-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)) +
(A*Log[x])/a + ((-(A*b) + a*C)*Log[a + b*x^2])/(2*a*b)
```

Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{Dx}{b}+\frac{A\ln(x)}{a}+\frac{\frac{\left(-b^{2}A+Cab\right)\ln\left(b\,x^{2}+a\right)}{2b}+\frac{\left(abB-Da^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{ba}$	73

```
[In] int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] D*x/b+A*ln(x)/a+1/b/a*(1/2*(-A*b^2+C*a*b)/b*ln(b*x^2+a)+(B*a*b-D*a^2)/(a*b)
^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.19

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)}\,dx$$
$$=\left[\frac{2\,Dabx+2\,Ab^2\log\left(x\right)-(Da-Bb)\sqrt{-ab}\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right)+\left(Cab-Ab^2\right)\log\left(bx^2+a\right)}{2\,ab^2},\ \frac{2\,Dabx+2\,A}{}\right.$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x, algorithm="fricas")
[Out] [1/2*(2*D*a*b*x + 2*A*b^2*log(x) - (D*a - B*b)*sqrt(-a*b)*log((b*x^2 + 2*sq
rt(-a*b)*x - a)/(b*x^2 + a)) + (C*a*b - A*b^2)*log(b*x^2 + a))/(a*b^2), 1/2
*(2*D*a*b*x + 2*A*b^2*log(x) - 2*(D*a - B*b)*sqrt(a*b)*arctan(sqrt(a*b)*x/a
) + (C*a*b - A*b^2)*log(b*x^2 + a))/(a*b^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)}\,dx=\text{Timed out}$$

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)}\,dx = \frac{Dx}{b} + \frac{A\log(x)}{a} - \frac{(Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{(Ca-Ab)\log(bx^2+a)}{2\,ab}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x, algorithm="maxima")
[Out] D*x/b + A*log(x)/a - (D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + 1/2*
(C*a - A*b)*log(b*x^2 + a)/(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)}\,dx = \frac{Dx}{b} + \frac{A\log(|x|)}{a} - \frac{(Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{(Ca-Ab)\log(bx^2+a)}{2\,ab}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x, algorithm="giac")
[Out] D*x/b + A*log(abs(x))/a - (D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) +
 1/2*(C*a - A*b)*log(b*x^2 + a)/(a*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{x\,\left(b\,x^2+a\right)}\,dx$$

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)),x)
[Out] int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)), x)
```

3.92 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)}\,dx$

Optimal result 574
Rubi [A] (verified) 574
Mathematica [A] (verified) 575
Maple [A] (verified) 576
Fricas [A] (verification not implemented) 576
Sympy [F(-1)] 576
Maxima [A] (verification not implemented) 577
Giac [A] (verification not implemented) 577
Mupad [B] (verification not implemented) 577

Optimal result

Integrand size = 28, antiderivative size = 76

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)}\,dx = -\frac{A}{ax}-\frac{(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{B\log(x)}{a}-\frac{(bB-aD)\log(a+bx^2)}{2ab}$$

```
[Out] -A/a/x+B*ln(x)/a-1/2*(B*b-D*a)*ln(b*x^2+a)/a/b-(A*b-C*a)*arctan(x*b^(1/2)/a
^(1/2))/a^(3/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1816, 649, 211, 266}

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)}\,dx = -\frac{(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}-\frac{(bB-aD)\log(a+bx^2)}{2ab}+\frac{B\log(x)}{a}$$

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(A/(a*x)) - ((A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]) +
(B*Log[x])/a - ((b*B - a*D)*Log[a + b*x^2])/(2*a*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{A}{ax^2}+\frac{B}{ax}+\frac{-Ab+aC-(bB-aD)x}{a\,(a+bx^2)}\right)dx\\
&= -\frac{A}{ax}+\frac{B\log(x)}{a}+\frac{\int \frac{-Ab+aC-(bB-aD)x}{a+bx^2}\,dx}{a}\\
&= -\frac{A}{ax}+\frac{B\log(x)}{a}+\frac{(-Ab+aC)\int \frac{1}{a+bx^2}\,dx}{a}+\frac{(-bB+aD)\int \frac{x}{a+bx^2}\,dx}{a}\\
&= -\frac{A}{ax}-\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{B\log(x)}{a}-\frac{(bB-aD)\log\left(a+bx^2\right)}{2ab}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\,(a+bx^2)}\,dx = -\frac{A}{ax}+\frac{(-Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{B\log(x)}{a}+\frac{(-bB+aD)\log\left(a+bx^2\right)}{2ab}$$

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)),x]
[Out] -(A/(a*x)) + ((-(A*b) + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b])
 + (B*Log[x])/a + ((-(b*B) + a*D)*Log[a + b*x^2])/(2*a*b)
```

Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{A}{ax}+\frac{B\ln(x)}{a}+\frac{\frac{(-Bb+Da)\ln(b\,x^2+a)}{2b}+\frac{(-Ab+Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{a}$	67

```
[In] int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -A/a/x+B*ln(x)/a+1/a*(1/2*(-B*b+D*a)/b*ln(b*x^2+a)+(-A*b+C*a)/(a*b)^(1/2)*a
rctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.17

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)}\,dx$$
$$=\left[\frac{2\,Babx\log\left(x\right)+(Ca-Ab)\sqrt{-ab}x\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right)-2\,Aab+(Da^2-Bab)x\log\left(bx^2+a\right)}{2\,a^2bx},\frac{2\,Babx\log\right.$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*B*a*b*x*log(x) + (C*a - A*b)*sqrt(-a*b)*x*log((b*x^2 + 2*sqrt(-a*b)
*x - a)/(b*x^2 + a)) - 2*A*a*b + (D*a^2 - B*a*b)*x*log(b*x^2 + a))/(a^2*b*x
), 1/2*(2*B*a*b*x*log(x) + 2*(C*a - A*b)*sqrt(a*b)*x*arctan(sqrt(a*b)*x/a)
- 2*A*a*b + (D*a^2 - B*a*b)*x*log(b*x^2 + a))/(a^2*b*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)}\,dx=\text{Timed out}$$

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)}\,dx = \frac{B\log\left(x\right)}{a} + \frac{\left(Ca-Ab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} + \frac{\left(Da-Bb\right)\log\left(bx^2+a\right)}{2\,ab} - \frac{A}{ax}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] B*log(x)/a + (C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*(D*a - B
*b)*log(b*x^2 + a)/(a*b) - A/(a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)}\,dx = \frac{B\log\left(|x|\right)}{a} + \frac{\left(Ca-Ab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} + \frac{\left(Da-Bb\right)\log\left(bx^2+a\right)}{2\,ab} - \frac{A}{ax}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] B*log(abs(x))/a + (C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*(D*
a - B*b)*log(b*x^2 + a)/(a*b) - A/(a*x)
```

Mupad [B] (verification not implemented)

Time = 5.62 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)}\,dx = \frac{\ln\left(b\,x^2+a\right)\,D}{2\,b} - \frac{A}{a\,x} - \frac{B\,\left(\ln\left(b\,x^2+a\right)-2\,\ln\left(x\right)\right)}{2\,a} - \frac{A\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}} + \frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x^2*(a + b*x^2)),x)
```

```
[Out] (log(a + b*x^2)*D)/(2*b) - A/(a*x) - (B*(log(a + b*x^2) - 2*log(x)))/(2*a)
- (A*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2) + (C*atan((b^(1/2)*x)/a^(1/
2)))/(a^(1/2)*b^(1/2))
```

3.93 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)}\,dx$

Optimal result 578
Rubi [A] (verified) 578
Mathematica [A] (verified) 579
Maple [A] (verified) 580
Fricas [A] (verification not implemented) 580
Sympy [F(-1)] 580
Maxima [A] (verification not implemented) 581
Giac [A] (verification not implemented) 581
Mupad [B] (verification not implemented) 581

Optimal result

Integrand size = 28, antiderivative size = 92

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)}\,dx = -\frac{A}{2ax^2}-\frac{B}{ax}-\frac{(bB-aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}} -\frac{(Ab-aC)\log(x)}{a^2}+\frac{(Ab-aC)\log(a+bx^2)}{2a^2}$$

```
[Out] -1/2*A/a/x^2-B/a/x-(A*b-C*a)*ln(x)/a^2+1/2*(A*b-C*a)*ln(b*x^2+a)/a^2-(B*b-D
*a)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1816, 649, 211, 266}

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)}\,dx = -\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bB-aD)}{a^{3/2}\sqrt{b}}+\frac{(Ab-aC)\log(a+bx^2)}{2a^2} -\frac{\log(x)(Ab-aC)}{a^2}-\frac{A}{2ax^2}-\frac{B}{ax}$$

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)),x]
```

```
[Out] -1/2*A/(a*x^2) - B/(a*x) - ((b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/
2)*Sqrt[b]) - ((A*b - a*C)*Log[x])/a^2 + ((A*b - a*C)*Log[a + b*x^2])/(2*a^
2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{A}{ax^3}+\frac{B}{ax^2}+\frac{-Ab+aC}{a^2x}+\frac{-a(bB-aD)+b(Ab-aC)x}{a^2\left(a+bx^2\right)}\right)dx \\
&= -\frac{A}{2ax^2}-\frac{B}{ax}-\frac{(Ab-aC)\log(x)}{a^2}+\frac{\int \frac{-a(bB-aD)+b(Ab-aC)x}{a+bx^2}\,dx}{a^2} \\
&= -\frac{A}{2ax^2}-\frac{B}{ax}-\frac{(Ab-aC)\log(x)}{a^2}+\frac{(b(Ab-aC))\int \frac{x}{a+bx^2}\,dx}{a^2}-\frac{(bB-aD)\int \frac{1}{a+bx^2}\,dx}{a} \\
&= -\frac{A}{2ax^2}-\frac{B}{ax}-\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{(Ab-aC)\log(x)}{a^2}+\frac{(Ab-aC)\log\left(a+bx^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)}\,dx = \frac{-\frac{aA}{x^2}-\frac{2aB}{x}+\frac{2\sqrt{a}(-bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+2(-Ab+aC)\log(x)+(Ab-aC)\log\left(a+bx^2\right)}{2a^2}$$

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)),x]
```

```
[Out] (-((a*A)/x^2) - (2*a*B)/x + (2*Sqrt[a]*(-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/Sqrt[b] + 2*(-(A*b) + a*C)*Log[x] + (A*b - a*C)*Log[a + b*x^2])/(2*
a^2)
```

Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{A}{2a\,x^2}-\frac{B}{ax}+\frac{(-Ab+Ca)\ln(x)}{a^2}+\frac{\frac{\left(b^2A-Cab\right)\ln\left(b\,x^2+a\right)}{2b}+\frac{\left(-abB+Da^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{a^2}$	89

```
[In] int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*A/a/x^2-B/a/x+1/a^2*(-A*b+C*a)*ln(x)+1/a^2*(1/2*(A*b^2-C*a*b)/b*ln(b*x
^2+a)+(-B*a*b+D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.23

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)}\,dx$$
$$=\left[-\frac{(Da-Bb)\sqrt{-ab}x^2\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)+2\,Babx+(Cab-Ab^2)x^2\log\left(bx^2+a\right)-2\,(Cab-Ab^2)x^2\log}{2\,a^2bx^2}\right.$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*((D*a - B*b)*sqrt(-a*b)*x^2*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 +
 a)) + 2*B*a*b*x + (C*a*b - A*b^2)*x^2*log(b*x^2 + a) - 2*(C*a*b - A*b^2)*x
^2*log(x) + A*a*b)/(a^2*b*x^2), 1/2*(2*(D*a - B*b)*sqrt(a*b)*x^2*arctan(sqr
t(a*b)*x/a) - 2*B*a*b*x - (C*a*b - A*b^2)*x^2*log(b*x^2 + a) + 2*(C*a*b - A
*b^2)*x^2*log(x) - A*a*b)/(a^2*b*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)}\,dx=\text{Timed out}$$

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)}\,dx = \frac{(Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{(Ca-Ab)\log(bx^2+a)}{2a^2} + \frac{(Ca-Ab)\log(x)}{a^2} - \frac{2Bx+A}{2ax^2}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x, algorithm="maxima")
[Out] (D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/2*(C*a - A*b)*log(b*x^2
 + a)/a^2 + (C*a - A*b)*log(x)/a^2 - 1/2*(2*B*x + A)/(a*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)}\,dx = \frac{(Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{(Ca-Ab)\log(bx^2+a)}{2a^2} + \frac{(Ca-Ab)\log(|x|)}{a^2} - \frac{2Bax+Aa}{2a^2x^2}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x, algorithm="giac")
[Out] (D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/2*(C*a - A*b)*log(b*x^2
 + a)/a^2 + (C*a - A*b)*log(abs(x))/a^2 - 1/2*(2*B*a*x + A*a)/(a^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 5.72 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,D}{\sqrt{a}\,\sqrt{b}} - \frac{B}{a\,x} - \frac{C\,(\ln(b\,x^2+a)-2\,\ln(x))}{2\,a} - \frac{A}{2\,a\,x^2} + \frac{A\,b\,\ln(b\,x^2+a)}{2\,a^2} - \frac{A\,b\,\ln(x)}{a^2} - \frac{B\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x^3*(a + b*x^2)),x)
[Out] (atan((b^(1/2)*x)/a^(1/2))*D)/(a^(1/2)*b^(1/2)) - B/(a*x) - (C*(log(a + b*x
^2) - 2*log(x)))/(2*a) - A/(2*a*x^2) + (A*b*log(a + b*x^2))/(2*a^2) - (A*b*
log(x))/a^2 - (B*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2)
```

3.94 $\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$

Optimal result 582
Rubi [A] (verified) 582
Mathematica [A] (verified) 584
Maple [A] (verified) 585
Fricas [A] (verification not implemented) 585
Sympy [B] (verification not implemented) 586
Maxima [A] (verification not implemented) 586
Giac [A] (verification not implemented) 587
Mupad [F(-1)] 588

Optimal result

Integrand size = 28, antiderivative size = 176

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{(3Ab-5aC)x}{2b^3}+\frac{(2bB-3aD)x^2}{2b^3}-\frac{(3Ab-5aC)x^3}{6ab^2}+\frac{Dx^4}{4b^2}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)}-\frac{\sqrt{a}(3Ab-5aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}-\frac{a(2bB-3aD)\log(a+bx^2)}{2b^4}$$

```
[Out] 1/2*(3*A*b-5*C*a)*x/b^3+1/2*(2*B*b-3*D*a)*x^2/b^3-1/6*(3*A*b-5*C*a)*x^3/a/b
^2+1/4*D*x^4/b^2-1/2*x^4*(a*(B-a*D/b)-(A*b-C*a)*x)/a/b/(b*x^2+a)-1/2*a*(2*B
*b-3*D*a)*ln(b*x^2+a)/b^4-1/2*(3*A*b-5*C*a)*arctan(x*b^(1/2)/a^(1/2))*a^(1/
2)/b^(7/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used

$= \{1818, 1816, 649, 211, 266\}$

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = -\frac{\sqrt{a}(3Ab-5aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}} + \frac{x(3Ab-5aC)}{2b^3} - \frac{x^3(3Ab-5aC)}{6ab^2} - \frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\,(a+bx^2)} - \frac{a(2bB-3aD)\log\left(a+bx^2\right)}{2b^4} + \frac{x^2(2bB-3aD)}{2b^3} + \frac{Dx^4}{4b^2}$$

```
[In] Int[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] ((3*A*b - 5*a*C)*x)/(2*b^3) + ((2*b*B - 3*a*D)*x^2)/(2*b^3) - ((3*A*b - 5*a
*C)*x^3)/(6*a*b^2) + (D*x^4)/(4*b^2) - (x^4*(a*(B - (a*D)/b) - (A*b - a*C)*
x))/(2*a*b*(a + b*x^2)) - (Sqrt[a]*(3*A*b - 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*b^(7/2)) - (a*(2*b*B - 3*a*D)*Log[a + b*x^2])/(2*b^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
```

```
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}-\frac{\int\frac{x^3\left(-4a\left(B-\frac{aD}{b}\right)+(3Ab-5aC)x-2aDx^2\right)}{a+bx^2}\,dx}{2ab}$$

$$=-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}-\frac{\int\left(-\frac{a(3Ab-5aC)}{b^2}-\frac{2a(2bB-3aD)x}{b^2}+\frac{(3Ab-5aC)x^2}{b}-\frac{2aDx^3}{b}+\frac{a^2(3Ab-5aC)+2a^2(2bB-3aD)x}{b^2(a+bx^2)}\right)dx}{2ab}$$

$$=\frac{(3Ab-5aC)x}{2b^3}+\frac{(2bB-3aD)x^2}{2b^3}-\frac{(3Ab-5aC)x^3}{6ab^2}+\frac{Dx^4}{4b^2}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}-\frac{\int\frac{a^2(3Ab-5aC)+2a^2(2bB-3aD)x}{a+bx^2}\,dx}{2ab^3}$$

$$=\frac{(3Ab-5aC)x}{2b^3}+\frac{(2bB-3aD)x^2}{2b^3}-\frac{(3Ab-5aC)x^3}{6ab^2}+\frac{Dx^4}{4b^2}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}-\frac{(a(3Ab-5aC))\int\frac{1}{a+bx^2}\,dx}{2b^3}-\frac{(a(2bB-3aD))\int\frac{x}{a+bx^2}\,dx}{b^3}$$

$$=\frac{(3Ab-5aC)x}{2b^3}+\frac{(2bB-3aD)x^2}{2b^3}-\frac{(3Ab-5aC)x^3}{6ab^2}+\frac{Dx^4}{4b^2}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}-\frac{\sqrt{a}(3Ab-5aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}-\frac{a(2bB-3aD)\log\left(a+bx^2\right)}{2b^4}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.79

$$\int\frac{x^4(A+Bx+Cx^2+Dx^3)}{\left(a+bx^2\right)^2}\,dx=\frac{12b(Ab-2aC)x+6b(bB-2aD)x^2+4b^2Cx^3+3b^2Dx^4+\frac{6a\left(a^2D+Ab^2x-ab(B+Cx)\right)}{a+bx^2}+6\sqrt{a}\sqrt{b}(-3Ab+5aC)}{12b^4}$$

```
[In] Integrate[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] (12*b*(A*b - 2*a*C)*x + 6*b*(b*B - 2*a*D)*x^2 + 4*b^2*C*x^3 + 3*b^2*D*x^4 +
 (6*a*(a^2*D + A*b^2*x - a*b*(B + C*x)))/(a + b*x^2) + 6*Sqrt[a]*Sqrt[b]*(-
3*A*b + 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 6*a*(-2*b*B + 3*a*D)*Log[a + b
*x^2])/(12*b^4)
```

Maple [A] (verified)

Time = 3.55 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.76

method	result
default	$\frac{\frac{1}{4}Db\,x^4+\frac{1}{3}bC\,x^3+\frac{1}{2}bB\,x^2-Da\,x^2+Abx-2Cax}{b^3}-\frac{a\left(\frac{\left(-\frac{Ab}{2}+\frac{Ca}{2}\right)x+\frac{a(Bb-Da)}{2b}}{b\,x^2+a}+\frac{(4Bb-6Da)\ln\left(b\,x^2+a\right)}{4b}+\frac{(3Ab-5Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^3}$

```
[In] int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/4*D*b*x^4+1/3*b*C*x^3+1/2*b*B*x^2-D*a*x^2+A*b*x-2*C*a*x)-a/b^3*(((
-1/2*A*b+1/2*C*a)*x+1/2*a*(B*b-D*a)/b)/(b*x^2+a)+1/4*(4*B*b-6*D*a)/b*ln(b*x
^2+a)+1/2*(3*A*b-5*C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 468, normalized size of antiderivative = 2.66

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$
$$=\left[\frac{3\,Db^3x^6+4\,Cb^3x^5-3\,(3\,Dab^2-2\,Bb^3)x^4+6\,Da^3-6\,Ba^2b-4\,(5\,Cab^2-3\,Ab^3)x^3-6\,(2\,Da^2b-Bab\right.$$

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(3*D*b^3*x^6 + 4*C*b^3*x^5 - 3*(3*D*a*b^2 - 2*B*b^3)*x^4 + 6*D*a^3 -
6*B*a^2*b - 4*(5*C*a*b^2 - 3*A*b^3)*x^3 - 6*(2*D*a^2*b - B*a*b^2)*x^2 - 3*(
5*C*a^2*b - 3*A*a*b^2 + (5*C*a*b^2 - 3*A*b^3)*x^2)*sqrt(-a/b)*log((b*x^2 -
2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 6*(5*C*a^2*b - 3*A*a*b^2)*x + 6*(3*D*a
^3 - 2*B*a^2*b + (3*D*a^2*b - 2*B*a*b^2)*x^2)*log(b*x^2 + a))/(b^5*x^2 + a*
b^4), 1/12*(3*D*b^3*x^6 + 4*C*b^3*x^5 - 3*(3*D*a*b^2 - 2*B*b^3)*x^4 + 6*D*a
^3 - 6*B*a^2*b - 4*(5*C*a*b^2 - 3*A*b^3)*x^3 - 6*(2*D*a^2*b - B*a*b^2)*x^2
+ 6*(5*C*a^2*b - 3*A*a*b^2 + (5*C*a*b^2 - 3*A*b^3)*x^2)*sqrt(a/b)*arctan(b*
x*sqrt(a/b)/a) - 6*(5*C*a^2*b - 3*A*a*b^2)*x + 6*(3*D*a^3 - 2*B*a^2*b + (3*
D*a^2*b - 2*B*a*b^2)*x^2)*log(b*x^2 + a))/(b^5*x^2 + a*b^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(160) = 320.

Time = 1.87 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.90

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$
$$= \frac{Cx^3}{3b^2}+\frac{Dx^4}{4b^2}+x^2\left(\frac{B}{2b^2}-\frac{Da}{b^3}\right)+x\left(\frac{A}{b^2}-\frac{2Ca}{b^3}\right)+\left(\frac{a(-2Bb+3Da)}{2b^4}\right.$$
$$\left.-\frac{\sqrt{-ab^9}(-3Ab+5Ca)}{4b^8}\right)\log\left(x+\frac{4Bab-6Da^2+4b^4\left(\frac{a(-2Bb+3Da)}{2b^4}-\frac{\sqrt{-ab^9}(-3Ab+5Ca)}{4b^8}\right)}{-3Ab^2+5Cab}\right)$$
$$+\left(\frac{a(-2Bb+3Da)}{2b^4}\right.$$
$$\left.+\frac{\sqrt{-ab^9}(-3Ab+5Ca)}{4b^8}\right)\log\left(x+\frac{4Bab-6Da^2+4b^4\left(\frac{a(-2Bb+3Da)}{2b^4}+\frac{\sqrt{-ab^9}(-3Ab+5Ca)}{4b^8}\right)}{-3Ab^2+5Cab}\right)$$
$$+\frac{-Ba^2b+Da^3+x(Aab^2-Ca^2b)}{2ab^4+2b^5x^2}$$

```
[In] integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] C*x**3/(3*b**2) + D*x**4/(4*b**2) + x**2*(B/(2*b**2) - D*a/b**3) + x*(A/b**
2 - 2*C*a/b**3) + (a*(-2*B*b + 3*D*a)/(2*b**4) - sqrt(-a*b**9)*(-3*A*b + 5*
C*a)/(4*b**8))*log(x + (4*B*a*b - 6*D*a**2 + 4*b**4*(a*(-2*B*b + 3*D*a)/(2*
b**4) - sqrt(-a*b**9)*(-3*A*b + 5*C*a)/(4*b**8)))/(-3*A*b**2 + 5*C*a*b)) +
(a*(-2*B*b + 3*D*a)/(2*b**4) + sqrt(-a*b**9)*(-3*A*b + 5*C*a)/(4*b**8))*log
(x + (4*B*a*b - 6*D*a**2 + 4*b**4*(a*(-2*B*b + 3*D*a)/(2*b**4) + sqrt(-a*b*
*9)*(-3*A*b + 5*C*a)/(4*b**8)))/(-3*A*b**2 + 5*C*a*b)) + (-B*a**2*b + D*a**
3 + x*(A*a*b**2 - C*a**2*b))/(2*a*b**4 + 2*b**5*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.85

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{Da^3-Ba^2b-(Ca^2b-Aab^2)x}{2\,(b^5x^2+ab^4)} + \frac{(5\,Ca^2-3\,Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3} + \frac{3\,Dbx^4+4\,Cbx^3-6\,(2\,Da-Bb)x^2-12\,(2\,Ca-Ab)x}{12\,b^3} + \frac{(3\,Da^2-2\,Bab)\log(bx^2+a)}{2\,b^4}$$

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(D*a^3 - B*a^2*b - (C*a^2*b - A*a*b^2)*x)/(b^5*x^2 + a*b^4) + 1/2*(5*C*
a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/12*(3*D*b*x^4 + 4*
C*b*x^3 - 6*(2*D*a - B*b)*x^2 - 12*(2*C*a - A*b)*x)/b^3 + 1/2*(3*D*a^2 - 2*
B*a*b)*log(b*x^2 + a)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.90

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{(5\,Ca^2-3\,Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3} + \frac{(3\,Da^2-2\,Bab)\log(bx^2+a)}{2\,b^4} + \frac{Da^3-Ba^2b-(Ca^2b-Aab^2)x}{2\,(bx^2+a)b^4} + \frac{3\,Db^6x^4+4\,Cb^6x^3-12\,Dab^5x^2+6\,Bb^6x^2-24\,Cab^5x+12\,Ab^6x}{12\,b^8}$$

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*C*a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/2*(3*D*a^
2 - 2*B*a*b)*log(b*x^2 + a)/b^4 + 1/2*(D*a^3 - B*a^2*b - (C*a^2*b - A*a*b^2
)*x)/((b*x^2 + a)*b^4) + 1/12*(3*D*b^6*x^4 + 4*C*b^6*x^3 - 12*D*a*b^5*x^2 +
 6*B*b^6*x^2 - 24*C*a*b^5*x + 12*A*b^6*x)/b^8
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx=\int \frac{x^4\,(A+B\,x+C\,x^2+x^3\,D)}{(b\,x^2+a)^2}\,dx$$

```
[In] int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
[Out] int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2, x)
```

3.95 $\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$

Optimal result 589
Rubi [A] (verified) 589
Mathematica [A] (verified) 591
Maple [A] (verified) 592
Fricas [A] (verification not implemented) 592
Sympy [B] (verification not implemented) 593
Maxima [A] (verification not implemented) 593
Giac [A] (verification not implemented) 594
Mupad [F(-1)] 594

Optimal result

Integrand size = 28, antiderivative size = 154

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{(3bB-5aD)x}{2b^3}-\frac{(Ab-2aC)x^2}{2ab^2}+\frac{Dx^3}{3b^2}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)}-\frac{\sqrt{a}(3bB-5aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{(Ab-2aC)\log(a+bx^2)}{2b^3}$$

```
[Out] 1/2*(3*B*b-5*D*a)*x/b^3-1/2*(A*b-2*C*a)*x^2/a/b^2+1/3*D*x^3/b^2-1/2*x^3*(a*
(B-a*D/b)-(A*b-C*a)*x)/a/b/(b*x^2+a)+1/2*(A*b-2*C*a)*ln(b*x^2+a)/b^3-1/2*(3
*B*b-5*D*a)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(7/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used

= {1818, 1816, 649, 211, 266}

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{(Ab-2aC)\log(a+bx^2)}{2b^3} - \frac{x^2(Ab-2aC)}{2ab^2} - \frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab(a+bx^2)} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(3bB-5aD)}{2b^{7/2}} + \frac{x(3bB-5aD)}{2b^3} + \frac{Dx^3}{3b^2}$$

```
[In] Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] ((3*b*B - 5*a*D)*x)/(2*b^3) - ((A*b - 2*a*C)*x^2)/(2*a*b^2) + (D*x^3)/(3*b^
2) - (x^3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(2*a*b*(a + b*x^2)) - (Sqrt[a]
*(3*b*B - 5*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2)) + ((A*b - 2*a*C)*
Log[a + b*x^2])/(2*b^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
```

```
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)}-\frac{\int \frac{x^2\left(-3a\left(B-\frac{aD}{b}\right)+2(Ab-2aC)x-2aDx^2\right)}{a+bx^2}\,dx}{2ab}\\
&= -\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)}\\
&\quad -\frac{\int\left(-\frac{a(3bB-5aD)}{b^2}+\frac{2(Ab-2aC)x}{b}-\frac{2aDx^2}{b}+\frac{a^2(3bB-5aD)-2ab(Ab-2aC)x}{b^2(a+bx^2)}\right)dx}{2ab}\\
&= \frac{(3bB-5aD)x}{2b^3}-\frac{(Ab-2aC)x^2}{2ab^2}+\frac{Dx^3}{3b^2}\\
&\quad -\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)}-\frac{\int \frac{a^2(3bB-5aD)-2ab(Ab-2aC)x}{a+bx^2}\,dx}{2ab^3}\\
&= \frac{(3bB-5aD)x}{2b^3}-\frac{(Ab-2aC)x^2}{2ab^2}+\frac{Dx^3}{3b^2}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)}\\
&\quad +\frac{(Ab-2aC)\int \frac{x}{a+bx^2}\,dx}{b^2}-\frac{(a(3bB-5aD))\int \frac{1}{a+bx^2}\,dx}{2b^3}\\
&= \frac{(3bB-5aD)x}{2b^3}-\frac{(Ab-2aC)x^2}{2ab^2}+\frac{Dx^3}{3b^2}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)}\\
&\quad -\frac{\sqrt{a}(3bB-5aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{(Ab-2aC)\log\left(a+bx^2\right)}{2b^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.83

$$
\begin{aligned}
\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx &= \frac{(bB-2aD)x}{b^3}+\frac{Cx^2}{2b^2}+\frac{Dx^3}{3b^2}+\frac{a(Ab+bBx-a(C+Dx))}{2b^3\,(a+bx^2)}\\
&\quad +\frac{\sqrt{a}(-3bB+5aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}\\
&\quad +\frac{(Ab-2aC)\log\left(a+bx^2\right)}{2b^3}
\end{aligned}
$$

```
[In] Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
[Out] ((b*B - 2*a*D)*x)/b^3 + (C*x^2)/(2*b^2) + (D*x^3)/(3*b^2) + (a*(A*b + b*B*x
 - a*(C + D*x)))/(2*b^3*(a + b*x^2)) + (Sqrt[a]*(-3*b*B + 5*a*D)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*b^(7/2)) + ((A*b - 2*a*C)*Log[a + b*x^2])/(2*b^3)
```

Maple [A] (verified)

Time = 3.45 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\frac{1}{3}Db\,x^{3}+\frac{1}{2}bC\,x^{2}+bBx-2Dax}{b^{3}}+\frac{\frac{\left(\frac{1}{2}abB-\frac{1}{2}Da^{2}\right)x+\frac{a(Ab-Ca)}{2}}{b\,x^{2}+a}+\frac{\left(2b^{2}A-4Cab\right)\ln\left(b\,x^{2}+a\right)}{4b}+\frac{\left(-3abB+5Da^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{b^{3}}$	124

```
[In] int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/3*D*b*x^3+1/2*b*C*x^2+b*B*x-2*D*a*x)+1/b^3*(((1/2*a*b*B-1/2*D*a^2)
*x+1/2*a*(A*b-C*a))/(b*x^2+a)+1/4*(2*A*b^2-4*C*a*b)/b*ln(b*x^2+a)+1/2*(-3*B
*a*b+5*D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.42

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$
$$=\left[\frac{4\,Db^2x^5+6\,Cb^2x^4+6\,Cabx^2-4\,(5\,Dab-3\,Bb^2)x^3-6\,Ca^2+6\,Aab+3\,(5\,Da^2-3\,Bab+(5\,Dab-3\,B}{12\,(b^4x}\right.$$

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*D*b^2*x^5 + 6*C*b^2*x^4 + 6*C*a*b*x^2 - 4*(5*D*a*b - 3*B*b^2)*x^3
- 6*C*a^2 + 6*A*a*b + 3*(5*D*a^2 - 3*B*a*b + (5*D*a*b - 3*B*b^2)*x^2)*sqrt(
-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 6*(5*D*a^2 - 3*B*a*
b)*x - 6*(2*C*a^2 - A*a*b + (2*C*a*b - A*b^2)*x^2)*log(b*x^2 + a))/(b^4*x^2
 + a*b^3), 1/6*(2*D*b^2*x^5 + 3*C*b^2*x^4 + 3*C*a*b*x^2 - 2*(5*D*a*b - 3*B*
b^2)*x^3 - 3*C*a^2 + 3*A*a*b + 3*(5*D*a^2 - 3*B*a*b + (5*D*a*b - 3*B*b^2)*x
^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 3*(5*D*a^2 - 3*B*a*b)*x - 3*(2*C*a^
2 - A*a*b + (2*C*a*b - A*b^2)*x^2)*log(b*x^2 + a))/(b^4*x^2 + a*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(134) = 268.

Time = 1.71 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.88

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{Cx^2}{2b^2}+\frac{Dx^3}{3b^2}+x\left(\frac{B}{b^2}-\frac{2Da}{b^3}\right)+\left(-\frac{-Ab+2Ca}{2b^3}-\frac{\sqrt{-ab^7}(-3Bb+5Da)}{4b^7}\right)\log\left(x+\frac{-2Ab+4Ca+4b^3\left(-\frac{-Ab+2Ca}{2b^3}-\frac{\sqrt{-ab^7}(-3Bb+5Da)}{4b^7}\right)}{-3Bb+5Da}\right)+\left(-\frac{-Ab+2Ca}{2b^3}+\frac{\sqrt{-ab^7}(-3Bb+5Da)}{4b^7}\right)\log\left(x+\frac{-2Ab+4Ca+4b^3\left(-\frac{-Ab+2Ca}{2b^3}+\frac{\sqrt{-ab^7}(-3Bb+5Da)}{4b^7}\right)}{-3Bb+5Da}\right)+\frac{Aab-Ca^2+x(Bab-Da^2)}{2ab^3+2b^4x^2}$$

```
[In] integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] C*x**2/(2*b**2) + D*x**3/(3*b**2) + x*(B/b**2 - 2*D*a/b**3) + (-(-A*b + 2*C
*a)/(2*b**3) - sqrt(-a*b**7)*(-3*B*b + 5*D*a)/(4*b**7))*log(x + (-2*A*b + 4
*C*a + 4*b**3*(-(-A*b + 2*C*a)/(2*b**3) - sqrt(-a*b**7)*(-3*B*b + 5*D*a)/(4
*b**7)))/(-3*B*b + 5*D*a)) + (-(-A*b + 2*C*a)/(2*b**3) + sqrt(-a*b**7)*(-3*
B*b + 5*D*a)/(4*b**7))*log(x + (-2*A*b + 4*C*a + 4*b**3*(-(-A*b + 2*C*a)/(2
*b**3) + sqrt(-a*b**7)*(-3*B*b + 5*D*a)/(4*b**7)))/(-3*B*b + 5*D*a)) + (A*a
*b - C*a**2 + x*(B*a*b - D*a**2))/(2*a*b**3 + 2*b**4*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.82

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = -\frac{Ca^2-Aab+(Da^2-Bab)x}{2\,(b^4x^2+ab^3)}-\frac{(2\,Ca-Ab)\log\left(bx^2+a\right)}{2\,b^3}+\frac{(5\,Da^2-3\,Bab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3}+\frac{2\,Dbx^3+3\,Cbx^2-6\,(2\,Da-Bb)x}{6\,b^3}$$

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(C*a^2 - A*a*b + (D*a^2 - B*a*b)*x)/(b^4*x^2 + a*b^3) - 1/2*(2*C*a - A
*b)*log(b*x^2 + a)/b^3 + 1/2*(5*D*a^2 - 3*B*a*b)*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*b^3) + 1/6*(2*D*b*x^3 + 3*C*b*x^2 - 6*(2*D*a - B*b)*x)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.85

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = -\frac{(2\,Ca-Ab)\log\left(bx^2+a\right)}{2\,b^3} + \frac{(5\,Da^2-3\,Bab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3} - \frac{Ca^2-Aab+(Da^2-Bab)x}{2\,(bx^2+a)b^3} + \frac{2\,Db^4x^3+3\,Cb^4x^2-12\,Dab^3x+6\,Bb^4x}{6\,b^6}$$

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*C*a - A*b)*log(b*x^2 + a)/b^3 + 1/2*(5*D*a^2 - 3*B*a*b)*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(C*a^2 - A*a*b + (D*a^2 - B*a*b)*x)/((b*x^
2 + a)*b^3) + 1/6*(2*D*b^4*x^3 + 3*C*b^4*x^2 - 12*D*a*b^3*x + 6*B*b^4*x)/b^
6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \int \frac{x^3\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(b\,x^2+a\right)}^2}\,dx$$

```
[In] int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

```
[Out] int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2, x)
```

3.96 $\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$

Optimal result 595
Rubi [A] (verified) 595
Mathematica [A] (verified) 597
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Optimal result

Integrand size = 28, antiderivative size = 134

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = -\frac{(Ab-3aC)x}{2ab^2}+\frac{Dx^2}{2b^2}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)} + \frac{(Ab-3aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}+\frac{(bB-2aD)\log(a+bx^2)}{2b^3}$$

```
[Out] -1/2*(A*b-3*C*a)*x/a/b^2+1/2*D*x^2/b^2-1/2*x^2*(a*(B-a*D/b)-(A*b-C*a)*x)/a/
b/(b*x^2+a)+1/2*(B*b-2*D*a)*ln(b*x^2+a)/b^3+1/2*(A*b-3*C*a)*arctan(x*b^(1/2
)/a^(1/2))/b^(5/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1818, 1816, 649, 211, 266}

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{(Ab-3aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}-\frac{x(Ab-3aC)}{2ab^2} - \frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\,(a+bx^2)} + \frac{(bB-2aD)\log(a+bx^2)}{2b^3}+\frac{Dx^2}{2b^2}$$

```
[In] Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] -1/2*((A*b - 3*a*C)*x)/(a*b^2) + (D*x^2)/(2*b^2) - (x^2*(a*(B - (a*D)/b) -
(A*b - a*C)*x))/(2*a*b*(a + b*x^2)) + ((A*b - 3*a*C)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(2*Sqrt[a]*b^(5/2)) + ((b*B - 2*a*D)*Log[a + b*x^2])/(2*b^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}-\frac{\int \frac{x\left(-2a\left(B-\frac{aD}{b}\right)+(Ab-3aC)x-2aDx^2\right)}{a+bx^2}\,dx}{2ab}$$

$$=-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}-\frac{\int \left(A-\frac{3aC}{b}-\frac{2aDx}{b}-\frac{a(Ab-3aC)+2a(bB-2aD)x}{b(a+bx^2)}\right)dx}{2ab}$$

$$=-\frac{(Ab-3aC)x}{2ab^2}+\frac{Dx^2}{2b^2}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}+\frac{\int \frac{a(Ab-3aC)+2a(bB-2aD)x}{a+bx^2}\,dx}{2ab^2}$$

$$= -\frac{(Ab-3aC)x}{2ab^2} + \frac{Dx^2}{2b^2} - \frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)} + \frac{(Ab-3aC)\int\frac{1}{a+bx^2}\,dx}{2b^2} + \frac{(bB-2aD)\int\frac{x}{a+bx^2}\,dx}{b^2}$$

$$= -\frac{(Ab-3aC)x}{2ab^2} + \frac{Dx^2}{2b^2} - \frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)} + \frac{(Ab-3aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}} + \frac{(bB-2aD)\log(a+bx^2)}{2b^3}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.75

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{2bCx+bDx^2+\frac{-a^2D-Ab^2x+ab(B+Cx)}{a+bx^2}+\frac{\sqrt{b}(Ab-3aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}+(bB-2aD)\log(a+bx^2)}{2b^3}$$

```
[In] Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] (2*b*C*x + b*D*x^2 + (-(a^2*D) - A*b^2*x + a*b*(B + C*x))/(a + b*x^2) + (Sq
rt[b]*(A*b - 3*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + (b*B - 2*a*D)*Lo
g[a + b*x^2])/(2*b^3)
```

Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{\frac{1}{2}Dx^2+Cx}{b^2}+\frac{\frac{\left(-\frac{Ab}{2}+\frac{Ca}{2}\right)x+\frac{a(Bb-Da)}{2b}}{b\,x^2+a}+\frac{(2Bb-4Da)\ln\left(b\,x^2+a\right)}{4b}+\frac{(Ab-3Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{b^2}$	103

```
[In] int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/2*D*x^2+C*x)+1/b^2*(((-1/2*A*b+1/2*C*a)*x+1/2*a*(B*b-D*a)/b)/(b*x^
2+a)+1/4*(2*B*b-4*D*a)/b*ln(b*x^2+a)+1/2*(A*b-3*C*a)/(a*b)^(1/2)*arctan(b*x
/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.66

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$
$$= \left[\frac{2\,Dab^2x^4+4\,Cab^2x^3+2\,Da^2bx^2-2\,Da^3+2\,Ba^2b+(3\,Ca^2-Aab+(3\,Cab-Ab^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\cdots}{bx\cdots}\right.}{4\,(ab^4x^2+a^2b^3)}\right.$$

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*D*a*b^2*x^4 + 4*C*a*b^2*x^3 + 2*D*a^2*b*x^2 - 2*D*a^3 + 2*B*a^2*b +
 (3*C*a^2 - A*a*b + (3*C*a*b - A*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-
a*b)*x - a)/(b*x^2 + a)) + 2*(3*C*a^2*b - A*a*b^2)*x - 2*(2*D*a^3 - B*a^2*b
 + (2*D*a^2*b - B*a*b^2)*x^2)*log(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3), 1/2*(D
*a*b^2*x^4 + 2*C*a*b^2*x^3 + D*a^2*b*x^2 - D*a^3 + B*a^2*b - (3*C*a^2 - A*a
*b + (3*C*a*b - A*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (3*C*a^2*b -
A*a*b^2)*x - (2*D*a^3 - B*a^2*b + (2*D*a^2*b - B*a*b^2)*x^2)*log(b*x^2 + a)
)/(a*b^4*x^2 + a^2*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(116) = 232.

Time = 1.60 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.12

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{Cx}{b^2}+\frac{Dx^2}{2b^2}+\left(-\frac{-Bb+2Da}{2b^3} - \frac{\sqrt{-ab^7}(-Ab+3Ca)}{4ab^6}\right)\log\left(x+\frac{2Bab-4Da^2-4ab^3\left(-\frac{-Bb+2Da}{2b^3}-\frac{\sqrt{-ab^7}(-Ab+3Ca)}{4ab^6}\right)}{-Ab^2+3Cab}\right) + \left(-\frac{-Bb+2Da}{2b^3} + \frac{\sqrt{-ab^7}(-Ab+3Ca)}{4ab^6}\right)\log\left(x+\frac{2Bab-4Da^2-4ab^3\left(-\frac{-Bb+2Da}{2b^3}+\frac{\sqrt{-ab^7}(-Ab+3Ca)}{4ab^6}\right)}{-Ab^2+3Cab}\right) + \frac{Bab-Da^2+x(-Ab^2+Cab)}{2ab^3+2b^4x^2}$$

```
[In] integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] C*x/b**2 + D*x**2/(2*b**2) + (-(-B*b + 2*D*a)/(2*b**3) - sqrt(-a*b**7)*(-A*
b + 3*C*a)/(4*a*b**6))*log(x + (2*B*a*b - 4*D*a**2 - 4*a*b**3*(-(-B*b + 2*D
*a)/(2*b**3) - sqrt(-a*b**7)*(-A*b + 3*C*a)/(4*a*b**6)))/(-A*b**2 + 3*C*a*b
)) + (-(-B*b + 2*D*a)/(2*b**3) + sqrt(-a*b**7)*(-A*b + 3*C*a)/(4*a*b**6))*l
og(x + (2*B*a*b - 4*D*a**2 - 4*a*b**3*(-(-B*b + 2*D*a)/(2*b**3) + sqrt(-a*b
**7)*(-A*b + 3*C*a)/(4*a*b**6)))/(-A*b**2 + 3*C*a*b)) + (B*a*b - D*a**2 + x
*(-A*b**2 + C*a*b))/(2*a*b**3 + 2*b**4*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.81

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = -\frac{Da^2-Bab-(Cab-Ab^2)x}{2\,(b^4x^2+ab^3)} - \frac{(3\,Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2} + \frac{Dx^2+2\,Cx}{2\,b^2} - \frac{(2\,Da-Bb)\log(bx^2+a)}{2\,b^3}$$

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(D*a^2 - B*a*b - (C*a*b - A*b^2)*x)/(b^4*x^2 + a*b^3) - 1/2*(3*C*a - A
*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*(D*x^2 + 2*C*x)/b^2 - 1/2*(
2*D*a - B*b)*log(b*x^2 + a)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = -\frac{(3\,Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2} - \frac{(2\,Da-Bb)\log(bx^2+a)}{2\,b^3} + \frac{Db^2x^2+2\,Cb^2x}{2\,b^4} - \frac{Da^2-Bab-(Cab-Ab^2)x}{2\,(bx^2+a)b^3}$$

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/2*(2*D*a - B*b
)*log(b*x^2 + a)/b^3 + 1/2*(D*b^2*x^2 + 2*C*b^2*x)/b^4 - 1/2*(D*a^2 - B*a*b
 - (C*a*b - A*b^2)*x)/((b*x^2 + a)*b^3)
```

Mupad [B] (verification not implemented)

Time = 5.74 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.13

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{B\,\ln\left(b\,x^2+a\right)}{2\,b^2}+\frac{x^2\,D}{2\,b^2}+\frac{C\,x}{b^2}-\frac{a^2\,D}{2\,b^3\,\left(b\,x^2+a\right)}+\frac{B\,a}{2\,b^2\,\left(b\,x^2+a\right)}-\frac{A\,x}{2\,b\,\left(b\,x^2+a\right)}+\frac{C\,a\,x}{2\,\left(b^3\,x^2+a\,b^2\right)}+\frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,b^{3/2}}-\frac{3\,C\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{5/2}}-\frac{a\,\ln\left(b\,x^2+a\right)\,D}{b^3}$$

```
[In] int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

```
[Out] (B*log(a + b*x^2))/(2*b^2) + (x^2*D)/(2*b^2) + (C*x)/b^2 - (a^2*D)/(2*b^3*(
a + b*x^2)) + (B*a)/(2*b^2*(a + b*x^2)) - (A*x)/(2*b*(a + b*x^2)) + (C*a*x)
/(2*(a*b^2 + b^3*x^2)) + (A*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(1/2)*b^(3/2))
- (3*C*a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*b^(5/2)) - (a*log(a + b*x^2)*D
)/b^3
```

3.97 $\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$

Optimal result 601
Rubi [A] (verified) 601
Mathematica [A] (verified) 603
Maple [A] (verified) 603
Fricas [A] (verification not implemented) 603
Sympy [B] (verification not implemented) 604
Maxima [A] (verification not implemented) 604
Giac [A] (verification not implemented) 605
Mupad [F(-1)] 605

Optimal result

Integrand size = 26, antiderivative size = 101

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{Dx}{b^2}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)} + \frac{(bB-3aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}+\frac{C\log(a+bx^2)}{2b^2}$$

```
[Out] D*x/b^2-1/2*x*(a*(B-a*D/b)-(A*b-C*a)*x)/a/b/(b*x^2+a)+1/2*C*ln(b*x^2+a)/b^2
+1/2*(B*b-3*D*a)*arctan(x*b^(1/2)/a^(1/2))/b^(5/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1818, 1824, 649, 211, 266}

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = -\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\,(a+bx^2)} + \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bB-3aD)}{2\sqrt{a}b^{5/2}}+\frac{C\log(a+bx^2)}{2b^2}+\frac{Dx}{b^2}$$

```
[In] Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] (D*x)/b^2 - (x*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(2*a*b*(a + b*x^2)) + ((b
*B - 3*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2)) + (C*Log[a + b
*x^2])/(2*b^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}-\frac{\int \frac{-a\left(B-\frac{aD}{b}\right)-2aCx-2aDx^2}{a+bx^2}\,dx}{2ab} \\
&= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}-\frac{\int \left(-\frac{2aD}{b}-\frac{a(bB-3aD)+2abCx}{b(a+bx^2)}\right)dx}{2ab} \\
&= \frac{Dx}{b^2}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}+\frac{\int \frac{a(bB-3aD)+2abCx}{a+bx^2}\,dx}{2ab^2} \\
&= \frac{Dx}{b^2}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}+\frac{C\int \frac{x}{a+bx^2}\,dx}{b}+\frac{(bB-3aD)\int \frac{1}{a+bx^2}\,dx}{2b^2} \\
&= \frac{Dx}{b^2}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}+\frac{(bB-3aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}+\frac{C\log\left(a+bx^2\right)}{2b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{Dx}{b^2}+\frac{-Ab+aC-bBx+aDx}{2b^2(a+bx^2)} - \frac{(-bB+3aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}+\frac{C\log(a+bx^2)}{2b^2}$$

```
[In] Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] (D*x)/b^2 + (-(A*b) + a*C - b*B*x + a*D*x)/(2*b^2*(a + b*x^2)) - ((-(b*B) +
 3*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2)) + (C*Log[a + b*x^2
])/(2*b^2)
```

Maple [A] (verified)

Time = 3.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{Dx}{b^2}+\frac{\frac{\left(-\frac{Bb}{2}+\frac{Da}{2}\right)x-\frac{Ab}{2}+\frac{Ca}{2}}{b\,x^2+a}+\frac{C\ln\left(b\,x^2+a\right)}{2}+\frac{(Bb-3Da)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{b^2}$	78

```
[In] int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] D*x/b^2+1/b^2*(((-1/2*B*b+1/2*D*a)*x-1/2*A*b+1/2*C*a)/(b*x^2+a)+1/2*C*ln(b*
x^2+a)+1/2*(B*b-3*D*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.84

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \left[\frac{4\,Dab^2x^3+2\,Ca^2b-2\,Aab^2-(3\,Da^2-Bab+(3\,Dab-Bb^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right)+2\,(3\,Da^2b}{4\,(ab^4x^2+a^2b^3)}\right.$$

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*D*a*b^2*x^3 + 2*C*a^2*b - 2*A*a*b^2 - (3*D*a^2 - B*a*b + (3*D*a*b -
 B*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(
```

```
3*D*a^2*b - B*a*b^2)*x + 2*(C*a*b^2*x^2 + C*a^2*b)*log(b*x^2 + a))/(a*b^4*x
^2 + a^2*b^3), 1/2*(2*D*a*b^2*x^3 + C*a^2*b - A*a*b^2 - (3*D*a^2 - B*a*b +
(3*D*a*b - B*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (3*D*a^2*b - B*a*b
^2)*x + (C*a*b^2*x^2 + C*a^2*b)*log(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(87) = 174$.

Time = 1.25 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.10

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$
$$= \frac{Dx}{b^2} + \left(\frac{C}{2b^2} - \frac{\sqrt{-ab^5}(-Bb+3Da)}{4ab^5}\right)\log\left(x + \frac{2Ca - 4ab^2\left(\frac{C}{2b^2} - \frac{\sqrt{-ab^5}(-Bb+3Da)}{4ab^5}\right)}{-Bb+3Da}\right) + \left(\frac{C}{2b^2} + \frac{\sqrt{-ab^5}(-Bb+3Da)}{4ab^5}\right)\log\left(x + \frac{2Ca - 4ab^2\left(\frac{C}{2b^2} + \frac{\sqrt{-ab^5}(-Bb+3Da)}{4ab^5}\right)}{-Bb+3Da}\right) + \frac{-Ab+Ca+x(-Bb+Da)}{2ab^2+2b^3x^2}$$

```
[In] integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] D*x/b**2 + (C/(2*b**2) - sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5))*log(x + (
2*C*a - 4*a*b**2*(C/(2*b**2) - sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5)))/(-
B*b + 3*D*a)) + (C/(2*b**2) + sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5))*log(
x + (2*C*a - 4*a*b**2*(C/(2*b**2) + sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5)
))/(-B*b + 3*D*a)) + (-A*b + C*a + x*(-B*b + D*a))/(2*a*b**2 + 2*b**3*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.83

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{Ca-Ab+(Da-Bb)x}{2\,(b^3x^2+ab^2)} + \frac{Dx}{b^2} + \frac{C\log\left(bx^2+a\right)}{2\,b^2} - \frac{(3\,Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2}$$

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(C*a - A*b + (D*a - B*b)*x)/(b^3*x^2 + a*b^2) + D*x/b^2 + 1/2*C*log(b*x
^2 + a)/b^2 - 1/2*(3*D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{Dx}{b^2}+\frac{C\log\left(bx^2+a\right)}{2\,b^2}-\frac{(3\,Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2}+\frac{Ca-Ab+(Da-Bb)x}{2\,(bx^2+a)b^2}$$

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] D*x/b^2 + 1/2*C*log(b*x^2 + a)/b^2 - 1/2*(3*D*a - B*b)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b^2) + 1/2*(C*a - A*b + (D*a - B*b)*x)/((b*x^2 + a)*b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \int \frac{x\,(A+B\,x+C\,x^2+x^3\,D)}{(b\,x^2+a)^2}\,dx$$

```
[In] int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

```
[Out] int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2, x)
```

3.98 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2} \, dx$

Optimal result 606
Rubi [A] (verified) 606
Mathematica [A] (verified) 607
Maple [A] (verified) 608
Fricas [A] (verification not implemented) 608
Sympy [B] (verification not implemented) 608
Maxima [A] (verification not implemented) 609
Giac [A] (verification not implemented) 610
Mupad [B] (verification not implemented) 610

Optimal result

Integrand size = 25, antiderivative size = 93

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2} \, dx = \frac{-a\left(B-\frac{aD}{b}\right)+(Ab-aC)x}{2ab(a+bx^2)} + \frac{(Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}} + \frac{D\log(a+bx^2)}{2b^2}$$

```
[Out] 1/2*(-a*(B-a*D/b)+(A*b-C*a)*x)/a/b/(b*x^2+a)+1/2*(A*b+C*a)*arctan(x*b^(1/2)
/a^(1/2))/a^(3/2)/b^(3/2)+1/2*D*ln(b*x^2+a)/b^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1828, 649, 211, 266}

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2} \, dx = \frac{(aC+Ab)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}} - \frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab(a+bx^2)} + \frac{D\log(a+bx^2)}{2b^2}$$

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^2,x]
```

```
[Out] -1/2*(a*(B - (a*D)/b) - (A*b - a*C)*x)/(a*b*(a + b*x^2)) + ((A*b + a*C)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) + (D*Log[a + b*x^2])/(2*b^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{2ab\,(a+bx^2)}-\frac{\int \frac{-\frac{Ab+aC}{b}-\frac{2aDx}{b}}{a+bx^2}\,dx}{2a}\\
&= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{2ab\,(a+bx^2)}+\frac{(Ab+aC)\int \frac{1}{a+bx^2}\,dx}{2ab}+\frac{D\int \frac{x}{a+bx^2}\,dx}{b}\\
&= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{2ab\,(a+bx^2)}+\frac{(Ab+aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{D\log\left(a+bx^2\right)}{2b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx=\frac{\frac{a^2D+Ab^2x-ab(B+Cx)}{a(a+bx^2)}+\frac{\sqrt{b}(Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}+D\log\left(a+bx^2\right)}{2b^2}$$

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^2,x]
[Out] ((a^2*D + A*b^2*x - a*b*(B + C*x))/(a*(a + b*x^2)) + (Sqrt[b]*(A*b + a*C)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2) + D*Log[a + b*x^2])/(2*b^2)
```

Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\frac{(Ab-Ca)x}{2ab}-\frac{Bb-Da}{2b^2}}{b\,x^2+a}+\frac{\frac{Da\ln\left(b\,x^2+a\right)}{b}+\frac{(Ab+Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{2ba}$	88

```
[In] int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*(A*b-C*a)/a/b*x-1/2*(B*b-D*a)/b^2)/(b*x^2+a)+1/2/b/a*(D*a/b*ln(b*x^2+a
)+(A*b+C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.76

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx$$
$$=\left[\frac{2\,Da^3-2\,Ba^2b-(Ca^2+Aab+(Cab+Ab^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)-2\,(Ca^2b-Aab^2)x+2\,(Da^2}{4\,(a^2b^3x^2+a^3b^2)}\right.$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*D*a^3 - 2*B*a^2*b - (C*a^2 + A*a*b + (C*a*b + A*b^2)*x^2)*sqrt(-a*b
)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*(C*a^2*b - A*a*b^2)*x +
 2*(D*a^2*b*x^2 + D*a^3)*log(b*x^2 + a))/(a^2*b^3*x^2 + a^3*b^2), 1/2*(D*a^
3 - B*a^2*b + (C*a^2 + A*a*b + (C*a*b + A*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a
*b)*x/a) - (C*a^2*b - A*a*b^2)*x + (D*a^2*b*x^2 + D*a^3)*log(b*x^2 + a))/(a
^2*b^3*x^2 + a^3*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(78) = 156$.

Time = 0.99 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.51

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx = \left(\frac{D}{2b^2}-\frac{\sqrt{-a^3b^5}(Ab+Ca)}{4a^3b^4}\right)\log\left(x+\frac{-2Da^2+4a^2b^2\left(\frac{D}{2b^2}-\frac{\sqrt{-a^3b^5}(Ab+Ca)}{4a^3b^4}\right)}{Ab^2+Cab}\right) + \left(\frac{D}{2b^2}+\frac{\sqrt{-a^3b^5}(Ab+Ca)}{4a^3b^4}\right)\log\left(x+\frac{-2Da^2+4a^2b^2\left(\frac{D}{2b^2}+\frac{\sqrt{-a^3b^5}(Ab+Ca)}{4a^3b^4}\right)}{Ab^2+Cab}\right) + \frac{-Bab+Da^2+x(Ab^2-Cab)}{2a^2b^2+2ab^3x^2}$$

```
[In] integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] (D/(2*b**2) - sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b**4))*log(x + (-2*D*a**
2 + 4*a**2*b**2*(D/(2*b**2) - sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b**4)))/
(A*b**2 + C*a*b)) + (D/(2*b**2) + sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b**4
))*log(x + (-2*D*a**2 + 4*a**2*b**2*(D/(2*b**2) + sqrt(-a**3*b**5)*(A*b + C
*a)/(4*a**3*b**4)))/(A*b**2 + C*a*b)) + (-B*a*b + D*a**2 + x*(A*b**2 - C*a*
b))/(2*a**2*b**2 + 2*a*b**3*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx = \frac{Da^2-Bab-(Cab-Ab^2)x}{2\,(ab^3x^2+a^2b^2)} + \frac{D\log(bx^2+a)}{2\,b^2} + \frac{(Ca+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(D*a^2 - B*a*b - (C*a*b - A*b^2)*x)/(a*b^3*x^2 + a^2*b^2) + 1/2*D*log(b
*x^2 + a)/b^2 + 1/2*(C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx = \frac{D\log(bx^2+a)}{2\,b^2} + \frac{(Ca+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab} - \frac{(Ca-Ab)x-\frac{Da^2-Bab}{b}}{2\,(bx^2+a)ab}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*D*log(b*x^2 + a)/b^2 + 1/2*(C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a*b) - 1/2*((C*a - A*b)*x - (D*a^2 - B*a*b)/b)/((b*x^2 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx = \frac{\left(\ln(b\,x^2+a)+\frac{a}{b\,x^2+a}\right)\,D}{2\,b^2} - \frac{B}{2\,b\,(b\,x^2+a)} + \frac{A\,x}{2\,a\,(b\,x^2+a)} - \frac{C\,x}{2\,b\,(b\,x^2+a)} + \frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}} + \frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,b^{3/2}}$$

```
[In] int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^2,x)
```

```
[Out] ((log(a + b*x^2) + a/(a + b*x^2))*D)/(2*b^2) - B/(2*b*(a + b*x^2)) + (A*x)/
(2*a*(a + b*x^2)) - (C*x)/(2*b*(a + b*x^2)) + (A*atan((b^(1/2)*x)/a^(1/2)))
/(2*a^(3/2)*b^(1/2)) + (C*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(1/2)*b^(3/2))
```

3.99 $\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^2}\,dx$

Optimal result 611
Rubi [A] (verified) 611
Mathematica [A] (verified) 613
Maple [A] (verified) 613
Fricas [A] (verification not implemented) 613
Sympy [F(-1)] 614
Maxima [A] (verification not implemented) 614
Giac [A] (verification not implemented) 614
Mupad [F(-1)] 615

Optimal result

Integrand size = 28, antiderivative size = 95

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^2}\,dx=\frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)}+\frac{(bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{A\log(x)}{a^2}-\frac{A\log\left(a+bx^2\right)}{2a^2}$$

```
[Out] 1/2*(A*b-C*a+(B*b-D*a)*x)/a/b/(b*x^2+a)+1/2*(B*b+D*a)*arctan(x*b^(1/2)/a^(1
/2))/a^(3/2)/b^(3/2)+A*ln(x)/a^2-1/2*A*ln(b*x^2+a)/a^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1819, 815, 649, 211, 266}

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^2}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(aD+bB)}{2a^{3/2}b^{3/2}}-\frac{A\log\left(a+bx^2\right)}{2a^2}+\frac{A\log(x)}{a^2}+\frac{x(bB-aD)-aC+Ab}{2ab\left(a+bx^2\right)}$$

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^2),x]
```

```
[Out] (A*b - a*C + (b*B - a*D)*x)/(2*a*b*(a + b*x^2)) + ((b*B + a*D)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) + (A*Log[x])/a^2 - (A*Log[a + b*x^2])/
(2*a^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)} - \frac{\int \frac{-2A-\frac{(bB+aD)x}{b}}{x(a+bx^2)}\,dx}{2a} \\
&= \frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)} - \frac{\int \left(-\frac{2A}{ax}+\frac{-abB-a^2D+2Ab^2x}{ab(a+bx^2)}\right)dx}{2a} \\
&= \frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)} + \frac{A\log(x)}{a^2} - \frac{\int \frac{-abB-a^2D+2Ab^2x}{a+bx^2}\,dx}{2a^2b} \\
&= \frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)} + \frac{A\log(x)}{a^2} - \frac{(Ab)\int \frac{x}{a+bx^2}\,dx}{a^2} + \frac{(bB+aD)\int \frac{1}{a+bx^2}\,dx}{2ab} \\
&= \frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)} + \frac{(bB+aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}} + \frac{A\log(x)}{a^2} - \frac{A\log\left(a+bx^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^2}\,dx = \frac{\frac{a(Ab+bBx-a(C+Dx))}{b(a+bx^2)}+\frac{\sqrt{a}(bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}+2A\log(x)-A\log(a+bx^2)}{2a^2}$$

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^2),x]
```

```
[Out] ((a*(A*b + b*B*x - a*(C + D*x)))/(b*(a + b*x^2)) + (Sqrt[a]*(b*B + a*D)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2) + 2*A*Log[x] - A*Log[a + b*x^2])/(2*a^2)
```

Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{A\ln(x)}{a^2}-\frac{\frac{-\frac{a(Bb-Da)x}{2b}-\frac{a(Ab-Ca)}{2b}}{bx^2+a}+\frac{bA\ln\left(bx^2+a\right)+\frac{\left(-abB-Da^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{2b}}{a^2}$	99

```
[In] int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] A*ln(x)/a^2-1/a^2*((-1/2*a*(B*b-D*a)/b*x-1/2*a*(A*b-C*a)/b)/(b*x^2+a)+1/2/b
*(b*A*ln(b*x^2+a)+(-B*a*b-D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 296, normalized size of antiderivative = 3.12

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^2}\,dx = \left[-\frac{2\,Ca^2b-2\,Aab^2+(Da^2+Bab+(Dab+Bb^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)+2\,(Da^2b-Bab^2)x+2}{4\,(a^2b^3x^2+a^3b^2)}\right.$$
$$-\frac{Ca^2b-Aab^2-(Da^2+Bab+(Dab+Bb^2)x^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)+(Da^2b-Bab^2)x+(Ab^3x^2+Aab}{2\,(a^2b^3x^2+a^3b^2)}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*C*a^2*b - 2*A*a*b^2 + (D*a^2 + B*a*b + (D*a*b + B*b^2)*x^2)*sqrt(-
a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(D*a^2*b - B*a*b^2)*
x + 2*(A*b^3*x^2 + A*a*b^2)*log(b*x^2 + a) - 4*(A*b^3*x^2 + A*a*b^2)*log(x)
)/(a^2*b^3*x^2 + a^3*b^2), -1/2*(C*a^2*b - A*a*b^2 - (D*a^2 + B*a*b + (D*a*
b + B*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (D*a^2*b - B*a*b^2)*x + (
A*b^3*x^2 + A*a*b^2)*log(b*x^2 + a) - 2*(A*b^3*x^2 + A*a*b^2)*log(x))/(a^2*
b^3*x^2 + a^3*b^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^2}\,dx = -\frac{Ca-Ab+(Da-Bb)x}{2\left(ab^2x^2+a^2b\right)}-\frac{A\log\left(bx^2+a\right)}{2\,a^2}+\frac{A\log\left(x\right)}{a^2}+\frac{(Da+Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(C*a - A*b + (D*a - B*b)*x)/(a*b^2*x^2 + a^2*b) - 1/2*A*log(b*x^2 + a)
/a^2 + A*log(x)/a^2 + 1/2*(D*a + B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^2}\,dx = -\frac{A\log\left(bx^2+a\right)}{2\,a^2}+\frac{A\log\left(|x|\right)}{a^2}+\frac{(Da+Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab}-\frac{Ca^2-Aab+(Da^2-Bab)x}{2\,(bx^2+a)a^2b}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*A*log(b*x^2 + a)/a^2 + A*log(abs(x))/a^2 + 1/2*(D*a + B*b)*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*a*b) - 1/2*(C*a^2 - A*a*b + (D*a^2 - B*a*b)*x)/((b*x^
2 + a)*a^2*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^2}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{x\,\left(b\,x^2+a\right)^2}\,dx$$

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^2),x)
```

```
[Out] int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^2), x)
```

3.100 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^2}\,dx$

Optimal result 616
Rubi [A] (verified) 616
Mathematica [A] (verified) 618
Maple [A] (verified) 618
Fricas [A] (verification not implemented) 618
Sympy [F(-1)] 619
Maxima [A] (verification not implemented) 619
Giac [A] (verification not implemented) 620
Mupad [B] (verification not implemented) 620

Optimal result

Integrand size = 28, antiderivative size = 110

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^2}\,dx = -\frac{A}{a^2x}+\frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab(a+bx^2)} - \frac{(3Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}+\frac{B\log(x)}{a^2}-\frac{B\log(a+bx^2)}{2a^2}$$

```
[Out] -A/a^2/x+1/2*(B*b-D*a-b*(A*b/a-C)*x)/a/b/(b*x^2+a)+B*ln(x)/a^2-1/2*B*ln(b*x
^2+a)/a^2-1/2*(3*A*b-C*a)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1819, 1816, 649, 211, 266}

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^2}\,dx = -\frac{(3Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{A}{a^2x}-\frac{B\log(a+bx^2)}{2a^2} + \frac{B\log(x)}{a^2}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{2ab(a+bx^2)}$$

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(A/(a^2*x)) + (b*B - a*D - b*((A*b)/a - C)*x)/(2*a*b*(a + b*x^2)) - ((3*A*
b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b]) + (B*Log[x])/a^2
- (B*Log[a + b*x^2])/(2*a^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)}-\frac{\int \frac{-2A-2Bx+\left(\frac{Ab}{a}-C\right)x^2}{x^2\left(a+bx^2\right)}\,dx}{2a}\\
&= \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)}-\frac{\int \left(-\frac{2A}{ax^2}-\frac{2B}{ax}+\frac{3Ab-aC+2bBx}{a\left(a+bx^2\right)}\right)dx}{2a}\\
&= -\frac{A}{a^2x}+\frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)}+\frac{B\log(x)}{a^2}-\frac{\int \frac{3Ab-aC+2bBx}{a+bx^2}\,dx}{2a^2}\\
&= -\frac{A}{a^2x}+\frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)}+\frac{B\log(x)}{a^2}-\frac{(bB)\int \frac{x}{a+bx^2}\,dx}{a^2}-\frac{(3Ab-aC)\int \frac{1}{a+bx^2}\,dx}{2a^2}
\end{aligned}
$$

$$= -\frac{A}{a^2x} + \frac{bB - aD - b\left(\frac{Ab}{a} - C\right)x}{2ab\,(a + bx^2)} - \frac{(3Ab - aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}} + \frac{B\log(x)}{a^2} - \frac{B\log\,(a + bx^2)}{2a^2}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x^2\,(a + bx^2)^2}\,dx = -\frac{A}{a^2x} + \frac{abB - a^2D - Ab^2x + abCx}{2a^2b\,(a + bx^2)} + \frac{(-3Ab + aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}} + \frac{B\log(x)}{a^2} - \frac{B\log\,(a + bx^2)}{2a^2}$$

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(A/(a^2*x)) + (a*b*B - a^2*D - A*b^2*x + a*b*C*x)/(2*a^2*b*(a + b*x^2)) +
((-3*A*b + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b]) + (B*Log[x
])/a^2 - (B*Log[a + b*x^2])/(2*a^2)
```

Maple [A] (verified)

Time = 3.55 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{A}{a^{2} x}+\frac{B \ln \left(x \right)}{a^{2}}-\frac{\frac{\left(\frac{A b}{2}-\frac{C a}{2}\right) x -\frac{a \left(B b -D a \right)}{2 b}}{b \,x^{2}+a}+\frac{B \ln \left(b \,x^{2}+a \right)}{2}+\frac{\left(3 A b -C a \right) \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}}}{a^{2}}$	96

```
[In] int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -A/a^2/x+B*ln(x)/a^2-1/a^2*(((1/2*A*b-1/2*C*a)*x-1/2*a*(B*b-D*a)/b)/(b*x^2+
a)+1/2*B*ln(b*x^2+a)+1/2*(3*A*b-C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 336, normalized size of antiderivative = 3.05

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^2}\,dx$$
$$= \left[-\frac{4\,Aa^2b-2\,(Ca^2b-3\,Aab^2)x^2-((Cab-3\,Ab^2)x^3+(Ca^2-3\,Aab)x)\sqrt{-ab}\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right)+2\,(\ldots}{4\,(a^3b^2x^3+a^4bx)}\right.$$
$$-\frac{2\,Aa^2b-(Ca^2b-3\,Aab^2)x^2-((Cab-3\,Ab^2)x^3+(Ca^2-3\,Aab)x)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)+(Da^3-Ba^2\ldots}{2\,(a^3b^2x^3+a^4bx)}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*A*a^2*b - 2*(C*a^2*b - 3*A*a*b^2)*x^2 - ((C*a*b - 3*A*b^2)*x^3 + (
C*a^2 - 3*A*a*b)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
) + 2*(D*a^3 - B*a^2*b)*x + 2*(B*a*b^2*x^3 + B*a^2*b*x)*log(b*x^2 + a) - 4*
(B*a*b^2*x^3 + B*a^2*b*x)*log(x))/(a^3*b^2*x^3 + a^4*b*x), -1/2*(2*A*a^2*b
- (C*a^2*b - 3*A*a*b^2)*x^2 - ((C*a*b - 3*A*b^2)*x^3 + (C*a^2 - 3*A*a*b)*x)
*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (D*a^3 - B*a^2*b)*x + (B*a*b^2*x^3 + B*a
^2*b*x)*log(b*x^2 + a) - 2*(B*a*b^2*x^3 + B*a^2*b*x)*log(x))/(a^3*b^2*x^3 +
 a^4*b*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^2}\,dx = -\frac{2\,Aab-(Cab-3\,Ab^2)x^2+(Da^2-Bab)x}{2\,(a^2b^2x^3+a^3bx)} - \frac{B\log(bx^2+a)}{2\,a^2} + \frac{B\log(x)}{a^2} + \frac{(Ca-3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*A*a*b - (C*a*b - 3*A*b^2)*x^2 + (D*a^2 - B*a*b)*x)/(a^2*b^2*x^3 + a
^3*b*x) - 1/2*B*log(b*x^2 + a)/a^2 + B*log(x)/a^2 + 1/2*(C*a - 3*A*b)*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^2}\,dx = -\frac{B\log\left(bx^2+a\right)}{2\,a^2}+\frac{B\log\left(|x|\right)}{a^2}+\frac{(Ca-3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2}+\frac{Cabx^2-3\,Ab^2x^2-Da^2x+Babx-2\,Aab}{2\,(bx^3+ax)a^2b}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*B*log(b*x^2 + a)/a^2 + B*log(abs(x))/a^2 + 1/2*(C*a - 3*A*b)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/2*(C*a*b*x^2 - 3*A*b^2*x^2 - D*a^2*x + B*a
*b*x - 2*A*a*b)/((b*x^3 + a*x)*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 6.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^2}\,dx = \frac{B}{2\,a\,\left(b\,x^2+a\right)}-\frac{\frac{A}{a}+\frac{3\,A\,b\,x^2}{2\,a^2}}{b\,x^3+a\,x}-\frac{B\,\ln\left(b\,x^2+a\right)}{2\,a^2}+\frac{B\,\ln\left(x\right)}{a^2}-\frac{D}{2\,b\,\left(b\,x^2+a\right)}+\frac{C\,x}{2\,a\,\left(b\,x^2+a\right)}-\frac{3\,A\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}+\frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x^2*(a + b*x^2)^2),x)
```

```
[Out] B/(2*a*(a + b*x^2)) - (A/a + (3*A*b*x^2)/(2*a^2))/(a*x + b*x^3) - (B*log(a
+ b*x^2))/(2*a^2) + (B*log(x))/a^2 - D/(2*b*(a + b*x^2)) + (C*x)/(2*a*(a +
b*x^2)) - (3*A*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(5/2)) + (C*atan((b^
(1/2)*x)/a^(1/2)))/(2*a^(3/2)*b^(1/2))
```

3.101 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx$

Optimal result 621
Rubi [A] (verified) 621
Mathematica [A] (verified) 623
Maple [A] (verified) 623
Fricas [A] (verification not implemented) 624
Sympy [F(-1)] 624
Maxima [A] (verification not implemented) 625
Giac [A] (verification not implemented) 625
Mupad [B] (verification not implemented) 626

Optimal result

Integrand size = 28, antiderivative size = 135

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx = -\frac{A}{2a^2x^2}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)} - \frac{(3bB-aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}} - \frac{(2Ab-aC)\log(x)}{a^3}+\frac{(2Ab-aC)\log\left(a+bx^2\right)}{2a^3}$$

```
[Out] -1/2*A/a^2/x^2-B/a^2/x+1/2*(-A*b/a+C-(b*B/a-D)*x)/a/(b*x^2+a)-(2*A*b-C*a)*l
n(x)/a^3+1/2*(2*A*b-C*a)*ln(b*x^2+a)/a^3-1/2*(3*B*b-D*a)*arctan(x*b^(1/2)/a
^(1/2))/a^(5/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1819, 1816, 649, 211, 266}

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx = -\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(3bB-aD)}{2a^{5/2}\sqrt{b}}+\frac{(2Ab-aC)\log\left(a+bx^2\right)}{2a^3} - \frac{\log(x)(2Ab-aC)}{a^3}-\frac{A}{2a^2x^2}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{2a\left(a+bx^2\right)}$$

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -1/2*A/(a^2*x^2) - B/(a^2*x) - ((A*b)/a - C + ((b*B)/a - D)*x)/(2*a*(a + b*
x^2)) - ((3*b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b]) - (
(2*A*b - a*C)*Log[x])/a^3 + ((2*A*b - a*C)*Log[a + b*x^2])/(2*a^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)}-\frac{\int \frac{-2A-2Bx+2\left(\frac{Ab}{a}-C\right)x^2+\left(\frac{bB}{a}-D\right)x^3}{x^3\left(a+bx^2\right)}\,dx}{2a}\\
&= -\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)}-\frac{\int \left(-\frac{2A}{ax^3}-\frac{2B}{ax^2}-\frac{2(-2Ab+aC)}{a^2x}+\frac{a(3bB-aD)-2b(2Ab-aC)x}{a^2\left(a+bx^2\right)}\right)dx}{2a}\\
&= -\frac{A}{2a^2x^2}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)}-\frac{(2Ab-aC)\log(x)}{a^3}-\frac{\int \frac{a(3bB-aD)-2b(2Ab-aC)x}{a+bx^2}\,dx}{2a^3}
\end{aligned}$$

$$= -\frac{A}{2a^2x^2} - \frac{B}{a^2x} - \frac{\frac{Ab}{a} - C + \left(\frac{bB}{a} - D\right)x}{2a\left(a+bx^2\right)} - \frac{(2Ab-aC)\log(x)}{a^3} + \frac{(b(2Ab-aC))\int \frac{x}{a+bx^2}\,dx}{a^3} - \frac{(3bB-aD)\int \frac{1}{a+bx^2}\,dx}{2a^2}$$

$$= -\frac{A}{2a^2x^2} - \frac{B}{a^2x} - \frac{\frac{Ab}{a} - C + \left(\frac{bB}{a} - D\right)x}{2a\left(a+bx^2\right)} - \frac{(3bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}} - \frac{(2Ab-aC)\log(x)}{a^3} + \frac{(2Ab-aC)\log\left(a+bx^2\right)}{2a^3}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx$$
$$= \frac{-\frac{aA}{x^2} - \frac{2aB}{x} + \frac{a(-Ab-bBx+a(C+Dx))}{a+bx^2} + \frac{\sqrt{a}(-3bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} + 2(-2Ab+aC)\log(x) + (2Ab-aC)\log(a+}{2a^3}$$

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] (-((a*A)/x^2) - (2*a*B)/x + (a*(-(A*b) - b*B*x + a*(C + D*x)))/(a + b*x^2)
+ (Sqrt[a]*(-3*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + 2*(-2*A*b
+ a*C)*Log[x] + (2*A*b - a*C)*Log[a + b*x^2])/(2*a^3)
```

Maple [A] (verified)

Time = 3.45 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.94

method	result
default	$-\frac{A}{2a^2x^2} - \frac{B}{a^2x} + \frac{(-2Ab+Ca)\ln(x)}{a^3} + \frac{\frac{\left(-\frac{1}{2}abB+\frac{1}{2}Da^2\right)x - \frac{a(Ab-Ca)}{2}}{b\,x^2+a} + \frac{\left(4b^2A-2Cab\right)\ln\left(b\,x^2+a\right)}{4b} + \frac{\left(-3abB+Da^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{a^3}$

```
[In] int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*A/a^2/x^2-B/a^2/x+(-2*A*b+C*a)/a^3*ln(x)+1/a^3*(((-1/2*a*b*B+1/2*D*a^2
)*x-1/2*a*(A*b-C*a))/(b*x^2+a)+1/4*(4*A*b^2-2*C*a*b)/b*ln(b*x^2+a)+1/2*(-3*
B*a*b+D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 441, normalized size of antiderivative = 3.27

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x^3 (a + bx^2)^2} dx$$
$$= \left[-\frac{4 Ba^2bx + 2 Aa^2b - 2 (Da^2b - 3 Bab^2)x^3 - 2 (Ca^2b - 2 Aab^2)x^2 + ((Dab - 3 Bb^2)x^4 + (Da^2 - 3 Bab)x^2 \right.$$
$$- \frac{2 Ba^2bx + Aa^2b - (Da^2b - 3 Bab^2)x^3 - (Ca^2b - 2 Aab^2)x^2 - ((Dab - 3 Bb^2)x^4 + (Da^2 - 3 Bab)x^2)\sqrt{a}}{}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*B*a^2*b*x + 2*A*a^2*b - 2*(D*a^2*b - 3*B*a*b^2)*x^3 - 2*(C*a^2*b -
 2*A*a*b^2)*x^2 + ((D*a*b - 3*B*b^2)*x^4 + (D*a^2 - 3*B*a*b)*x^2)*sqrt(-a*b
)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*((C*a*b^2 - 2*A*b^3)*x^
4 + (C*a^2*b - 2*A*a*b^2)*x^2)*log(b*x^2 + a) - 4*((C*a*b^2 - 2*A*b^3)*x^4
+ (C*a^2*b - 2*A*a*b^2)*x^2)*log(x))/(a^3*b^2*x^4 + a^4*b*x^2), -1/2*(2*B*a
^2*b*x + A*a^2*b - (D*a^2*b - 3*B*a*b^2)*x^3 - (C*a^2*b - 2*A*a*b^2)*x^2 -
((D*a*b - 3*B*b^2)*x^4 + (D*a^2 - 3*B*a*b)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*
x/a) + ((C*a*b^2 - 2*A*b^3)*x^4 + (C*a^2*b - 2*A*a*b^2)*x^2)*log(b*x^2 + a)
 - 2*((C*a*b^2 - 2*A*b^3)*x^4 + (C*a^2*b - 2*A*a*b^2)*x^2)*log(x))/(a^3*b^2
*x^4 + a^4*b*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x^3 (a + bx^2)^2} dx = \text{Timed out}$$

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx = \frac{(Da-3\,Bb)x^3-2\,Bax+(Ca-2\,Ab)x^2-Aa}{2\,(a^2bx^4+a^3x^2)} + \frac{(Da-3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2} - \frac{(Ca-2\,Ab)\log\left(bx^2+a\right)}{2\,a^3} + \frac{(Ca-2\,Ab)\log\left(x\right)}{a^3}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((D*a - 3*B*b)*x^3 - 2*B*a*x + (C*a - 2*A*b)*x^2 - A*a)/(a^2*b*x^4 + a^
3*x^2) + 1/2*(D*a - 3*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(C*a
 - 2*A*b)*log(b*x^2 + a)/a^3 + (C*a - 2*A*b)*log(x)/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx = \frac{(Da-3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2} - \frac{(Ca-2\,Ab)\log\left(bx^2+a\right)}{2\,a^3} + \frac{(Ca-2\,Ab)\log\left(|x|\right)}{a^3} - \frac{2\,Ba^2x-(Da^2-3\,Bab)x^3+Aa^2-(Ca^2-2\,Aab)x^2}{2\,(bx^2+a)a^3x^2}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(D*a - 3*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(C*a - 2*A*b)
*log(b*x^2 + a)/a^3 + (C*a - 2*A*b)*log(abs(x))/a^3 - 1/2*(2*B*a^2*x - (D*a
^2 - 3*B*a*b)*x^3 + A*a^2 - (C*a^2 - 2*A*a*b)*x^2)/((b*x^2 + a)*a^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 6.13 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx = \frac{C}{2\,a\,\left(b\,x^2+a\right)}-\frac{\frac{A}{2\,a}+\frac{A\,b\,x^2}{a^2}}{b\,x^4+a\,x^2}-\frac{\frac{B}{a}+\frac{3\,B\,b\,x^2}{2\,a^2}}{b\,x^3+a\,x}-\frac{C\,\ln\left(b\,x^2+a\right)}{2\,a^2}+\frac{C\,\ln\left(x\right)}{a^2}+\frac{A\,b\,\ln\left(b\,x^2+a\right)}{a^3}-\frac{2\,A\,b\,\ln\left(x\right)}{a^3}+\frac{x\,D\,{}_2\mathrm{F}_1\left(\frac{1}{2},2;\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{a^2}-\frac{3\,B\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x^3*(a + b*x^2)^2),x)
```

```
[Out] C/(2*a*(a + b*x^2)) - (A/(2*a) + (A*b*x^2)/a^2)/(a*x^2 + b*x^4) - (B/a + (3
*B*b*x^2)/(2*a^2))/(a*x + b*x^3) - (C*log(a + b*x^2))/(2*a^2) + (C*log(x))/
a^2 + (A*b*log(a + b*x^2))/a^3 - (2*A*b*log(x))/a^3 + (x*D*hypergeom([1/2,
2], 3/2, -(b*x^2)/a))/a^2 - (3*B*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(5
/2))
```

3.102 $\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} dx$

Optimal result 627
Rubi [A] (verified) 628
Mathematica [A] (verified) 630
Maple [A] (verified) 630
Fricas [A] (verification not implemented) 630
Sympy [B] (verification not implemented) 631
Maxima [A] (verification not implemented) 632
Giac [A] (verification not implemented) 632
Mupad [B] (verification not implemented) 633

Optimal result

Integrand size = 28, antiderivative size = 185

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} dx = -\frac{3(Ab-5aC)x}{8ab^3} - \frac{(bB-3aD)x^2}{2ab^3} - \frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab(a+bx^2)^2} + \frac{x^3(Ab-5aC+4(bB-2aD)x)}{8ab^2(a+bx^2)} + \frac{3(Ab-5aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}} + \frac{(bB-3aD)\log(a+bx^2)}{2b^4}$$

```
[Out] -3/8*(A*b-5*C*a)*x/a/b^3-1/2*(B*b-3*D*a)*x^2/a/b^3-1/4*x^4*(a*(B-a*D/b)-(A*
b-C*a)*x)/a/b/(b*x^2+a)^2+1/8*x^3*(A*b-5*C*a+4*(B*b-2*D*a)*x)/a/b^2/(b*x^2+
a)+1/2*(B*b-3*D*a)*ln(b*x^2+a)/b^4+3/8*(A*b-5*C*a)*arctan(x*b^(1/2)/a^(1/2)
)/b^(7/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1818, 815, 649, 211, 266}

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{3(Ab-5aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}} - \frac{3x(Ab-5aC)}{8ab^3} + \frac{x^3(4x(bB-2aD)-5aC+Ab)}{8ab^2(a+bx^2)} - \frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab(a+bx^2)^2} + \frac{(bB-3aD)\log(a+bx^2)}{2b^4} - \frac{x^2(bB-3aD)}{2ab^3}$$

```
[In] Int[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] (-3*(A*b - 5*a*C)*x)/(8*a*b^3) - ((b*B - 3*a*D)*x^2)/(2*a*b^3) - (x^4*(a*(B
 - (a*D)/b) - (A*b - a*C)*x))/(4*a*b*(a + b*x^2)^2) + (x^3*(A*b - 5*a*C + 4
*(b*B - 2*a*D)*x))/(8*a*b^2*(a + b*x^2)) + (3*(A*b - 5*a*C)*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(8*Sqrt[a]*b^(7/2)) + ((b*B - 3*a*D)*Log[a + b*x^2])/(2*b^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{\int \frac{x^3\left(-4a\left(B-\frac{aD}{b}\right)+(Ab-5aC)x-4aDx^2\right)}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= -\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}+\frac{x^3(Ab-5aC+4(bB-2aD)x)}{8ab^2\left(a+bx^2\right)}+\frac{\int \frac{x^2(-3a(Ab-5aC)-8a(bB-3aD)x)}{a+bx^2}\,dx}{8a^2b^2}$$

$$= -\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}+\frac{x^3(Ab-5aC+4(bB-2aD)x)}{8ab^2\left(a+bx^2\right)}+\frac{\int \left(-\frac{3a(Ab-5aC)}{b}-\frac{8a(bB-3aD)x}{b}+\frac{3a^2(Ab-5aC)+8a^2(bB-3aD)x}{b\left(a+bx^2\right)}\right)dx}{8a^2b^2}$$

$$= -\frac{3(Ab-5aC)x}{8ab^3}-\frac{(bB-3aD)x^2}{2ab^3}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}+\frac{x^3(Ab-5aC+4(bB-2aD)x)}{8ab^2\left(a+bx^2\right)}+\frac{\int \frac{3a^2(Ab-5aC)+8a^2(bB-3aD)x}{a+bx^2}\,dx}{8a^2b^3}$$

$$= -\frac{3(Ab-5aC)x}{8ab^3}-\frac{(bB-3aD)x^2}{2ab^3}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}+\frac{x^3(Ab-5aC+4(bB-2aD)x)}{8ab^2\left(a+bx^2\right)}+\frac{(3(Ab-5aC))\int \frac{1}{a+bx^2}\,dx}{8b^3}+\frac{(bB-3aD)\int \frac{x}{a+bx^2}\,dx}{b^3}$$

$$= -\frac{3(Ab-5aC)x}{8ab^3}-\frac{(bB-3aD)x^2}{2ab^3}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}+\frac{x^3(Ab-5aC+4(bB-2aD)x)}{8ab^2\left(a+bx^2\right)}+\frac{3(Ab-5aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}}+\frac{(bB-3aD)\log\left(a+bx^2\right)}{2b^4}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.75

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$=\frac{8bCx+4bDx^2+\frac{8abB-12a^2D-5Ab^2x+9abCx}{a+bx^2}+\frac{2a(a^2D+Ab^2x-ab(B+Cx))}{(a+bx^2)^2}+\frac{3\sqrt{b}(Ab-5aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}+4(bB-3aD)\,\mathrm{l}}{8b^4}$$

```
[In] Integrate[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] (8*b*C*x + 4*b*D*x^2 + (8*a*b*B - 12*a^2*D - 5*A*b^2*x + 9*a*b*C*x)/(a + b*
x^2) + (2*a*(a^2*D + A*b^2*x - a*b*(B + C*x)))/(a + b*x^2)^2 + (3*Sqrt[b]*(
A*b - 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + 4*(b*B - 3*a*D)*Log[a +
 b*x^2])/(8*b^4)
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.76

method	result
default	$\frac{\frac{1}{2}Dx^2+Cx}{b^3}+\frac{\frac{\left(-\frac{5}{8}b^2A+\frac{9}{8}Cab\right)x^3+\left(abB-\frac{3}{2}Da^2\right)x^2-\frac{a(3Ab-7Ca)x}{8}+\frac{a^2(3Bb-5Da)}{4b}}{\left(b\,x^2+a\right)^2}+\frac{(8Bb-24Da)\ln\left(b\,x^2+a\right)}{16b}+\frac{(3Ab-15Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}}{b^3}$

```
[In] int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/2*D*x^2+C*x)+1/b^3*(((-5/8*b^2*A+9/8*C*a*b)*x^3+(a*b*B-3/2*D*a^2)*
x^2-1/8*a*(3*A*b-7*C*a)*x+1/4*a^2*(3*B*b-5*D*a)/b)/(b*x^2+a)^2+1/16*(8*B*b-
24*D*a)/b*ln(b*x^2+a)+1/8*(3*A*b-15*C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)
))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 574, normalized size of antiderivative = 3.10

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$=\left[\frac{8\,Dab^3x^6+16\,Cab^3x^5+16\,Da^2b^2x^4-20\,Da^4+12\,Ba^3b+10\,(5\,Ca^2b^2-Aab^3)x^3-16\,(Da^3b-Ba^2b^2)x}{}\right.$$

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*D*a*b^3*x^6 + 16*C*a*b^3*x^5 + 16*D*a^2*b^2*x^4 - 20*D*a^4 + 12*B*
a^3*b + 10*(5*C*a^2*b^2 - A*a*b^3)*x^3 - 16*(D*a^3*b - B*a^2*b^2)*x^2 + 3*(
(5*C*a*b^2 - A*b^3)*x^4 + 5*C*a^3 - A*a^2*b + 2*(5*C*a^2*b - A*a*b^2)*x^2)*
sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(5*C*a^3*b - A
*a^2*b^2)*x - 8*(3*D*a^4 - B*a^3*b + (3*D*a^2*b^2 - B*a*b^3)*x^4 + 2*(3*D*a
^3*b - B*a^2*b^2)*x^2)*log(b*x^2 + a))/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4
), 1/8*(4*D*a*b^3*x^6 + 8*C*a*b^3*x^5 + 8*D*a^2*b^2*x^4 - 10*D*a^4 + 6*B*a^
3*b + 5*(5*C*a^2*b^2 - A*a*b^3)*x^3 - 8*(D*a^3*b - B*a^2*b^2)*x^2 - 3*((5*C
*a*b^2 - A*b^3)*x^4 + 5*C*a^3 - A*a^2*b + 2*(5*C*a^2*b - A*a*b^2)*x^2)*sqrt
(a*b)*arctan(sqrt(a*b)*x/a) + 3*(5*C*a^3*b - A*a^2*b^2)*x - 4*(3*D*a^4 - B*
a^3*b + (3*D*a^2*b^2 - B*a*b^3)*x^4 + 2*(3*D*a^3*b - B*a^2*b^2)*x^2)*log(b*
x^2 + a))/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(172) = 344$.

Time = 98.08 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.93

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{Cx}{b^3} + \frac{Dx^2}{2b^3} + \left(-\frac{-Bb+3Da}{2b^4} - \frac{3\sqrt{-ab^9}(-Ab+5Ca)}{16ab^8}\right)\log\left(x + \frac{8Bab-24Da^2-16ab^4\left(-\frac{-Bb+3Da}{2b^4}-\frac{3\sqrt{-ab^9}(-Ab+5Ca)}{16ab^8}\right)}{-3Ab^2+15Cab}\right) + \left(-\frac{-Bb+3Da}{2b^4} + \frac{3\sqrt{-ab^9}(-Ab+5Ca)}{16ab^8}\right)\log\left(x + \frac{8Bab-24Da^2-16ab^4\left(-\frac{-Bb+3Da}{2b^4}+\frac{3\sqrt{-ab^9}(-Ab+5Ca)}{16ab^8}\right)}{-3Ab^2+15Cab}\right) + \frac{6Ba^2b-10Da^3+x^3(-5Ab^3+9Cab^2)+x^2\cdot(8Bab^2-12Da^2b)+x(-3Aab^2+7Ca^2b)}{8a^2b^4+16ab^5x^2+8b^6x^4}$$

```
[In] integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] C*x/b**3 + D*x**2/(2*b**3) + (-(-B*b + 3*D*a)/(2*b**4) - 3*sqrt(-a*b**9)*(-
A*b + 5*C*a)/(16*a*b**8))*log(x + (8*B*a*b - 24*D*a**2 - 16*a*b**4*(-(-B*b
+ 3*D*a)/(2*b**4) - 3*sqrt(-a*b**9)*(-A*b + 5*C*a)/(16*a*b**8)))/(-3*A*b**2
 + 15*C*a*b)) + (-(-B*b + 3*D*a)/(2*b**4) + 3*sqrt(-a*b**9)*(-A*b + 5*C*a)/
(16*a*b**8))*log(x + (8*B*a*b - 24*D*a**2 - 16*a*b**4*(-(-B*b + 3*D*a)/(2*b
**4) + 3*sqrt(-a*b**9)*(-A*b + 5*C*a)/(16*a*b**8)))/(-3*A*b**2 + 15*C*a*b))
 + (6*B*a**2*b - 10*D*a**3 + x**3*(-5*A*b**3 + 9*C*a*b**2) + x**2*(8*B*a*b*
*2 - 12*D*a**2*b) + x*(-3*A*a*b**2 + 7*C*a**2*b))/(8*a**2*b**4 + 16*a*b**5*
x**2 + 8*b**6*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.89

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= -\frac{10\,Da^3-6\,Ba^2b-(9\,Cab^2-5\,Ab^3)x^3+4\,(3\,Da^2b-2\,Bab^2)x^2-(7\,Ca^2b-3\,Aab^2)x}{8\,(b^6x^4+2\,ab^5x^2+a^2b^4)}$$
$$-\frac{3\,(5\,Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3}+\frac{Dx^2+2\,Cx}{2\,b^3}-\frac{(3\,Da-Bb)\log\left(bx^2+a\right)}{2\,b^4}$$

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(10*D*a^3 - 6*B*a^2*b - (9*C*a*b^2 - 5*A*b^3)*x^3 + 4*(3*D*a^2*b - 2*B
*a*b^2)*x^2 - (7*C*a^2*b - 3*A*a*b^2)*x)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
- 3/8*(5*C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/2*(D*x^2 + 2*
C*x)/b^3 - 1/2*(3*D*a - B*b)*log(b*x^2 + a)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.85

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= -\frac{3\,(5\,Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3}-\frac{(3\,Da-Bb)\log\left(bx^2+a\right)}{2\,b^4}+\frac{Db^3x^2+2\,Cb^3x}{2\,b^6}$$
$$-\frac{10\,Da^3-6\,Ba^2b-(9\,Cab^2-5\,Ab^3)x^3+4\,(3\,Da^2b-2\,Bab^2)x^2-(7\,Ca^2b-3\,Aab^2)x}{8\,(bx^2+a)^2b^4}$$

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3/8*(5*C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(3*D*a - B*b
)*log(b*x^2 + a)/b^4 + 1/2*(D*b^3*x^2 + 2*C*b^3*x)/b^6 - 1/8*(10*D*a^3 - 6*
B*a^2*b - (9*C*a*b^2 - 5*A*b^3)*x^3 + 4*(3*D*a^2*b - 2*B*a*b^2)*x^2 - (7*C*
a^2*b - 3*A*a*b^2)*x)/((b*x^2 + a)^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 6.17 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.25

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{\frac{7\,C\,a^2\,x}{8}+\frac{9\,C\,b\,a\,x^3}{8}}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4} - \frac{\frac{5\,A\,x^3}{8\,b}+\frac{3\,A\,a\,x}{8\,b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4} + \frac{\frac{3\,B\,a^2}{4\,b^3}+\frac{B\,a\,x^2}{b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4} - \frac{D\left(3\,a\,\ln\left(b\,x^2+a\right)-b\,x^2+\frac{3\,a^2}{b\,x^2+a}-\frac{a^3}{2\,(b\,x^2+a)^2}\right)}{2\,b^4} + \frac{B\,\ln\left(b\,x^2+a\right)}{2\,b^3} + \frac{C\,x}{b^3} + \frac{3\,A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,\sqrt{a}\,b^{5/2}} - \frac{15\,C\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,b^{7/2}}$$

```
[In] int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

```
[Out] ((7*C*a^2*x)/8 + (9*C*a*b*x^3)/8)/(a^2*b^3 + b^5*x^4 + 2*a*b^4*x^2) - ((5*A
*x^3)/(8*b) + (3*A*a*x)/(8*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) + ((3*B*a^2)/(
4*b^3) + (B*a*x^2)/b^2)/(a^2 + b^2*x^4 + 2*a*b*x^2) - (D*(3*a*log(a + b*x^2
) - b*x^2 + (3*a^2)/(a + b*x^2) - a^3/(2*(a + b*x^2)^2)))/(2*b^4) + (B*log(
a + b*x^2))/(2*b^3) + (C*x)/b^3 + (3*A*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(1/2
)*b^(5/2)) - (15*C*a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*b^(7/2))
```

3.103 $\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$

Optimal result 634
Rubi [A] (verified) 634
Mathematica [A] (verified) 636
Maple [A] (verified) 636
Fricas [A] (verification not implemented) 637
Sympy [B] (verification not implemented) 637
Maxima [A] (verification not implemented) 638
Giac [A] (verification not implemented) 638
Mupad [F(-1)] 639

Optimal result

Integrand size = 28, antiderivative size = 155

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = -\frac{3(bB-5aD)x}{8ab^3} - \frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\,(a+bx^2)^2} - \frac{x^2(4aC-(3bB-7aD)x)}{8ab^2\,(a+bx^2)} + \frac{3(bB-5aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}} + \frac{C\log(a+bx^2)}{2b^3}$$

```
[Out] -3/8*(B*b-5*D*a)*x/a/b^3-1/4*x^3*(a*(B-a*D/b)-(A*b-C*a)*x)/a/b/(b*x^2+a)^2-
1/8*x^2*(4*C*a-(3*B*b-7*D*a)*x)/a/b^2/(b*x^2+a)+1/2*C*ln(b*x^2+a)/b^3+3/8*(
B*b-5*D*a)*arctan(x*b^(1/2)/a^(1/2))/b^(7/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1818, 788, 649, 211, 266}

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = -\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\,(a+bx^2)^2} + \frac{3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bB-5aD)}{8\sqrt{a}b^{7/2}} - \frac{3x(bB-5aD)}{8ab^3} + \frac{C\log(a+bx^2)}{2b^3} - \frac{x^2(4aC-x(3bB-7aD))}{8ab^2\,(a+bx^2)}$$

```
[In] Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] (-3*(b*B - 5*a*D)*x)/(8*a*b^3) - (x^3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(4
*a*b*(a + b*x^2)^2) - (x^2*(4*a*C - (3*b*B - 7*a*D)*x))/(8*a*b^2*(a + b*x^2
)) + (3*(b*B - 5*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(7/2)) + (C
*Log[a + b*x^2])/(2*b^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{\int \frac{x^2\left(-3a\left(B-\frac{aD}{b}\right)-4aCx-4aDx^2\right)}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$=-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{x^2(4aC-(3bB-7aD)x)}{8ab^2\left(a+bx^2\right)}+\frac{\int \frac{x\left(8a^2C-3a(bB-5aD)x\right)}{a+bx^2}\,dx}{8a^2b^2}$$

$$
\begin{aligned}
&= -\frac{3(bB-5aD)x}{8ab^3} - \frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} \\
&\quad - \frac{x^2(4aC-(3bB-7aD)x)}{8ab^2\left(a+bx^2\right)} + \frac{\int \frac{3a^2(bB-5aD)+8a^2bCx}{a+bx^2}\,dx}{8a^2b^3} \\
&= -\frac{3(bB-5aD)x}{8ab^3} - \frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} \\
&\quad - \frac{x^2(4aC-(3bB-7aD)x)}{8ab^2\left(a+bx^2\right)} + \frac{C\int \frac{x}{a+bx^2}\,dx}{b^2} + \frac{(3(bB-5aD))\int \frac{1}{a+bx^2}\,dx}{8b^3} \\
&= -\frac{3(bB-5aD)x}{8ab^3} - \frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} \\
&\quad - \frac{x^2(4aC-(3bB-7aD)x)}{8ab^2\left(a+bx^2\right)} + \frac{3(bB-5aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}} + \frac{C\log\left(a+bx^2\right)}{2b^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.81

$$
\begin{aligned}
\int \frac{x^3\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx &= \frac{Dx}{b^3} + \frac{-4Ab+8aC-5bBx+9aDx}{8b^3\left(a+bx^2\right)} \\
&\quad + \frac{a(Ab+bBx-a(C+Dx))}{4b^3\left(a+bx^2\right)^2} \\
&\quad + \frac{3(bB-5aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}} + \frac{C\log\left(a+bx^2\right)}{2b^3}
\end{aligned}
$$

```
[In] Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] (D*x)/b^3 + (-4*A*b + 8*a*C - 5*b*B*x + 9*a*D*x)/(8*b^3*(a + b*x^2)) + (a*(
A*b + b*B*x - a*(C + D*x)))/(4*b^3*(a + b*x^2)^2) + (3*(b*B - 5*a*D)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(7/2)) + (C*Log[a + b*x^2])/(2*b^3)
```

Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{Dx}{b^{3}}+\frac{\frac{\left(-\frac{5}{8}B\,b^{2}+\frac{9}{8}Dab\right)x^{3}+\left(-\frac{1}{2}b^{2}A+Cab\right)x^{2}-\frac{a(3Bb-7Da)x}{8}-\frac{abA}{4}+\frac{3C\,a^{2}}{4}}{\left(b\,x^{2}+a\right)^{2}}+\frac{C\ln\left(b\,x^{2}+a\right)}{2}+\frac{(3Bb-15Da)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}}{b^{3}}$	115

```
[In] int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] D/b^3*x+1/b^3*(((-5/8*B*b^2+9/8*D*a*b)*x^3+(-1/2*b^2*A+C*a*b)*x^2-1/8*a*(3*
B*b-7*D*a)*x-1/4*a*b*A+3/4*C*a^2)/(b*x^2+a)^2+1/2*C*ln(b*x^2+a)+1/8*(3*B*b-
15*D*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 480, normalized size of antiderivative = 3.10

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= \left[\frac{16\,Dab^3x^5 + 12\,Ca^3b - 4\,Aa^2b^2 + 10\,(5\,Da^2b^2 - Bab^3)x^3 + 8\,(2\,Ca^2b^2 - Aab^3)x^2 - 3\,((5\,Dab^2 - Bb^3)x}{} \right.$$

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*D*a*b^3*x^5 + 12*C*a^3*b - 4*A*a^2*b^2 + 10*(5*D*a^2*b^2 - B*a*b^
3)*x^3 + 8*(2*C*a^2*b^2 - A*a*b^3)*x^2 - 3*((5*D*a*b^2 - B*b^3)*x^4 + 5*D*a
^3 - B*a^2*b + 2*(5*D*a^2*b - B*a*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(
-a*b)*x - a)/(b*x^2 + a)) + 6*(5*D*a^3*b - B*a^2*b^2)*x + 8*(C*a*b^3*x^4 +
2*C*a^2*b^2*x^2 + C*a^3*b)*log(b*x^2 + a))/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3
*b^4), 1/8*(8*D*a*b^3*x^5 + 6*C*a^3*b - 2*A*a^2*b^2 + 5*(5*D*a^2*b^2 - B*a*
b^3)*x^3 + 4*(2*C*a^2*b^2 - A*a*b^3)*x^2 - 3*((5*D*a*b^2 - B*b^3)*x^4 + 5*D
*a^3 - B*a^2*b + 2*(5*D*a^2*b - B*a*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/
a) + 3*(5*D*a^3*b - B*a^2*b^2)*x + 4*(C*a*b^3*x^4 + 2*C*a^2*b^2*x^2 + C*a^3
*b)*log(b*x^2 + a))/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(139) = 278$.

Time = 87.26 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.82

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= \frac{Dx}{b^3} + \left(\frac{C}{2b^3} - \frac{3\sqrt{-ab^7}(-Bb+5Da)}{16ab^7}\right)\log\left(x + \frac{8Ca - 16ab^3\left(\frac{C}{2b^3} - \frac{3\sqrt{-ab^7}(-Bb+5Da)}{16ab^7}\right)}{-3Bb+15Da}\right)$$
$$+ \left(\frac{C}{2b^3} + \frac{3\sqrt{-ab^7}(-Bb+5Da)}{16ab^7}\right)\log\left(x + \frac{8Ca - 16ab^3\left(\frac{C}{2b^3} + \frac{3\sqrt{-ab^7}(-Bb+5Da)}{16ab^7}\right)}{-3Bb+15Da}\right)$$
$$+ \frac{-2Aab + 6Ca^2 + x^3(-5Bb^2 + 9Dab) + x^2(-4Ab^2 + 8Cab) + x(-3Bab + 7Da^2)}{8a^2b^3 + 16ab^4x^2 + 8b^5x^4}$$

```
[In] integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] D*x/b**3 + (C/(2*b**3) - 3*sqrt(-a*b**7)*(-B*b + 5*D*a)/(16*a*b**7))*log(x
+ (8*C*a - 16*a*b**3*(C/(2*b**3) - 3*sqrt(-a*b**7)*(-B*b + 5*D*a)/(16*a*b**
7)))/(-3*B*b + 15*D*a)) + (C/(2*b**3) + 3*sqrt(-a*b**7)*(-B*b + 5*D*a)/(16*
a*b**7))*log(x + (8*C*a - 16*a*b**3*(C/(2*b**3) + 3*sqrt(-a*b**7)*(-B*b + 5
*D*a)/(16*a*b**7)))/(-3*B*b + 15*D*a)) + (-2*A*a*b + 6*C*a**2 + x**3*(-5*B*
b**2 + 9*D*a*b) + x**2*(-4*A*b**2 + 8*C*a*b) + x*(-3*B*a*b + 7*D*a**2))/(8*
a**2*b**3 + 16*a*b**4*x**2 + 8*b**5*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.88

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{(9\,Dab-5\,Bb^2)x^3+6\,Ca^2-2\,Aab+4\,(2\,Cab-Ab^2)x^2+(7\,Da^2-3\,Bab)x}{8\,(b^5x^4+2\,ab^4x^2+a^2b^3)} + \frac{Dx}{b^3} + \frac{C\log\left(bx^2+a\right)}{2\,b^3} - \frac{3\,(5\,Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3}$$

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((9*D*a*b - 5*B*b^2)*x^3 + 6*C*a^2 - 2*A*a*b + 4*(2*C*a*b - A*b^2)*x^2
+ (7*D*a^2 - 3*B*a*b)*x)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3) + D*x/b^3 + 1/2*
C*log(b*x^2 + a)/b^3 - 3/8*(5*D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b
^3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.79

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{Dx}{b^3} + \frac{C\log\left(bx^2+a\right)}{2\,b^3} - \frac{3\,(5\,Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3} + \frac{(9\,Dab-5\,Bb^2)x^3+6\,Ca^2-2\,Aab+4\,(2\,Cab-Ab^2)x^2+(7\,Da^2-3\,Bab)x}{8\,(bx^2+a)^2b^3}$$

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] D*x/b^3 + 1/2*C*log(b*x^2 + a)/b^3 - 3/8*(5*D*a - B*b)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b^3) + 1/8*((9*D*a*b - 5*B*b^2)*x^3 + 6*C*a^2 - 2*A*a*b + 4*(2
*C*a*b - A*b^2)*x^2 + (7*D*a^2 - 3*B*a*b)*x)/((b*x^2 + a)^2*b^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx=\int \frac{x^3\,(A+B\,x+C\,x^2+x^3\,D)}{(b\,x^2+a)^3}\,dx$$

```
[In] int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

```
[Out] int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3, x)
```

3.104 $\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} dx$

Optimal result 640
Rubi [A] (verified) 640
Mathematica [A] (verified) 642
Maple [A] (verified) 642
Fricas [A] (verification not implemented) 643
Sympy [B] (verification not implemented) 643
Maxima [A] (verification not implemented) 644
Giac [A] (verification not implemented) 644
Mupad [B] (verification not implemented) 645

Optimal result

Integrand size = 28, antiderivative size = 136

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} dx = -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\,(a+bx^2)^2} - \frac{x(Ab+3aC-2(bB-3aD)x)}{8ab^2\,(a+bx^2)} + \frac{(Ab+3aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}} + \frac{D\log\left(a+bx^2\right)}{2b^3}$$

```
[Out] -1/4*x^2*(a*(B-a*D/b)-(A*b-C*a)*x)/a/b/(b*x^2+a)^2-1/8*x*(A*b+3*C*a-2*(B*b-
3*D*a)*x)/a/b^2/(b*x^2+a)+1/8*(A*b+3*C*a)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)
/b^(5/2)+1/2*D*ln(b*x^2+a)/b^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1818, 649, 211, 266}

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} dx = \frac{(3aC+Ab)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}} - \frac{x(-2x(bB-3aD)+3aC+Ab)}{8ab^2\,(a+bx^2)} - \frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\,(a+bx^2)^2} + \frac{D\log\left(a+bx^2\right)}{2b^3}$$

```
[In] Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] -1/4*(x^2*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)^2) - (x*(A*b
+ 3*a*C - 2*(b*B - 3*a*D)*x))/(8*a*b^2*(a + b*x^2)) + ((A*b + 3*a*C)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2)) + (D*Log[a + b*x^2])/(2*b^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{\int \frac{x\left(-2a\left(B-\frac{aD}{b}\right)-(Ab+3aC)x-4aDx^2\right)}{\left(a+bx^2\right)^2}\,dx}{4ab}\\
&= -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{x(Ab+3aC-2(bB-3aD)x)}{8ab^2\left(a+bx^2\right)}+\frac{\int \frac{a(Ab+3aC)+8a^2Dx}{a+bx^2}\,dx}{8a^2b^2}\\
&= -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{x(Ab+3aC-2(bB-3aD)x)}{8ab^2\left(a+bx^2\right)}\\
&\quad+\frac{(Ab+3aC)\int \frac{1}{a+bx^2}\,dx}{8ab^2}+\frac{D\int \frac{x}{a+bx^2}\,dx}{b^2}
\end{aligned}$$

$$= -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} - \frac{x(Ab+3aC-2(bB-3aD)x)}{8ab^2\left(a+bx^2\right)} + \frac{(Ab+3aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}} + \frac{D\log\left(a+bx^2\right)}{2b^3}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.90

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{\left(a+bx^2\right)^3}\,dx = \frac{\frac{-2a^2D-2Ab^2x+2ab(B+Cx)}{(a+bx^2)^2} + \frac{8a^2D+Ab^2x-ab(4B+5Cx)}{a(a+bx^2)} + \frac{\sqrt{b}(Ab+3aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}} + 4D\log\left(a+bx^2\right)}{8b^3}$$

```
[In] Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] ((-2*a^2*D - 2*A*b^2*x + 2*a*b*(B + C*x))/(a + b*x^2)^2 + (8*a^2*D + A*b^2*
x - a*b*(4*B + 5*C*x))/(a*(a + b*x^2)) + (Sqrt[b]*(A*b + 3*a*C)*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/a^(3/2) + 4*D*Log[a + b*x^2])/(8*b^3)
```

Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{\frac{(Ab-5Ca)x^3}{8ab}-\frac{(Bb-2Da)x^2}{2b^2}-\frac{(Ab+3Ca)x}{8b^2}-\frac{a(Bb-3Da)}{4b^3}}{(b\,x^2+a)^2}+\frac{\frac{4Da\ln\left(b\,x^2+a\right)}{b}+\frac{(Ab+3Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{8a\,b^2}$	123

```
[In] int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(A*b-5*C*a)/a/b*x^3-1/2*(B*b-2*D*a)/b^2*x^2-1/8*(A*b+3*C*a)/b^2*x-1/4*
a*(B*b-3*D*a)/b^3)/(b*x^2+a)^2+1/8/a/b^2*(4*D*a/b*ln(b*x^2+a)+(A*b+3*C*a)/(
a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 447, normalized size of antiderivative = 3.29

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= \left[\frac{12\,Da^4-4\,Ba^3b-2\,(5\,Ca^2b^2-Aab^3)x^3+8\,(2\,Da^3b-Ba^2b^2)x^2-((3\,Cab^2+Ab^3)x^4+3\,Ca^3+Aa^2b}{16\,(a^2b}\right.$$

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(12*D*a^4 - 4*B*a^3*b - 2*(5*C*a^2*b^2 - A*a*b^3)*x^3 + 8*(2*D*a^3*b
- B*a^2*b^2)*x^2 - ((3*C*a*b^2 + A*b^3)*x^4 + 3*C*a^3 + A*a^2*b + 2*(3*C*a^
2*b + A*a*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
) - 2*(3*C*a^3*b + A*a^2*b^2)*x + 8*(D*a^2*b^2*x^4 + 2*D*a^3*b*x^2 + D*a^4)
*log(b*x^2 + a))/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3), 1/8*(6*D*a^4 - 2*
B*a^3*b - (5*C*a^2*b^2 - A*a*b^3)*x^3 + 4*(2*D*a^3*b - B*a^2*b^2)*x^2 + ((3
*C*a*b^2 + A*b^3)*x^4 + 3*C*a^3 + A*a^2*b + 2*(3*C*a^2*b + A*a*b^2)*x^2)*sq
rt(a*b)*arctan(sqrt(a*b)*x/a) - (3*C*a^3*b + A*a^2*b^2)*x + 4*(D*a^2*b^2*x^
4 + 2*D*a^3*b*x^2 + D*a^4)*log(b*x^2 + a))/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a
^4*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(119) = 238.

Time = 74.79 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.24

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= \left(\frac{D}{2b^3}-\frac{\sqrt{-a^3b^7}(Ab+3Ca)}{16a^3b^6}\right)\log\left(x+\frac{-8Da^2+16a^2b^3\left(\frac{D}{2b^3}-\frac{\sqrt{-a^3b^7}(Ab+3Ca)}{16a^3b^6}\right)}{Ab^2+3Cab}\right)$$
$$+\left(\frac{D}{2b^3}+\frac{\sqrt{-a^3b^7}(Ab+3Ca)}{16a^3b^6}\right)\log\left(x+\frac{-8Da^2+16a^2b^3\left(\frac{D}{2b^3}+\frac{\sqrt{-a^3b^7}(Ab+3Ca)}{16a^3b^6}\right)}{Ab^2+3Cab}\right)$$
$$+\frac{-2Ba^2b+6Da^3+x^3(Ab^3-5Cab^2)+x^2(-4Bab^2+8Da^2b)+x(-Aab^2-3Ca^2b)}{8a^3b^3+16a^2b^4x^2+8ab^5x^4}$$

```
[In] integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] (D/(2*b**3) - sqrt(-a**3*b**7)*(A*b + 3*C*a)/(16*a**3*b**6))*log(x + (-8*D*
a**2 + 16*a**2*b**3*(D/(2*b**3) - sqrt(-a**3*b**7)*(A*b + 3*C*a)/(16*a**3*b
```

```
**6)))/(A*b**2 + 3*C*a*b)) + (D/(2*b**3) + sqrt(-a**3*b**7)*(A*b + 3*C*a)/(
16*a**3*b**6))*log(x + (-8*D*a**2 + 16*a**2*b**3*(D/(2*b**3) + sqrt(-a**3*b
**7)*(A*b + 3*C*a)/(16*a**3*b**6)))/(A*b**2 + 3*C*a*b)) + (-2*B*a**2*b + 6*
D*a**3 + x**3*(A*b**3 - 5*C*a*b**2) + x**2*(-4*B*a*b**2 + 8*D*a**2*b) + x*(
-A*a*b**2 - 3*C*a**2*b))/(8*a**3*b**3 + 16*a**2*b**4*x**2 + 8*a*b**5*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{6\,Da^3-2\,Ba^2b-(5\,Cab^2-Ab^3)x^3+4\,(2\,Da^2b-Bab^2)x^2-(3\,Ca^2b+Aab^2)x}{8\,(ab^5x^4+2\,a^2b^4x^2+a^3b^3)} + \frac{D\log(bx^2+a)}{2\,b^3} + \frac{(3\,Ca+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^2}$$

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(6*D*a^3 - 2*B*a^2*b - (5*C*a*b^2 - A*b^3)*x^3 + 4*(2*D*a^2*b - B*a*b^2
)*x^2 - (3*C*a^2*b + A*a*b^2)*x)/(a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3) + 1/
2*D*log(b*x^2 + a)/b^3 + 1/8*(3*C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.94

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{D\log(bx^2+a)}{2\,b^3} + \frac{(3\,Ca+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^2} - \frac{(5\,Cab-Ab^2)x^3-4\,(2\,Da^2-Bab)x^2+(3\,Ca^2+Aab)x-\frac{2\,(3\,Da^3-Ba^2b)}{b}}{8\,(bx^2+a)^2ab^2}$$

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*D*log(b*x^2 + a)/b^3 + 1/8*(3*C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*
b)*a*b^2) - 1/8*((5*C*a*b - A*b^2)*x^3 - 4*(2*D*a^2 - B*a*b)*x^2 + (3*C*a^2
 + A*a*b)*x - 2*(3*D*a^3 - B*a^2*b)/b)/((b*x^2 + a)^2*a*b^2)
```

Mupad [B] (verification not implemented)

Time = 6.03 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.43

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{\frac{A\,x^3}{8\,a}-\frac{A\,x}{8\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{\frac{B\,x^2}{2\,b}+\frac{B\,a}{4\,b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{\frac{5\,C\,x^3}{8\,b}+\frac{3\,C\,a\,x}{8\,b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{D\left(\ln\left(b\,x^2+a\right)+\frac{2\,a}{b\,x^2+a}-\frac{a^2}{2\,{\left(b\,x^2+a\right)}^2}\right)}{2\,b^3}+\frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,b^{3/2}}+\frac{3\,C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,\sqrt{a}\,b^{5/2}}$$

```
[In] int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

```
[Out] ((A*x^3)/(8*a) - (A*x)/(8*b))/(a^2 + b^2*x^4 + 2*a*b*x^2) - ((B*x^2)/(2*b)
+ (B*a)/(4*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - ((5*C*x^3)/(8*b) + (3*C*a*x)
/(8*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) + (D*(log(a + b*x^2) + (2*a)/(a + b*x
^2) - a^2/(2*(a + b*x^2)^2)))/(2*b^3) + (A*atan((b^(1/2)*x)/a^(1/2)))/(8*a^
(3/2)*b^(3/2)) + (3*C*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(1/2)*b^(5/2))
```

3.105 $\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} dx$

Optimal result 646
Rubi [A] (verified) 646
Mathematica [A] (verified) 648
Maple [A] (verified) 648
Fricas [A] (verification not implemented) 648
Sympy [A] (verification not implemented) 649
Maxima [A] (verification not implemented) 649
Giac [A] (verification not implemented) 650
Mupad [F(-1)] 650

Optimal result

Integrand size = 26, antiderivative size = 119

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} dx = -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2} - \frac{2(Ab+aC)-(bB-5aD)x}{8ab^2\left(a+bx^2\right)} + \frac{(bB+3aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}$$

```
[Out] -1/4*x*(a*(B-a*D/b)-(A*b-C*a)*x)/a/b/(b*x^2+a)^2+1/8*(-2*A*b-2*C*a+(B*b-5*D
*a)*x)/a/b^2/(b*x^2+a)+1/8*(B*b+3*D*a)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^
(5/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1818, 1828, 12, 211}

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(3aD+bB)}{8a^{3/2}b^{5/2}} - \frac{2(aC+Ab)-x(bB-5aD)}{8ab^2\left(a+bx^2\right)} - \frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

```
[In] Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] -1/4*(x*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)^2) - (2*(A*b +
a*C) - (b*B - 5*a*D)*x)/(8*a*b^2*(a + b*x^2)) + ((b*B + 3*a*D)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{\int \frac{-a\left(B-\frac{aD}{b}\right)-2(Ab+aC)x-4aDx^2}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{2(Ab+aC)-(bB-5aD)x}{8ab^2\left(a+bx^2\right)}+\frac{\int \frac{a\left(B+\frac{3aD}{b}\right)}{a+bx^2}\,dx}{8a^2b}$$

$$= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{2(Ab+aC)-(bB-5aD)x}{8ab^2\left(a+bx^2\right)}+\frac{(bB+3aD)\int \frac{1}{a+bx^2}\,dx}{8ab^2}$$

$$= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{2(Ab+aC)-(bB-5aD)x}{8ab^2\left(a+bx^2\right)}+\frac{(bB+3aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.83

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{\frac{\sqrt{b}(b^2Bx^3-a^2(2C+3Dx)-ab(2A+x(B+4Cx+5Dx^2)))}{a(a+bx^2)^2}+\frac{(bB+3aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}}{8b^{5/2}}$$

[In] Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]

[Out] ((Sqrt[b]*(b^2*B*x^3 - a^2*(2*C + 3*D*x) - a*b*(2*A + x*(B + 4*C*x + 5*D*x^
2))))/(a*(a + b*x^2)^2) + ((b*B + 3*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/
2))/(8*b^(5/2))

Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\frac{(Bb-5Da)x^3}{8ab}-\frac{Cx^2}{2b}-\frac{(Bb+3Da)x}{8b^2}-\frac{Ab+Ca}{4b^2}}{(bx^2+a)^2}+\frac{(Bb+3Da)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8b^2a\sqrt{ab}}$	97

[In] int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)

[Out] (1/8*(B*b-5*D*a)/a/b*x^3-1/2*C*x^2/b-1/8*(B*b+3*D*a)/b^2*x-1/4*(A*b+C*a)/b^
2)/(b*x^2+a)^2+1/8*(B*b+3*D*a)/b^2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 357, normalized size of antiderivative = 3.00

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \left[-\frac{8\,Ca^2b^2x^2+4\,Ca^3b+4\,Aa^2b^2+2\,(5\,Da^2b^2-Bab^3)x^3+((3\,Dab^2+Bb^3)x^4+3\,Da^3+Ba^2b+2\,(3\,Da}{16\,(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3)}\right.$$

$$-\frac{4\,Ca^2b^2x^2+2\,Ca^3b+2\,Aa^2b^2+(5\,Da^2b^2-Bab^3)x^3-((3\,Dab^2+Bb^3)x^4+3\,Da^3+Ba^2b+2\,(3\,Da^2b}{8\,(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3)}$$

[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")

```
[Out] [-1/16*(8*C*a^2*b^2*x^2 + 4*C*a^3*b + 4*A*a^2*b^2 + 2*(5*D*a^2*b^2 - B*a*b^
3)*x^3 + ((3*D*a*b^2 + B*b^3)*x^4 + 3*D*a^3 + B*a^2*b + 2*(3*D*a^2*b + B*a*
b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(3*D
*a^3*b + B*a^2*b^2)*x)/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3), -1/8*(4*C*a
^2*b^2*x^2 + 2*C*a^3*b + 2*A*a^2*b^2 + (5*D*a^2*b^2 - B*a*b^3)*x^3 - ((3*D*
a*b^2 + B*b^3)*x^4 + 3*D*a^3 + B*a^2*b + 2*(3*D*a^2*b + B*a*b^2)*x^2)*sqrt(
a*b)*arctan(sqrt(a*b)*x/a) + (3*D*a^3*b + B*a^2*b^2)*x)/(a^2*b^5*x^4 + 2*a^
3*b^4*x^2 + a^4*b^3)]
```

Sympy [A] (verification not implemented)

Time = 7.84 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.50

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{a^3b^5}}(Bb+3Da)\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^3b^5}}(Bb+3Da)\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{16} + \frac{-2Aab-2Ca^2-4Cabx^2+x^3(Bb^2-5Dab)+x(-Bab-3Da^2)}{8a^3b^2+16a^2b^3x^2+8ab^4x^4}$$

```
[In] integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**3*b**5))*(B*b + 3*D*a)*log(-a**2*b**2*sqrt(-1/(a**3*b**5)) + x
)/16 + sqrt(-1/(a**3*b**5))*(B*b + 3*D*a)*log(a**2*b**2*sqrt(-1/(a**3*b**5)
) + x)/16 + (-2*A*a*b - 2*C*a**2 - 4*C*a*b*x**2 + x**3*(B*b**2 - 5*D*a*b) +
 x*(-B*a*b - 3*D*a**2))/(8*a**3*b**2 + 16*a**2*b**3*x**2 + 8*a*b**4*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = -\frac{4\,Cabx^2+(5\,Dab-Bb^2)x^3+2\,Ca^2+2\,Aab+(3\,Da^2+Bab)x}{8\,(ab^4x^4+2\,a^2b^3x^2+a^3b^2)} + \frac{(3\,Da+Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^2}$$

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(4*C*a*b*x^2 + (5*D*a*b - B*b^2)*x^3 + 2*C*a^2 + 2*A*a*b + (3*D*a^2 +
B*a*b)*x)/(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2) + 1/8*(3*D*a + B*b)*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.82

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{(3\,Da+Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^2} - \frac{5\,Dabx^3-Bb^2x^3+4\,Cabx^2+3\,Da^2x+Babx+2\,Ca^2+2\,Aab}{8\,(bx^2+a)^2ab^2}$$

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*D*a + B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2) - 1/8*(5*D*a*b*x^
3 - B*b^2*x^3 + 4*C*a*b*x^2 + 3*D*a^2*x + B*a*b*x + 2*C*a^2 + 2*A*a*b)/((b*
x^2 + a)^2*a*b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \int \frac{x\,(A+B\,x+C\,x^2+x^3\,D)}{(b\,x^2+a)^3}\,dx$$

```
[In] int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

```
[Out] int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3, x)
```

3.106 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx$

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Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx = \frac{-a\left(B-\frac{aD}{b}\right)+(Ab-aC)x}{4ab(a+bx^2)^2} - \frac{4a^2D-b(3Ab+aC)x}{8a^2b^2(a+bx^2)} + \frac{(3Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

```
[Out] 1/4*(-a*(B-a*D/b)+(A*b-C*a)*x)/a/b/(b*x^2+a)^2+1/8*(-4*a^2*D+b*(3*A*b+C*a)*
x)/a^2/b^2/(b*x^2+a)+1/8*(3*A*b+C*a)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(3
/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1828, 653, 211}

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx = \frac{(aC+3Ab)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}} - \frac{4a^2D-bx(aC+3Ab)}{8a^2b^2(a+bx^2)} - \frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab(a+bx^2)^2}$$

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^3,x]
```

```
[Out] -1/4*(a*(B - (a*D)/b) - (A*b - a*C)*x)/(a*b*(a + b*x^2)^2) - (4*a^2*D - b*(
3*A*b + a*C)*x)/(8*a^2*b^2*(a + b*x^2)) + ((3*A*b + a*C)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(8*a^(5/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{4ab\left(a+bx^2\right)^2}-\frac{\int \frac{-3A-\frac{aC}{b}-\frac{4aDx}{b}}{\left(a+bx^2\right)^2}\,dx}{4a} \\
&= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{4ab\left(a+bx^2\right)^2}-\frac{4a^2D-b(3Ab+aC)x}{8a^2b^2\left(a+bx^2\right)}+\frac{(3Ab+aC)\int \frac{1}{a+bx^2}\,dx}{8a^2b} \\
&= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{4ab\left(a+bx^2\right)^2}-\frac{4a^2D-b(3Ab+aC)x}{8a^2b^2\left(a+bx^2\right)}+\frac{(3Ab+aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^3}\,dx = \frac{\frac{\sqrt{a}\left(-2a^3D+3Ab^3x^3+ab^2x\left(5A+Cx^2\right)-a^2b(2B+x(C+4Dx))\right)}{\left(a+bx^2\right)^2}+\sqrt{b}(3Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^2}$$

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^3,x]
[Out] ((Sqrt[a]*(-2*a^3*D + 3*A*b^3*x^3 + a*b^2*x*(5*A + C*x^2) - a^2*b*(2*B + x*
(C + 4*D*x))))/(a + b*x^2)^2 + Sqrt[b]*(3*A*b + a*C)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(8*a^(5/2)*b^2)
```

Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\frac{(3Ab+Ca)x^3}{8a^2}-\frac{Dx^2}{2b}+\frac{(5Ab-Ca)x}{8ab}-\frac{Bb+Da}{4b^2}}{(b\,x^2+a)^2}+\frac{(3Ab+Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2b\sqrt{ab}}$	98

```
[In] int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(3*A*b+C*a)/a^2*x^3-1/2*D*x^2/b+1/8*(5*A*b-C*a)/a/b*x-1/4*(B*b+D*a)/b^
2)/(b*x^2+a)^2+1/8*(3*A*b+C*a)/a^2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.98

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx$$

$$= \left[-\frac{8\,Da^3bx^2+4\,Da^4+4\,Ba^3b-2\,(Ca^2b^2+3\,Aab^3)x^3+((Cab^2+3\,Ab^3)x^4+Ca^3+3\,Aa^2b+2\,(Ca^2b+}{16\,(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2)}\right.$$

$$-\frac{4\,Da^3bx^2+2\,Da^4+2\,Ba^3b-(Ca^2b^2+3\,Aab^3)x^3-((Cab^2+3\,Ab^3)x^4+Ca^3+3\,Aa^2b+2\,(Ca^2b+3}{8\,(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2)}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(8*D*a^3*b*x^2 + 4*D*a^4 + 4*B*a^3*b - 2*(C*a^2*b^2 + 3*A*a*b^3)*x^3
 + ((C*a*b^2 + 3*A*b^3)*x^4 + C*a^3 + 3*A*a^2*b + 2*(C*a^2*b + 3*A*a*b^2)*x
^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(C*a^3*b -
 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2 + a^5*b^2), -1/8*(4*D*a^3*b*x
^2 + 2*D*a^4 + 2*B*a^3*b - (C*a^2*b^2 + 3*A*a*b^3)*x^3 - ((C*a*b^2 + 3*A*b^
3)*x^4 + C*a^3 + 3*A*a^2*b + 2*(C*a^2*b + 3*A*a*b^2)*x^2)*sqrt(a*b)*arctan(
sqrt(a*b)*x/a) + (C*a^3*b - 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2 +
a^5*b^2)]
```

Sympy [A] (verification not implemented)

Time = 3.41 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.59

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{a^5b^3}}\cdot(3Ab+Ca)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^5b^3}}\cdot(3Ab+Ca)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16} + \frac{-2Ba^2b-2Da^3-4Da^2bx^2+x^3\cdot(3Ab^3+Cab^2)+x(5Aab^2-Ca^2b)}{8a^4b^2+16a^3b^3x^2+8a^2b^4x^4}$$

```
[In] integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*b**3))*(3*A*b + C*a)*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/1
6 + sqrt(-1/(a**5*b**3))*(3*A*b + C*a)*log(a**3*b*sqrt(-1/(a**5*b**3)) + x)
/16 + (-2*B*a**2*b - 2*D*a**3 - 4*D*a**2*b*x**2 + x**3*(3*A*b**3 + C*a*b**2
) + x*(5*A*a*b**2 - C*a**2*b))/(8*a**4*b**2 + 16*a**3*b**3*x**2 + 8*a**2*b*
*4*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx = -\frac{4\,Da^2bx^2+2\,Da^3+2\,Ba^2b-(Cab^2+3\,Ab^3)x^3+(Ca^2b-5\,Aab^2)x}{8\,(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2)} + \frac{(Ca+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(4*D*a^2*b*x^2 + 2*D*a^3 + 2*B*a^2*b - (C*a*b^2 + 3*A*b^3)*x^3 + (C*a^
2*b - 5*A*a*b^2)*x)/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2) + 1/8*(C*a + 3*
A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx = \frac{(Ca+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}a^2b} + \frac{Cab^2x^3+3\,Ab^3x^3-4\,Da^2bx^2-Ca^2bx+5\,Aab^2x-2\,Da^3-2\,Ba^2b}{8\,(bx^2+a)^2a^2b^2}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(C*a + 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(C*a*b^2*x^
3 + 3*A*b^3*x^3 - 4*D*a^2*b*x^2 - C*a^2*b*x + 5*A*a*b^2*x - 2*D*a^3 - 2*B*a
^2*b)/((b*x^2 + a)^2*a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 5.96 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.41

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx = \frac{\frac{C\,x^3}{8\,a}-\frac{C\,x}{8\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4} + \frac{\frac{5\,A\,x}{8\,a}+\frac{3\,A\,b\,x^3}{8\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4} - \frac{B}{4\,b\,(a^2+2\,a\,b\,x^2+b^2\,x^4)} - \frac{(2\,b\,x^2+a)\,D}{4\,b^2\,(b\,x^2+a)^2} + \frac{3\,A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}} + \frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,b^{3/2}}$$

```
[In] int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^3,x)
```

```
[Out] ((C*x^3)/(8*a) - (C*x)/(8*b))/(a^2 + b^2*x^4 + 2*a*b*x^2) + ((5*A*x)/(8*a)
+ (3*A*b*x^3)/(8*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - B/(4*b*(a^2 + b^2*x^4
+ 2*a*b*x^2)) - ((a + 2*b*x^2)*D)/(4*b^2*(a + b*x^2)^2) + (3*A*atan((b^(1/2
)*x)/a^(1/2)))/(8*a^(5/2)*b^(1/2)) + (C*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(3/
2)*b^(3/2))
```

3.107 $\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^3}\,dx$

Optimal result . . . 656
Rubi [A] (verified) . . . 656
Mathematica [A] (verified) . . . 658
Maple [A] (verified) . . . 658
Fricas [B] (verification not implemented) . . . 659
Sympy [F(-1)] . . . 659
Maxima [A] (verification not implemented) . . . 660
Giac [A] (verification not implemented) . . . 660
Mupad [F(-1)] . . . 661

Optimal result

Integrand size = 28, antiderivative size = 130

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^3}\,dx=\frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2}+\frac{4Ab+(3bB+aD)x}{8a^2b\left(a+bx^2\right)}+\frac{(3bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}+\frac{A\log(x)}{a^3}-\frac{A\log\left(a+bx^2\right)}{2a^3}$$

```
[Out] 1/4*(A*b-C*a+(B*b-D*a)*x)/a/b/(b*x^2+a)^2+1/8*(4*A*b+(3*B*b+D*a)*x)/a^2/b/(
b*x^2+a)+1/8*(3*B*b+D*a)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)+A*ln(x)/
a^3-1/2*A*ln(b*x^2+a)/a^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1819, 837, 815, 649, 211, 266}

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^3}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(aD+3bB)}{8a^{5/2}b^{3/2}}-\frac{A\log\left(a+bx^2\right)}{2a^3}+\frac{A\log(x)}{a^3}+\frac{x(aD+3bB)+4Ab}{8a^2b\left(a+bx^2\right)}+\frac{x(bB-aD)-aC+Ab}{4ab\left(a+bx^2\right)^2}$$

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^3),x]
```

```
[Out] (A*b - a*C + (b*B - a*D)*x)/(4*a*b*(a + b*x^2)^2) + (4*A*b + (3*b*B + a*D)*
x)/(8*a^2*b*(a + b*x^2)) + ((3*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a
^(5/2)*b^(3/2)) + (A*Log[x])/a^3 - (A*Log[a + b*x^2])/(2*a^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{Ab - aC + (bB - aD)x}{4ab\left(a + bx^2\right)^2} - \frac{\int \frac{-4A - \frac{(3bB + aD)x}{b}}{x(a+bx^2)^2}\,dx}{4a}$$

$$= \frac{Ab - aC + (bB - aD)x}{4ab\left(a + bx^2\right)^2} + \frac{4Ab + (3bB + aD)x}{8a^2b\left(a + bx^2\right)} + \frac{\int \frac{8aAb + a(3bB + aD)x}{x(a + bx^2)}\,dx}{8a^3b}$$

$$= \frac{Ab - aC + (bB - aD)x}{4ab\left(a + bx^2\right)^2} + \frac{4Ab + (3bB + aD)x}{8a^2b\left(a + bx^2\right)} + \frac{\int \left(\frac{8Ab}{x} + \frac{3abB + a^2D - 8Ab^2x}{a + bx^2}\right)dx}{8a^3b}$$

$$= \frac{Ab - aC + (bB - aD)x}{4ab\left(a + bx^2\right)^2} + \frac{4Ab + (3bB + aD)x}{8a^2b\left(a + bx^2\right)} + \frac{A\log(x)}{a^3} + \frac{\int \frac{3abB + a^2D - 8Ab^2x}{a + bx^2}\,dx}{8a^3b}$$

$$= \frac{Ab - aC + (bB - aD)x}{4ab\left(a + bx^2\right)^2} + \frac{4Ab + (3bB + aD)x}{8a^2b\left(a + bx^2\right)} + \frac{A\log(x)}{a^3} - \frac{(Ab)\int \frac{x}{a + bx^2}\,dx}{a^3} + \frac{(3bB + aD)\int \frac{1}{a + bx^2}\,dx}{8a^2b}$$

$$= \frac{Ab - aC + (bB - aD)x}{4ab\left(a + bx^2\right)^2} + \frac{4Ab + (3bB + aD)x}{8a^2b\left(a + bx^2\right)} + \frac{(3bB + aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}} + \frac{A\log(x)}{a^3} - \frac{A\log\left(a + bx^2\right)}{2a^3}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.90

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x\left(a + bx^2\right)^3}\,dx = \frac{\frac{a(4Ab + 3bBx + aDx)}{b(a + bx^2)} + \frac{2a^2(Ab + bBx - a(C + Dx))}{b(a + bx^2)^2} + \frac{\sqrt{a}(3bB + aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}} + 8A\log(x) - 4A\log\left(a + bx^2\right)}{8a^3}$$

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^3),x]
```

```
[Out] ((a*(4*A*b + 3*b*B*x + a*D*x))/(b*(a + b*x^2)) + (2*a^2*(A*b + b*B*x - a*(C
 + D*x)))/(b*(a + b*x^2)^2) + (Sqrt[a]*(3*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/b^(3/2) + 8*A*Log[x] - 4*A*Log[a + b*x^2])/(8*a^3)
```

Maple [A] (verified)

Time = 3.45 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{A\ln(x)}{a^3} - \frac{\frac{\left(-\frac{3}{8}abB - \frac{1}{8}Da^2\right)x^3 - \frac{aAb\,x^2}{2} - \frac{a^2(5Bb - Da)x}{8b} - \frac{a^2(3Ab - Ca)}{4b}}{\left(b\,x^2 + a\right)^2} + \frac{4bA\ln\left(b\,x^2 + a\right) + \frac{\left(-3abB - Da^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{8b}}{a^3}$	130

```
[In] int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] A*ln(x)/a^3-1/a^3*(((-3/8*a*b*B-1/8*D*a^2)*x^3-1/2*a*A*b*x^2-1/8*a^2*(5*B*b
-D*a)/b*x-1/4*a^2*(3*A*b-C*a)/b)/(b*x^2+a)^2+1/8/b*(4*b*A*ln(b*x^2+a)+(-3*B
*a*b-D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(114) = 228$.

Time = 0.32 (sec) , antiderivative size = 488, normalized size of antiderivative = 3.75

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x(a + bx^2)^3} dx$$

$$= \left[\frac{8 Aab^3x^2 - 4 Ca^3b + 12 Aa^2b^2 + 2 (Da^2b^2 + 3 Bab^3)x^3 - ((Dab^2 + 3 Bb^3)x^4 + Da^3 + 3 Ba^2b + 2 (Da^2b}{} \right.$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*A*a*b^3*x^2 - 4*C*a^3*b + 12*A*a^2*b^2 + 2*(D*a^2*b^2 + 3*B*a*b^3)
*x^3 - ((D*a*b^2 + 3*B*b^3)*x^4 + D*a^3 + 3*B*a^2*b + 2*(D*a^2*b + 3*B*a*b^
2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*(D*a^3
*b - 5*B*a^2*b^2)*x - 8*(A*b^4*x^4 + 2*A*a*b^3*x^2 + A*a^2*b^2)*log(b*x^2 +
 a) + 16*(A*b^4*x^4 + 2*A*a*b^3*x^2 + A*a^2*b^2)*log(x))/(a^3*b^4*x^4 + 2*a
^4*b^3*x^2 + a^5*b^2), 1/8*(4*A*a*b^3*x^2 - 2*C*a^3*b + 6*A*a^2*b^2 + (D*a^
2*b^2 + 3*B*a*b^3)*x^3 + ((D*a*b^2 + 3*B*b^3)*x^4 + D*a^3 + 3*B*a^2*b + 2*(
D*a^2*b + 3*B*a*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (D*a^3*b - 5*B*
a^2*b^2)*x - 4*(A*b^4*x^4 + 2*A*a*b^3*x^2 + A*a^2*b^2)*log(b*x^2 + a) + 8*(
A*b^4*x^4 + 2*A*a*b^3*x^2 + A*a^2*b^2)*log(x))/(a^3*b^4*x^4 + 2*a^4*b^3*x^2
 + a^5*b^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x(a + bx^2)^3} dx = \text{Timed out}$$

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^3}\,dx = \frac{4\,Ab^2x^2+(Dab+3\,Bb^2)x^3-2\,Ca^2+6\,Aab-(Da^2-5\,Bab)x}{8\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)} - \frac{A\log(bx^2+a)}{2\,a^3} + \frac{A\log(x)}{a^3} + \frac{(Da+3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x, algorithm="maxima")
[Out] 1/8*(4*A*b^2*x^2 + (D*a*b + 3*B*b^2)*x^3 - 2*C*a^2 + 6*A*a*b - (D*a^2 - 5*B
*a*b)*x)/(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b) - 1/2*A*log(b*x^2 + a)/a^3 +
 A*log(x)/a^3 + 1/8*(D*a + 3*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^3}\,dx = -\frac{A\log(bx^2+a)}{2\,a^3} + \frac{A\log(|x|)}{a^3} + \frac{(Da+3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b} + \frac{4\,Aab^2x^2-2\,Ca^3+6\,Aa^2b+(Da^2b+3\,Bab^2)x^3-(Da^3-5\,Ba^2b)x}{8\,(bx^2+a)^2a^3b}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x, algorithm="giac")
[Out] -1/2*A*log(b*x^2 + a)/a^3 + A*log(abs(x))/a^3 + 1/8*(D*a + 3*B*b)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(4*A*a*b^2*x^2 - 2*C*a^3 + 6*A*a^2*b +
 (D*a^2*b + 3*B*a*b^2)*x^3 - (D*a^3 - 5*B*a^2*b)*x)/((b*x^2 + a)^2*a^3*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^3}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{x\,\left(b\,x^2+a\right)^3}\,dx$$

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^3),x)
[Out] int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^3), x)
```

3.108 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^3}\,dx$

Optimal result 662
Rubi [A] (verified) 662
Mathematica [A] (verified) 664
Maple [A] (verified) 664
Fricas [B] (verification not implemented) 665
Sympy [F(-1)] 665
Maxima [A] (verification not implemented) 666
Giac [A] (verification not implemented) 666
Mupad [B] (verification not implemented) 667

Optimal result

Integrand size = 28, antiderivative size = 144

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^3}\,dx = -\frac{A}{a^3x}+\frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2}+\frac{4B-\left(\frac{7Ab}{a}-3C\right)x}{8a^2\left(a+bx^2\right)} - \frac{3(5Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}+\frac{B\log(x)}{a^3}-\frac{B\log\left(a+bx^2\right)}{2a^3}$$

```
[Out] -A/a^3/x+1/4*(B*b-D*a-b*(A*b/a-C)*x)/a/b/(b*x^2+a)^2+1/8*(4*B-(7*A*b/a-3*C)
*x)/a^2/(b*x^2+a)+B*ln(x)/a^3-1/2*B*ln(b*x^2+a)/a^3-3/8*(5*A*b-C*a)*arctan(
x*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1819, 1816, 649, 211, 266}

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^3}\,dx = -\frac{3(5Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}-\frac{A}{a^3x}-\frac{B\log\left(a+bx^2\right)}{2a^3} +\frac{B\log(x)}{a^3}+\frac{4B-x\left(\frac{7Ab}{a}-3C\right)}{8a^2\left(a+bx^2\right)}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{4ab\left(a+bx^2\right)^2}$$

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(A/(a^3*x)) + (b*B - a*D - b*((A*b)/a - C)*x)/(4*a*b*(a + b*x^2)^2) + (4*B
 - ((7*A*b)/a - 3*C)*x)/(8*a^2*(a + b*x^2)) - (3*(5*A*b - a*C)*ArcTan[(Sqrt
```

```
[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]) + (B*Log[x])/a^3 - (B*Log[a + b*x^2])/
(2*a^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bB - aD - b\left(\frac{Ab}{a} - C\right)x}{4ab\left(a+bx^2\right)^2} - \frac{\int \frac{-4A-4Bx+3\left(\frac{Ab}{a}-C\right)x^2}{x^2\left(a+bx^2\right)^2}\,dx}{4a} \\
&= \frac{bB - aD - b\left(\frac{Ab}{a} - C\right)x}{4ab\left(a+bx^2\right)^2} + \frac{4B - \left(\frac{7Ab}{a} - 3C\right)x}{8a^2\left(a+bx^2\right)} + \frac{\int \frac{8A+8Bx-\left(\frac{7Ab}{a}-3C\right)x^2}{x^2\left(a+bx^2\right)}\,dx}{8a^2} \\
&= \frac{bB - aD - b\left(\frac{Ab}{a} - C\right)x}{4ab\left(a+bx^2\right)^2} + \frac{4B - \left(\frac{7Ab}{a} - 3C\right)x}{8a^2\left(a+bx^2\right)} + \frac{\int \left(\frac{8A}{ax^2} + \frac{8B}{ax} + \frac{-15Ab+3aC-8bBx}{a\left(a+bx^2\right)}\right)dx}{8a^2}
\end{aligned}$$

$$= -\frac{A}{a^3 x} + \frac{bB - aD - b\left(\frac{Ab}{a} - C\right) x}{4ab\,(a + bx^2)^2} + \frac{4B - \left(\frac{7Ab}{a} - 3C\right) x}{8a^2\,(a + bx^2)} + \frac{B\log(x)}{a^3} + \frac{\int \frac{-15Ab + 3aC - 8bBx}{a + bx^2}\,dx}{8a^3}$$

$$= -\frac{A}{a^3 x} + \frac{bB - aD - b\left(\frac{Ab}{a} - C\right) x}{4ab\,(a + bx^2)^2} + \frac{4B - \left(\frac{7Ab}{a} - 3C\right) x}{8a^2\,(a + bx^2)} + \frac{B\log(x)}{a^3} - \frac{(bB)\int \frac{x}{a + bx^2}\,dx}{a^3} - \frac{(3(5Ab - aC))\int \frac{1}{a + bx^2}\,dx}{8a^3}$$

$$= -\frac{A}{a^3 x} + \frac{bB - aD - b\left(\frac{Ab}{a} - C\right) x}{4ab\,(a + bx^2)^2} + \frac{4B - \left(\frac{7Ab}{a} - 3C\right) x}{8a^2\,(a + bx^2)} - \frac{3(5Ab - aC)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}} + \frac{B\log(x)}{a^3} - \frac{B\log(a + bx^2)}{2a^3}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.98

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x^2\,(a + bx^2)^3}\,dx = -\frac{A}{a^3 x} + \frac{abB - a^2 D - Ab^2 x + abCx}{4a^2 b\,(a + bx^2)^2} + \frac{4aB - 7Abx + 3aCx}{8a^3\,(a + bx^2)} + \frac{3(-5Ab + aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}} + \frac{B\log(x)}{a^3} - \frac{B\log(a + bx^2)}{2a^3}$$

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(A/(a^3*x)) + (a*b*B - a^2*D - A*b^2*x + a*b*C*x)/(4*a^2*b*(a + b*x^2)^2)
+ (4*a*B - 7*A*b*x + 3*a*C*x)/(8*a^3*(a + b*x^2)) + (3*(-5*A*b + a*C)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]) + (B*Log[x])/a^3 - (B*Log[a + b
*x^2])/(2*a^3)
```

Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{A}{a^3 x} + \frac{B\ln(x)}{a^3} - \frac{\frac{\left(\frac{7}{8}b^2 A - \frac{3}{8}Cab\right)x^3 - \frac{Bab\,x^2}{2} + \frac{a(9Ab - 5Ca)x}{8} - \frac{a^2(3Bb - Da)}{4b}}{(b\,x^2 + a)^2} + \frac{B\ln\left(b\,x^2 + a\right)}{2} + \frac{(15Ab - 3Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}}{a^3}$	125

```
[In] int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -A/a^3/x+B*ln(x)/a^3-1/a^3*(((7/8*b^2*A-3/8*C*a*b)*x^3-1/2*B*a*b*x^2+1/8*a*
(9*A*b-5*C*a)*x-1/4*a^2*(3*B*b-D*a)/b)/(b*x^2+a)^2+1/2*B*ln(b*x^2+a)+1/8*(1
5*A*b-3*C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(124) = 248$.

Time = 0.31 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.64

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^3}\,dx$$
$$= \left[\frac{8\,Ba^2b^2x^3 - 16\,Aa^3b + 6\,(Ca^2b^2 - 5\,Aab^3)x^4 + 10\,(Ca^3b - 5\,Aa^2b^2)x^2 + 3\,((Cab^2 - 5\,Ab^3)x^5 + 2\,(Ca^2b\right.$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*B*a^2*b^2*x^3 - 16*A*a^3*b + 6*(C*a^2*b^2 - 5*A*a*b^3)*x^4 + 10*(C
*a^3*b - 5*A*a^2*b^2)*x^2 + 3*((C*a*b^2 - 5*A*b^3)*x^5 + 2*(C*a^2*b - 5*A*a
*b^2)*x^3 + (C*a^3 - 5*A*a^2*b)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x -
 a)/(b*x^2 + a)) - 4*(D*a^4 - 3*B*a^3*b)*x - 8*(B*a*b^3*x^5 + 2*B*a^2*b^2*x
^3 + B*a^3*b*x)*log(b*x^2 + a) + 16*(B*a*b^3*x^5 + 2*B*a^2*b^2*x^3 + B*a^3*
b*x)*log(x))/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x), 1/8*(4*B*a^2*b^2*x^3
- 8*A*a^3*b + 3*(C*a^2*b^2 - 5*A*a*b^3)*x^4 + 5*(C*a^3*b - 5*A*a^2*b^2)*x^2
 + 3*((C*a*b^2 - 5*A*b^3)*x^5 + 2*(C*a^2*b - 5*A*a*b^2)*x^3 + (C*a^3 - 5*A*
a^2*b)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - 2*(D*a^4 - 3*B*a^3*b)*x - 4*(B*
a*b^3*x^5 + 2*B*a^2*b^2*x^3 + B*a^3*b*x)*log(b*x^2 + a) + 8*(B*a*b^3*x^5 +
2*B*a^2*b^2*x^3 + B*a^3*b*x)*log(x))/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x
)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^3}\,dx$$
$$=\frac{4\,Bab^2x^3+3\,(Cab^2-5\,Ab^3)x^4-8\,Aa^2b+5\,(Ca^2b-5\,Aab^2)x^2-2\,(Da^3-3\,Ba^2b)x}{8\,(a^3b^3x^5+2\,a^4b^2x^3+a^5bx)}$$
$$-\frac{B\log\left(bx^2+a\right)}{2\,a^3}+\frac{B\log\left(x\right)}{a^3}+\frac{3\,(Ca-5\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*B*a*b^2*x^3 + 3*(C*a*b^2 - 5*A*b^3)*x^4 - 8*A*a^2*b + 5*(C*a^2*b - 5
*A*a*b^2)*x^2 - 2*(D*a^3 - 3*B*a^2*b)*x)/(a^3*b^3*x^5 + 2*a^4*b^2*x^3 + a^5
*b*x) - 1/2*B*log(b*x^2 + a)/a^3 + B*log(x)/a^3 + 3/8*(C*a - 5*A*b)*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^3}\,dx$$
$$=-\frac{B\log\left(bx^2+a\right)}{2\,a^3}+\frac{B\log\left(|x|\right)}{a^3}+\frac{3\,(Ca-5\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3}$$
$$+\frac{4\,Bab^2x^3+3\,(Cab^2-5\,Ab^3)x^4-8\,Aa^2b+5\,(Ca^2b-5\,Aab^2)x^2-2\,(Da^3-3\,Ba^2b)x}{8\,\left(bx^2+a\right)^2a^3bx}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*B*log(b*x^2 + a)/a^3 + B*log(abs(x))/a^3 + 3/8*(C*a - 5*A*b)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/8*(4*B*a*b^2*x^3 + 3*(C*a*b^2 - 5*A*b^3)*x
^4 - 8*A*a^2*b + 5*(C*a^2*b - 5*A*a*b^2)*x^2 - 2*(D*a^3 - 3*B*a^2*b)*x)/((b
*x^2 + a)^2*a^3*b*x)
```

Mupad [B] (verification not implemented)

Time = 6.26 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.40

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^3}\,dx=\frac{\frac{3\,B}{4\,a}+\frac{B\,b\,x^2}{2\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{\frac{5\,C\,x}{8\,a}+\frac{3\,C\,b\,x^3}{8\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{\frac{A}{a}+\frac{25\,A\,b\,x^2}{8\,a^2}+\frac{15\,A\,b^2\,x^4}{8\,a^3}}{a^2\,x+2\,a\,b\,x^3+b^2\,x^5}-\frac{D}{4\,b\,\left(b\,x^2+a\right)^2}-\frac{B\,\ln\left(b\,x^2+a\right)}{2\,a^3}+\frac{B\,\ln\left(x\right)}{a^3}-\frac{15\,A\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{7/2}}+\frac{3\,C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}}$$

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x^2*(a + b*x^2)^3),x)
```

```
[Out] ((3*B)/(4*a) + (B*b*x^2)/(2*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) + ((5*C*x)/(8
*a) + (3*C*b*x^3)/(8*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - (A/a + (25*A*b*x^2
)/(8*a^2) + (15*A*b^2*x^4)/(8*a^3))/(a^2*x + b^2*x^5 + 2*a*b*x^3) - D/(4*b*
(a + b*x^2)^2) - (B*log(a + b*x^2))/(2*a^3) + (B*log(x))/a^3 - (15*A*b^(1/2
)*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(7/2)) + (3*C*atan((b^(1/2)*x)/a^(1/2)))/
(8*a^(5/2)*b^(1/2))
```

3.109 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)^3}\,dx$

Optimal result 668
Rubi [A] (verified) 668
Mathematica [A] (verified) 670
Maple [A] (verified) 671
Fricas [B] (verification not implemented) 671
Sympy [F(-1)] 672
Maxima [A] (verification not implemented) 672
Giac [A] (verification not implemented) 672
Mupad [B] (verification not implemented) 673

Optimal result

Integrand size = 28, antiderivative size = 174

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)^3}\,dx = -\frac{A}{2a^3x^2}-\frac{B}{a^3x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a(a+bx^2)^2} - \frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3(a+bx^2)} - \frac{3(5bB-aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}} - \frac{(3Ab-aC)\log(x)}{a^4}+\frac{(3Ab-aC)\log(a+bx^2)}{2a^4}$$

```
[Out] -1/2*A/a^3/x^2-B/a^3/x+1/4*(-A*b/a+C-(b*B/a-D)*x)/a/(b*x^2+a)^2+1/8*(-8*A*b
+4*C*a-(7*B*b-3*D*a)*x)/a^3/(b*x^2+a)-(3*A*b-C*a)*ln(x)/a^4+1/2*(3*A*b-C*a)
*ln(b*x^2+a)/a^4-3/8*(5*B*b-D*a)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1819, 1816, 649, 211, 266}

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)^3}\,dx = -\frac{3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(5bB-aD)}{8a^{7/2}\sqrt{b}}+\frac{(3Ab-aC)\log(a+bx^2)}{2a^4} - \frac{\log(x)(3Ab-aC)}{a^4}-\frac{4(2Ab-aC)+x(7bB-3aD)}{8a^3(a+bx^2)} - \frac{A}{2a^3x^2}-\frac{B}{a^3x}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{4a(a+bx^2)^2}$$

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -1/2*A/(a^3*x^2) - B/(a^3*x) - ((A*b)/a - C + ((b*B)/a - D)*x)/(4*a*(a + b*
x^2)^2) - (4*(2*A*b - a*C) + (7*b*B - 3*a*D)*x)/(8*a^3*(a + b*x^2)) - (3*(5
*b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]) - ((3*A*b - a*
C)*Log[x])/a^4 + ((3*A*b - a*C)*Log[a + b*x^2])/(2*a^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{\frac{Ab}{a} - C + \left(\frac{bB}{a} - D\right)x}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{-4A-4Bx+4\left(\frac{Ab}{a}-C\right)x^2+3\left(\frac{bB}{a}-D\right)x^3}{x^3\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= -\frac{\frac{Ab}{a} - C + \left(\frac{bB}{a} - D\right)x}{4a\left(a+bx^2\right)^2} - \frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)} + \frac{\int \frac{8A+8Bx-8\left(\frac{2Ab}{a}-C\right)x^2-\left(\frac{7bB}{a}-3D\right)x^3}{x^3(a+bx^2)}\,dx}{8a^2}$$

$$= -\frac{\frac{Ab}{a} - C + \left(\frac{bB}{a} - D\right)x}{4a\left(a+bx^2\right)^2} - \frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)} + \frac{\int \left(\frac{8A}{ax^3}+\frac{8B}{ax^2}+\frac{8(-3Ab+aC)}{a^2x}+\frac{-3a(5bB-aD)+8b(3Ab-aC)x}{a^2(a+bx^2)}\right)dx}{8a^2}$$

$$= -\frac{A}{2a^3x^2} - \frac{B}{a^3x} - \frac{\frac{Ab}{a} - C + \left(\frac{bB}{a} - D\right)x}{4a\left(a+bx^2\right)^2} - \frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)} - \frac{(3Ab-aC)\log(x)}{a^4} + \frac{\int \frac{-3a(5bB-aD)+8b(3Ab-aC)x}{a+bx^2}\,dx}{8a^4}$$

$$= -\frac{A}{2a^3x^2} - \frac{B}{a^3x} - \frac{\frac{Ab}{a} - C + \left(\frac{bB}{a} - D\right)x}{4a\left(a+bx^2\right)^2} - \frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)} - \frac{(3Ab-aC)\log(x)}{a^4} + \frac{(b(3Ab-aC))\int \frac{x}{a+bx^2}\,dx}{a^4} - \frac{(3(5bB-aD))\int \frac{1}{a+bx^2}\,dx}{8a^3}$$

$$= -\frac{A}{2a^3x^2} - \frac{B}{a^3x} - \frac{\frac{Ab}{a} - C + \left(\frac{bB}{a} - D\right)x}{4a\left(a+bx^2\right)^2} - \frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)} - \frac{3(5bB-aD)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}} - \frac{(3Ab-aC)\log(x)}{a^4} + \frac{(3Ab-aC)\log\left(a+bx^2\right)}{2a^4}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3}\,dx$$

$$= \frac{-\frac{4aA}{x^2} - \frac{8aB}{x} + \frac{a(-8Ab+4aC-7bBx+3aDx)}{a+bx^2} + \frac{2a^2(-Ab-bBx+a(C+Dx))}{(a+bx^2)^2} + \frac{3\sqrt{a}(-5bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} + 8(-3Ab+aC)\,\mathrm{lo}}{8a^4}$$

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] ((-4*a*A)/x^2 - (8*a*B)/x + (a*(-8*A*b + 4*a*C - 7*b*B*x + 3*a*D*x))/(a + b
*x^2) + (2*a^2*(-(A*b) - b*B*x + a*(C + D*x)))/(a + b*x^2)^2 + (3*Sqrt[a]*(
-5*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + 8*(-3*A*b + a*C)*Log[x
] + 4*(3*A*b - a*C)*Log[a + b*x^2])/(8*a^4)
```

Maple [A] (verified)

Time = 3.60 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.97

method	result
default	$-\frac{A}{2a^3x^2}-\frac{B}{a^3x}+\frac{(-3Ab+Ca)\ln(x)}{a^4}+\frac{\frac{\left(-\frac{7}{8}a\,b^2B+\frac{3}{8}Da^2b\right)x^3+\left(-a\,b^2A+\frac{1}{2}C\,a^2b\right)x^2-\frac{a^2(9Bb-5Da)x}{8}-\frac{5a^2bA}{4}+\frac{3C\,a^3}{4}}{\left(b\,x^2+a\right)^2}+\frac{\left(24b^2A-8C\right.}{}}{a^4}$

```
[In] int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*A/a^3/x^2-B/a^3/x+(-3*A*b+C*a)/a^4*ln(x)+1/a^4*(((-7/8*a*b^2*B+3/8*D*a
^2*b)*x^3+(-a*b^2*A+1/2*C*a^2*b)*x^2-1/8*a^2*(9*B*b-5*D*a)*x-5/4*a^2*b*A+3/
4*C*a^3)/(b*x^2+a)^2+1/16*(24*A*b^2-8*C*a*b)/b*ln(b*x^2+a)+1/8*(-15*B*a*b+3
*D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(147) = 294.

Time = 0.34 (sec) , antiderivative size = 696, normalized size of antiderivative = 4.00

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3}\,dx$$

$$=\left[-\frac{16\,Ba^3bx-6\,(Da^2b^2-5\,Bab^3)x^5+8\,Aa^3b-8\,(Ca^2b^2-3\,Aab^3)x^4-10\,(Da^3b-5\,Ba^2b^2)x^3-12\,(C}{}\right.$$

$$-\frac{8\,Ba^3bx-3\,(Da^2b^2-5\,Bab^3)x^5+4\,Aa^3b-4\,(Ca^2b^2-3\,Aab^3)x^4-5\,(Da^3b-5\,Ba^2b^2)x^3-6\,(Ca^3b}{}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*B*a^3*b*x - 6*(D*a^2*b^2 - 5*B*a*b^3)*x^5 + 8*A*a^3*b - 8*(C*a^2
*b^2 - 3*A*a*b^3)*x^4 - 10*(D*a^3*b - 5*B*a^2*b^2)*x^3 - 12*(C*a^3*b - 3*A*
a^2*b^2)*x^2 + 3*((D*a*b^2 - 5*B*b^3)*x^6 + 2*(D*a^2*b - 5*B*a*b^2)*x^4 + (
D*a^3 - 5*B*a^2*b)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2
+ a)) + 8*((C*a*b^3 - 3*A*b^4)*x^6 + 2*(C*a^2*b^2 - 3*A*a*b^3)*x^4 + (C*a^3
*b - 3*A*a^2*b^2)*x^2)*log(b*x^2 + a) - 16*((C*a*b^3 - 3*A*b^4)*x^6 + 2*(C*
a^2*b^2 - 3*A*a*b^3)*x^4 + (C*a^3*b - 3*A*a^2*b^2)*x^2)*log(x))/(a^4*b^3*x^
6 + 2*a^5*b^2*x^4 + a^6*b*x^2), -1/8*(8*B*a^3*b*x - 3*(D*a^2*b^2 - 5*B*a*b^
3)*x^5 + 4*A*a^3*b - 4*(C*a^2*b^2 - 3*A*a*b^3)*x^4 - 5*(D*a^3*b - 5*B*a^2*b
^2)*x^3 - 6*(C*a^3*b - 3*A*a^2*b^2)*x^2 - 3*((D*a*b^2 - 5*B*b^3)*x^6 + 2*(D
*a^2*b - 5*B*a*b^2)*x^4 + (D*a^3 - 5*B*a^2*b)*x^2)*sqrt(a*b)*arctan(sqrt(a*
b)*x/a) + 4*((C*a*b^3 - 3*A*b^4)*x^6 + 2*(C*a^2*b^2 - 3*A*a*b^3)*x^4 + (C*a
^3*b - 3*A*a^2*b^2)*x^2)*log(b*x^2 + a) - 8*((C*a*b^3 - 3*A*b^4)*x^6 + 2*(C
```

```
*a^2*b^2 - 3*A*a*b^3)*x^4 + (C*a^3*b - 3*A*a^2*b^2)*x^2)*log(x))/(a^4*b^3*x
^6 + 2*a^5*b^2*x^4 + a^6*b*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3}\,dx =\frac{3\,(Dab-5\,Bb^2)x^5+4\,(Cab-3\,Ab^2)x^4-8\,Ba^2x+5\,(Da^2-5\,Bab)x^3-4\,Aa^2+6\,(Ca^2-3\,Aab)x^2}{8\,(a^3b^2x^6+2\,a^4bx^4+a^5x^2)} +\frac{3\,(Da-5\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3}-\frac{(Ca-3\,Ab)\log\left(bx^2+a\right)}{2\,a^4}+\frac{(Ca-3\,Ab)\log\left(x\right)}{a^4}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x, algorithm="maxima")
[Out] 1/8*(3*(D*a*b - 5*B*b^2)*x^5 + 4*(C*a*b - 3*A*b^2)*x^4 - 8*B*a^2*x + 5*(D*a
^2 - 5*B*a*b)*x^3 - 4*A*a^2 + 6*(C*a^2 - 3*A*a*b)*x^2)/(a^3*b^2*x^6 + 2*a^4
*b*x^4 + a^5*x^2) + 3/8*(D*a - 5*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
 - 1/2*(C*a - 3*A*b)*log(b*x^2 + a)/a^4 + (C*a - 3*A*b)*log(x)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3}\,dx =\frac{3\,(Da-5\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3}-\frac{(Ca-3\,Ab)\log\left(bx^2+a\right)}{2\,a^4}+\frac{(Ca-3\,Ab)\log\left(|x|\right)}{a^4} +\frac{3\,Dabx^5-15\,Bb^2x^5+4\,Cabx^4-12\,Ab^2x^4+5\,Da^2x^3-25\,Babx^3+6\,Ca^2x^2-18\,Aabx^2-8\,Ba^2x-4\,A}{8\,\left(bx^3+ax\right)^2a^3}$$

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/8*(D*a - 5*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/2*(C*a - 3*A*b)
*log(b*x^2 + a)/a^4 + (C*a - 3*A*b)*log(abs(x))/a^4 + 1/8*(3*D*a*b*x^5 - 15
*B*b^2*x^5 + 4*C*a*b*x^4 - 12*A*b^2*x^4 + 5*D*a^2*x^3 - 25*B*a*b*x^3 + 6*C*
a^2*x^2 - 18*A*a*b*x^2 - 8*B*a^2*x - 4*A*a^2)/((b*x^3 + a*x)^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 6.39 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3}\,dx = \frac{\frac{3\,C}{4\,a}+\frac{C\,b\,x^2}{2\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{\frac{A}{2\,a}+\frac{9\,A\,b\,x^2}{4\,a^2}+\frac{3\,A\,b^2\,x^4}{2\,a^3}}{a^2\,x^2+2\,a\,b\,x^4+b^2\,x^6} - \frac{\frac{B}{a}+\frac{25\,B\,b\,x^2}{8\,a^2}+\frac{15\,B\,b^2\,x^4}{8\,a^3}}{a^2\,x+2\,a\,b\,x^3+b^2\,x^5}-\frac{C\,\ln\left(b\,x^2+a\right)}{2\,a^3} + \frac{C\,\ln\left(x\right)}{a^3}+\frac{3\,A\,b\,\ln\left(b\,x^2+a\right)}{2\,a^4}-\frac{3\,A\,b\,\ln\left(x\right)}{a^4} + \frac{x\,D\,{}_2\mathrm{F}_1\left(\frac{1}{2},3;\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{a^3}-\frac{15\,B\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{7/2}}$$

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x^3*(a + b*x^2)^3),x)
```

```
[Out] ((3*C)/(4*a) + (C*b*x^2)/(2*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - (A/(2*a) +
(9*A*b*x^2)/(4*a^2) + (3*A*b^2*x^4)/(2*a^3))/(a^2*x^2 + b^2*x^6 + 2*a*b*x^4
) - (B/a + (25*B*b*x^2)/(8*a^2) + (15*B*b^2*x^4)/(8*a^3))/(a^2*x + b^2*x^5
+ 2*a*b*x^3) - (C*log(a + b*x^2))/(2*a^3) + (C*log(x))/a^3 + (3*A*b*log(a +
 b*x^2))/(2*a^4) - (3*A*b*log(x))/a^4 + (x*D*hypergeom([1/2, 3], 3/2, -(b*x
^2)/a))/a^3 - (15*B*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(7/2))
```

3.110 $\int \frac{-x+4x^3}{(5+x^2)^2} dx$

Optimal result 674
Rubi [A] (verified) 674
Mathematica [A] (verified) 675
Maple [A] (verified) 675
Fricas [A] (verification not implemented) 676
Sympy [A] (verification not implemented) 676
Maxima [A] (verification not implemented) 677
Giac [A] (verification not implemented) 677
Mupad [B] (verification not implemented) 677

Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \frac{-x+4x^3}{(5+x^2)^2} dx = \frac{21}{2(5+x^2)} + 2\log(5+x^2)$$

```
[Out] 21/2/(x^2+5)+2*ln(x^2+5)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1607, 455, 45}

$$\int \frac{-x+4x^3}{(5+x^2)^2} dx = \frac{21}{2(x^2+5)} + 2\log(x^2+5)$$

```
[In] Int[(-x + 4*x^3)/(5 + x^2)^2,x]
```

```
[Out] 21/(2*(5 + x^2)) + 2*Log[5 + x^2]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x(-1+4x^2)}{(5+x^2)^2}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{-1+4x}{(5+x)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{21}{(5+x)^2}+\frac{4}{5+x}\right)\,dx, x, x^2\right)\\ &= \frac{21}{2\,(5+x^2)}+2\log\left(5+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{-x+4x^3}{(5+x^2)^2}\,dx = \frac{21}{2\,(5+x^2)}+2\log\left(5+x^2\right)$$

```
[In] Integrate[(-x + 4*x^3)/(5 + x^2)^2,x]
[Out] 21/(2*(5 + x^2)) + 2*Log[5 + x^2]
```

Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{21}{2(x^2+5)}+2\ln\left(x^2+5\right)$	19
norman	$\frac{21}{2(x^2+5)}+2\ln\left(x^2+5\right)$	19
risch	$\frac{21}{2(x^2+5)}+2\ln\left(x^2+5\right)$	19
meijerg	$-\frac{21x^2}{50\left(1+\frac{x^2}{5}\right)}+2\ln\left(1+\frac{x^2}{5}\right)$	26
parallelrisch	$\frac{4\ln(x^2+5)x^2+21+20\ln(x^2+5)}{2x^2+10}$	31

```
[In] int((4*x^3-x)/(x^2+5)^2,x,method=_RETURNVERBOSE)
[Out] 21/2/(x^2+5)+2*ln(x^2+5)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{-x+4x^3}{(5+x^2)^2}\,dx=\frac{4\left(x^2+5\right)\log\left(x^2+5\right)+21}{2\left(x^2+5\right)}$$

```
[In] integrate((4*x^3-x)/(x^2+5)^2,x, algorithm="fricas")
[Out] 1/2*(4*(x^2 + 5)*log(x^2 + 5) + 21)/(x^2 + 5)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{-x+4x^3}{(5+x^2)^2}\,dx=2\log\left(x^2+5\right)+\frac{21}{2x^2+10}$$

```
[In] integrate((4*x**3-x)/(x**2+5)**2,x)
[Out] 2*log(x**2 + 5) + 21/(2*x**2 + 10)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{-x+4x^3}{(5+x^2)^2}\,dx = \frac{21}{2\,(x^2+5)} + 2\,\log\left(x^2+5\right)$$

```
[In] integrate((4*x^3-x)/(x^2+5)^2,x, algorithm="maxima")
[Out] 21/2/(x^2 + 5) + 2*log(x^2 + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{-x+4x^3}{(5+x^2)^2}\,dx = -\frac{4\,x^2-1}{2\,(x^2+5)} + 2\,\log\left(x^2+5\right)$$

```
[In] integrate((4*x^3-x)/(x^2+5)^2,x, algorithm="giac")
[Out] -1/2*(4*x^2 - 1)/(x^2 + 5) + 2*log(x^2 + 5)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{-x+4x^3}{(5+x^2)^2}\,dx = 2\,\ln\left(x^2+5\right) + \frac{21}{2\,\left(x^2+5\right)}$$

```
[In] int(-(x - 4*x^3)/(x^2 + 5)^2,x)
[Out] 2*log(x^2 + 5) + 21/(2*(x^2 + 5))
```

3.111 $\int \frac{-x+x^3}{\sqrt{-2+x^2}} dx$

Optimal result 678
Rubi [A] (verified) 678
Mathematica [A] (verified) 679
Maple [A] (verified) 679
Fricas [A] (verification not implemented) 680
Sympy [A] (verification not implemented) 680
Maxima [A] (verification not implemented) 681
Giac [A] (verification not implemented) 681
Mupad [B] (verification not implemented) 681

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{-x+x^3}{\sqrt{-2+x^2}} dx = \sqrt{-2+x^2} + \frac{1}{3}\left(-2+x^2\right)^{3/2}$$

```
[Out] 1/3*(x^2-2)^(3/2)+(x^2-2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1607, 455, 45}

$$\int \frac{-x+x^3}{\sqrt{-2+x^2}} dx = \frac{1}{3}\left(x^2-2\right)^{3/2} + \sqrt{x^2-2}$$

```
[In] Int[(-x + x^3)/Sqrt[-2 + x^2],x]
```

```
[Out] Sqrt[-2 + x^2] + (-2 + x^2)^(3/2)/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x(-1+x^2)}{\sqrt{-2+x^2}}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{-1+x}{\sqrt{-2+x}}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{\sqrt{-2+x}}+\sqrt{-2+x}\right)\,dx, x, x^2\right) \\
&= \sqrt{-2+x^2}+\frac{1}{3}\left(-2+x^2\right)^{3/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx = \frac{1}{3}\sqrt{-2+x^2}\left(1+x^2\right)$$

```
[In] Integrate[(-x + x^3)/Sqrt[-2 + x^2],x]
[Out] (Sqrt[-2 + x^2]*(1 + x^2))/3
```

Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{(x^2+1)\sqrt{x^2-2}}{3}$	15
risch	$\frac{(x^2+1)\sqrt{x^2-2}}{3}$	15
pseudoelliptic	$\frac{(x^2+1)\sqrt{x^2-2}}{3}$	15
trager	$\left(\frac{x^2}{3}+\frac{1}{3}\right)\sqrt{x^2-2}$	16
default	$\frac{x^2\sqrt{x^2-2}}{3}+\frac{\sqrt{x^2-2}}{3}$	23
meijerg	$\frac{\sqrt{2}\,\sqrt{-\operatorname{signum}\left(-1+\frac{x^2}{2}\right)}\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\,(2x^2+8)\sqrt{1-\frac{x^2}{2}}}{6}\right)}{\sqrt{\pi}\,\sqrt{\operatorname{signum}\left(-1+\frac{x^2}{2}\right)}}+\frac{\sqrt{2}\,\sqrt{-\operatorname{signum}\left(-1+\frac{x^2}{2}\right)}\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{1-\frac{x^2}{2}}\right)}{2\sqrt{\pi}\,\sqrt{\operatorname{signum}\left(-1+\frac{x^2}{2}\right)}}$	108

```
[In] int((x^3-x)/(x^2-2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(x^2+1)*(x^2-2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx=\frac{1}{3}\left(x^2+1\right)\sqrt{x^2-2}$$

```
[In] integrate((x^3-x)/(x^2-2)^(1/2),x, algorithm="fricas")
[Out] 1/3*(x^2 + 1)*sqrt(x^2 - 2)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx=\frac{x^2\sqrt{x^2-2}}{3}+\frac{\sqrt{x^2-2}}{3}$$

```
[In] integrate((x**3-x)/(x**2-2)**(1/2),x)
[Out] x**2*sqrt(x**2 - 2)/3 + sqrt(x**2 - 2)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx = \frac{1}{3}\sqrt{x^2-2}x^2 + \frac{1}{3}\sqrt{x^2-2}$$

```
[In] integrate((x^3-x)/(x^2-2)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(x^2 - 2)*x^2 + 1/3*sqrt(x^2 - 2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx = \frac{1}{3}\left(x^2-2\right)^{\frac{3}{2}} + \sqrt{x^2-2}$$

```
[In] integrate((x^3-x)/(x^2-2)^(1/2),x, algorithm="giac")
[Out] 1/3*(x^2 - 2)^(3/2) + sqrt(x^2 - 2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx = \frac{(x^2+1)\sqrt{x^2-2}}{3}$$

```
[In] int(-(x - x^3)/(x^2 - 2)^(1/2),x)
[Out] ((x^2 + 1)*(x^2 - 2)^(1/2))/3
```

3.112 $\int \frac{-x^2+2x^4}{1+2x^2} dx$

Optimal result 682
Rubi [A] (verified) 682
Mathematica [A] (verified) 683
Maple [A] (verified) 684
Fricas [A] (verification not implemented) 684
Sympy [A] (verification not implemented) 684
Maxima [A] (verification not implemented) 685
Giac [A] (verification not implemented) 685
Mupad [B] (verification not implemented) 685

Optimal result

Integrand size = 21, antiderivative size = 25

$$\int \frac{-x^2+2x^4}{1+2x^2} dx = -x+\frac{x^3}{3}+\frac{\arctan\left(\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] -x+1/3*x^3+1/2*arctan(x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1607, 470, 327, 209}

$$\int \frac{-x^2+2x^4}{1+2x^2} dx = \frac{\arctan\left(\sqrt{2}x\right)}{\sqrt{2}}+\frac{x^3}{3}-x$$

```
[In] Int[(-x^2 + 2*x^4)/(1 + 2*x^2),x]
```

```
[Out] -x + x^3/3 + ArcTan[Sqrt[2]*x]/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^2(-1+2x^2)}{1+2x^2}\,dx \\
&= \frac{x^3}{3} - 2\int \frac{x^2}{1+2x^2}\,dx \\
&= -x + \frac{x^3}{3} + \int \frac{1}{1+2x^2}\,dx \\
&= -x + \frac{x^3}{3} + \frac{\tan^{-1}\left(\sqrt{2}x\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{-x^2+2x^4}{1+2x^2}\,dx = -x + \frac{x^3}{3} + \frac{\arctan\left(\sqrt{2}x\right)}{\sqrt{2}}$$

```
[In] Integrate[(-x^2 + 2*x^4)/(1 + 2*x^2),x]
[Out] -x + x^3/3 + ArcTan[Sqrt[2]*x]/Sqrt[2]
```

Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
default	$-x+\frac{x^{3}}{3}+\frac{\arctan\left(\sqrt{2}\, x\right)\sqrt{2}}{2}$	21
risch	$-x+\frac{x^{3}}{3}+\frac{\arctan\left(\sqrt{2}\, x\right)\sqrt{2}}{2}$	21
meijerg	$\frac{\sqrt{2}\left(-\frac{2x\sqrt{2}\left(-10x^{2}+15\right)}{15}+2\arctan\left(\sqrt{2}\, x\right)\right)}{8}-\frac{\sqrt{2}\left(2\sqrt{2}\, x-2\arctan\left(\sqrt{2}\, x\right)\right)}{8}$	49

```
[In] int((2*x^4-x^2)/(2*x^2+1),x,method=_RETURNVERBOSE)
[Out] -x+1/3*x^3+1/2*arctan(2^(1/2)*x)*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{-x^2+2x^4}{1+2x^2}\,dx=\frac{1}{3}\,x^3+\frac{1}{2}\,\sqrt{2}\arctan\left(\sqrt{2}x\right)-x$$

```
[In] integrate((2*x^4-x^2)/(2*x^2+1),x, algorithm="fricas")
[Out] 1/3*x^3 + 1/2*sqrt(2)*arctan(sqrt(2)*x) - x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{-x^2+2x^4}{1+2x^2}\,dx=\frac{x^3}{3}-x+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x\right)}{2}$$

```
[In] integrate((2*x**4-x**2)/(2*x**2+1),x)
[Out] x**3/3 - x + sqrt(2)*atan(sqrt(2)*x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{-x^2+2x^4}{1+2x^2}\,dx = \frac{1}{3}\,x^3 + \frac{1}{2}\,\sqrt{2}\arctan\left(\sqrt{2}x\right) - x$$

```
[In] integrate((2*x^4-x^2)/(2*x^2+1),x, algorithm="maxima")
[Out] 1/3*x^3 + 1/2*sqrt(2)*arctan(sqrt(2)*x) - x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{-x^2+2x^4}{1+2x^2}\,dx = \frac{1}{3}\,x^3 + \frac{1}{2}\,\sqrt{2}\arctan\left(\sqrt{2}x\right) - x$$

```
[In] integrate((2*x^4-x^2)/(2*x^2+1),x, algorithm="giac")
[Out] 1/3*x^3 + 1/2*sqrt(2)*arctan(sqrt(2)*x) - x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{-x^2+2x^4}{1+2x^2}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\right)}{2} - x + \frac{x^3}{3}$$

```
[In] int(-(x^2 - 2*x^4)/(2*x^2 + 1),x)
[Out] (2^(1/2)*atan(2^(1/2)*x))/2 - x + x^3/3
```

3.113 $\int \frac{x^3+x^4}{1+x^2}\,dx$

Optimal result 686
Rubi [A] (verified) 686
Mathematica [A] (verified) 687
Maple [A] (verified) 688
Fricas [A] (verification not implemented) 688
Sympy [A] (verification not implemented) 688
Maxima [A] (verification not implemented) 689
Giac [A] (verification not implemented) 689
Mupad [B] (verification not implemented) 689

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{x^3+x^4}{1+x^2}\,dx = -x+\frac{x^2}{2}+\frac{x^3}{3}+\arctan(x)-\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] -x+1/2*x^2+1/3*x^3+arctan(x)-1/2*ln(x^2+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1607, 815, 649, 209, 266}

$$\int \frac{x^3+x^4}{1+x^2}\,dx = \arctan(x)+\frac{x^3}{3}+\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)-x$$

```
[In] Int[(x^3 + x^4)/(1 + x^2),x]
```

```
[Out] -x + x^2/2 + x^3/3 + ArcTan[x] - Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^3(1+x)}{1+x^2}\,dx \\
&= \int \left(-1+x+x^2+\frac{1-x}{1+x^2}\right) dx \\
&= -x+\frac{x^2}{2}+\frac{x^3}{3}+\int \frac{1-x}{1+x^2}\,dx \\
&= -x+\frac{x^2}{2}+\frac{x^3}{3}+\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx \\
&= -x+\frac{x^2}{2}+\frac{x^3}{3}+\tan^{-1}(x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{x^3+x^4}{1+x^2}\,dx=-x+\frac{x^2}{2}+\frac{x^3}{3}+\arctan(x)-\frac{1}{2}\log\left(1+x^2\right)$$

```
[In] Integrate[(x^3 + x^4)/(1 + x^2),x]
[Out] -x + x^2/2 + x^3/3 + ArcTan[x] - Log[1 + x^2]/2
```

Maple [A] (verified)

Time = 3.44 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$-x+\frac{x^{2}}{2}+\frac{x^{3}}{3}+\arctan\left(x\right)-\frac{\ln\left(x^{2}+1\right)}{2}$	25
risch	$-x+\frac{x^{2}}{2}+\frac{x^{3}}{3}+\arctan\left(x\right)-\frac{\ln\left(x^{2}+1\right)}{2}$	25
meijerg	$-\frac{x\left(-5x^{2}+15\right)}{15}+\arctan\left(x\right)+\frac{x^{2}}{2}-\frac{\ln\left(x^{2}+1\right)}{2}$	27
parallelrisch	$\frac{x^{3}}{3}+\frac{x^{2}}{2}-x-\frac{\ln(x-i)}{2}-\frac{i\ln(x-i)}{2}-\frac{\ln(x+i)}{2}+\frac{i\ln(x+i)}{2}$	45

```
[In] int((x^4+x^3)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -x+1/2*x^2+1/3*x^3+arctan(x)-1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int\frac{x^{3}+x^{4}}{1+x^{2}}\,dx=\frac{1}{3}\,x^{3}+\frac{1}{2}\,x^{2}-x+\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^{2}+1\right)$$

```
[In] integrate((x^4+x^3)/(x^2+1),x, algorithm="fricas")
[Out] 1/3*x^3 + 1/2*x^2 - x + arctan(x) - 1/2*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int\frac{x^{3}+x^{4}}{1+x^{2}}\,dx=\frac{x^{3}}{3}+\frac{x^{2}}{2}-x-\frac{\log\left(x^{2}+1\right)}{2}+\operatorname{atan}\left(x\right)$$

```
[In] integrate((x**4+x**3)/(x**2+1),x)
[Out] x**3/3 + x**2/2 - x - log(x**2 + 1)/2 + atan(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{x^3+x^4}{1+x^2}\,dx = \frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 - x + \arctan\left(x\right) - \frac{1}{2}\,\log\left(x^2+1\right)$$

```
[In] integrate((x^4+x^3)/(x^2+1),x, algorithm="maxima")
[Out] 1/3*x^3 + 1/2*x^2 - x + arctan(x) - 1/2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{x^3+x^4}{1+x^2}\,dx = \frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 - x + \arctan\left(x\right) - \frac{1}{2}\,\log\left(x^2+1\right)$$

```
[In] integrate((x^4+x^3)/(x^2+1),x, algorithm="giac")
[Out] 1/3*x^3 + 1/2*x^2 - x + arctan(x) - 1/2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{x^3+x^4}{1+x^2}\,dx = \mathrm{atan}(x) - \frac{\ln\left(x^2+1\right)}{2} - x + \frac{x^2}{2} + \frac{x^3}{3}$$

```
[In] int((x^3 + x^4)/(x^2 + 1),x)
[Out] atan(x) - log(x^2 + 1)/2 - x + x^2/2 + x^3/3
```

3.114 $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{a+bx^2} dx$

Optimal result 690
Rubi [A] (verified) 690
Mathematica [A] (verified) 692
Maple [A] (verified) 692
Fricas [A] (verification not implemented) 693
Sympy [A] (verification not implemented) 693
Maxima [A] (verification not implemented) 694
Giac [A] (verification not implemented) 694
Mupad [B] (verification not implemented) 695

Optimal result

Integrand size = 30, antiderivative size = 210

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{a+bx^2} dx = \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{b^6} - \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^5}{5b^4} + \frac{(b^2d-abe+a^2f)\,x^7}{7b^3} + \frac{(be-af)x^9}{9b^2} + \frac{fx^{11}}{11b} - \frac{a^{5/2}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{13/2}}$$

```
[Out] a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^6-1/3*a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*
c)*x^3/b^5+1/5*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^5/b^4+1/7*(a^2*f-a*b*e+b^2*
d)*x^7/b^3+1/9*(-a*f+b*e)*x^9/b^2+1/11*f*x^11/b-a^(5/2)*(-a^3*f+a^2*b*e-a*b
^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/b^(13/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used

$= \{1816, 211\}$

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = \frac{x^7(a^2f-abe+b^2d)}{7b^3} + \frac{a^2x(a^3(-f)+a^2be-ab^2d+b^3c)}{b^6} - \frac{ax^3(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^5} + \frac{x^5(a^3(-f)+a^2be-ab^2d+b^3c)}{5b^4} - \frac{a^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{b^{13/2}} + \frac{x^9(be-af)}{9b^2} + \frac{fx^{11}}{11b}$$

```
[In] Int[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^6 - (a*(b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*x^3)/(3*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^5)/(5*b
^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e - a*f)*x^9)/(9*b^2) + (
f*x^11)/(11*b) - (a^(5/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/b^(13/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a^2(b^3c-ab^2d+a^2be-a^3f)}{b^6} - \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^4}{b^4} + \frac{(b^2d-abe+a^2f)\,x^6}{b^3} + \frac{(be-af)x^8}{b^2} + \frac{fx^{10}}{b} + \frac{-a^3b^3c+a^4b^2d-a^5be+a^6f}{b^6\,(a+bx^2)}\right)dx \\
&= \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{b^6} - \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^5}{5b^4} + \frac{(b^2d-abe+a^2f)\,x^7}{7b^3} + \frac{(be-af)x^9}{9b^2} + \frac{fx^{11}}{11b} - \frac{(a^3(b^3c-ab^2d+a^2be-a^3f))\int\frac{1}{a+bx^2}\,dx}{b^6}
\end{aligned}$$

$$= \frac{a^2(b^3c - ab^2d + a^2be - a^3f)\,x}{b^6} - \frac{a(b^3c - ab^2d + a^2be - a^3f)\,x^3}{3b^5} + \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^5}{5b^4} + \frac{(b^2d - abe + a^2f)\,x^7}{7b^3} + \frac{(be - af)x^9}{9b^2} + \frac{fx^{11}}{11b} - \frac{a^{5/2}(b^3c - ab^2d + a^2be - a^3f)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{13/2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00

$$\int \frac{x^6(c + dx^2 + ex^4 + fx^6)}{a + bx^2}\,dx = -\frac{a^2(-b^3c + ab^2d - a^2be + a^3f)\,x}{b^6} + \frac{a(-b^3c + ab^2d - a^2be + a^3f)\,x^3}{3b^5} + \frac{(b^3c - ab^2d + a^2be - a^3f)\,x^5}{5b^4} + \frac{(b^2d - abe + a^2f)\,x^7}{7b^3} + \frac{(be - af)x^9}{9b^2} + \frac{fx^{11}}{11b} + \frac{a^{5/2}(-b^3c + ab^2d - a^2be + a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{13/2}}$$

```
[In] Integrate[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] -((a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/b^6) + (a*(-(b^3*c) + a*b^
2*d - a^2*b*e + a^3*f)*x^3)/(3*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
x^5)/(5*b^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e - a*f)*x^9)/(9
*b^2) + (f*x^11)/(11*b) + (a^(5/2)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(13/2)
```

Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.11

method	result
default	$-\frac{-\frac{1}{11} f\,x^{11} b^5+\frac{1}{9} a\,b^4 f\,x^9-\frac{1}{9} b^5 e\,x^9-\frac{1}{7} a^2 b^3 f\,x^7+\frac{1}{7} a\,b^4 e\,x^7-\frac{1}{7} b^5 d\,x^7+\frac{1}{5} a^3 b^2 f\,x^5-\frac{1}{5} a^2 b^3 e\,x^5+\frac{1}{5} a\,b^4 d\,x^5-\frac{1}{5} b^5 c\,x^5-\frac{1}{3} a^4 b f\,x^3+\frac{1}{3} a^3 b^2}{b^6}$
risch	$-\frac{\sqrt{-ab}\,a^2\ln(-\sqrt{-ab}\,x+a)c}{2b^4}-\frac{\sqrt{-ab}\,a^5\ln(\sqrt{-ab}\,x+a)f}{2b^7}-\frac{\sqrt{-ab}\,a^3\ln(\sqrt{-ab}\,x+a)d}{2b^5}+\frac{e\,x^9}{9b}+\frac{d\,x^7}{7b}+\frac{c\,x^5}{5b}+\frac{\sqrt{-ab}\,a^2\ln(}{2b}$

```
[In] int(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^6*(-1/11*f*x^11*b^5+1/9*a*b^4*f*x^9-1/9*b^5*e*x^9-1/7*a^2*b^3*f*x^7+1/
7*a*b^4*e*x^7-1/7*b^5*d*x^7+1/5*a^3*b^2*f*x^5-1/5*a^2*b^3*e*x^5+1/5*a*b^4*d
```

```
*x^5-1/5*b^5*c*x^5-1/3*a^4*b*f*x^3+1/3*a^3*b^2*e*x^3-1/3*a^2*b^3*d*x^3+1/3*
a*b^4*c*x^3+a^5*f*x-a^4*b*e*x+a^3*b^2*d*x-a^2*b^3*c*x)+a^3*(a^3*f-a^2*b*e+a
*b^2*d-b^3*c)/b^6/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.15

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = \left[\frac{630\,b^5fx^{11}+770\,(b^5e-ab^4f)x^9+990\,(b^5d-ab^4e+a^2b^3f)x^7+1386\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^5-}{}\right.$$

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
[Out] [1/6930*(630*b^5*f*x^11 + 770*(b^5*e - a*b^4*f)*x^9 + 990*(b^5*d - a*b^4*e
+ a^2*b^3*f)*x^7 + 1386*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^5 - 231
0*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^3 - 3465*(a^2*b^3*c - a^3*b
^2*d + a^4*b*e - a^5*f)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^
2 + a)) + 6930*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x)/b^6, 1/3465*(31
5*b^5*f*x^11 + 385*(b^5*e - a*b^4*f)*x^9 + 495*(b^5*d - a*b^4*e + a^2*b^3*f
)*x^7 + 693*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^5 - 1155*(a*b^4*c -
 a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^3 - 3465*(a^2*b^3*c - a^3*b^2*d + a^4*b
*e - a^5*f)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 3465*(a^2*b^3*c - a^3*b^2*d
 + a^4*b*e - a^5*f)*x)/b^6]
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.83

$$\begin{aligned}\int \frac{x^6(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx &= x^9\left(-\frac{af}{9b^2}+\frac{e}{9b}\right)+x^7\left(\frac{a^2f}{7b^3}-\frac{ae}{7b^2}+\frac{d}{7b}\right)+x^5\left(-\frac{a^3f}{5b^4}+\frac{a^2e}{5b^3}-\frac{ad}{5b^2}+\frac{c}{5b}\right)\\ &\quad+x^3\left(\frac{a^4f}{3b^5}-\frac{a^3e}{3b^4}+\frac{a^2d}{3b^3}-\frac{ac}{3b^2}\right)+x\left(-\frac{a^5f}{b^6}+\frac{a^4e}{b^5}-\frac{a^3d}{b^4}+\frac{a^2c}{b^3}\right)\\ &\quad-\frac{\sqrt{-\frac{a^5}{b^{13}}}(a^3f-a^2be+ab^2d-b^3c)\log\left(-\frac{b^6\sqrt{-\frac{a^5}{b^{13}}}(a^3f-a^2be+ab^2d-b^3c)}{a^5f-a^4be+a^3b^2d-a^2b^3c}+x\right)}{2}\\ &\quad+\frac{\sqrt{-\frac{a^5}{b^{13}}}(a^3f-a^2be+ab^2d-b^3c)\log\left(\frac{b^6\sqrt{-\frac{a^5}{b^{13}}}(a^3f-a^2be+ab^2d-b^3c)}{a^5f-a^4be+a^3b^2d-a^2b^3c}+x\right)}{2}+\frac{fx^{11}}{11b}\end{aligned}$$

```
[In] integrate(x**6*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] x**9*(-a*f/(9*b**2) + e/(9*b)) + x**7*(a**2*f/(7*b**3) - a*e/(7*b**2) + d/(
7*b)) + x**5*(-a**3*f/(5*b**4) + a**2*e/(5*b**3) - a*d/(5*b**2) + c/(5*b))
+ x**3*(a**4*f/(3*b**5) - a**3*e/(3*b**4) + a**2*d/(3*b**3) - a*c/(3*b**2))
 + x*(-a**5*f/b**6 + a**4*e/b**5 - a**3*d/b**4 + a**2*c/b**3) - sqrt(-a**5/
b**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-b**6*sqrt(-a**5/b**13)*
(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**5*f - a**4*b*e + a**3*b**2*d -
a**2*b**3*c) + x)/2 + sqrt(-a**5/b**13)*(a**3*f - a**2*b*e + a*b**2*d - b**
3*c)*log(b**6*sqrt(-a**5/b**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a*
*5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c) + x)/2 + f*x**11/(11*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.01

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = -\frac{(a^3b^3c-a^4b^2d+a^5be-a^6f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^6} + \frac{315\,b^5fx^{11}+385\,(b^5e-ab^4f)x^9+495\,(b^5d-ab^4e+a^2b^3f)x^7+693\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^5-1}{3465\,b^6}$$

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(a^3*b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*b^6) + 1/3465*(315*b^5*f*x^11 + 385*(b^5*e - a*b^4*f)*x^9 + 495*(b^5*d - a
*b^4*e + a^2*b^3*f)*x^7 + 693*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^5
 - 1155*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^3 + 3465*(a^2*b^3*c -
 a^3*b^2*d + a^4*b*e - a^5*f)*x)/b^6
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.16

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = -\frac{(a^3b^3c-a^4b^2d+a^5be-a^6f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^6} + \frac{315\,b^{10}fx^{11}+385\,b^{10}ex^9-385\,ab^9fx^9+495\,b^{10}dx^7-495\,ab^9ex^7+495\,a^2b^8fx^7+693\,b^{10}cx^5-693\,ab^9dx}{}$$

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a^3*b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*b^6) + 1/3465*(315*b^10*f*x^11 + 385*b^10*e*x^9 - 385*a*b^9*f*x^9 + 495*b^
```

```
10*d*x^7 - 495*a*b^9*e*x^7 + 495*a^2*b^8*f*x^7 + 693*b^10*c*x^5 - 693*a*b^9
*d*x^5 + 693*a^2*b^8*e*x^5 - 693*a^3*b^7*f*x^5 - 1155*a*b^9*c*x^3 + 1155*a^
2*b^8*d*x^3 - 1155*a^3*b^7*e*x^3 + 1155*a^4*b^6*f*x^3 + 3465*a^2*b^8*c*x -
3465*a^3*b^7*d*x + 3465*a^4*b^6*e*x - 3465*a^5*b^5*f*x)/b^11
```

Mupad [B] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.38

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$$
$$= x^9\left(\frac{e}{9\,b}-\frac{a\,f}{9\,b^2}\right)+x^7\left(\frac{d}{7\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{7\,b}\right)+x^5\left(\frac{c}{5\,b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{5\,b}\right)$$
$$+\frac{f\,x^{11}}{11\,b}+\frac{a^{5/2}\,\mathrm{atan}\left(\frac{a^{5/2}\,\sqrt{b}\,x\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{f\,a^6-e\,a^5\,b+d\,a^4\,b^2-c\,a^3\,b^3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{b^{13/2}}$$
$$-\frac{a\,x^3\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{3\,b}+\frac{a^2\,x\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{b^2}$$

```
[In] int((x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x)
```

```
[Out] x^9*(e/(9*b) - (a*f)/(9*b^2)) + x^7*(d/(7*b) - (a*(e/b - (a*f)/b^2))/(7*b))
 + x^5*(c/(5*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(5*b)) + (f*x^11)/(11
*b) + (a^(5/2)*atan((a^(5/2)*b^(1/2)*x*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))
/(a^6*f - a^3*b^3*c + a^4*b^2*d - a^5*b*e))*(b^3*c - a^3*f - a*b^2*d + a^2*
b*e))/b^(13/2) - (a*x^3*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/(3*b
) + (a^2*x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/b^2
```

3.115 $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{a+bx^2} dx$

Optimal result 696
Rubi [A] (verified) 696
Mathematica [A] (verified) 698
Maple [A] (verified) 698
Fricas [A] (verification not implemented) 699
Sympy [B] (verification not implemented) 699
Maxima [A] (verification not implemented) 700
Giac [A] (verification not implemented) 700
Mupad [B] (verification not implemented) 701

Optimal result

Integrand size = 30, antiderivative size = 172

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^4} + \frac{(b^2d-abe+a^2f)\,x^5}{5b^3} + \frac{(be-af)x^7}{7b^2} + \frac{fx^9}{9b} + \frac{a^{3/2}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}$$

```
[Out] -a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^5+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*
x^3/b^4+1/5*(a^2*f-a*b*e+b^2*d)*x^5/b^3+1/7*(-a*f+b*e)*x^7/b^2+1/9*f*x^9/b+
a^(3/2)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/b^(11/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1816, 211}

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = \frac{x^5(a^2f-abe+b^2d)}{5b^3} - \frac{ax(a^3(-f)+a^2be-ab^2d+b^3c)}{b^5} + \frac{x^3(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^4} + \frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{b^{11/2}} + \frac{x^7(be-af)}{7b^2} + \frac{fx^9}{9b}$$

```
[In] Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^5) + ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*x^3)/(3*b^4) + ((b^2*d - a*b*e + a^2*f)*x^5)/(5*b^3) + ((b*e -
 a*f)*x^7)/(7*b^2) + (f*x^9)/(9*b) + (a^(3/2)*(b^3*c - a*b^2*d + a^2*b*e -
a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{a(b^3c-ab^2d+a^2be-a^3f)}{b^5}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{b^4}\right.\\
&\quad\left.+\frac{(b^2d-abe+a^2f)\,x^4}{b^3}+\frac{(be-af)x^6}{b^2}+\frac{fx^8}{b}+\frac{a^2b^3c-a^3b^2d+a^4be-a^5f}{b^5\,(a+bx^2)}\right)dx\\
&= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{b^5}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^4}\\
&\quad+\frac{(b^2d-abe+a^2f)\,x^5}{5b^3}+\frac{(be-af)x^7}{7b^2}+\frac{fx^9}{9b}\\
&\quad+\frac{(a^2(b^3c-ab^2d+a^2be-a^3f))\int\frac{1}{a+bx^2}\,dx}{b^5}\\
&= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{b^5}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^4}\\
&\quad+\frac{(b^2d-abe+a^2f)\,x^5}{5b^3}+\frac{(be-af)x^7}{7b^2}+\frac{fx^9}{9b}\\
&\quad+\frac{a^{3/2}(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.94

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$$
$$= \frac{x(315a^4f-105a^3b(3e+fx^2)+21a^2b^2(15d+5ex^2+3fx^4)-3ab^3(105c+35dx^2+21ex^4+15fx^6)+b^4x}{315b^5}$$
$$-\frac{a^{3/2}(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}$$

```
[In] Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] (x*(315*a^4*f - 105*a^3*b*(3*e + f*x^2) + 21*a^2*b^2*(15*d + 5*e*x^2 + 3*f*
x^4) - 3*a*b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6) + b^4*x^2*(105*c +
63*d*x^2 + 45*e*x^4 + 35*f*x^6)))/(315*b^5) - (a^(3/2)*(-(b^3*c) + a*b^2*d
- a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

Maple [A] (verified)

Time = 3.56 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.08

method	result
default	$\frac{\frac{1}{9}fx^9b^4-\frac{1}{7}ab^3fx^7+\frac{1}{7}b^4ex^7+\frac{1}{5}a^2b^2fx^5-\frac{1}{5}ab^3ex^5+\frac{1}{5}b^4dx^5-\frac{1}{3}a^3bfx^3+\frac{1}{3}a^2b^2ex^3-\frac{1}{3}ab^3dx^3+\frac{1}{3}b^4cx^3+a^4fx-a^3bex+a^2b^2dx-a}{b^5}$
risch	$\frac{fx^9}{9b}-\frac{afx^7}{7b^2}+\frac{ex^7}{7b}+\frac{a^2fx^5}{5b^3}-\frac{aex^5}{5b^2}+\frac{dx^5}{5b}-\frac{a^3fx^3}{3b^4}+\frac{a^2ex^3}{3b^3}-\frac{adx^3}{3b^2}+\frac{cx^3}{3b}+\frac{a^4fx}{b^5}-\frac{a^3ex}{b^4}+\frac{a^2dx}{b^3}-\frac{acx}{b^2}+$

```
[In] int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(1/9*f*x^9*b^4-1/7*a*b^3*f*x^7+1/7*b^4*e*x^7+1/5*a^2*b^2*f*x^5-1/5*a*
b^3*e*x^5+1/5*b^4*d*x^5-1/3*a^3*b*f*x^3+1/3*a^2*b^2*e*x^3-1/3*a*b^3*d*x^3+1
/3*b^4*c*x^3+a^4*f*x-a^3*b*e*x+a^2*b^2*d*x-a*b^3*c*x)-a^2*(a^3*f-a^2*b*e+a*
b^2*d-b^3*c)/b^5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.14

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$$
$$= \left[\frac{70\,b^4fx^9 + 90\,(b^4e-ab^3f)x^7 + 126\,(b^4d-ab^3e+a^2b^2f)x^5 + 210\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^3 - 315\,(}{630\,b^5}\right.$$

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/630*(70*b^4*f*x^9 + 90*(b^4*e - a*b^3*f)*x^7 + 126*(b^4*d - a*b^3*e + a^
2*b^2*f)*x^5 + 210*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^3 - 315*(a*b^3
*c - a^2*b^2*d + a^3*b*e - a^4*f)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b)
- a)/(b*x^2 + a)) - 630*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/b^5, 1/3
15*(35*b^4*f*x^9 + 45*(b^4*e - a*b^3*f)*x^7 + 63*(b^4*d - a*b^3*e + a^2*b^2
*f)*x^5 + 105*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^3 + 315*(a*b^3*c -
a^2*b^2*d + a^3*b*e - a^4*f)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 315*(a*b^3
*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/b^5]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(167) = 334.

Time = 0.45 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.96

$$\begin{aligned}\int \frac{x^4(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx &= x^7\left(-\frac{af}{7b^2}+\frac{e}{7b}\right)+x^5\left(\frac{a^2f}{5b^3}-\frac{ae}{5b^2}+\frac{d}{5b}\right)\\ &+x^3\left(-\frac{a^3f}{3b^4}+\frac{a^2e}{3b^3}-\frac{ad}{3b^2}+\frac{c}{3b}\right)+x\left(\frac{a^4f}{b^5}-\frac{a^3e}{b^4}+\frac{a^2d}{b^3}-\frac{ac}{b^2}\right)\\ &+\frac{\sqrt{-\frac{a^3}{b^{11}}}\,(a^3f-a^2be+ab^2d-b^3c)\log\left(-\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}\,(a^3f-a^2be+ab^2d-b^3c)}{a^4f-a^3be+a^2b^2d-ab^3c}+x\right)}{2}\\ &-\frac{\sqrt{-\frac{a^3}{b^{11}}}\,(a^3f-a^2be+ab^2d-b^3c)\log\left(\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}\,(a^3f-a^2be+ab^2d-b^3c)}{a^4f-a^3be+a^2b^2d-ab^3c}+x\right)}{2}+\frac{fx^9}{9b}\end{aligned}$$

```
[In] integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] x**7*(-a*f/(7*b**2) + e/(7*b)) + x**5*(a**2*f/(5*b**3) - a*e/(5*b**2) + d/(
5*b)) + x**3*(-a**3*f/(3*b**4) + a**2*e/(3*b**3) - a*d/(3*b**2) + c/(3*b))
+ x*(a**4*f/b**5 - a**3*e/b**4 + a**2*d/b**3 - a*c/b**2) + sqrt(-a**3/b**11
)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-b**5*sqrt(-a**3/b**11)*(a**3
*f - a**2*b*e + a*b**2*d - b**3*c)/(a**4*f - a**3*b*e + a**2*b**2*d - a*b**
3*c) + x)/2 - sqrt(-a**3/b**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log
(b**5*sqrt(-a**3/b**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**4*f - a
**3*b*e + a**2*b**2*d - a*b**3*c) + x)/2 + f*x**9/(9*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = \frac{(a^2b^3c-a^3b^2d+a^4be-a^5f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^5} + \frac{35\,b^4fx^9+45\,(b^4e-ab^3f)x^7+63\,(b^4d-ab^3e+a^2b^2f)x^5+105\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^3-315\,(ab}{315\,b^5}$$

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
b^5) + 1/315*(35*b^4*f*x^9 + 45*(b^4*e - a*b^3*f)*x^7 + 63*(b^4*d - a*b^3*e
 + a^2*b^2*f)*x^5 + 105*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^3 - 315*(
a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/b^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.13

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = \frac{(a^2b^3c-a^3b^2d+a^4be-a^5f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^5} + \frac{35\,b^8fx^9+45\,b^8ex^7-45\,ab^7fx^7+63\,b^8dx^5-63\,ab^7ex^5+63\,a^2b^6fx^5+105\,b^8cx^3-105\,ab^7dx^3+105\,a^2b}{315\,b^9}$$

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
b^5) + 1/315*(35*b^8*f*x^9 + 45*b^8*e*x^7 - 45*a*b^7*f*x^7 + 63*b^8*d*x^5 -
 63*a*b^7*e*x^5 + 63*a^2*b^6*f*x^5 + 105*b^8*c*x^3 - 105*a*b^7*d*x^3 + 105*
a^2*b^6*e*x^3 - 105*a^3*b^5*f*x^3 - 315*a*b^7*c*x + 315*a^2*b^6*d*x - 315*a
^3*b^5*e*x + 315*a^4*b^4*f*x)/b^9
```

Mupad [B] (verification not implemented)

Time = 5.92 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.41

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$$
$$= x^7\left(\frac{e}{7\,b}-\frac{a\,f}{7\,b^2}\right)+x^5\left(\frac{d}{5\,b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{5\,b}\right)$$
$$+x^3\left(\frac{c}{3\,b}-\frac{a\,\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{3\,b}\right)+\frac{f\,x^9}{9\,b}-\frac{a\,x\,\left(\frac{c}{b}-\frac{a\,\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{b}$$
$$-\frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{f\,a^5-e\,a^4\,b+d\,a^3\,b^2-c\,a^2\,b^3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{b^{11/2}}$$

```
[In] int((x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x)
```

```
[Out] x^7*(e/(7*b) - (a*f)/(7*b^2)) + x^5*(d/(5*b) - (a*(e/b - (a*f)/b^2))/(5*b))
 + x^3*(c/(3*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(3*b)) + (f*x^9)/(9*b
) - (a*x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/b - (a^(3/2)*atan((
a^(3/2)*b^(1/2)*x*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a^5*f - a^2*b^3*c +
 a^3*b^2*d - a^4*b*e))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/b^(11/2)
```

3.116 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{a+bx^2} \, dx$

Optimal result 702
Rubi [A] (verified) 702
Mathematica [A] (verified) 703
Maple [A] (verified) 704
Fricas [A] (verification not implemented) 704
Sympy [A] (verification not implemented) 705
Maxima [A] (verification not implemented) 705
Giac [A] (verification not implemented) 706
Mupad [B] (verification not implemented) 706

Optimal result

Integrand size = 30, antiderivative size = 136

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{a+bx^2} \, dx = \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{b^4} + \frac{(b^2d-abe+a^2f)\,x^3}{3b^3} + \frac{(be-af)x^5}{5b^2} + \frac{fx^7}{7b} - \frac{\sqrt{a}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}$$

```
[Out] (-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^4+1/3*(a^2*f-a*b*e+b^2*d)*x^3/b^3+1/5*(-
a*f+b*e)*x^5/b^2+1/7*f*x^7/b-(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(x*b^(1/2
)/a^(1/2))*a^(1/2)/b^(9/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1816, 211}

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{a+bx^2} \, dx = \frac{x^3(a^2f-abe+b^2d)}{3b^3} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{b^{9/2}} + \frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{b^4} + \frac{x^5(be-af)}{5b^2} + \frac{fx^7}{7b}$$

```
[In] Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4 + ((b^2*d - a*b*e + a^2*f)*x^3)
/(3*b^3) + ((b*e - a*f)*x^5)/(5*b^2) + (f*x^7)/(7*b) - (Sqrt[a]*(b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{b^3c-ab^2d+a^2be-a^3f}{b^4}+\frac{(b^2d-abe+a^2f)\,x^2}{b^3}+\frac{(be-af)x^4}{b^2}+\frac{fx^6}{b}\right.\\
&\qquad \left.+\frac{-ab^3c+a^2b^2d-a^3be+a^4f}{b^4\,(a+bx^2)}\right)dx\\
&= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{b^4}+\frac{(b^2d-abe+a^2f)\,x^3}{3b^3}\\
&\quad +\frac{(be-af)x^5}{5b^2}+\frac{fx^7}{7b}-\frac{(a(b^3c-ab^2d+a^2be-a^3f))\int\frac{1}{a+bx^2}\,dx}{b^4}\\
&= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{b^4}+\frac{(b^2d-abe+a^2f)\,x^3}{3b^3}+\frac{(be-af)x^5}{5b^2}\\
&\quad +\frac{fx^7}{7b}-\frac{\sqrt{a}(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.94

$$\begin{aligned}
&\int \frac{x^2(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx\\
&=\frac{x(-105a^3f+35a^2b(3e+fx^2)-7ab^2(15d+5ex^2+3fx^4)+b^3(105c+35dx^2+21ex^4+15fx^6))}{105b^4}\\
&\quad+\frac{\sqrt{a}(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}
\end{aligned}$$

```
[In] Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] (x*(-105*a^3*f + 35*a^2*b*(3*e + f*x^2) - 7*a*b^2*(15*d + 5*e*x^2 + 3*f*x^4
) + b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6)))/(105*b^4) + (Sqrt[a]*(-(
b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

method	result
default	$-\frac{-\frac{1}{7} f\,x^{7} b^{3}+\frac{1}{5} a\,b^{2} f\,x^{5}-\frac{1}{5} b^{3} e\,x^{5}-\frac{1}{3} a^{2} b f\,x^{3}+\frac{1}{3} a\,b^{2} e\,x^{3}-\frac{1}{3} b^{3} d\,x^{3}+f\,a^{3} x-a^{2} b e x+a\,b^{2} d x-b^{3} c x}{b^{4}}+\frac{a\left(f\,a^{3}-a^{2} b e+a\,b^{2} d-b^{3} c\right) \arctan\left(\right.}{b^{4} \sqrt{a b}}$
risch	$\frac{f\,x^{7}}{7 b}-\frac{a f\,x^{5}}{5 b^{2}}+\frac{e\,x^{5}}{5 b}+\frac{a^{2} f\,x^{3}}{3 b^{3}}-\frac{a e\,x^{3}}{3 b^{2}}+\frac{d\,x^{3}}{3 b}-\frac{f\,a^{3} x}{b^{4}}+\frac{a^{2} e x}{b^{3}}-\frac{a d x}{b^{2}}+\frac{c x}{b}+\frac{\sqrt{-a b}\,\ln\left(-\sqrt{-a b}\,x+a\right) f\,a^{3}}{2 b^{5}}-\frac{\sqrt{-a b}\,\ln}{}$

```
[In] int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(-1/7*f*x^7*b^3+1/5*a*b^2*f*x^5-1/5*b^3*e*x^5-1/3*a^2*b*f*x^3+1/3*a*
b^2*e*x^3-1/3*b^3*d*x^3+f*a^3*x-a^2*b*e*x+a*b^2*d*x-b^3*c*x)+a*(a^3*f-a^2*b
*e+a*b^2*d-b^3*c)/b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.10

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$$
$$=\left[\frac{30\,b^3fx^7+42\,(b^3e-ab^2f)x^5+70\,(b^3d-ab^2e+a^2bf)x^3-105\,(b^3c-ab^2d+a^2be-a^3f)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+}{}\right.}{210\,b^4}\right.$$

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(30*b^3*f*x^7 + 42*(b^3*e - a*b^2*f)*x^5 + 70*(b^3*d - a*b^2*e + a^2
*b*f)*x^3 - 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(-a/b)*log((b*x^2 +
 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 210*(b^3*c - a*b^2*d + a^2*b*e - a^3*
f)*x)/b^4, 1/105*(15*b^3*f*x^7 + 21*(b^3*e - a*b^2*f)*x^5 + 35*(b^3*d - a*b
^2*e + a^2*b*f)*x^3 - 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(a/b)*arc
tan(b*x*sqrt(a/b)/a) + 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4]
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.36

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = x^5\left(-\frac{af}{5b^2}+\frac{e}{5b}\right)+x^3\left(\frac{a^2f}{3b^3}-\frac{ae}{3b^2}+\frac{d}{3b}\right) + x\left(-\frac{a^3f}{b^4}+\frac{a^2e}{b^3}-\frac{ad}{b^2}+\frac{c}{b}\right) - \frac{\sqrt{-\frac{a}{b^9}}(a^3f-a^2be+ab^2d-b^3c)\log\left(-b^4\sqrt{-\frac{a}{b^9}}+x\right)}{2} + \frac{\sqrt{-\frac{a}{b^9}}(a^3f-a^2be+ab^2d-b^3c)\log\left(b^4\sqrt{-\frac{a}{b^9}}+x\right)}{2} + \frac{fx^7}{7b}$$

```
[In] integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] x**5*(-a*f/(5*b**2) + e/(5*b)) + x**3*(a**2*f/(3*b**3) - a*e/(3*b**2) + d/(
3*b)) + x*(-a**3*f/b**4 + a**2*e/b**3 - a*d/b**2 + c/b) - sqrt(-a/b**9)*(a*
*3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-b**4*sqrt(-a/b**9) + x)/2 + sqrt(
-a/b**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(b**4*sqrt(-a/b**9) + x
)/2 + f*x**7/(7*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.98

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = -\frac{(ab^3c-a^2b^2d+a^3be-a^4f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{15\,b^3fx^7+21\,(b^3e-ab^2f)x^5+35\,(b^3d-ab^2e+a^2bf)x^3+105\,(b^3c-ab^2d+a^2be-a^3f)x}{105\,b^4}$$

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b
^4) + 1/105*(15*b^3*f*x^7 + 21*(b^3*e - a*b^2*f)*x^5 + 35*(b^3*d - a*b^2*e
+ a^2*b*f)*x^3 + 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.09

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = -\frac{(ab^3c-a^2b^2d+a^3be-a^4f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{15\,b^6fx^7+21\,b^6ex^5-21\,ab^5fx^5+35\,b^6dx^3-35\,ab^5ex^3+35\,a^2b^4fx^3+105\,b^6cx-105\,ab^5dx+105\,a^2b^4e}{105\,b^7}$$

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b
^4) + 1/105*(15*b^6*f*x^7 + 21*b^6*e*x^5 - 21*a*b^5*f*x^5 + 35*b^6*d*x^3 -
35*a*b^5*e*x^3 + 35*a^2*b^4*f*x^3 + 105*b^6*c*x - 105*a*b^5*d*x + 105*a^2*b
^4*e*x - 105*a^3*b^3*f*x)/b^7
```

Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.42

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = x^5\left(\frac{e}{5\,b}-\frac{a\,f}{5\,b^2}\right)+x^3\left(\frac{d}{3\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{3\,b}\right)+x\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right) + \frac{f\,x^7}{7\,b}+\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{f\,a^4-e\,a^3\,b+d\,a^2\,b^2-c\,a\,b^3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{b^{9/2}}$$

```
[In] int((x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x)
```

```
[Out] x^5*(e/(5*b) - (a*f)/(5*b^2)) + x^3*(d/(3*b) - (a*(e/b - (a*f)/b^2))/(3*b))
 + x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b) + (f*x^7)/(7*b) + (a^(1/
2)*atan((a^(1/2)*b^(1/2)*x*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a^4*f + a^
2*b^2*d - a*b^3*c - a^3*b*e))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/b^(9/2)
```

3.117 $\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2} dx$

Optimal result 707
Rubi [A] (verified) 707
Mathematica [A] (verified) 708
Maple [A] (verified) 708
Fricas [A] (verification not implemented) 709
Sympy [A] (verification not implemented) 709
Maxima [A] (verification not implemented) 710
Giac [A] (verification not implemented) 710
Mupad [B] (verification not implemented) 711

Optimal result

Integrand size = 27, antiderivative size = 100

$$\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2} dx = \frac{(b^2d-abe+a^2f)\,x}{b^3}+\frac{(be-af)x^3}{3b^2}+\frac{fx^5}{5b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}$$

```
[Out] (a^2*f-a*b*e+b^2*d)*x/b^3+1/3*(-a*f+b*e)*x^3/b^2+1/5*f*x^5/b+(-a^3*f+a^2*b*
e-a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/b^(7/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1824, 211}

$$\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2} dx = \frac{x(a^2f-abe+b^2d)}{b^3}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{a}b^{7/2}}+\frac{x^3(be-af)}{3b^2}+\frac{fx^5}{5b}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2),x]
```

```
[Out] ((b^2*d - a*b*e + a^2*f)*x)/b^3 + ((b*e - a*f)*x^3)/(3*b^2) + (f*x^5)/(5*b)
 + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[
a]*b^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{b^2d-abe+a^2f}{b^3}+\frac{(be-af)x^2}{b^2}+\frac{fx^4}{b}+\frac{b^3c-ab^2d+a^2be-a^3f}{b^3\,(a+bx^2)}\right)dx \\
&= \frac{(b^2d-abe+a^2f)\,x}{b^3}+\frac{(be-af)x^3}{3b^2}+\frac{fx^5}{5b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\int\frac{1}{a+bx^2}\,dx}{b^3} \\
&= \frac{(b^2d-abe+a^2f)\,x}{b^3}+\frac{(be-af)x^3}{3b^2}+\frac{fx^5}{5b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2}\,dx = \frac{x(15a^2f-5ab(3e+fx^2)+b^2(15d+5ex^2+3fx^4))}{15b^3} + \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2),x]
[Out] (x*(15*a^2*f - 5*a*b*(3*e + f*x^2) + b^2*(15*d + 5*e*x^2 + 3*f*x^4)))/(15*b
^3) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sq
rt[a]*b^(7/2))
```

Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

method	result
default	$\frac{\frac{1}{5}f\,x^5b^2-\frac{1}{3}abf\,x^3+\frac{1}{3}b^2e\,x^3+a^2fx-abex+b^2dx}{b^3}+\frac{(-f\,a^3+a^2be-a\,b^2d+b^3c)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$
risch	$\frac{f\,x^5}{5b}-\frac{af\,x^3}{3b^2}+\frac{e\,x^3}{3b}+\frac{a^2fx}{b^3}-\frac{aex}{b^2}+\frac{dx}{b}-\frac{\ln(bx-\sqrt{-ab})f\,a^3}{2b^3\sqrt{-ab}}+\frac{\ln(bx-\sqrt{-ab})a^2e}{2b^2\sqrt{-ab}}-\frac{\ln(bx-\sqrt{-ab})ad}{2b\sqrt{-ab}}+\frac{\ln(bx-\sqrt{-a}}{2\sqrt{-ab}}$

```
[In] int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/5*f*x^5*b^2-1/3*a*b*f*x^3+1/3*b^2*e*x^3+a^2*f*x-a*b*e*x+b^2*d*x)+(
-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.36

$$\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2}\,dx = \left[\frac{6\,ab^3fx^5+10\,(ab^3e-a^2b^2f)x^3+15\,(b^3c-ab^2d+a^2be-a^3f)\sqrt{-ab}\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right)+30\,(ab^3d-a}{30\,ab^4}\right.$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*a*b^3*f*x^5 + 10*(a*b^3*e - a^2*b^2*f)*x^3 + 15*(b^3*c - a*b^2*d +
 a^2*b*e - a^3*f)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a))
+ 30*(a*b^3*d - a^2*b^2*e + a^3*b*f)*x)/(a*b^4), 1/15*(3*a*b^3*f*x^5 + 5*(a
*b^3*e - a^2*b^2*f)*x^3 + 15*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(a*b)*
arctan(sqrt(a*b)*x/a) + 15*(a*b^3*d - a^2*b^2*e + a^3*b*f)*x)/(a*b^4)]
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.60

$$\begin{aligned}\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2}\,dx &= x^3\left(-\frac{af}{3b^2}+\frac{e}{3b}\right)+x\left(\frac{a^2f}{b^3}-\frac{ae}{b^2}+\frac{d}{b}\right)\\ &+\frac{\sqrt{-\frac{1}{ab^7}}(a^3f-a^2be+ab^2d-b^3c)\log\left(-ab^3\sqrt{-\frac{1}{ab^7}}+x\right)}{2}\\ &-\frac{\sqrt{-\frac{1}{ab^7}}(a^3f-a^2be+ab^2d-b^3c)\log\left(ab^3\sqrt{-\frac{1}{ab^7}}+x\right)}{2}\\ &+\frac{fx^5}{5b}\end{aligned}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] x**3*(-a*f/(3*b**2) + e/(3*b)) + x*(a**2*f/b**3 - a*e/b**2 + d/b) + sqrt(-1
/(a*b**7))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a*b**3*sqrt(-1/(a*b
**7)) + x)/2 - sqrt(-1/(a*b**7))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*lo
g(a*b**3*sqrt(-1/(a*b**7)) + x)/2 + f*x**5/(5*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2}\,dx = \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,b^2fx^5+5\,(b^2e-abf)x^3+15\,(b^2d-abe+a^2f)x}{15\,b^3}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) +
 1/15*(3*b^2*f*x^5 + 5*(b^2*e - a*b*f)*x^3 + 15*(b^2*d - a*b*e + a^2*f)*x)/
b^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2}\,dx = \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,b^4fx^5+5\,b^4ex^3-5\,ab^3fx^3+15\,b^4dx-15\,ab^3ex+15\,a^2b^2fx}{15\,b^5}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) +
 1/15*(3*b^4*f*x^5 + 5*b^4*e*x^3 - 5*a*b^3*f*x^3 + 15*b^4*d*x - 15*a*b^3*e*
x + 15*a^2*b^2*f*x)/b^5
```

Mupad [B] (verification not implemented)

Time = 5.72 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2}\,dx = x^3\left(\frac{e}{3\,b}-\frac{a\,f}{3\,b^2}\right)+x\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)+\frac{f\,x^5}{5\,b}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{\sqrt{a}\,b^{7/2}}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2),x)
```

```
[Out] x^3*(e/(3*b) - (a*f)/(3*b^2)) + x*(d/b - (a*(e/b - (a*f)/b^2))/b) + (f*x^5)
/(5*b) + (atan((b^(1/2)*x)/a^(1/2))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a
^(1/2)*b^(7/2))
```

3.118 $\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)}\,dx$

Optimal result 712
Rubi [A] (verified) 712
Mathematica [A] (verified) 713
Maple [A] (verified) 713
Fricas [A] (verification not implemented) 714
Sympy [B] (verification not implemented) 714
Maxima [A] (verification not implemented) 715
Giac [A] (verification not implemented) 715
Mupad [B] (verification not implemented) 715

Optimal result

Integrand size = 30, antiderivative size = 84

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)}\,dx = -\frac{c}{ax}+\frac{(be-af)x}{b^2}+\frac{fx^3}{3b}-\frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}$$

```
[Out] -c/a/x+(-a*f+b*e)*x/b^2+1/3*f*x^3/b-(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(x
*b^(1/2)/a^(1/2))/a^(3/2)/b^(5/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1816, 211}

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)}\,dx = -\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{a^{3/2}b^{5/2}}+\frac{x(be-af)}{b^2}-\frac{c}{ax}+\frac{fx^3}{3b}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c/(a*x)) + ((b*e - a*f)*x)/b^2 + (f*x^3)/(3*b) - ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{be-af}{b^2}+\frac{c}{ax^2}+\frac{fx^2}{b}+\frac{-b^3c+ab^2d-a^2be+a^3f}{ab^2\left(a+bx^2\right)}\right)dx \\
&= -\frac{c}{ax}+\frac{(be-af)x}{b^2}+\frac{fx^3}{3b}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)\int \frac{1}{a+bx^2}\,dx}{ab^2} \\
&= -\frac{c}{ax}+\frac{(be-af)x}{b^2}+\frac{fx^3}{3b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)}\,dx = -\frac{c}{ax}+\frac{(be-af)x}{b^2}+\frac{fx^3}{3b}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)),x]
[Out] -(c/(a*x)) + ((b*e - a*f)*x)/b^2 + (f*x^3)/(3*b) + ((-(b^3*c) + a*b^2*d - a
^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

method	result
default	$-\frac{-\frac{1}{3} f\,x^3 b+a f x-b e x}{b^2}+\frac{\left(f\,a^3-a^2 b e+a\,b^2 d-b^3 c\right)\arctan\left(\frac{b x}{\sqrt{a b}}\right)}{a\,b^2\sqrt{a b}}-\frac{c}{a x}$
risch	$\frac{f\,x^3}{3b}-\frac{a f x}{b^2}+\frac{e x}{b}-\frac{c}{a x}-\frac{a^2\ln\left(-\sqrt{-a b}\,x+a\right) f}{2b^2\sqrt{-a b}}+\frac{a\ln\left(-\sqrt{-a b}\,x+a\right) e}{2b\sqrt{-a b}}-\frac{\ln\left(-\sqrt{-a b}\,x+a\right) d}{2\sqrt{-a b}}+\frac{b\ln\left(-\sqrt{-a b}\,x+a\right) c}{2\sqrt{-a b}\,a}+\frac{a^2 \mathrm{l}$

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^2*(-1/3*f*x^3*b+a*f*x-b*e*x)+1/a/b^2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/(a*
b)^(1/2)*arctan(b*x/(a*b)^(1/2))-c/a/x
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.51

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^2 (a + bx^2)} \, dx = \left[\frac{2 a^2b^2fx^4 - 6 ab^3c + 3 (b^3c - ab^2d + a^2be - a^3f)\sqrt{-ab}x \log\left(\frac{bx^2 - 2\sqrt{-ab}x - a}{bx^2 + a}\right) + 6 (a^2b^2e - a^3bf)x^2}{6 a^2b^3x}, \frac{a^2b^2f x}{} \right.$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a^2*b^2*f*x^4 - 6*a*b^3*c + 3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*s
qrt(-a*b)*x*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(a^2*b^2*e -
a^3*b*f)*x^2)/(a^2*b^3*x), 1/3*(a^2*b^2*f*x^4 - 3*a*b^3*c - 3*(b^3*c - a*b^
2*d + a^2*b*e - a^3*f)*sqrt(a*b)*x*arctan(sqrt(a*b)*x/a) + 3*(a^2*b^2*e - a
^3*b*f)*x^2)/(a^2*b^3*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(71) = 142.

Time = 0.51 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.79

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^2 (a + bx^2)} \, dx = x\left(-\frac{af}{b^2} + \frac{e}{b}\right) - \frac{\sqrt{-\frac{1}{a^3b^5}}(a^3f - a^2be + ab^2d - b^3c)\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{a^3b^5}}(a^3f - a^2be + ab^2d - b^3c)\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}} + x\right)}{2} + \frac{fx^3}{3b} - \frac{c}{ax}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a),x)
```

```
[Out] x*(-a*f/b**2 + e/b) - sqrt(-1/(a**3*b**5))*(a**3*f - a**2*b*e + a*b**2*d -
b**3*c)*log(-a**2*b**2*sqrt(-1/(a**3*b**5)) + x)/2 + sqrt(-1/(a**3*b**5))*(
a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a**2*b**2*sqrt(-1/(a**3*b**5)) +
 x)/2 + f*x**3/(3*b) - c/(a*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)}\,dx = \frac{bfx^3+3(be-af)x}{3b^2} - \frac{c}{ax} - \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}ab^2}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*(b*f*x^3 + 3*(b*e - a*f)*x)/b^2 - c/(a*x) - (b^3*c - a*b^2*d + a^2*b*e
- a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)}\,dx = -\frac{c}{ax} - \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}ab^2} + \frac{b^2fx^3+3b^2ex-3abfx}{3b^3}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -c/(a*x) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(
a*b)*a*b^2) + 1/3*(b^2*f*x^3 + 3*b^2*e*x - 3*a*b*f*x)/b^3
```

Mupad [B] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)}\,dx = x\left(\frac{e}{b}-\frac{a\,f}{b^2}\right) - \frac{c}{a\,x} + \frac{f\,x^3}{3\,b} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^{3/2}\,b^{5/2}}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)),x)
```

```
[Out] x*(e/b - (a*f)/b^2) - c/(a*x) + (f*x^3)/(3*b) - (atan((b^(1/2)*x)/a^(1/2))*
(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a^(3/2)*b^(5/2))
```

3.119 $\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)}\,dx$

Optimal result 716
Rubi [A] (verified) 716
Mathematica [A] (verified) 717
Maple [A] (verified) 717
Fricas [A] (verification not implemented) 718
Sympy [B] (verification not implemented) 718
Maxima [A] (verification not implemented) 719
Giac [A] (verification not implemented) 719
Mupad [B] (verification not implemented) 719

Optimal result

Integrand size = 30, antiderivative size = 82

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)}\,dx = -\frac{c}{3ax^3}+\frac{bc-ad}{a^2x}+\frac{fx}{b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}$$

```
[Out] -1/3*c/a/x^3+(-a*d+b*c)/a^2/x+f*x/b+(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(x
*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1816, 211}

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)}\,dx = \frac{bc-ad}{a^2x}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{a^{5/2}b^{3/2}}-\frac{c}{3ax^3}+\frac{fx}{b}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)),x]
```

```
[Out] -1/3*c/(a*x^3) + (b*c - a*d)/(a^2*x) + (f*x)/b + ((b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{f}{b}+\frac{c}{ax^4}+\frac{-bc+ad}{a^2x^2}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^2b\,(a+bx^2)}\right)dx \\
&= -\frac{c}{3ax^3}+\frac{bc-ad}{a^2x}+\frac{fx}{b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\int\frac{1}{a+bx^2}\,dx}{a^2b} \\
&= -\frac{c}{3ax^3}+\frac{bc-ad}{a^2x}+\frac{fx}{b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\,(a+bx^2)}\,dx = -\frac{c}{3ax^3}+\frac{bc-ad}{a^2x}+\frac{fx}{b} - \frac{(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)),x]
[Out] -1/3*c/(a*x^3) + (b*c - a*d)/(a^2*x) + (f*x)/b - ((-(b^3*c) + a*b^2*d - a^2
*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

method	result
default	$\frac{fx}{b}-\frac{c}{3a\,x^3}-\frac{ad-bc}{a^2x}+\frac{(-f\,a^3+a^2be-a\,b^2d+b^3c)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^2b\sqrt{ab}}$
risch	$\frac{fx}{b}+\frac{-\frac{(ad-bc)b\,x^2}{a^2}-\frac{cb}{3a}}{b\,x^3}-\frac{a\ln(-\sqrt{-ab}\,x-a)f}{2b\sqrt{-ab}}+\frac{\ln(-\sqrt{-ab}\,x-a)e}{2\sqrt{-ab}}-\frac{b\ln(-\sqrt{-ab}\,x-a)d}{2\sqrt{-ab}\,a}+\frac{b^2\ln(-\sqrt{-ab}\,x-a)c}{2\sqrt{-ab}\,a^2}+\frac{a\ln(-$

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] f*x/b-1/3*c/a/x^3-(a*d-b*c)/a^2/x+1/a^2/b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/(a
*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.63

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)}\,dx$$
$$=\left[\frac{6\,a^3bfx^4+3\,(b^3c-ab^2d+a^2be-a^3f)\sqrt{-ab}x^3\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right)-2\,a^2b^2c+6\,(ab^3c-a^2b^2d)x^2}{6\,a^3b^2x^3},\ \frac{3\,a^3b}{}\right.$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(6*a^3*b*f*x^4 + 3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(-a*b)*x^3*
log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*a^2*b^2*c + 6*(a*b^3*c -
a^2*b^2*d)*x^2)/(a^3*b^2*x^3), 1/3*(3*a^3*b*f*x^4 + 3*(b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*sqrt(a*b)*x^3*arctan(sqrt(a*b)*x/a) - a^2*b^2*c + 3*(a*b^3*c
 - a^2*b^2*d)*x^2)/(a^3*b^2*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(71) = 142.

Time = 0.99 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.84

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)}\,dx=\frac{\sqrt{-\frac{1}{a^5b^3}}(a^3f-a^2be+ab^2d-b^3c)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{2}-\frac{\sqrt{-\frac{1}{a^5b^3}}(a^3f-a^2be+ab^2d-b^3c)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{2}+\frac{fx}{b}+\frac{-ac+x^2(-3ad+3bc)}{3a^2x^3}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a**5*b**3))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**3*b*sq
rt(-1/(a**5*b**3)) + x)/2 - sqrt(-1/(a**5*b**3))*(a**3*f - a**2*b*e + a*b**
2*d - b**3*c)*log(a**3*b*sqrt(-1/(a**5*b**3)) + x)/2 + f*x/b + (-a*c + x**2
*(-3*a*d + 3*b*c))/(3*a**2*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)}\,dx = \frac{fx}{b} + \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2b} + \frac{3\,(bc-ad)x^2-ac}{3\,a^2x^3}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] f*x/b + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a^2*b) + 1/3*(3*(b*c - a*d)*x^2 - a*c)/(a^2*x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)}\,dx = \frac{fx}{b} + \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2b} + \frac{3\,bcx^2-3\,adx^2-ac}{3\,a^2x^3}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] f*x/b + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a^2*b) + 1/3*(3*b*c*x^2 - 3*a*d*x^2 - a*c)/(a^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)}\,dx = \frac{f\,x}{b} - \frac{\frac{b\,c}{3\,a}+\frac{b\,x^2\,(a\,d-b\,c)}{a^2}}{b\,x^3} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^{5/2}\,b^{3/2}}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)),x)
```

```
[Out] (f*x)/b - ((b*c)/(3*a) + (b*x^2*(a*d - b*c))/a^2)/(b*x^3) + (atan((b^(1/2)*
x)/a^(1/2))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a^(5/2)*b^(3/2))
```

3.120 $\int \frac{c+dx^2+ex^4+fx^6}{x^6(a+bx^2)}\,dx$

Optimal result 720
Rubi [A] (verified) 720
Mathematica [A] (verified) 721
Maple [A] (verified) 721
Fricas [A] (verification not implemented) 722
Sympy [A] (verification not implemented) 722
Maxima [A] (verification not implemented) 723
Giac [A] (verification not implemented) 723
Mupad [B] (verification not implemented) 724

Optimal result

Integrand size = 30, antiderivative size = 104

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6(a+bx^2)}\,dx = -\frac{c}{5ax^5}+\frac{bc-ad}{3a^2x^3}-\frac{b^2c-abd+a^2e}{a^3x} - \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}\sqrt{b}}$$

```
[Out] -1/5*c/a/x^5+1/3*(-a*d+b*c)/a^2/x^3+(-a^2*e+a*b*d-b^2*c)/a^3/x-(-a^3*f+a^2*
b*e-a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1816, 211}

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6(a+bx^2)}\,dx = \frac{bc-ad}{3a^2x^3}-\frac{a^2e-abd+b^2c}{a^3x} - \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{a^{7/2}\sqrt{b}}-\frac{c}{5ax^5}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)),x]
```

```
[Out] -1/5*c/(a*x^5) + (b*c - a*d)/(3*a^2*x^3) - (b^2*c - a*b*d + a^2*e)/(a^3*x)
- ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7/2
)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{c}{ax^6} + \frac{-bc+ad}{a^2x^4} + \frac{b^2c-abd+a^2e}{a^3x^2} + \frac{-b^3c+ab^2d-a^2be+a^3f}{a^3(a+bx^2)}\right) dx \\ &= -\frac{c}{5ax^5} + \frac{bc-ad}{3a^2x^3} - \frac{b^2c-abd+a^2e}{a^3x} + \frac{(-b^3c+ab^2d-a^2be+a^3f)\int \frac{1}{a+bx^2}\, dx}{a^3} \\ &= -\frac{c}{5ax^5} + \frac{bc-ad}{3a^2x^3} - \frac{b^2c-abd+a^2e}{a^3x} - \frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6(a+bx^2)}\, dx = -\frac{c}{5ax^5} + \frac{bc-ad}{3a^2x^3} + \frac{-b^2c+abd-a^2e}{a^3x} + \frac{(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}\sqrt{b}}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)),x]
[Out] -1/5*c/(a*x^5) + (b*c - a*d)/(3*a^2*x^3) + (-(b^2*c) + a*b*d - a^2*e)/(a^3*
x) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
a^(7/2)*Sqrt[b])
```

Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

method	result
default	$-\frac{c}{5a\,x^{5}}-\frac{ad-bc}{3a^{2}x^{3}}-\frac{a^{2}e-abd+b^{2}c}{a^{3}x}+\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^{3}\sqrt{ab}}$
risch	$\frac{-\frac{\left(a^{2}e-abd+b^{2}c\right)x^{4}}{a^{3}}-\frac{(ad-bc)x^{2}}{3a^{2}}-\frac{c}{5a}}{x^{5}}-\frac{\ln\left(-\sqrt{-ab}\,x+a\right)f}{2\sqrt{-ab}}+\frac{\ln\left(-\sqrt{-ab}\,x+a\right)be}{2\sqrt{-ab}\,a}-\frac{\ln\left(-\sqrt{-ab}\,x+a\right)b^{2}d}{2\sqrt{-ab}\,a^{2}}+\frac{\ln\left(-\sqrt{-ab}\,x+a\right)b^{3}c}{2\sqrt{-ab}\,a^{3}}$

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] -1/5*c/a/x^5-1/3*(a*d-b*c)/a^2/x^3-(a^2*e-a*b*d+b^2*c)/a^3/x+(a^3*f-a^2*b*e
+a*b^2*d-b^3*c)/a^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.37

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\,(a+bx^2)}\,dx$$
$$=\left[\frac{15\,(b^3c-ab^2d+a^2be-a^3f)\sqrt{-ab}x^5\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)-6\,a^3bc-30\,(ab^3c-a^2b^2d+a^3be)x^4+10\,(a^2b}{30\,a^4bx^5}\right.$$
$$-\frac{15\,(b^3c-ab^2d+a^2be-a^3f)\sqrt{ab}x^5\arctan\left(\frac{\sqrt{ab}x}{a}\right)+3\,a^3bc+15\,(ab^3c-a^2b^2d+a^3be)x^4-5\,(a^2b^2c-a^3}{15\,a^4bx^5}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a),x, algorithm="fricas")
[Out] [1/30*(15*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(-a*b)*x^5*log((b*x^2 - 2
*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 6*a^3*b*c - 30*(a*b^3*c - a^2*b^2*d + a^3
*b*e)*x^4 + 10*(a^2*b^2*c - a^3*b*d)*x^2)/(a^4*b*x^5), -1/15*(15*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f)*sqrt(a*b)*x^5*arctan(sqrt(a*b)*x/a) + 3*a^3*b*c +
 15*(a*b^3*c - a^2*b^2*d + a^3*b*e)*x^4 - 5*(a^2*b^2*c - a^3*b*d)*x^2)/(a^4
*b*x^5)]
```

Sympy [A] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.61

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\,(a+bx^2)}\,dx=-\frac{\sqrt{-\frac{1}{a^7b}}(a^3f-a^2be+ab^2d-b^3c)\log\left(-a^4\sqrt{-\frac{1}{a^7b}}+x\right)}{2}$$
$$+\frac{\sqrt{-\frac{1}{a^7b}}(a^3f-a^2be+ab^2d-b^3c)\log\left(a^4\sqrt{-\frac{1}{a^7b}}+x\right)}{2}$$
$$+\frac{-3a^2c+x^4(-15a^2e+15abd-15b^2c)+x^2(-5a^2d+5abc)}{15a^3x^5}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a**7*b))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**4*sqrt(-
1/(a**7*b)) + x)/2 + sqrt(-1/(a**7*b))*(a**3*f - a**2*b*e + a*b**2*d - b**3
*c)*log(a**4*sqrt(-1/(a**7*b)) + x)/2 + (-3*a**2*c + x**4*(-15*a**2*e + 15*
a*b*d - 15*b**2*c) + x**2*(-5*a**2*d + 5*a*b*c))/(15*a**3*x**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{15\left(b^2c-abd+a^2e\right)x^4+3\,a^2c-5\left(abc-a^2d\right)x^2}{15\,a^3x^5}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
- 1/15*(15*(b^2*c - a*b*d + a^2*e)*x^4 + 3*a^2*c - 5*(a*b*c - a^2*d)*x^2)/(
a^3*x^5)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{15\,b^2cx^4-15\,abdx^4+15\,a^2ex^4-5\,abcx^2+5\,a^2dx^2+3\,a^2c}{15\,a^3x^5}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
- 1/15*(15*b^2*c*x^4 - 15*a*b*d*x^4 + 15*a^2*e*x^4 - 5*a*b*c*x^2 + 5*a^2*d*
x^2 + 3*a^2*c)/(a^3*x^5)
```

Mupad [B] (verification not implemented)

Time = 5.99 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\,(a+bx^2)}\,dx=-\frac{\frac{c}{5\,a}+\frac{x^2\,(a\,d-b\,c)}{3\,a^2}+\frac{x^4\,(e\,a^2-d\,a\,b+c\,b^2)}{a^3}}{x^5}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)}{a^{7/2}\,\sqrt{b}}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)),x)
```

```
[Out] - (c/(5*a) + (x^2*(a*d - b*c))/(3*a^2) + (x^4*(b^2*c + a^2*e - a*b*d))/a^3)
/x^5 - (atan((b^(1/2)*x)/a^(1/2))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a^(
7/2)*b^(1/2))
```

3.121 $\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)}\,dx$

Optimal result . . . 725
Rubi [A] (verified) . . . 725
Mathematica [A] (verified) . . . 726
Maple [A] (verified) . . . 727
Fricas [A] (verification not implemented) . . . 727
Sympy [B] (verification not implemented) . . . 728
Maxima [A] (verification not implemented) . . . 728
Giac [A] (verification not implemented) . . . 729
Mupad [B] (verification not implemented) . . . 729

Optimal result

Integrand size = 30, antiderivative size = 137

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)}\,dx = -\frac{c}{7ax^7}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{3a^3x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x}+\frac{\sqrt{b}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{9/2}}$$

```
[Out] -1/7*c/a/x^7+1/5*(-a*d+b*c)/a^2/x^5+1/3*(-a^2*e+a*b*d-b^2*c)/a^3/x^3+(-a^3*
f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x+(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(x*b^(1
/2)/a^(1/2))*b^(1/2)/a^(9/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1816, 211}

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)}\,dx = \frac{bc-ad}{5a^2x^5}-\frac{a^2e-abd+b^2c}{3a^3x^3}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{a^{9/2}}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{a^4x}-\frac{c}{7ax^7}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)),x]
```

```
[Out] -1/7*c/(a*x^7) + (b*c - a*d)/(5*a^2*x^5) - (b^2*c - a*b*d + a^2*e)/(3*a^3*x
^3) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) + (Sqrt[b]*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c}{ax^8}+\frac{-bc+ad}{a^2x^6}+\frac{b^2c-abd+a^2e}{a^3x^4}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^2} - \frac{b(-b^3c+ab^2d-a^2be+a^3f)}{a^4\,(a+bx^2)}\right)dx \\
&= -\frac{c}{7ax^7}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{3a^3x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x} + \frac{(b(b^3c-ab^2d+a^2be-a^3f))\int\frac{1}{a+bx^2}\,dx}{a^4} \\
&= -\frac{c}{7ax^7}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{3a^3x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x} + \frac{\sqrt{b}(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{9/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\,(a+bx^2)}\,dx = -\frac{c}{7ax^7}+\frac{bc-ad}{5a^2x^5}+\frac{-b^2c+abd-a^2e}{3a^3x^3} + \frac{b^3c-ab^2d+a^2be-a^3f}{a^4x} - \frac{\sqrt{b}(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{9/2}}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)),x]
[Out] -1/7*c/(a*x^7) + (b*c - a*d)/(5*a^2*x^5) + (-(b^2*c) + a*b*d - a^2*e)/(3*a^
3*x^3) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) - (Sqrt[b]*(-(b^3*c) +
 a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.94

method	result
default	$-\frac{c}{7a\,x^{7}}-\frac{ad-bc}{5a^{2}x^{5}}-\frac{a^{2}e-abd+b^{2}c}{3a^{3}x^{3}}-\frac{f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c}{a^{4}x}-\frac{b\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^{4}\sqrt{ab}}$
risch	$\frac{-\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)x^{6}}{a^{4}}-\frac{\left(a^{2}e-abd+b^{2}c\right)x^{4}}{3a^{3}}-\frac{(ad-bc)x^{2}}{5a^{2}}-\frac{c}{7a}}{x^{7}}+\left(\sum_{\_R=\text{RootOf}\left(a^{9}\_Z^{2}+a^{6}b\,f^{2}-2a^{5}b^{2}ef+2a^{4}b^{3}df+a^{4}b^{3}e^{2}-2a^{3}b^{4}c\right.}\right.$

[In] int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a),x,method=_RETURNVERBOSE)

[Out] -1/7*c/a/x^7-1/5*(a*d-b*c)/a^2/x^5-1/3*(a^2*e-a*b*d+b^2*c)/a^3/x^3-(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/x-b*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.13

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)}\,dx$$

$$=\left[-\frac{105\left(b^3c-ab^2d+a^2be-a^3f\right)x^7\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-210\left(b^3c-ab^2d+a^2be-a^3f\right)x^6+70\,(a}{210\,a^4x^7}\right.$$

[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a),x, algorithm="fricas")

[Out] [-1/210*(105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^7*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 210*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^6 + 70*(a*b^2*c - a^2*b*d + a^3*e)*x^4 + 30*a^3*c - 42*(a^2*b*c - a^3*d)*x^2)/(a^4*x^7), 1/105*(105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^7*sqrt(b/a)*arctan(x*sqrt(b/a)) + 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^6 - 35*(a*b^2*c - a^2*b*d + a^3*e)*x^4 - 15*a^3*c + 21*(a^2*b*c - a^3*d)*x^2)/(a^4*x^7)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(128) = 256.

Time = 5.86 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.20

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)}\,dx = \frac{\sqrt{-\frac{b}{a^9}}(a^3f-a^2be+ab^2d-b^3c)\log\left(-\frac{a^5\sqrt{-\frac{b}{a^9}}(a^3f-a^2be+ab^2d-b^3c)}{a^3bf-a^2b^2e+ab^3d-b^4c}+x\right)}{2} - \frac{\sqrt{-\frac{b}{a^9}}(a^3f-a^2be+ab^2d-b^3c)\log\left(\frac{a^5\sqrt{-\frac{b}{a^9}}(a^3f-a^2be+ab^2d-b^3c)}{a^3bf-a^2b^2e+ab^3d-b^4c}+x\right)}{2} + \frac{-15a^3c+x^6(-105a^3f+105a^2be-105ab^2d+105b^3c)+x^4(-35a^3e+35a^2bd-35ab^2c)+x^2(-21a^3d+}{105a^4x^7}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a),x)
```

```
[Out] sqrt(-b/a**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**5*sqrt(-b/a**
9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b*f - a**2*b**2*e + a*b**3
*d - b**4*c) + x)/2 - sqrt(-b/a**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)
*log(a**5*sqrt(-b/a**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b*f -
 a**2*b**2*e + a*b**3*d - b**4*c) + x)/2 + (-15*a**3*c + x**6*(-105*a**3*f
+ 105*a**2*b*e - 105*a*b**2*d + 105*b**3*c) + x**4*(-35*a**3*e + 35*a**2*b*
d - 35*a*b**2*c) + x**2*(-21*a**3*d + 21*a**2*b*c))/(105*a**4*x**7)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)}\,dx = \frac{(b^4c-ab^3d+a^2b^2e-a^3bf)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} + \frac{105\,(b^3c-ab^2d+a^2be-a^3f)x^6-35\,(ab^2c-a^2bd+a^3e)x^4-15\,a^3c+21\,(a^2bc-a^3d)x^2}{105\,a^4x^7}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^
4) + 1/105*(105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^6 - 35*(a*b^2*c - a^2
*b*d + a^3*e)*x^4 - 15*a^3*c + 21*(a^2*b*c - a^3*d)*x^2)/(a^4*x^7)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.08

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)}\,dx = \frac{(b^4c-ab^3d+a^2b^2e-a^3bf)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} + \frac{105\,b^3cx^6-105\,ab^2dx^6+105\,a^2bex^6-105\,a^3fx^6-35\,ab^2cx^4+35\,a^2bdx^4-35\,a^3ex^4+21\,a^2bcx^2-21}{105\,a^4x^7}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^
4) + 1/105*(105*b^3*c*x^6 - 105*a*b^2*d*x^6 + 105*a^2*b*e*x^6 - 105*a^3*f*x
^6 - 35*a*b^2*c*x^4 + 35*a^2*b*d*x^4 - 35*a^3*e*x^4 + 21*a^2*b*c*x^2 - 21*a
^3*d*x^2 - 15*a^3*c)/(a^4*x^7)
```

Mupad [B] (verification not implemented)

Time = 6.02 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)}\,dx = \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^{9/2}} - \frac{\frac{c}{7\,a}-\frac{x^6\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^4}+\frac{x^2\,\left(a\,d-b\,c\right)}{5\,a^2}+\frac{x^4\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{3\,a^3}}{x^7}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)),x)
```

```
[Out] (b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^(
9/2) - (c/(7*a) - (x^6*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^4 + (x^2*(a*d
 - b*c))/(5*a^2) + (x^4*(b^2*c + a^2*e - a*b*d))/(3*a^3))/x^7
```

3.122 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)}\,dx$

Optimal result 730
Rubi [A] (verified) 730
Mathematica [A] (verified) 731
Maple [A] (verified) 732
Fricas [A] (verification not implemented) 732
Sympy [B] (verification not implemented) 733
Maxima [A] (verification not implemented) 733
Giac [A] (verification not implemented) 734
Mupad [B] (verification not implemented) 734

Optimal result

Integrand size = 30, antiderivative size = 175

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)}\,dx = -\frac{c}{9ax^9}+\frac{bc-ad}{7a^2x^7}-\frac{b^2c-abd+a^2e}{5a^3x^5}+\frac{b^3c-ab^2d+a^2be-a^3f}{3a^4x^3}-\frac{b(b^3c-ab^2d+a^2be-a^3f)}{a^5x}-\frac{b^{3/2}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{11/2}}$$

```
[Out] -1/9*c/a/x^9+1/7*(-a*d+b*c)/a^2/x^7+1/5*(-a^2*e+a*b*d-b^2*c)/a^3/x^5+1/3*(-
a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^3-b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^5/x
-b^(3/2)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(11/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1816, 211}

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)}\,dx = \frac{bc-ad}{7a^2x^7}-\frac{a^2e-abd+b^2c}{5a^3x^5}-\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{a^{11/2}}-\frac{b(a^3(-f)+a^2be-ab^2d+b^3c)}{a^5x}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3a^4x^3}-\frac{c}{9ax^9}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)),x]
```

```
[Out] -1/9*c/(a*x^9) + (b*c - a*d)/(7*a^2*x^7) - (b^2*c - a*b*d + a^2*e)/(5*a^3*x
^5) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*x^3) - (b*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f))/(a^5*x) - (b^(3/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(11/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{c}{ax^{10}}+\frac{-bc+ad}{a^2x^8}+\frac{b^2c-abd+a^2e}{a^3x^6}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^4}\right.\\
&\qquad\left.-\frac{b(-b^3c+ab^2d-a^2be+a^3f)}{a^5x^2}+\frac{b^2(-b^3c+ab^2d-a^2be+a^3f)}{a^5(a+bx^2)}\right)dx\\
&= -\frac{c}{9ax^9}+\frac{bc-ad}{7a^2x^7}-\frac{b^2c-abd+a^2e}{5a^3x^5}+\frac{b^3c-ab^2d+a^2be-a^3f}{3a^4x^3}\\
&\quad-\frac{b(b^3c-ab^2d+a^2be-a^3f)}{a^5x}-\frac{(b^2(b^3c-ab^2d+a^2be-a^3f))\int\frac{1}{a+bx^2}\,dx}{a^5}\\
&= -\frac{c}{9ax^9}+\frac{bc-ad}{7a^2x^7}-\frac{b^2c-abd+a^2e}{5a^3x^5}+\frac{b^3c-ab^2d+a^2be-a^3f}{3a^4x^3}\\
&\quad-\frac{b(b^3c-ab^2d+a^2be-a^3f)}{a^5x}-\frac{b^{3/2}(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{11/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.99

$$\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)}\,dx &= -\frac{c}{9ax^9}+\frac{bc-ad}{7a^2x^7}+\frac{-b^2c+abd-a^2e}{5a^3x^5}\\
&\quad+\frac{b^3c-ab^2d+a^2be-a^3f}{3a^4x^3}+\frac{b(-b^3c+ab^2d-a^2be+a^3f)}{a^5x}\\
&\quad+\frac{b^{3/2}(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{11/2}}
\end{aligned}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)),x]
```

```
[Out] -1/9*c/(a*x^9) + (b*c - a*d)/(7*a^2*x^7) + (-(b^2*c) + a*b*d - a^2*e)/(5*a^
3*x^5) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*x^3) + (b*(-(b^3*c) + a
*b^2*d - a^2*b*e + a^3*f))/(a^5*x) + (b^(3/2)*(-(b^3*c) + a*b^2*d - a^2*b*e
 + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(11/2)
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.93

method	result
default	$-\frac{c}{9a\,x^9}-\frac{ad-bc}{7a^2x^7}-\frac{a^2e-abd+b^2c}{5a^3x^5}-\frac{f\,a^3-a^2be+a\,b^2d-b^3c}{3a^4x^3}+\frac{b(f\,a^3-a^2be+a\,b^2d-b^3c)}{a^5x}+\frac{b^2(f\,a^3-a^2be+a\,b^2d-b^3c)\arctan(}{a^5\sqrt{ab}}$
risch	$\frac{\frac{b\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)x^8}{a^5}-\frac{\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)x^6}{3a^4}-\frac{\left(a^2e-abd+b^2c\right)x^4}{5a^3}-\frac{(ad-bc)x^2}{7a^2}-\frac{c}{9a}}{x^9}+\frac{\sqrt{-ab}\,b\ln\left(-bx-\sqrt{-ab}\right)f}{2a^3}-\frac{\sqrt{-ab}\,b^2}{}$

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*c/a/x^9-1/7*(a*d-b*c)/a^2/x^7-1/5*(a^2*e-a*b*d+b^2*c)/a^3/x^5-1/3*(a^3
*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/x^3+b*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5/x+b^2
*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.14

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)}\,dx$$

$$= \left[-\frac{315\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^9\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+630\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^8-21}{630\,a^5x^9}\right.$$

$$-\frac{315\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^9\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+315\left(b^4c-ab^3d+a^2b^2e-a^3bf\right)x^8-105\left(ab^3c-\right.}{315\,a^5x^9}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/630*(315*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^9*sqrt(-b/a)*log((b*
x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 630*(b^4*c - a*b^3*d + a^2*b^2*e
 - a^3*b*f)*x^8 - 210*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^6 + 70*a^4*
```

```
c + 126*(a^2*b^2*c - a^3*b*d + a^4*e)*x^4 - 90*(a^3*b*c - a^4*d)*x^2)/(a^5*
x^9), -1/315*(315*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^9*sqrt(b/a)*arc
tan(x*sqrt(b/a)) + 315*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^8 - 105*(a
*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^6 + 35*a^4*c + 63*(a^2*b^2*c - a^3*
b*d + a^4*e)*x^4 - 45*(a^3*b*c - a^4*d)*x^2)/(a^5*x^9)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(167) = 334$.

Time = 24.35 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.02

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^{10}(a + bx^2)}\,dx = -\frac{\sqrt{-\frac{b^3}{a^{11}}}(a^3f - a^2be + ab^2d - b^3c)\log\left(-\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}(a^3f-a^2be+ab^2d-b^3c)}{a^3b^2f-a^2b^3e+ab^4d-b^5c} + x\right)}{2} + \frac{\sqrt{-\frac{b^3}{a^{11}}}(a^3f - a^2be + ab^2d - b^3c)\log\left(\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}(a^3f-a^2be+ab^2d-b^3c)}{a^3b^2f-a^2b^3e+ab^4d-b^5c} + x\right)}{2} + \frac{-35a^4c + x^8\cdot(315a^3bf - 315a^2b^2e + 315ab^3d - 315b^4c) + x^6(-105a^4f + 105a^3be - 105a^2b^2d + 105ab}{315a^5x^9}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a),x)
```

```
[Out] -sqrt(-b**3/a**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**6*sqrt(-
b**3/a**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b**2*f - a**2*b**
3*e + a*b**4*d - b**5*c) + x)/2 + sqrt(-b**3/a**11)*(a**3*f - a**2*b*e + a*
b**2*d - b**3*c)*log(a**6*sqrt(-b**3/a**11)*(a**3*f - a**2*b*e + a*b**2*d -
 b**3*c)/(a**3*b**2*f - a**2*b**3*e + a*b**4*d - b**5*c) + x)/2 + (-35*a**4
*c + x**8*(315*a**3*b*f - 315*a**2*b**2*e + 315*a*b**3*d - 315*b**4*c) + x*
*6*(-105*a**4*f + 105*a**3*b*e - 105*a**2*b**2*d + 105*a*b**3*c) + x**4*(-6
3*a**4*e + 63*a**3*b*d - 63*a**2*b**2*c) + x**2*(-45*a**4*d + 45*a**3*b*c))
/(315*a**5*x**9)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^{10}(a + bx^2)}\,dx = -\frac{(b^5c - ab^4d + a^2b^3e - a^3b^2f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^5} - \frac{315\,(b^4c - ab^3d + a^2b^2e - a^3bf)x^8 - 105\,(ab^3c - a^2b^2d + a^3be - a^4f)x^6 + 35\,a^4c + 63\,(a^2b^2c - a^3bd + a}{315\,a^5x^9}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a^5) - 1/315*(315*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^8 - 105*(a*b^3
*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^6 + 35*a^4*c + 63*(a^2*b^2*c - a^3*b*d
+ a^4*e)*x^4 - 45*(a^3*b*c - a^4*d)*x^2)/(a^5*x^9)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.13

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)}\,dx = -\frac{\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^5} - \frac{315\,b^4cx^8-315\,ab^3dx^8+315\,a^2b^2ex^8-315\,a^3bfx^8-105\,ab^3cx^6+105\,a^2b^2dx^6-105\,a^3bex^6+105\,a^4fx}{315\,a^5x^9}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a^5) - 1/315*(315*b^4*c*x^8 - 315*a*b^3*d*x^8 + 315*a^2*b^2*e*x^8 - 315*a^
3*b*f*x^8 - 105*a*b^3*c*x^6 + 105*a^2*b^2*d*x^6 - 105*a^3*b*e*x^6 + 105*a^4
*f*x^6 + 63*a^2*b^2*c*x^4 - 63*a^3*b*d*x^4 + 63*a^4*e*x^4 - 45*a^3*b*c*x^2
+ 45*a^4*d*x^2 + 35*a^4*c)/(a^5*x^9)
```

Mupad [B] (verification not implemented)

Time = 5.96 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)}\,dx = -\frac{\frac{c}{9\,a}-\frac{x^6\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^4}+\frac{x^2\,\left(a\,d-b\,c\right)}{7\,a^2}+\frac{x^4\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{5\,a^3}+\frac{b\,x^8\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^5}}{x^9} - \frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^{11/2}}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)),x)
```

```
[Out] - (c/(9*a) - (x^6*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^4) + (x^2*(a*d
- b*c))/(7*a^2) + (x^4*(b^2*c + a^2*e - a*b*d))/(5*a^3) + (b*x^8*(b^3*c - a
^3*f - a*b^2*d + a^2*b*e))/a^5)/x^9 - (b^(3/2)*atan((b^(1/2)*x)/a^(1/2))*(b
^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^(11/2)
```

3.123 $\int \frac{c+dx^2+ex^4+fx^6}{x^{12}(a+bx^2)}\,dx$

Optimal result 735
Rubi [A] (verified) 735
Mathematica [A] (verified) 737
Maple [A] (verified) 737
Fricas [A] (verification not implemented) 738
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Mupad [B] (verification not implemented) 740

Optimal result

Integrand size = 30, antiderivative size = 211

$$\begin{aligned}\int \frac{c+dx^2+ex^4+fx^6}{x^{12}\left(a+bx^2\right)}\,dx = &-\frac{c}{11ax^{11}}+\frac{bc-ad}{9a^2x^9}-\frac{b^2c-abd+a^2e}{7a^3x^7}\\ &+\frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5}-\frac{b(b^3c-ab^2d+a^2be-a^3f)}{3a^5x^3}\\ &+\frac{b^2(b^3c-ab^2d+a^2be-a^3f)}{a^6x}\\ &+\frac{b^{5/2}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{13/2}}\end{aligned}$$

```
[Out] -1/11*c/a/x^11+1/9*(-a*d+b*c)/a^2/x^9+1/7*(-a^2*e+a*b*d-b^2*c)/a^3/x^7+1/5*
(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^5-1/3*b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)
/a^5/x^3+b^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^6/x+b^(5/2)*(-a^3*f+a^2*b*e-a
*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(13/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used

$= \{1816, 211\}$

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{12}\left(a+bx^2\right)}\,dx = \frac{bc-ad}{9a^2x^9} - \frac{a^2e-abd+b^2c}{7a^3x^7} + \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{13/2}} + \frac{b^2(a^3(-f)+a^2be-ab^2d+b^3c)}{a^6x} - \frac{b(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^5x^3} + \frac{a^3(-f)+a^2be-ab^2d+b^3c}{5a^4x^5} - \frac{c}{11ax^{11}}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^12*(a + b*x^2)),x]
```

```
[Out] -1/11*c/(a*x^11) + (b*c - a*d)/(9*a^2*x^9) - (b^2*c - a*b*d + a^2*e)/(7*a^3
*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) - (b*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f))/(3*a^5*x^3) + (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*
f))/(a^6*x) + (b^(5/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/a^(13/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \int \left(\frac{c}{ax^{12}} + \frac{-bc+ad}{a^2x^{10}} + \frac{b^2c-abd+a^2e}{a^3x^8} + \frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^6} - \frac{b(-b^3c+ab^2d-a^2be+a^3f)}{a^5x^4} + \frac{b^2(-b^3c+ab^2d-a^2be+a^3f)}{a^6x^2} - \frac{b^3(-b^3c+ab^2d-a^2be+a^3f)}{a^6\left(a+bx^2\right)}\right) dx$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{9a^2x^9} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} - \frac{b(b^3c-ab^2d+a^2be-a^3f)}{3a^5x^3} + \frac{b^2(b^3c-ab^2d+a^2be-a^3f)}{a^6x} + \frac{(b^3(b^3c-ab^2d+a^2be-a^3f))\int\frac{1}{a+bx^2}\,dx}{a^6}$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{9a^2x^9} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} - \frac{b(b^3c-ab^2d+a^2be-a^3f)}{3a^5x^3} + \frac{b^2(b^3c-ab^2d+a^2be-a^3f)}{a^6x} + \frac{b^{5/2}(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{13/2}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{12}(a+bx^2)}\,dx = -\frac{c}{11ax^{11}} + \frac{bc-ad}{9a^2x^9} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} + \frac{b(-b^3c+ab^2d-a^2be+a^3f)}{3a^5x^3} + \frac{b^2(b^3c-ab^2d+a^2be-a^3f)}{a^6x} + \frac{b^{5/2}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{13/2}}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^12*(a + b*x^2)),x]
```

```
[Out] -1/11*c/(a*x^11) + (b*c - a*d)/(9*a^2*x^9) - (b^2*c - a*b*d + a^2*e)/(7*a^3
*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) + (b*(-(b^3*c) + a*
b^2*d - a^2*b*e + a^3*f))/(3*a^5*x^3) + (b^2*(b^3*c - a*b^2*d + a^2*b*e - a
^3*f))/(a^6*x) + (b^(5/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/a^(13/2)
```

Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.95

method	result
default	$-\frac{c}{11a\,x^{11}} - \frac{ad-bc}{9a^2x^9} - \frac{a^2e-abd+b^2c}{7a^3x^7} - \frac{f\,a^3-a^2be+a\,b^2d-b^3c}{5a^4x^5} - \frac{b^2\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)}{a^6x} + \frac{b\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)}{3a^5x^3} -$
risch	$\frac{-\frac{b^2\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)x^{10}}{a^6} + \frac{b\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)x^8}{3a^5} - \frac{\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)x^6}{5a^4} - \frac{\left(a^2e-abd+b^2c\right)x^4}{7a^3} - \frac{(ad-bc)x^2}{9a^2} - \frac{c}{11a}}{x^{11}} + \sqrt{-}$

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^12/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/11*c/a/x^11-1/9*(a*d-b*c)/a^2/x^9-1/7*(a^2*e-a*b*d+b^2*c)/a^3/x^7-1/5*(a
^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/x^5-b^2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^6/x
+1/3*b*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5/x^3-b^3*(a^3*f-a^2*b*e+a*b^2*d-b^3
*c)/a^6/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.17

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{12}(a+bx^2)}\,dx = \left[-\frac{3465\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^{11}\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-6930\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^{1}}{}\right.$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^12/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/6930*(3465*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^11*sqrt(-b/a)*lo
g((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 6930*(b^5*c - a*b^4*d + a^2
*b^3*e - a^3*b^2*f)*x^10 + 2310*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)
*x^8 - 1386*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x^6 + 630*a^5*c + 990
*(a^3*b^2*c - a^4*b*d + a^5*e)*x^4 - 770*(a^4*b*c - a^5*d)*x^2)/(a^6*x^11),
 1/3465*(3465*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^11*sqrt(b/a)*arct
an(x*sqrt(b/a)) + 3465*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^10 - 115
5*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^8 + 693*(a^2*b^3*c - a^3*b^
2*d + a^4*b*e - a^5*f)*x^6 - 315*a^5*c - 495*(a^3*b^2*c - a^4*b*d + a^5*e)*
x^4 + 385*(a^4*b*c - a^5*d)*x^2)/(a^6*x^11)]
```

Sympy [A] (verification not implemented)

Time = 38.92 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.89

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{12}(a+bx^2)}\,dx = \frac{\sqrt{-\frac{b^5}{a^{13}}}(a^3f-a^2be+ab^2d-b^3c)\log\left(-\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}(a^3f-a^2be+ab^2d-b^3c)}{a^3b^3f-a^2b^4e+ab^5d-b^6c}+x\right)}{2} - \frac{\sqrt{-\frac{b^5}{a^{13}}}(a^3f-a^2be+ab^2d-b^3c)\log\left(\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}(a^3f-a^2be+ab^2d-b^3c)}{a^3b^3f-a^2b^4e+ab^5d-b^6c}+x\right)}{2} + \frac{-315a^5c+x^{10}(-3465a^3b^2f+3465a^2b^3e-3465ab^4d+3465b^5c)+x^8\cdot(1155a^4bf-1155a^3b^2e+1155a^2}{}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**12/(b*x**2+a),x)
```

```
[Out] sqrt(-b**5/a**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**7*sqrt(-b
**5/a**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b**3*f - a**2*b**4
```

```
*e + a*b**5*d - b**6*c) + x)/2 - sqrt(-b**5/a**13)*(a**3*f - a**2*b*e + a*b
**2*d - b**3*c)*log(a**7*sqrt(-b**5/a**13)*(a**3*f - a**2*b*e + a*b**2*d -
b**3*c)/(a**3*b**3*f - a**2*b**4*e + a*b**5*d - b**6*c) + x)/2 + (-315*a**5
*c + x**10*(-3465*a**3*b**2*f + 3465*a**2*b**3*e - 3465*a*b**4*d + 3465*b**
5*c) + x**8*(1155*a**4*b*f - 1155*a**3*b**2*e + 1155*a**2*b**3*d - 1155*a*b
**4*c) + x**6*(-693*a**5*f + 693*a**4*b*e - 693*a**3*b**2*d + 693*a**2*b**3
*c) + x**4*(-495*a**5*e + 495*a**4*b*d - 495*a**3*b**2*c) + x**2*(-385*a**5
*d + 385*a**4*b*c))/(3465*a**6*x**11)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{12}(a+bx^2)}\,dx=\frac{(b^6c-ab^5d+a^2b^4e-a^3b^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^6}+\frac{3465\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^{10}-1155\,(ab^4c-a^2b^3d+a^3b^2e-a^4bf)x^8+693\,(a^2b^3c-a^3b^2d+a}{3465\,a^6x^{11}}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^12/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^6*c - a*b^5*d + a^2*b^4*e - a^3*b^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
a^6) + 1/3465*(3465*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^10 - 1155*(
a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^8 + 693*(a^2*b^3*c - a^3*b^2*d
 + a^4*b*e - a^5*f)*x^6 - 315*a^5*c - 495*(a^3*b^2*c - a^4*b*d + a^5*e)*x^4
 + 385*(a^4*b*c - a^5*d)*x^2)/(a^6*x^11)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.16

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{12}(a+bx^2)}\,dx=\frac{(b^6c-ab^5d+a^2b^4e-a^3b^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^6}+\frac{3465\,b^5cx^{10}-3465\,ab^4dx^{10}+3465\,a^2b^3ex^{10}-3465\,a^3b^2fx^{10}-1155\,ab^4cx^8+1155\,a^2b^3dx^8-1155\,a^3b^2}{}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^12/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^6*c - a*b^5*d + a^2*b^4*e - a^3*b^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
a^6) + 1/3465*(3465*b^5*c*x^10 - 3465*a*b^4*d*x^10 + 3465*a^2*b^3*e*x^10 -
3465*a^3*b^2*f*x^10 - 1155*a*b^4*c*x^8 + 1155*a^2*b^3*d*x^8 - 1155*a^3*b^2*
e*x^8 + 1155*a^4*b*f*x^8 + 693*a^2*b^3*c*x^6 - 693*a^3*b^2*d*x^6 + 693*a^4*
b*e*x^6 - 693*a^5*f*x^6 - 495*a^3*b^2*c*x^4 + 495*a^4*b*d*x^4 - 495*a^5*e*x
^4 + 385*a^4*b*c*x^2 - 385*a^5*d*x^2 - 315*a^5*c)/(a^6*x^11)
```

Mupad [B] (verification not implemented)

Time = 5.86 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{12}\,(a+bx^2)}\,dx = \frac{b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^{13/2}}$$
$$-\frac{\frac{c}{11\,a}-\frac{x^6\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{5\,a^4}+\frac{x^2\,\left(a\,d-b\,c\right)}{9\,a^2}+\frac{x^4\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{7\,a^3}+\frac{b\,x^8\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^5}-\frac{b^2\,x^{10}\,\left(-f\,a^3+e\,a^2\,b\right.}{a^6}}{x^{11}}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^12*(a + b*x^2)),x)
```

```
[Out] (b^(5/2)*atan((b^(1/2)*x)/a^(1/2))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^(
13/2) - (c/(11*a) - (x^6*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(5*a^4) + (x^
2*(a*d - b*c))/(9*a^2) + (x^4*(b^2*c + a^2*e - a*b*d))/(7*a^3) + (b*x^8*(b^
3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^5) - (b^2*x^10*(b^3*c - a^3*f - a*b^
2*d + a^2*b*e))/a^6)/x^11
```

3.124 $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$

Optimal result 741
Rubi [A] (verified) 742
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Optimal result

Integrand size = 30, antiderivative size = 240

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx = -\frac{a(5b^3c-7ab^2d+9a^2be-11a^3f)\,x}{2b^6} + \frac{(5b^3c-7ab^2d+9a^2be-11a^3f)\,x^3}{6b^5} - \frac{(5b^3c-7ab^2d+9a^2be-11a^3f)\,x^5}{10ab^4} + \frac{(be-2af)x^7}{7b^3} + \frac{fx^9}{9b^2} + \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^7}{2a\,(a+bx^2)} + \frac{a^{3/2}(5b^3c-7ab^2d+9a^2be-11a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{13/2}}$$

```
[Out] -1/2*a*(-11*a^3*f+9*a^2*b*e-7*a*b^2*d+5*b^3*c)*x/b^6+1/6*(-11*a^3*f+9*a^2*b
*e-7*a*b^2*d+5*b^3*c)*x^3/b^5-1/10*(-11*a^3*f+9*a^2*b*e-7*a*b^2*d+5*b^3*c)*
x^5/a/b^4+1/7*(-2*a*f+b*e)*x^7/b^3+1/9*f*x^9/b^2+1/2*(c-a*(a^2*f-a*b*e+b^2*
d)/b^3)*x^7/a/(b*x^2+a)+1/2*a^(3/2)*(-11*a^3*f+9*a^2*b*e-7*a*b^2*d+5*b^3*c)
*arctan(x*b^(1/2)/a^(1/2))/b^(13/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1818, 1599, 1275, 211}

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx = \frac{x^7\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a\,(a+bx^2)} - \frac{ax(-11a^3f+9a^2be-7ab^2d+5b^3c)}{2b^6} + \frac{x^3(-11a^3f+9a^2be-7ab^2d+5b^3c)}{6b^5} - \frac{x^5(-11a^3f+9a^2be-7ab^2d+5b^3c)}{10ab^4} + \frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(-11a^3f+9a^2be-7ab^2d+5b^3c)}{2b^{13/2}} + \frac{x^7(be-2af)}{7b^3} + \frac{fx^9}{9b^2}$$

```
[In] Int[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] -1/2*(a*(5*b^3*c - 7*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*x)/b^6 + ((5*b^3*c - 7
*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*x^3)/(6*b^5) - ((5*b^3*c - 7*a*b^2*d + 9*a
^2*b*e - 11*a^3*f)*x^5)/(10*a*b^4) + ((b*e - 2*a*f)*x^7)/(7*b^3) + (f*x^9)/
(9*b^2) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^7)/(2*a*(a + b*x^2)) + (
a^(3/2)*(5*b^3*c - 7*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(2*b^(13/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
```

```
Q[r - p]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^7}{2a\left(a+bx^2\right)}-\frac{\int \frac{x^5\left(\left(5bc-7ad+\frac{7a^2e}{b}-\frac{7a^3f}{b^2}\right)x-2a\left(e-\frac{af}{b}\right)x^3-2afx^5\right)}{a+bx^2}\,dx}{2ab}$$

$$= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^7}{2a\left(a+bx^2\right)}-\frac{\int \frac{x^6\left(5bc-7ad+\frac{7a^2e}{b}-\frac{7a^3f}{b^2}-2a\left(e-\frac{af}{b}\right)x^2-2afx^4\right)}{a+bx^2}\,dx}{2ab}$$

$$= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^7}{2a\left(a+bx^2\right)} - \frac{\int \left(\frac{a^2(5b^3c-7ab^2d+9a^2be-11a^3f)}{b^5}-\frac{a(5b^3c-7ab^2d+9a^2be-11a^3f)x^2}{b^4}+\frac{(5b^3c-7ab^2d+9a^2be-11a^3f)x^4}{b^3}-\frac{2a(be-2af)x^6}{b^2}\right.}{2ab}$$

$$= -\frac{a(5b^3c-7ab^2d+9a^2be-11a^3f)\,x}{2b^6}+\frac{(5b^3c-7ab^2d+9a^2be-11a^3f)\,x^3}{6b^5} - \frac{(5b^3c-7ab^2d+9a^2be-11a^3f)\,x^5}{10ab^4}+\frac{(be-2af)x^7}{7b^3}+\frac{fx^9}{9b^2} + \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^7}{2a\left(a+bx^2\right)}+\frac{\left(a^2(5b^3c-7ab^2d+9a^2be-11a^3f)\right)\int \frac{1}{a+bx^2}\,dx}{2b^6}$$

$$= -\frac{a(5b^3c-7ab^2d+9a^2be-11a^3f)\,x}{2b^6}+\frac{(5b^3c-7ab^2d+9a^2be-11a^3f)\,x^3}{6b^5} - \frac{(5b^3c-7ab^2d+9a^2be-11a^3f)\,x^5}{10ab^4}+\frac{(be-2af)x^7}{7b^3}+\frac{fx^9}{9b^2} + \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^7}{2a\left(a+bx^2\right)}+\frac{a^{3/2}(5b^3c-7ab^2d+9a^2be-11a^3f)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{13/2}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.95

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx = \frac{a(-2b^3c+3ab^2d-4a^2be+5a^3f)\,x}{b^6} + \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^3}{3b^5} + \frac{(b^2d-2abe+3a^2f)\,x^5}{5b^4} + \frac{(be-2af)x^7}{7b^3} + \frac{fx^9}{9b^2} - \frac{(a^2b^3c-a^3b^2d+a^4be-a^5f)\,x}{2b^6\,(a+bx^2)} - \frac{a^{3/2}(-5b^3c+7ab^2d-9a^2be+11a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{13/2}}$$

```
[In] Integrate[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] (a*(-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f)*x)/b^6 + ((b^3*c - 2*a*b^2*
d + 3*a^2*b*e - 4*a^3*f)*x^3)/(3*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*x^5)/(
5*b^4) + ((b*e - 2*a*f)*x^7)/(7*b^3) + (f*x^9)/(9*b^2) - ((a^2*b^3*c - a^3*
b^2*d + a^4*b*e - a^5*f)*x)/(2*b^6*(a + b*x^2)) - (a^(3/2)*(-5*b^3*c + 7*a*
b^2*d - 9*a^2*b*e + 11*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(13/2))
```

Maple [A] (verified)

Time = 3.59 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.95

method	result
default	$\frac{\frac{1}{9}f\,x^9b^4-\frac{2}{7}a\,b^3f\,x^7+\frac{1}{7}b^4e\,x^7+\frac{3}{5}a^2b^2f\,x^5-\frac{2}{5}a\,b^3e\,x^5+\frac{1}{5}b^4d\,x^5-\frac{4}{3}a^3bf\,x^3+a^2b^2e\,x^3-\frac{2}{3}a\,b^3d\,x^3+\frac{1}{3}b^4c\,x^3+5a^4fx-4a^3bex+3a^2b^2dx-}{b^6}$
risch	$\frac{f\,x^9}{9b^2}-\frac{2af\,x^7}{7b^3}+\frac{e\,x^7}{7b^2}+\frac{3a^2f\,x^5}{5b^4}-\frac{2ae\,x^5}{5b^3}+\frac{d\,x^5}{5b^2}-\frac{4a^3f\,x^3}{3b^5}+\frac{a^2e\,x^3}{b^4}-\frac{2ad\,x^3}{3b^3}+\frac{c\,x^3}{3b^2}+\frac{5a^4fx}{b^6}-\frac{4a^3ex}{b^5}+\frac{3a^2dx}{b^4}$

```
[In] int(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^6*(1/9*f*x^9*b^4-2/7*a*b^3*f*x^7+1/7*b^4*e*x^7+3/5*a^2*b^2*f*x^5-2/5*a*
b^3*e*x^5+1/5*b^4*d*x^5-4/3*a^3*b*f*x^3+a^2*b^2*e*x^3-2/3*a*b^3*d*x^3+1/3*b
^4*c*x^3+5*a^4*f*x-4*a^3*b*e*x+3*a^2*b^2*d*x-2*a*b^3*c*x)-a^2/b^6*((-1/2*f*
a^3+1/2*a^2*b*e-1/2*a*b^2*d+1/2*b^3*c)*x/(b*x^2+a)+1/2*(11*a^3*f-9*a^2*b*e+
7*a*b^2*d-5*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 572, normalized size of antiderivative = 2.38

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$$

$$= \left[\frac{140\,b^5fx^{11}+20\,(9\,b^5e-11\,ab^4f)x^9+36\,(7\,b^5d-9\,ab^4e+11\,a^2b^3f)x^7+84\,(5\,b^5c-7\,ab^4d+9\,a^2b^3e-1}{}\right.$$

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/1260*(140*b^5*f*x^11 + 20*(9*b^5*e - 11*a*b^4*f)*x^9 + 36*(7*b^5*d - 9*a
*b^4*e + 11*a^2*b^3*f)*x^7 + 84*(5*b^5*c - 7*a*b^4*d + 9*a^2*b^3*e - 11*a^3
*b^2*f)*x^5 - 420*(5*a*b^4*c - 7*a^2*b^3*d + 9*a^3*b^2*e - 11*a^4*b*f)*x^3
- 315*(5*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11*a^5*f + (5*a*b^4*c - 7*a^
2*b^3*d + 9*a^3*b^2*e - 11*a^4*b*f)*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt
(-a/b) - a)/(b*x^2 + a)) - 630*(5*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11*
a^5*f)*x)/(b^7*x^2 + a*b^6), 1/630*(70*b^5*f*x^11 + 10*(9*b^5*e - 11*a*b^4*
f)*x^9 + 18*(7*b^5*d - 9*a*b^4*e + 11*a^2*b^3*f)*x^7 + 42*(5*b^5*c - 7*a*b^
4*d + 9*a^2*b^3*e - 11*a^3*b^2*f)*x^5 - 210*(5*a*b^4*c - 7*a^2*b^3*d + 9*a^
3*b^2*e - 11*a^4*b*f)*x^3 + 315*(5*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11
*a^5*f + (5*a*b^4*c - 7*a^2*b^3*d + 9*a^3*b^2*e - 11*a^4*b*f)*x^2)*sqrt(a/b
)*arctan(b*x*sqrt(a/b)/a) - 315*(5*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11
*a^5*f)*x)/(b^7*x^2 + a*b^6)]
```

Sympy [A] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.85

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$$
$$= x^7\left(-\frac{2af}{7b^3}+\frac{e}{7b^2}\right)+x^5\cdot\left(\frac{3a^2f}{5b^4}-\frac{2ae}{5b^3}+\frac{d}{5b^2}\right)+x^3\left(-\frac{4a^3f}{3b^5}+\frac{a^2e}{b^4}-\frac{2ad}{3b^3}+\frac{c}{3b^2}\right)$$
$$+x\left(\frac{5a^4f}{b^6}-\frac{4a^3e}{b^5}+\frac{3a^2d}{b^4}-\frac{2ac}{b^3}\right)+\frac{x(a^5f-a^4be+a^3b^2d-a^2b^3c)}{2ab^6+2b^7x^2}$$
$$+\frac{\sqrt{-\frac{a^3}{b^{13}}}\cdot(11a^3f-9a^2be+7ab^2d-5b^3c)\log\left(-\frac{b^6\sqrt{-\frac{a^3}{b^{13}}}\cdot(11a^3f-9a^2be+7ab^2d-5b^3c)}{11a^4f-9a^3be+7a^2b^2d-5ab^3c}+x\right)}{4}$$
$$-\frac{\sqrt{-\frac{a^3}{b^{13}}}\cdot(11a^3f-9a^2be+7ab^2d-5b^3c)\log\left(\frac{b^6\sqrt{-\frac{a^3}{b^{13}}}\cdot(11a^3f-9a^2be+7ab^2d-5b^3c)}{11a^4f-9a^3be+7a^2b^2d-5ab^3c}+x\right)}{4}$$
$$+\frac{fx^9}{9b^2}$$

```
[In] integrate(x**6*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x**7*(-2*a*f/(7*b**3) + e/(7*b**2)) + x**5*(3*a**2*f/(5*b**4) - 2*a*e/(5*b*
*3) + d/(5*b**2)) + x**3*(-4*a**3*f/(3*b**5) + a**2*e/b**4 - 2*a*d/(3*b**3)
 + c/(3*b**2)) + x*(5*a**4*f/b**6 - 4*a**3*e/b**5 + 3*a**2*d/b**4 - 2*a*c/b
**3) + x*(a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c)/(2*a*b**6 + 2*b**7
*x**2) + sqrt(-a**3/b**13)*(11*a**3*f - 9*a**2*b*e + 7*a*b**2*d - 5*b**3*c)
*log(-b**6*sqrt(-a**3/b**13)*(11*a**3*f - 9*a**2*b*e + 7*a*b**2*d - 5*b**3*
c)/(11*a**4*f - 9*a**3*b*e + 7*a**2*b**2*d - 5*a*b**3*c) + x)/4 - sqrt(-a**
3/b**13)*(11*a**3*f - 9*a**2*b*e + 7*a*b**2*d - 5*b**3*c)*log(b**6*sqrt(-a*
*3/b**13)*(11*a**3*f - 9*a**2*b*e + 7*a*b**2*d - 5*b**3*c)/(11*a**4*f - 9*a
**3*b*e + 7*a**2*b**2*d - 5*a*b**3*c) + x)/4 + f*x**9/(9*b**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.95

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$$
$$= -\frac{(a^2b^3c-a^3b^2d+a^4be-a^5f)x}{2\,(b^7x^2+ab^6)}+\frac{(5\,a^2b^3c-7\,a^3b^2d+9\,a^4be-11\,a^5f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^6}$$
$$+\frac{35\,b^4fx^9+45\,(b^4e-2\,ab^3f)x^7+63\,(b^4d-2\,ab^3e+3\,a^2b^2f)x^5+105\,(b^4c-2\,ab^3d+3\,a^2b^2e-4\,a^3bf)x^3}{315\,b^6}$$

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x/(b^7*x^2 + a*b^6) + 1/2*(5
*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11*a^5*f)*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*b^6) + 1/315*(35*b^4*f*x^9 + 45*(b^4*e - 2*a*b^3*f)*x^7 + 63*(b^4*d
- 2*a*b^3*e + 3*a^2*b^2*f)*x^5 + 105*(b^4*c - 2*a*b^3*d + 3*a^2*b^2*e - 4*a
^3*b*f)*x^3 - 315*(2*a*b^3*c - 3*a^2*b^2*d + 4*a^3*b*e - 5*a^4*f)*x)/b^6
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.02

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx = \frac{(5\,a^2b^3c-7\,a^3b^2d+9\,a^4be-11\,a^5f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^6} - \frac{a^2b^3cx-a^3b^2dx+a^4bex-a^5fx}{2\,(bx^2+a)b^6} + \frac{35\,b^{16}fx^9+45\,b^{16}ex^7-90\,ab^{15}fx^7+63\,b^{16}dx^5-126\,ab^{15}ex^5+189\,a^2b^{14}fx^5+105\,b^{16}cx^3-210\,ab^{15}dx}{315\,b^{18}}$$

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11*a^5*f)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b^6) - 1/2*(a^2*b^3*c*x - a^3*b^2*d*x + a^4*b*e*x - a^5*f*x)/(
(b*x^2 + a)*b^6) + 1/315*(35*b^16*f*x^9 + 45*b^16*e*x^7 - 90*a*b^15*f*x^7 +
 63*b^16*d*x^5 - 126*a*b^15*e*x^5 + 189*a^2*b^14*f*x^5 + 105*b^16*c*x^3 - 2
10*a*b^15*d*x^3 + 315*a^2*b^14*e*x^3 - 420*a^3*b^13*f*x^3 - 630*a*b^15*c*x
+ 945*a^2*b^14*d*x - 1260*a^3*b^13*e*x + 1575*a^4*b^12*f*x)/b^18
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.72

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$$

$$= x^7\left(\frac{e}{7\,b^2}-\frac{2\,a\,f}{7\,b^3}\right)-x\left(\frac{2\,a\left(\frac{c}{b^2}-\frac{a^2\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b^2}+\frac{2\,a\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{b}\right)}{b}-\frac{a^2\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{b^2}\right)-x^5\left(\frac{a^2\,f}{5\,b^4}-\frac{d}{5\,b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{5\,b}\right)$$
$$+x^3\left(\frac{c}{3\,b^2}-\frac{a^2\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{3\,b^2}+\frac{2\,a\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{3\,b}\right)$$
$$+\frac{f\,x^9}{9\,b^2}+\frac{x\left(\frac{f\,a^5}{2}-\frac{e\,a^4\,b}{2}+\frac{d\,a^3\,b^2}{2}-\frac{c\,a^2\,b^3}{2}\right)}{b^7\,x^2+a\,b^6}$$
$$-\frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,(-11\,f\,a^3+9\,e\,a^2\,b-7\,d\,a\,b^2+5\,c\,b^3)}{11\,f\,a^5-9\,e\,a^4\,b+7\,d\,a^3\,b^2-5\,c\,a^2\,b^3}\right)\,(-11\,f\,a^3+9\,e\,a^2\,b-7\,d\,a\,b^2+5\,c\,b^3)}{2\,b^{13/2}}$$

```
[In] int((x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x)
```

```
[Out] x^7*(e/(7*b^2) - (2*a*f)/(7*b^3)) - x*((2*a*(c/b^2 - (a^2*(e/b^2 - (2*a*f)/
b^3))/b^2 + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b))
/b - (a^2*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b^2) - x^5
*((a^2*f)/(5*b^4) - d/(5*b^2) + (2*a*(e/b^2 - (2*a*f)/b^3))/(5*b)) + x^3*(c
/(3*b^2) - (a^2*(e/b^2 - (2*a*f)/b^3))/(3*b^2) + (2*a*((a^2*f)/b^4 - d/b^2
+ (2*a*(e/b^2 - (2*a*f)/b^3))/b))/(3*b)) + (f*x^9)/(9*b^2) + (x*((a^5*f)/2
- (a^2*b^3*c)/2 + (a^3*b^2*d)/2 - (a^4*b*e)/2))/(a*b^6 + b^7*x^2) - (a^(3/2
)*atan((a^(3/2)*b^(1/2)*x*(5*b^3*c - 11*a^3*f - 7*a*b^2*d + 9*a^2*b*e))/(11
*a^5*f - 5*a^2*b^3*c + 7*a^3*b^2*d - 9*a^4*b*e))*(5*b^3*c - 11*a^3*f - 7*a*
b^2*d + 9*a^2*b*e))/(2*b^(13/2))
```

3.125 $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2} dx$

Optimal result 749
Rubi [A] (verified) 749
Mathematica [A] (verified) 751
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Fricas [A] (verification not implemented) 752
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Optimal result

Integrand size = 30, antiderivative size = 202

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2} dx = \frac{(3b^3c-5ab^2d+7a^2be-9a^3f)\,x}{2b^5} - \frac{(3b^3c-5ab^2d+7a^2be-9a^3f)\,x^3}{6ab^4} + \frac{(be-2af)x^5}{5b^3} + \frac{fx^7}{7b^2} + \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^5}{2a\,(a+bx^2)} - \frac{\sqrt{a}(3b^3c-5ab^2d+7a^2be-9a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}$$

```
[Out] 1/2*(-9*a^3*f+7*a^2*b*e-5*a*b^2*d+3*b^3*c)*x/b^5-1/6*(-9*a^3*f+7*a^2*b*e-5*
a*b^2*d+3*b^3*c)*x^3/a/b^4+1/5*(-2*a*f+b*e)*x^5/b^3+1/7*f*x^7/b^2+1/2*(c-a*
(a^2*f-a*b*e+b^2*d)/b^3)*x^5/a/(b*x^2+a)-1/2*(-9*a^3*f+7*a^2*b*e-5*a*b^2*d+
3*b^3*c)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(11/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used

$= \{1818, 1599, 1275, 211\}$

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx = \frac{x^5\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a\,(a+bx^2)} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(-9a^3f+7a^2be-5ab^2d+3b^3c)}{2b^{11/2}} + \frac{x(-9a^3f+7a^2be-5ab^2d+3b^3c)}{2b^5} - \frac{x^3(-9a^3f+7a^2be-5ab^2d+3b^3c)}{6ab^4} + \frac{x^5(be-2af)}{5b^3} + \frac{fx^7}{7b^2}$$

```
[In] Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] ((3*b^3*c - 5*a*b^2*d + 7*a^2*b*e - 9*a^3*f)*x)/(2*b^5) - ((3*b^3*c - 5*a*b
^2*d + 7*a^2*b*e - 9*a^3*f)*x^3)/(6*a*b^4) + ((b*e - 2*a*f)*x^5)/(5*b^3) +
(f*x^7)/(7*b^2) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^5)/(2*a*(a + b*x
^2)) - (Sqrt[a]*(3*b^3*c - 5*a*b^2*d + 7*a^2*b*e - 9*a^3*f)*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(2*b^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
```

```
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(c - \frac{a(b^2d-abe+a^2f)}{b^3}\right)x^5}{2a\left(a+bx^2\right)} - \frac{\int \frac{x^3\left(\left(3bc-5ad+\frac{5a^2e}{b}-\frac{5a^3f}{b^2}\right)x-2a\left(e-\frac{af}{b}\right)x^3-2afx^5\right)}{a+bx^2}\,dx}{2ab}$$

$$= \frac{\left(c - \frac{a(b^2d-abe+a^2f)}{b^3}\right)x^5}{2a\left(a+bx^2\right)} - \frac{\int \frac{x^4\left(3bc-5ad+\frac{5a^2e}{b}-\frac{5a^3f}{b^2}-2a\left(e-\frac{af}{b}\right)x^2-2afx^4\right)}{a+bx^2}\,dx}{2ab}$$

$$= \frac{\left(c - \frac{a(b^2d-abe+a^2f)}{b^3}\right)x^5}{2a\left(a+bx^2\right)} - \frac{\int \left(-\frac{a(3b^3c-5ab^2d+7a^2be-9a^3f)}{b^4} + \frac{(3b^3c-5ab^2d+7a^2be-9a^3f)x^2}{b^3} - \frac{2a(be-2af)x^4}{b^2} - \frac{2afx^6}{b} + \frac{3a^2b^3c-5a^3b^2d+7a^4be}{b^4(a+bx^2)}\right.}{2ab}$$

$$= \frac{\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)x}{2b^5} - \frac{\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)x^3}{6ab^4} + \frac{(be-2af)x^5}{5b^3} + \frac{fx^7}{7b^2} + \frac{\left(c - \frac{a(b^2d-abe+a^2f)}{b^3}\right)x^5}{2a\left(a+bx^2\right)} - \frac{\left(a\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)\right)\int \frac{1}{a+bx^2}\,dx}{2b^5}$$

$$= \frac{\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)x}{2b^5} - \frac{\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)x^3}{6ab^4} + \frac{(be-2af)x^5}{5b^3} + \frac{fx^7}{7b^2} + \frac{\left(c - \frac{a(b^2d-abe+a^2f)}{b^3}\right)x^5}{2a\left(a+bx^2\right)} - \frac{\sqrt{a}\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.93

$$\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^2}\,dx = \frac{\left(b^3c-2ab^2d+3a^2be-4a^3f\right)x}{b^5} + \frac{\left(b^2d-2abe+3a^2f\right)x^3}{3b^4} + \frac{(be-2af)x^5}{5b^3} + \frac{fx^7}{7b^2} + \frac{\left(ab^3c-a^2b^2d+a^3be-a^4f\right)x}{2b^5\left(a+bx^2\right)} + \frac{\sqrt{a}\left(-3b^3c+5ab^2d-7a^2be+9a^3f\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}$$

```
[In] Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x)/b^5 + ((b^2*d - 2*a*b*e + 3*a
^2*f)*x^3)/(3*b^4) + ((b*e - 2*a*f)*x^5)/(5*b^3) + (f*x^7)/(7*b^2) + ((a*b^
3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/(2*b^5*(a + b*x^2)) + (Sqrt[a]*(-3*b^
3*c + 5*a*b^2*d - 7*a^2*b*e + 9*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(1
1/2))
```

Maple [A] (verified)

Time = 3.44 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.90

method	result
default	$-\frac{-\frac{1}{7} f\, x^{7} b^{3}+\frac{2}{5} a\, b^{2} f\, x^{5}-\frac{1}{5} b^{3} e\, x^{5}-a^{2} b f\, x^{3}+\frac{2}{3} a\, b^{2} e\, x^{3}-\frac{1}{3} b^{3} d\, x^{3}+4 f\, a^{3} x-3 a^{2} b e x+2 a\, b^{2} d x-b^{3} c x}{b^{5}}+\frac{a \left(\frac{\left(-\frac{1}{2} f\, a^{3}+\frac{1}{2} a^{2} b e-\frac{1}{2} a\, b^{2} d+\frac{1}{2} b^{3}}{b\, x^{2}+a}\right.}{}$
risch	$\frac{f\, x^{7}}{7 b^{2}}-\frac{2 a f\, x^{5}}{5 b^{3}}+\frac{e\, x^{5}}{5 b^{2}}+\frac{a^{2} f\, x^{3}}{b^{4}}-\frac{2 a e\, x^{3}}{3 b^{3}}+\frac{d\, x^{3}}{3 b^{2}}-\frac{4 f\, a^{3} x}{b^{5}}+\frac{3 a^{2} e x}{b^{4}}-\frac{2 a d x}{b^{3}}+\frac{c x}{b^{2}}+\frac{\left(-\frac{1}{2} a^{4} f+\frac{1}{2} a^{3} b e-\frac{1}{2} a^{2} b^{2} d+\frac{1}{2} a\, b^{3} c\right)}{b^{5} \left(b\, x^{2}+a\right)}$

```
[In] int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^5*(-1/7*f*x^7*b^3+2/5*a*b^2*f*x^5-1/5*b^3*e*x^5-a^2*b*f*x^3+2/3*a*b^2*
e*x^3-1/3*b^3*d*x^3+4*f*a^3*x-3*a^2*b*e*x+2*a*b^2*d*x-b^3*c*x)+a/b^5*((-1/2
*f*a^3+1/2*a^2*b*e-1/2*a*b^2*d+1/2*b^3*c)*x/(b*x^2+a)+1/2*(9*a^3*f-7*a^2*b*
e+5*a*b^2*d-3*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.37

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$$

$$=\left[\frac{60\,b^4fx^9+12\,(7\,b^4e-9\,ab^3f)x^7+28\,(5\,b^4d-7\,ab^3e+9\,a^2b^2f)x^5+140\,(3\,b^4c-5\,ab^3d+7\,a^2b^2e-9\,a^3b}{}\right.$$

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(60*b^4*f*x^9 + 12*(7*b^4*e - 9*a*b^3*f)*x^7 + 28*(5*b^4*d - 7*a*b^3
*e + 9*a^2*b^2*f)*x^5 + 140*(3*b^4*c - 5*a*b^3*d + 7*a^2*b^2*e - 9*a^3*b*f)
*x^3 - 105*(3*a*b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a^4*f + (3*b^4*c - 5*a*
b^3*d + 7*a^2*b^2*e - 9*a^3*b*f)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a
/b) - a)/(b*x^2 + a)) + 210*(3*a*b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a^4*f)
*x)/(b^6*x^2 + a*b^5), 1/210*(30*b^4*f*x^9 + 6*(7*b^4*e - 9*a*b^3*f)*x^7 +
```

```
14*(5*b^4*d - 7*a*b^3*e + 9*a^2*b^2*f)*x^5 + 70*(3*b^4*c - 5*a*b^3*d + 7*a^
2*b^2*e - 9*a^3*b*f)*x^3 - 105*(3*a*b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a^4
*f + (3*b^4*c - 5*a*b^3*d + 7*a^2*b^2*e - 9*a^3*b*f)*x^2)*sqrt(a/b)*arctan(
b*x*sqrt(a/b)/a) + 105*(3*a*b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a^4*f)*x)/(
b^6*x^2 + a*b^5)]
```

Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.27

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx = x^5\left(-\frac{2af}{5b^3}+\frac{e}{5b^2}\right)+x^3\left(\frac{a^2f}{b^4}-\frac{2ae}{3b^3}+\frac{d}{3b^2}\right) + x\left(-\frac{4a^3f}{b^5}+\frac{3a^2e}{b^4}-\frac{2ad}{b^3}+\frac{c}{b^2}\right)+\frac{x(-a^4f+a^3be-a^2b^2d+ab^3c)}{2ab^5+2b^6x^2} - \frac{\sqrt{-\frac{a}{b^{11}}}\cdot(9a^3f-7a^2be+5ab^2d-3b^3c)\log\left(-b^5\sqrt{-\frac{a}{b^{11}}}+x\right)}{4} + \frac{\sqrt{-\frac{a}{b^{11}}}\cdot(9a^3f-7a^2be+5ab^2d-3b^3c)\log\left(b^5\sqrt{-\frac{a}{b^{11}}}+x\right)}{4}+\frac{fx^7}{7b^2}$$

```
[In] integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x**5*(-2*a*f/(5*b**3) + e/(5*b**2)) + x**3*(a**2*f/b**4 - 2*a*e/(3*b**3) +
d/(3*b**2)) + x*(-4*a**3*f/b**5 + 3*a**2*e/b**4 - 2*a*d/b**3 + c/b**2) + x*
(-a**4*f + a**3*b*e - a**2*b**2*d + a*b**3*c)/(2*a*b**5 + 2*b**6*x**2) - sq
rt(-a/b**11)*(9*a**3*f - 7*a**2*b*e + 5*a*b**2*d - 3*b**3*c)*log(-b**5*sqrt
(-a/b**11) + x)/4 + sqrt(-a/b**11)*(9*a**3*f - 7*a**2*b*e + 5*a*b**2*d - 3*
b**3*c)*log(b**5*sqrt(-a/b**11) + x)/4 + f*x**7/(7*b**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.91

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx = \frac{(ab^3c-a^2b^2d+a^3be-a^4f)x}{2\,(b^6x^2+ab^5)} - \frac{(3\,ab^3c-5\,a^2b^2d+7\,a^3be-9\,a^4f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^5} + \frac{15\,b^3fx^7+21\,(b^3e-2\,ab^2f)x^5+35\,(b^3d-2\,ab^2e+3\,a^2bf)x^3+105\,(b^3c-2\,ab^2d+3\,a^2be-4\,a^3f)x}{105\,b^5}$$

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x/(b^6*x^2 + a*b^5) - 1/2*(3*a*
b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a^4*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*b^5) + 1/105*(15*b^3*f*x^7 + 21*(b^3*e - 2*a*b^2*f)*x^5 + 35*(b^3*d - 2*a*
b^2*e + 3*a^2*b*f)*x^3 + 105*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x)/b
^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.97

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$$
$$= -\frac{(3\,ab^3c-5\,a^2b^2d+7\,a^3be-9\,a^4f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^5}+\frac{ab^3cx-a^2b^2dx+a^3bex-a^4fx}{2\,(bx^2+a)b^5}$$
$$+\frac{15\,b^{12}fx^7+21\,b^{12}ex^5-42\,ab^{11}fx^5+35\,b^{12}dx^3-70\,ab^{11}ex^3+105\,a^2b^{10}fx^3+105\,b^{12}cx-210\,ab^{11}dx+3}{105\,b^{14}}$$

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*a*b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a^4*f)*arctan(b*x/sqrt(a*b))/
(sqrt(a*b)*b^5) + 1/2*(a*b^3*c*x - a^2*b^2*d*x + a^3*b*e*x - a^4*f*x)/((b*x
^2 + a)*b^5) + 1/105*(15*b^12*f*x^7 + 21*b^12*e*x^5 - 42*a*b^11*f*x^5 + 35*
b^12*d*x^3 - 70*a*b^11*e*x^3 + 105*a^2*b^10*f*x^3 + 105*b^12*c*x - 210*a*b^
11*d*x + 315*a^2*b^10*e*x - 420*a^3*b^9*f*x)/b^14
```

Mupad [B] (verification not implemented)

Time = 5.78 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.43

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$$
$$= x^5\left(\frac{e}{5\,b^2}-\frac{2\,a\,f}{5\,b^3}\right)+x\left(\frac{c}{b^2}-\frac{a^2\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b^2}+\frac{2\,a\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{b}\right)$$
$$-x^3\left(\frac{a^2\,f}{3\,b^4}-\frac{d}{3\,b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{3\,b}\right)-\frac{x\left(\frac{f\,a^4}{2}-\frac{e\,a^3\,b}{2}+\frac{d\,a^2\,b^2}{2}-\frac{c\,a\,b^3}{2}\right)}{b^6\,x^2+a\,b^5}+\frac{f\,x^7}{7\,b^2}$$
$$+\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,\left(-9\,f\,a^3+7\,e\,a^2\,b-5\,d\,a\,b^2+3\,c\,b^3\right)}{9\,f\,a^4-7\,e\,a^3\,b+5\,d\,a^2\,b^2-3\,c\,a\,b^3}\right)\,\left(-9\,f\,a^3+7\,e\,a^2\,b-5\,d\,a\,b^2+3\,c\,b^3\right)}{2\,b^{11/2}}$$

```
[In] int((x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x)
```

```
[Out] x^5*(e/(5*b^2) - (2*a*f)/(5*b^3)) + x*(c/b^2 - (a^2*(e/b^2 - (2*a*f)/b^3))/
b^2 + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b) - x^3*
((a^2*f)/(3*b^4) - d/(3*b^2) + (2*a*(e/b^2 - (2*a*f)/b^3))/(3*b)) - (x*((a^
4*f)/2 + (a^2*b^2*d)/2 - (a*b^3*c)/2 - (a^3*b*e)/2))/(a*b^5 + b^6*x^2) + (f
*x^7)/(7*b^2) + (a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(3*b^3*c - 9*a^3*f - 5*a*b
^2*d + 7*a^2*b*e))/(9*a^4*f + 5*a^2*b^2*d - 3*a*b^3*c - 7*a^3*b*e))*(3*b^3*
c - 9*a^3*f - 5*a*b^2*d + 7*a^2*b*e))/(2*b^(11/2))
```

3.126 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$

Optimal result 756
Rubi [A] (verified) 756
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Optimal result

Integrand size = 30, antiderivative size = 163

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx = -\frac{(b^3c-3ab^2d+5a^2be-7a^3f)\,x}{2ab^4} + \frac{(be-2af)x^3}{3b^3} + \frac{fx^5}{5b^2} + \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^3}{2a\,(a+bx^2)} + \frac{(b^3c-3ab^2d+5a^2be-7a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{9/2}}$$

```
[Out] -1/2*(-7*a^3*f+5*a^2*b*e-3*a*b^2*d+b^3*c)*x/a/b^4+1/3*(-2*a*f+b*e)*x^3/b^3+
1/5*f*x^5/b^2+1/2*(c-a*(a^2*f-a*b*e+b^2*d)/b^3)*x^3/a/(b*x^2+a)+1/2*(-7*a^3
*f+5*a^2*b*e-3*a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/b^(9/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1818, 1599, 1275, 211}

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx = \frac{x^3\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a\,(a+bx^2)} + \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(-7a^3f+5a^2be-3ab^2d+b^3c)}{2\sqrt{a}b^{9/2}} - \frac{x(-7a^3f+5a^2be-3ab^2d+b^3c)}{2ab^4} + \frac{x^3(be-2af)}{3b^3} + \frac{fx^5}{5b^2}$$

```
[In] Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] -1/2*((b^3*c - 3*a*b^2*d + 5*a^2*b*e - 7*a^3*f)*x)/(a*b^4) + ((b*e - 2*a*f)
*x^3)/(3*b^3) + (f*x^5)/(5*b^2) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^
3)/(2*a*(a + b*x^2)) + ((b^3*c - 3*a*b^2*d + 5*a^2*b*e - 7*a^3*f)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(c - \frac{a(b^2d-abe+a^2f)}{b^3}\right)x^3}{2a\,(a+bx^2)} - \frac{\int \frac{x\left(\left(bc-3ad+\frac{3a^2e}{b}-\frac{3a^3f}{b^2}\right)x-2a\left(e-\frac{af}{b}\right)x^3-2afx^5\right)}{a+bx^2}\,dx}{2ab}$$

$$= \frac{\left(c - \frac{a(b^2d-abe+a^2f)}{b^3}\right)x^3}{2a\,(a+bx^2)} - \frac{\int \frac{x^2\left(bc-3ad+\frac{3a^2e}{b}-\frac{3a^3f}{b^2}-2a\left(e-\frac{af}{b}\right)x^2-2afx^4\right)}{a+bx^2}\,dx}{2ab}$$

$$= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^3}{2a\left(a+bx^2\right)} - \frac{\int \left(c-\frac{a(3b^2d-5abe+7a^2f)}{b^3}-\frac{2a(be-2af)x^2}{b^2}-\frac{2afx^4}{b}+\frac{-ab^3c+3a^2b^2d-5a^3be+7a^4f}{b^3(a+bx^2)}\right)dx}{2ab}$$

$$= -\frac{\left(b^3c-3ab^2d+5a^2be-7a^3f\right)x}{2ab^4}+\frac{(be-2af)x^3}{3b^3}+\frac{fx^5}{5b^2} + \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^3}{2a\left(a+bx^2\right)}+\frac{\left(b^3c-3ab^2d+5a^2be-7a^3f\right)\int \frac{1}{a+bx^2}dx}{2b^4}$$

$$= -\frac{\left(b^3c-3ab^2d+5a^2be-7a^3f\right)x}{2ab^4}+\frac{(be-2af)x^3}{3b^3}+\frac{fx^5}{5b^2} + \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^3}{2a\left(a+bx^2\right)}+\frac{\left(b^3c-3ab^2d+5a^2be-7a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{9/2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.91

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}dx = \frac{\left(b^2d-2abe+3a^2f\right)x}{b^4}+\frac{(be-2af)x^3}{3b^3} + \frac{fx^5}{5b^2}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2b^4\left(a+bx^2\right)} - \frac{\left(-b^3c+3ab^2d-5a^2be+7a^3f\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{9/2}}$$

```
[In] Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x)/b^4 + ((b*e - 2*a*f)*x^3)/(3*b^3) + (f*x^5)
/(5*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(2*b^4*(a + b*x^2)) - ((
-(b^3*c) + 3*a*b^2*d - 5*a^2*b*e + 7*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2
*Sqrt[a]*b^(9/2))
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.85

method	result
default	$\frac{\frac{1}{5} f\, x^{5} b^{2}-\frac{2}{3} a b f\, x^{3}+\frac{1}{3} b^{2} e\, x^{3}+3 a^{2} f x -2 a b e x +b^{2} d x}{b^{4}}-\frac{\frac{\left(-\frac{1}{2} f\, a^{3}+\frac{1}{2} a^{2} b e -\frac{1}{2} a\, b^{2} d +\frac{1}{2} b^{3} c \right) x}{b\, x^{2}+a}+\frac{\left(7 f\, a^{3}-5 a^{2} b e +3 a\, b^{2} d -b^{3} c \right) \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}}}{b^{4}}$
risch	$\frac{f\, x^{5}}{5 b^{2}}-\frac{2 a f\, x^{3}}{3 b^{3}}+\frac{e\, x^{3}}{3 b^{2}}+\frac{3 a^{2} f x}{b^{4}}-\frac{2 a e x}{b^{3}}+\frac{d x}{b^{2}}+\frac{\left(\frac{1}{2} f\, a^{3}-\frac{1}{2} a^{2} b e +\frac{1}{2} a\, b^{2} d -\frac{1}{2} b^{3} c \right) x}{b^{4} \left(b\, x^{2}+a \right)}-\frac{7 \ln \left(b x -\sqrt{-a b}\right) f\, a^{3}}{4 b^{4} \sqrt{-a b}}+\frac{5 \ln \left(b x -\sqrt{-a}\right.}{4 b^{3} \sqrt{-a}}$

```
[In] int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/5*f*x^5*b^2-2/3*a*b*f*x^3+1/3*b^2*e*x^3+3*a^2*f*x-2*a*b*e*x+b^2*d*
x)-1/b^4*((-1/2*f*a^3+1/2*a^2*b*e-1/2*a*b^2*d+1/2*b^3*c)*x/(b*x^2+a)+1/2*(7
*a^3*f-5*a^2*b*e+3*a*b^2*d-b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 418, normalized size of antiderivative = 2.56

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$$

$$= \left[\frac{12\,ab^4fx^7+4\,(5\,ab^4e-7\,a^2b^3f)x^5+20\,(3\,ab^4d-5\,a^2b^3e+7\,a^3b^2f)x^3+15\,(ab^3c-3\,a^2b^2d+5\,a^3be-7}{60\,(a}\right.$$

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*a*b^4*f*x^7 + 4*(5*a*b^4*e - 7*a^2*b^3*f)*x^5 + 20*(3*a*b^4*d - 5
*a^2*b^3*e + 7*a^3*b^2*f)*x^3 + 15*(a*b^3*c - 3*a^2*b^2*d + 5*a^3*b*e - 7*a
^4*f + (b^4*c - 3*a*b^3*d + 5*a^2*b^2*e - 7*a^3*b*f)*x^2)*sqrt(-a*b)*log((b
*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 30*(a*b^4*c - 3*a^2*b^3*d + 5*a^3
*b^2*e - 7*a^4*b*f)*x)/(a*b^6*x^2 + a^2*b^5), 1/30*(6*a*b^4*f*x^7 + 2*(5*a*
b^4*e - 7*a^2*b^3*f)*x^5 + 10*(3*a*b^4*d - 5*a^2*b^3*e + 7*a^3*b^2*f)*x^3 +
 15*(a*b^3*c - 3*a^2*b^2*d + 5*a^3*b*e - 7*a^4*f + (b^4*c - 3*a*b^3*d + 5*a
^2*b^2*e - 7*a^3*b*f)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - 15*(a*b^4*c -
3*a^2*b^3*d + 5*a^3*b^2*e - 7*a^4*b*f)*x)/(a*b^6*x^2 + a^2*b^5)]
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.36

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx = x^3\left(-\frac{2af}{3b^3}+\frac{e}{3b^2}\right)+x\left(\frac{3a^2f}{b^4}-\frac{2ae}{b^3}+\frac{d}{b^2}\right)+\frac{x(a^3f-a^2be+ab^2d-b^3c)}{2ab^4+2b^5x^2} + \frac{\sqrt{-\frac{1}{ab^9}}\cdot(7a^3f-5a^2be+3ab^2d-b^3c)\log\left(-ab^4\sqrt{-\frac{1}{ab^9}}+x\right)}{4} - \frac{\sqrt{-\frac{1}{ab^9}}\cdot(7a^3f-5a^2be+3ab^2d-b^3c)\log\left(ab^4\sqrt{-\frac{1}{ab^9}}+x\right)}{4}+\frac{fx^5}{5b^2}$$

```
[In] integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x**3*(-2*a*f/(3*b**3) + e/(3*b**2)) + x*(3*a**2*f/b**4 - 2*a*e/b**3 + d/b**
2) + x*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(2*a*b**4 + 2*b**5*x**2) + s
qrt(-1/(a*b**9))*(7*a**3*f - 5*a**2*b*e + 3*a*b**2*d - b**3*c)*log(-a*b**4*
sqrt(-1/(a*b**9)) + x)/4 - sqrt(-1/(a*b**9))*(7*a**3*f - 5*a**2*b*e + 3*a*b
**2*d - b**3*c)*log(a*b**4*sqrt(-1/(a*b**9)) + x)/4 + f*x**5/(5*b**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.86

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx = -\frac{(b^3c-ab^2d+a^2be-a^3f)x}{2\,(b^5x^2+ab^4)} + \frac{(b^3c-3\,ab^2d+5\,a^2be-7\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^4} + \frac{3\,b^2fx^5+5\,(b^2e-2\,abf)x^3+15\,(b^2d-2\,abe+3\,a^2f)x}{15\,b^4}$$

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x/(b^5*x^2 + a*b^4) + 1/2*(b^3*c -
 3*a*b^2*d + 5*a^2*b*e - 7*a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1
/15*(3*b^2*f*x^5 + 5*(b^2*e - 2*a*b*f)*x^3 + 15*(b^2*d - 2*a*b*e + 3*a^2*f)
*x)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.91

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx = \frac{(b^3c-3\,ab^2d+5\,a^2be-7\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^4} - \frac{b^3cx-ab^2dx+a^2bex-a^3fx}{2\,(bx^2+a)b^4} + \frac{3\,b^8fx^5+5\,b^8ex^3-10\,ab^7fx^3+15\,b^8dx-30\,ab^7ex+45\,a^2b^6fx}{15\,b^{10}}$$

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c - 3*a*b^2*d + 5*a^2*b*e - 7*a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*b^4) - 1/2*(b^3*c*x - a*b^2*d*x + a^2*b*e*x - a^3*f*x)/((b*x^2 + a)*b^4
) + 1/15*(3*b^8*f*x^5 + 5*b^8*e*x^3 - 10*a*b^7*f*x^3 + 15*b^8*d*x - 30*a*b^
7*e*x + 45*a^2*b^6*f*x)/b^10
```

Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.94

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx = x^3\left(\frac{e}{3\,b^2}-\frac{2\,a\,f}{3\,b^3}\right) - x\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right) - \frac{x\left(-\frac{f\,a^3}{2}+\frac{e\,a^2\,b}{2}-\frac{d\,a\,b^2}{2}+\frac{c\,b^3}{2}\right)}{b^5\,x^2+a\,b^4} + \frac{f\,x^5}{5\,b^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-7\,f\,a^3+5\,e\,a^2\,b-3\,d\,a\,b^2+c\,b^3\right)}{2\,\sqrt{a}\,b^{9/2}}$$

```
[In] int((x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x)
```

```
[Out] x^3*(e/(3*b^2) - (2*a*f)/(3*b^3)) - x*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 -
(2*a*f)/b^3))/b) - (x*((b^3*c)/2 - (a^3*f)/2 - (a*b^2*d)/2 + (a^2*b*e)/2))/
(a*b^4 + b^5*x^2) + (f*x^5)/(5*b^2) + (atan((b^(1/2)*x)/a^(1/2))*(b^3*c - 7
*a^3*f - 3*a*b^2*d + 5*a^2*b*e))/(2*a^(1/2)*b^(9/2))
```

3.127 $\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^2}\,dx$

Optimal result . . . 762
Rubi [A] (verified) . . . 762
Mathematica [A] (verified) . . . 764
Maple [A] (verified) . . . 764
Fricas [A] (verification not implemented) . . . 764
Sympy [A] (verification not implemented) . . . 765
Maxima [A] (verification not implemented) . . . 765
Giac [A] (verification not implemented) . . . 766
Mupad [B] (verification not implemented) . . . 766

Optimal result

Integrand size = 27, antiderivative size = 118

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^2}\,dx = \frac{(be-2af)x}{b^3}+\frac{fx^3}{3b^2}+\frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x}{2a\,(a+bx^2)} + \frac{(b^3c+ab^2d-3a^2be+5a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}$$

```
[Out] (-2*a*f+b*e)*x/b^3+1/3*f*x^3/b^2+1/2*(c-a*(a^2*f-a*b*e+b^2*d)/b^3)*x/a/(b*x
^2+a)+1/2*(5*a^3*f-3*a^2*b*e+a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/
2)/b^(7/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1828, 1167, 211}

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^2}\,dx = \frac{x\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a\,(a+bx^2)} + \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(5a^3f-3a^2be+ab^2d+b^3c)}{2a^{3/2}b^{7/2}} + \frac{x(be-2af)}{b^3}+\frac{fx^3}{3b^2}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x)/b^3 + (f*x^3)/(3*b^2) + ((c - (a*(b^2*d - a*b*e + a^2*f))
/b^3)*x)/(2*a*(a + b*x^2)) + ((b^3*c + a*b^2*d - 3*a^2*b*e + 5*a^3*f)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{2a\left(a+bx^2\right)}-\frac{\int\frac{-\frac{b^3c+ab^2d-a^2be+a^3f}{b^3}-\frac{2a(be-af)x^2}{b^2}-\frac{2afx^4}{b}}{a+bx^2}\,dx}{2a}$$

$$=\frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{2a\left(a+bx^2\right)}-\frac{\int\left(-\frac{2a(be-2af)}{b^3}-\frac{2afx^2}{b^2}+\frac{-b^3c-ab^2d+3a^2be-5a^3f}{b^3(a+bx^2)}\right)dx}{2a}$$

$$=\frac{(be-2af)x}{b^3}+\frac{fx^3}{3b^2}+\frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{2a\left(a+bx^2\right)}+\frac{\left(b^3c+ab^2d-3a^2be+5a^3f\right)\int\frac{1}{a+bx^2}\,dx}{2ab^3}$$

$$=\frac{(be-2af)x}{b^3}+\frac{fx^3}{3b^2}+\frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{2a\left(a+bx^2\right)}+\frac{\left(b^3c+ab^2d-3a^2be+5a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^2}\,dx = \frac{(be-2af)x}{b^3}+\frac{fx^3}{3b^2}-\frac{(-b^3c+ab^2d-a^2be+a^3f)\,x}{2ab^3\,(a+bx^2)} + \frac{(b^3c+ab^2d-3a^2be+5a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x)/b^3 + (f*x^3)/(3*b^2) - ((-(b^3*c) + a*b^2*d - a^2*b*e +
a^3*f)*x)/(2*a*b^3*(a + b*x^2)) + ((b^3*c + a*b^2*d - 3*a^2*b*e + 5*a^3*f)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(7/2))
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

method	result
default	$-\frac{-\frac{1}{3}f\,x^3b+2afx-bex}{b^3}+\frac{-\frac{\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)x}{2a\left(b\,x^2+a\right)}+\frac{\left(5f\,a^3-3a^2be+a\,b^2d+b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{b^3}$
risch	$\frac{f\,x^3}{3b^2}-\frac{2afx}{b^3}+\frac{ex}{b^2}-\frac{(f\,a^3-a^2be+a\,b^2d-b^3c)x}{2a\,b^3(b\,x^2+a)}-\frac{5a^2\ln\left(bx+\sqrt{-ab}\right)f}{4b^3\sqrt{-ab}}+\frac{3a\ln\left(bx+\sqrt{-ab}\right)e}{4b^2\sqrt{-ab}}-\frac{\ln\left(bx+\sqrt{-ab}\right)d}{4b\sqrt{-ab}}-\frac{c\ln(bx+\sqrt{\ }}{4\sqrt{-a}}$

```
[In] int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^3*(-1/3*f*x^3*b+2*a*f*x-b*e*x)+1/b^3*(-1/2*(a^3*f-a^2*b*e+a*b^2*d-b^3*
c)/a*x/(b*x^2+a)+1/2*(5*a^3*f-3*a^2*b*e+a*b^2*d+b^3*c)/a/(a*b)^(1/2)*arctan
(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.08

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^2}\,dx$$

$$= \left[\frac{4\,a^2b^3fx^5+4\,(3\,a^2b^3e-5\,a^3b^2f)x^3-3\,(ab^3c+a^2b^2d-3\,a^3be+5\,a^4f+(b^4c+ab^3d-3\,a^2b^2e+5\,a^3bf)x}{12\,(a^2b^5x^2+a^3b^4)}\right.$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*a^2*b^3*f*x^5 + 4*(3*a^2*b^3*e - 5*a^3*b^2*f)*x^3 - 3*(a*b^3*c + a
^2*b^2*d - 3*a^3*b*e + 5*a^4*f + (b^4*c + a*b^3*d - 3*a^2*b^2*e + 5*a^3*b*f
)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(a*b^4*
c - a^2*b^3*d + 3*a^3*b^2*e - 5*a^4*b*f)*x)/(a^2*b^5*x^2 + a^3*b^4), 1/6*(2
*a^2*b^3*f*x^5 + 2*(3*a^2*b^3*e - 5*a^3*b^2*f)*x^3 + 3*(a*b^3*c + a^2*b^2*d
 - 3*a^3*b*e + 5*a^4*f + (b^4*c + a*b^3*d - 3*a^2*b^2*e + 5*a^3*b*f)*x^2)*s
qrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(a*b^4*c - a^2*b^3*d + 3*a^3*b^2*e - 5*a
^4*b*f)*x)/(a^2*b^5*x^2 + a^3*b^4)]
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.70

$$\begin{aligned}\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^2}\,dx &= x\left(-\frac{2af}{b^3}+\frac{e}{b^2}\right)+\frac{x(-a^3f+a^2be-ab^2d+b^3c)}{2a^2b^3+2ab^4x^2} \\ &\quad -\frac{\sqrt{-\frac{1}{a^3b^7}}\cdot(5a^3f-3a^2be+ab^2d+b^3c)\log\left(-a^2b^3\sqrt{-\frac{1}{a^3b^7}}+x\right)}{4} \\ &\quad +\frac{\sqrt{-\frac{1}{a^3b^7}}\cdot(5a^3f-3a^2be+ab^2d+b^3c)\log\left(a^2b^3\sqrt{-\frac{1}{a^3b^7}}+x\right)}{4}+\frac{fx^3}{3b^2}\end{aligned}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x*(-2*a*f/b**3 + e/b**2) + x*(-a**3*f + a**2*b*e - a*b**2*d + b**3*c)/(2*a*
*2*b**3 + 2*a*b**4*x**2) - sqrt(-1/(a**3*b**7))*(5*a**3*f - 3*a**2*b*e + a*
b**2*d + b**3*c)*log(-a**2*b**3*sqrt(-1/(a**3*b**7)) + x)/4 + sqrt(-1/(a**3
*b**7))*(5*a**3*f - 3*a**2*b*e + a*b**2*d + b**3*c)*log(a**2*b**3*sqrt(-1/(
a**3*b**7)) + x)/4 + f*x**3/(3*b**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^2}\,dx &= \frac{(b^3c-ab^2d+a^2be-a^3f)x}{2\,(ab^4x^2+a^2b^3)}+\frac{bfx^3+3\,(be-2\,af)x}{3\,b^3} \\ &\quad +\frac{(b^3c+ab^2d-3\,a^2be+5\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^3}\end{aligned}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x/(a*b^4*x^2 + a^2*b^3) + 1/3*(b*f*
x^3 + 3*(b*e - 2*a*f)*x)/b^3 + 1/2*(b^3*c + a*b^2*d - 3*a^2*b*e + 5*a^3*f)*
arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^3)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.04

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^2}\,dx = \frac{(b^3c+ab^2d-3\,a^2be+5\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^3} + \frac{b^3cx-ab^2dx+a^2bex-a^3fx}{2\,(bx^2+a)ab^3} + \frac{b^4fx^3+3\,b^4ex-6\,ab^3fx}{3\,b^6}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c + a*b^2*d - 3*a^2*b*e + 5*a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a*b^3) + 1/2*(b^3*c*x - a*b^2*d*x + a^2*b*e*x - a^3*f*x)/((b*x^2 + a)*a*b
^3) + 1/3*(b^4*f*x^3 + 3*b^4*e*x - 6*a*b^3*f*x)/b^6
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^2}\,dx = x\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)+\frac{f\,x^3}{3\,b^2}+\frac{x\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)}{2\,a\,\left(b^4\,x^2+a\,b^3\right)} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(5\,f\,a^3-3\,e\,a^2\,b+d\,a\,b^2+c\,b^3\right)}{2\,a^{3/2}\,b^{7/2}}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^2,x)
```

```
[Out] x*(e/b^2 - (2*a*f)/b^3) + (f*x^3)/(3*b^2) + (x*(b^3*c - a^3*f - a*b^2*d + a
^2*b*e))/(2*a*(a*b^3 + b^4*x^2)) + (atan((b^(1/2)*x)/a^(1/2))*(b^3*c + 5*a^
3*f + a*b^2*d - 3*a^2*b*e))/(2*a^(3/2)*b^(7/2))
```

3.128 $\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)^2}\,dx$

Optimal result . . . 767
Rubi [A] (verified) . . . 767
Mathematica [A] (verified) . . . 769
Maple [A] (verified) . . . 769
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Giac [A] (verification not implemented) . . . 771
Mupad [B] (verification not implemented) . . . 771

Optimal result

Integrand size = 30, antiderivative size = 112

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)^2}\,dx = -\frac{c}{a^2x}+\frac{fx}{b^2}-\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{2a(a+bx^2)}-\frac{(3b^3c-ab^2d-a^2be+3a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}b^{5/2}}$$

```
[Out] -c/a^2/x+f*x/b^2-1/2*(b*c/a-d+a*e/b-a^2*f/b^2)*x/a/(b*x^2+a)-1/2*(3*a^3*f-a
^2*b*e-a*b^2*d+3*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(5/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1819, 1275, 211}

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)^2}\,dx = -\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{2a(a+bx^2)}-\frac{c}{a^2x}-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(3a^3f-a^2be-ab^2d+3b^3c)}{2a^{5/2}b^{5/2}}+\frac{fx}{b^2}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c/(a^2*x)) + (f*x)/b^2 - (((b*c)/a - d + (a*e)/b - (a^2*f)/b^2)*x)/(2*a*(
a + b*x^2)) - ((3*b^3*c - a*b^2*d - a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(5/2)*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{2a\left(a+bx^2\right)}-\frac{\int \frac{-2c+\left(\frac{bc}{a}-d-\frac{ae}{b}+\frac{a^2f}{b^2}\right)x^2-\frac{2afx^4}{b}}{x^2(a+bx^2)}\,dx}{2a}\\
&= -\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{2a\left(a+bx^2\right)}-\frac{\int \left(-\frac{2af}{b^2}-\frac{2c}{ax^2}+\frac{3b^3c-ab^2d-a^2be+3a^3f}{ab^2(a+bx^2)}\right)dx}{2a}\\
&= -\frac{c}{a^2x}+\frac{fx}{b^2}-\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{2a\left(a+bx^2\right)}-\frac{\left(3b^3c-ab^2d-a^2be+3a^3f\right)\int \frac{1}{a+bx^2}\,dx}{2a^2b^2}\\
&= -\frac{c}{a^2x}+\frac{fx}{b^2}-\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{2a\left(a+bx^2\right)}-\frac{\left(3b^3c-ab^2d-a^2be+3a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}b^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^2}\,dx = -\frac{c}{a^2x}+\frac{fx}{b^2}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{2a^2b^2\left(a+bx^2\right)} - \frac{\left(3b^3c-ab^2d-a^2be+3a^3f\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}b^{5/2}}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c/(a^2*x)) + (f*x)/b^2 + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(2*a^
2*b^2*(a + b*x^2)) - ((3*b^3*c - a*b^2*d - a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(2*a^(5/2)*b^(5/2))
```

Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

method	result
default	$\frac{fx}{b^2}-\frac{c}{a^2x}-\frac{\frac{\left(-\frac{1}{2}f\,a^3+\frac{1}{2}a^2be-\frac{1}{2}a\,b^2d+\frac{1}{2}b^3c\right)x}{b\,x^2+a}+\frac{\left(3f\,a^3-a^2be-a\,b^2d+3b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{a^2b^2}$
risch	$\frac{fx}{b^2}+\frac{\frac{\left(f\,a^3-a^2be+a\,b^2d-3b^3c\right)x^2}{2a^2}-\frac{b^2c}{a}}{b^2x\left(b\,x^2+a\right)}-\frac{3a\ln\left(-\sqrt{-ab}\,x-a\right)f}{4b^2\sqrt{-ab}}+\frac{\ln\left(-\sqrt{-ab}\,x-a\right)e}{4b\sqrt{-ab}}+\frac{\ln\left(-\sqrt{-ab}\,x-a\right)d}{4\sqrt{-ab}\,a}-\frac{3b\ln\left(-\sqrt{-ab}\,x-\right.}{4\sqrt{-ab}\,a^2}$

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] f*x/b^2-c/a^2/x-1/a^2/b^2*((-1/2*f*a^3+1/2*a^2*b*e-1/2*a*b^2*d+1/2*b^3*c)*x
/(b*x^2+a)+1/2*(3*a^3*f-a^2*b*e-a*b^2*d+3*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*
b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 354, normalized size of antiderivative = 3.16

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^2}\,dx$$

$$= \left[\frac{4\,a^3b^2fx^4-4\,a^2b^3c-2\,(3\,ab^4c-a^2b^3d+a^3b^2e-3\,a^4bf)x^2-((3\,b^4c-ab^3d-a^2b^2e+3\,a^3bf)x^3+(3\,a}{4\,(a^3b^4x^3+a^4b^3x)}\right.$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^3*b^2*f*x^4 - 4*a^2*b^3*c - 2*(3*a*b^4*c - a^2*b^3*d + a^3*b^2*e
- 3*a^4*b*f)*x^2 - ((3*b^4*c - a*b^3*d - a^2*b^2*e + 3*a^3*b*f)*x^3 + (3*a*
b^3*c - a^2*b^2*d - a^3*b*e + 3*a^4*f)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a
*b)*x - a)/(b*x^2 + a)))/(a^3*b^4*x^3 + a^4*b^3*x), 1/2*(2*a^3*b^2*f*x^4 -
2*a^2*b^3*c - (3*a*b^4*c - a^2*b^3*d + a^3*b^2*e - 3*a^4*b*f)*x^2 - ((3*b^4
*c - a*b^3*d - a^2*b^2*e + 3*a^3*b*f)*x^3 + (3*a*b^3*c - a^2*b^2*d - a^3*b*
e + 3*a^4*f)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^4*x^3 + a^4*b^3*x)]
```

Sympy [A] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.76

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^2}\,dx = \frac{\sqrt{-\frac{1}{a^5b^5}}\cdot\left(3a^3f-a^2be-ab^2d+3b^3c\right)\log\left(-a^3b^2\sqrt{-\frac{1}{a^5b^5}}+x\right)}{4} - \frac{\sqrt{-\frac{1}{a^5b^5}}\cdot\left(3a^3f-a^2be-ab^2d+3b^3c\right)\log\left(a^3b^2\sqrt{-\frac{1}{a^5b^5}}+x\right)}{4} + \frac{-2ab^2c+x^2(a^3f-a^2be+ab^2d-3b^3c)}{2a^3b^2x+2a^2b^3x^3} + \frac{fx}{b^2}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a)**2,x)
```

```
[Out] sqrt(-1/(a**5*b**5))*(3*a**3*f - a**2*b*e - a*b**2*d + 3*b**3*c)*log(-a**3*
b**2*sqrt(-1/(a**5*b**5)) + x)/4 - sqrt(-1/(a**5*b**5))*(3*a**3*f - a**2*b*
e - a*b**2*d + 3*b**3*c)*log(a**3*b**2*sqrt(-1/(a**5*b**5)) + x)/4 + (-2*a*
b**2*c + x**2*(a**3*f - a**2*b*e + a*b**2*d - 3*b**3*c))/(2*a**3*b**2*x + 2
*a**2*b**3*x**3) + f*x/b**2
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^2}\,dx = -\frac{2\,ab^2c+(3\,b^3c-ab^2d+a^2be-a^3f)x^2}{2\,(a^2b^3x^3+a^3b^2x)} + \frac{fx}{b^2} - \frac{(3\,b^3c-ab^2d-a^2be+3\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2b^2}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*b^2*c + (3*b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a^2*b^3*x^3 +
 a^3*b^2*x) + f*x/b^2 - 1/2*(3*b^3*c - a*b^2*d - a^2*b*e + 3*a^3*f)*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^2}\,dx = \frac{fx}{b^2} - \frac{\left(3\,b^3c-ab^2d-a^2be+3\,a^3f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2b^2} - \frac{3\,b^3cx^2-ab^2dx^2+a^2bex^2-a^3fx^2+2\,ab^2c}{2\,(bx^3+ax)a^2b^2}$$

[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="giac")

[Out] f*x/b^2 - 1/2*(3*b^3*c - a*b^2*d - a^2*b*e + 3*a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^2) - 1/2*(3*b^3*c*x^2 - a*b^2*d*x^2 + a^2*b*e*x^2 - a^3*f*x^2 + 2*a*b^2*c)/((b*x^3 + a*x)*a^2*b^2)

Mupad [B] (verification not implemented)

Time = 6.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^2}\,dx = \frac{f\,x}{b^2} - \frac{\frac{x^2\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+3\,c\,b^3\right)}{2\,a^2}+\frac{b^2\,c}{a}}{b^3\,x^3+a\,b^2\,x} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(3\,f\,a^3-e\,a^2\,b-d\,a\,b^2+3\,c\,b^3\right)}{2\,a^{5/2}\,b^{5/2}}$$

[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^2),x)

[Out] (f*x)/b^2 - ((x^2*(3*b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(2*a^2) + (b^2*c)/a)/(b^3*x^3 + a*b^2*x) - (atan((b^(1/2)*x)/a^(1/2))*(3*b^3*c + 3*a^3*f - a*b^2*d - a^2*b*e))/(2*a^(5/2)*b^(5/2))

3.129 $\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)^2} dx$

Optimal result 772
Rubi [A] (verified) 772
Mathematica [A] (verified) 774
Maple [A] (verified) 774
Fricas [A] (verification not implemented) 774
Sympy [A] (verification not implemented) 775
Maxima [A] (verification not implemented) 776
Giac [A] (verification not implemented) 776
Mupad [B] (verification not implemented) 777

Optimal result

Integrand size = 30, antiderivative size = 121

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)^2} dx = -\frac{c}{3a^2x^3} + \frac{2bc-ad}{a^3x} + \frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{2a(a+bx^2)} + \frac{(5b^3c-3ab^2d+a^2be+a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}b^{3/2}}$$

```
[Out] -1/3*c/a^2/x^3+(-a*d+2*b*c)/a^3/x+1/2*(b^2*c/a^2-b*d/a+e-a*f/b)*x/a/(b*x^2+
a)+1/2*(a^3*f+a^2*b*e-3*a*b^2*d+5*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/
b^(3/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1819, 1275, 211}

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)^2} dx = \frac{2bc-ad}{a^3x} + \frac{x\left(\frac{b^2c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{2a(a+bx^2)} - \frac{c}{3a^2x^3} + \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(a^3f+a^2be-3ab^2d+5b^3c)}{2a^{7/2}b^{3/2}}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -1/3*c/(a^2*x^3) + (2*b*c - a*d)/(a^3*x) + (((b^2*c)/a^2 - (b*d)/a + e - (a
*f)/b)*x)/(2*a*(a + b*x^2)) + ((5*b^3*c - 3*a*b^2*d + a^2*b*e + a^3*f)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{2a\left(a+bx^2\right)}-\frac{\int \frac{-2c+2\left(\frac{bc}{a}-d\right)x^2+\left(-\frac{b^2c}{a^2}+\frac{bd}{a}-e-\frac{af}{b}\right)x^4}{x^4(a+bx^2)}\,dx}{2a}$$

$$=\frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{2a\left(a+bx^2\right)}-\frac{\int \left(-\frac{2c}{ax^4}-\frac{2(-2bc+ad)}{a^2x^2}+\frac{-5b^3c+3ab^2d-a^2be-a^3f}{a^2b(a+bx^2)}\right)dx}{2a}$$

$$=-\frac{c}{3a^2x^3}+\frac{2bc-ad}{a^3x}+\frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{2a\left(a+bx^2\right)}+\frac{\left(5b^3c-3ab^2d+a^2be+a^3f\right)\int \frac{1}{a+bx^2}\,dx}{2a^3b}$$

$$=-\frac{c}{3a^2x^3}+\frac{2bc-ad}{a^3x}+\frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{2a\left(a+bx^2\right)}+\frac{\left(5b^3c-3ab^2d+a^2be+a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}b^{3/2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)^2}\,dx = -\frac{c}{3a^2x^3}+\frac{2bc-ad}{a^3x}-\frac{(-b^3c+ab^2d-a^2be+a^3f)\,x}{2a^3b\,(a+bx^2)} + \frac{(5b^3c-3ab^2d+a^2be+a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}b^{3/2}}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -1/3*c/(a^2*x^3) + (2*b*c - a*d)/(a^3*x) - ((-(b^3*c) + a*b^2*d - a^2*b*e +
 a^3*f)*x)/(2*a^3*b*(a + b*x^2)) + ((5*b^3*c - 3*a*b^2*d + a^2*b*e + a^3*f)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*b^(3/2))
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.96

method	result
default	$-\frac{c}{3a^2x^3}-\frac{ad-2bc}{a^3x}+\frac{-\frac{\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)x}{2b\left(b\,x^2+a\right)}+\frac{\left(f\,a^3+a^2be-3a\,b^2d+5b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2b\sqrt{ab}}}{a^3}$
risch	$\frac{-\frac{\left(f\,a^3-a^2be+3a\,b^2d-5b^3c\right)x^4}{2a^3b}-\frac{(3ad-5bc)x^2}{3a^2}-\frac{c}{3a}}{x^3(b\,x^2+a)}-\frac{\ln(-\sqrt{-ab}\,x+a)f}{4\sqrt{-ab}\,b}-\frac{\ln(-\sqrt{-ab}\,x+a)e}{4\sqrt{-ab}\,a}+\frac{3b\ln(-\sqrt{-ab}\,x+a)d}{4\sqrt{-ab}\,a^2}-\frac{5b^2\ln(-\sqrt{\cdots}}{4\sqrt{-\cdots}}$

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*c/a^2/x^3-(a*d-2*b*c)/a^3/x+1/a^3*(-1/2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/
b*x/(b*x^2+a)+1/2*(a^3*f+a^2*b*e-3*a*b^2*d+5*b^3*c)/b/(a*b)^(1/2)*arctan(b*
x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 378, normalized size of antiderivative = 3.12

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^2}\,dx$$

$$= \left[-\frac{4\,a^3b^2c-6\,(5\,ab^4c-3\,a^2b^3d+a^3b^2e-a^4bf)x^4-4\,(5\,a^2b^3c-3\,a^3b^2d)x^2+3\,((5\,b^4c-3\,ab^3d+a^2b^2e-}{12\,(a^4b^3x^5+a^5b^2x^3)}\right.$$

$$-\frac{2\,a^3b^2c-3\,(5\,ab^4c-3\,a^2b^3d+a^3b^2e-a^4bf)x^4-2\,(5\,a^2b^3c-3\,a^3b^2d)x^2-3\,((5\,b^4c-3\,ab^3d+a^2b^2e+}{6\,(a^4b^3x^5+a^5b^2x^3)}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a^3*b^2*c - 6*(5*a*b^4*c - 3*a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^4
 - 4*(5*a^2*b^3*c - 3*a^3*b^2*d)*x^2 + 3*((5*b^4*c - 3*a*b^3*d + a^2*b^2*e
+ a^3*b*f)*x^5 + (5*a*b^3*c - 3*a^2*b^2*d + a^3*b*e + a^4*f)*x^3)*sqrt(-a*b
)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*b^3*x^5 + a^5*b^2*x^3
), -1/6*(2*a^3*b^2*c - 3*(5*a*b^4*c - 3*a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^
4 - 2*(5*a^2*b^3*c - 3*a^3*b^2*d)*x^2 - 3*((5*b^4*c - 3*a*b^3*d + a^2*b^2*e
 + a^3*b*f)*x^5 + (5*a*b^3*c - 3*a^2*b^2*d + a^3*b*e + a^4*f)*x^3)*sqrt(a*b
)*arctan(sqrt(a*b)*x/a))/(a^4*b^3*x^5 + a^5*b^2*x^3)]
```

Sympy [A] (verification not implemented)

Time = 5.30 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.75

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^2}\,dx$$
$$= -\frac{\sqrt{-\frac{1}{a^7b^3}}\,(a^3f+a^2be-3ab^2d+5b^3c)\log\left(-a^4b\sqrt{-\frac{1}{a^7b^3}}+x\right)}{4}$$
$$+\frac{\sqrt{-\frac{1}{a^7b^3}}\,(a^3f+a^2be-3ab^2d+5b^3c)\log\left(a^4b\sqrt{-\frac{1}{a^7b^3}}+x\right)}{4}$$
$$+\frac{-2a^2bc+x^4(-3a^3f+3a^2be-9ab^2d+15b^3c)+x^2(-6a^2bd+10ab^2c)}{6a^4bx^3+6a^3b^2x^5}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**7*b**3))*(a**3*f + a**2*b*e - 3*a*b**2*d + 5*b**3*c)*log(-a**4
*b*sqrt(-1/(a**7*b**3)) + x)/4 + sqrt(-1/(a**7*b**3))*(a**3*f + a**2*b*e -
3*a*b**2*d + 5*b**3*c)*log(a**4*b*sqrt(-1/(a**7*b**3)) + x)/4 + (-2*a**2*b*
c + x**4*(-3*a**3*f + 3*a**2*b*e - 9*a*b**2*d + 15*b**3*c) + x**2*(-6*a**2*
b*d + 10*a*b**2*c))/(6*a**4*b*x**3 + 6*a**3*b**2*x**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)^2}\,dx = \frac{3\,(5\,b^3c-3\,ab^2d+a^2be-a^3f)x^4-2\,a^2bc+2\,(5\,ab^2c-3\,a^2bd)x^2}{6\,(a^3b^2x^5+a^4bx^3)} + \frac{(5\,b^3c-3\,ab^2d+a^2be+a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3b}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="maxima")
[Out] 1/6*(3*(5*b^3*c - 3*a*b^2*d + a^2*b*e - a^3*f)*x^4 - 2*a^2*b*c + 2*(5*a*b^2
*c - 3*a^2*b*d)*x^2)/(a^3*b^2*x^5 + a^4*b*x^3) + 1/2*(5*b^3*c - 3*a*b^2*d +
 a^2*b*e + a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)^2}\,dx = \frac{(5\,b^3c-3\,ab^2d+a^2be+a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3b} + \frac{b^3cx-ab^2dx+a^2bex-a^3fx}{2\,(bx^2+a)a^3b} + \frac{6\,bcx^2-3\,adx^2-ac}{3\,a^3x^3}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="giac")
[Out] 1/2*(5*b^3*c - 3*a*b^2*d + a^2*b*e + a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a^3*b) + 1/2*(b^3*c*x - a*b^2*d*x + a^2*b*e*x - a^3*f*x)/((b*x^2 + a)*a^3
*b) + 1/3*(6*b*c*x^2 - 3*a*d*x^2 - a*c)/(a^3*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^2}\,dx=\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(f\,a^3+e\,a^2\,b-3\,d\,a\,b^2+5\,c\,b^3\right)}{2\,a^{7/2}\,b^{3/2}}-\frac{\frac{c}{3\,a}+\frac{x^2\,\left(3\,a\,d-5\,b\,c\right)}{3\,a^2}-\frac{x^4\,\left(-f\,a^3+e\,a^2\,b-3\,d\,a\,b^2+5\,c\,b^3\right)}{2\,a^3\,b}}{b\,x^5+a\,x^3}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^2),x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(5*b^3*c + a^3*f - 3*a*b^2*d + a^2*b*e))/(2*a^(7
/2)*b^(3/2)) - (c/(3*a) + (x^2*(3*a*d - 5*b*c))/(3*a^2) - (x^4*(5*b^3*c - a
^3*f - 3*a*b^2*d + a^2*b*e))/(2*a^3*b))/(a*x^3 + b*x^5)
```

3.130 $\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^2}\,dx$

Optimal result . . . 778
Rubi [A] (verified) . . . 778
Mathematica [A] (verified) . . . 780
Maple [A] (verified) . . . 780
Fricas [A] (verification not implemented) . . . 781
Sympy [A] (verification not implemented) . . . 781
Maxima [A] (verification not implemented) . . . 782
Giac [A] (verification not implemented) . . . 782
Mupad [B] (verification not implemented) . . . 783

Optimal result

Integrand size = 30, antiderivative size = 152

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^2}\,dx = -\frac{c}{5a^2x^5}+\frac{2bc-ad}{3a^3x^3}-\frac{3b^2c-2abd+a^2e}{a^4x} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^4\left(a+bx^2\right)} - \frac{\left(7b^3c-5ab^2d+3a^2be-a^3f\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}\sqrt{b}}$$

```
[Out] -1/5*c/a^2/x^5+1/3*(-a*d+2*b*c)/a^3/x^3+(-a^2*e+2*a*b*d-3*b^2*c)/a^4/x-1/2*
(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^4/(b*x^2+a)-1/2*(-a^3*f+3*a^2*b*e-5*a*b^
2*d+7*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(9/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1819, 1816, 211}

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^2}\,dx = \frac{2bc-ad}{3a^3x^3}-\frac{c}{5a^2x^5}-\frac{a^2e-2abd+3b^2c}{a^4x} - \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+3a^2be-5ab^2d+7b^3c\right)}{2a^{9/2}\sqrt{b}} - \frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2a^4\left(a+bx^2\right)}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^2),x]
```

```
[Out] -1/5*c/(a^2*x^5) + (2*b*c - a*d)/(3*a^3*x^3) - (3*b^2*c - 2*a*b*d + a^2*e)/
(a^4*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(2*a^4*(a + b*x^2)) - ((7
*b^3*c - 5*a*b^2*d + 3*a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(
9/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

integral

$$= -\frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{2a^4\,(a + bx^2)} - \frac{\int \frac{-2c+2\left(\frac{bc}{a}-d\right)x^2-\frac{2\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{a^3}}{x^6(a+bx^2)}\,dx}{2a}$$

$$= -\frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{2a^4\,(a + bx^2)} - \frac{\int \left(-\frac{2c}{ax^6} - \frac{2(-2bc+ad)}{a^2x^4} - \frac{2\left(3b^2c-2abd+a^2e\right)}{a^3x^2} + \frac{7b^3c-5ab^2d+3a^2be-a^3f}{a^3(a+bx^2)}\right)dx}{2a}$$

$$= -\frac{c}{5a^2x^5} + \frac{2bc - ad}{3a^3x^3} - \frac{3b^2c - 2abd + a^2e}{a^4x} - \frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{2a^4\,(a + bx^2)} - \frac{(7b^3c - 5ab^2d + 3a^2be - a^3f)\int \frac{1}{a+bx^2}\,dx}{2a^4}$$

$$= -\frac{c}{5a^2x^5} + \frac{2bc-ad}{3a^3x^3} - \frac{3b^2c-2abd+a^2e}{a^4x} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{2a^4\,(a+bx^2)} - \frac{(7b^3c-5ab^2d+3a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\,(a+bx^2)^2}\,dx = -\frac{c}{5a^2x^5} + \frac{2bc-ad}{3a^3x^3} + \frac{-3b^2c+2abd-a^2e}{a^4x} + \frac{(-b^3c+ab^2d-a^2be+a^3f)\,x}{2a^4\,(a+bx^2)} + \frac{(-7b^3c+5ab^2d-3a^2be+a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}\sqrt{b}}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^2),x]
```

```
[Out] -1/5*c/(a^2*x^5) + (2*b*c - a*d)/(3*a^3*x^3) + (-3*b^2*c + 2*a*b*d - a^2*e)
/(a^4*x) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(2*a^4*(a + b*x^2)) +
 ((-7*b^3*c + 5*a*b^2*d - 3*a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
2*a^(9/2)*Sqrt[b])
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.90

method	result
default	$-\frac{c}{5a^2x^5}-\frac{ad-2bc}{3a^3x^3}-\frac{a^2e-2abd+3b^2c}{a^4x}+\frac{\frac{\left(\frac{1}{2}fa^3-\frac{1}{2}a^2be+\frac{1}{2}ab^2d-\frac{1}{2}b^3c\right)x}{bx^2+a}+\frac{\left(fa^3-3a^2be+5ab^2d-7b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{a^4}$
risch	$\frac{\frac{\left(fa^3-3a^2be+5ab^2d-7b^3c\right)x^6}{2a^4}-\frac{\left(3a^2e-5abd+7b^2c\right)x^4}{3a^3}-\frac{(5ad-7bc)x^2}{15a^2}-\frac{c}{5a}}{x^5(bx^2+a)}+ \Big(\sum_{\_R=\mathrm{RootOf}\left(a^9\_Z^2b+a^6f^2-6a^5bef+10a^4b^2df+9a^4b^2e^2-14\right.}$

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*c/a^2/x^5-1/3*(a*d-2*b*c)/a^3/x^3-(a^2*e-2*a*b*d+3*b^2*c)/a^4/x+1/a^4*
((1/2*f*a^3-1/2*a^2*b*e+1/2*a*b^2*d-1/2*b^3*c)*x/(b*x^2+a)+1/2*(a^3*f-3*a^2
*b*e+5*a*b^2*d-7*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.88

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^2}\,dx$$
$$= \left[-\frac{30\,(7\,ab^4c-5\,a^2b^3d+3\,a^3b^2e-a^4bf)x^6+12\,a^4bc+20\,(7\,a^2b^3c-5\,a^3b^2d+3\,a^4be)x^4-4\,(7\,a^3b^2c-5}{60\,(}\right.$$
$$-\frac{15\,(7\,ab^4c-5\,a^2b^3d+3\,a^3b^2e-a^4bf)x^6+6\,a^4bc+10\,(7\,a^2b^3c-5\,a^3b^2d+3\,a^4be)x^4-2\,(7\,a^3b^2c-5\,a}{30\,(a^5b^2}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(30*(7*a*b^4*c - 5*a^2*b^3*d + 3*a^3*b^2*e - a^4*b*f)*x^6 + 12*a^4*b
*c + 20*(7*a^2*b^3*c - 5*a^3*b^2*d + 3*a^4*b*e)*x^4 - 4*(7*a^3*b^2*c - 5*a^
4*b*d)*x^2 - 15*((7*b^4*c - 5*a*b^3*d + 3*a^2*b^2*e - a^3*b*f)*x^7 + (7*a*b
^3*c - 5*a^2*b^2*d + 3*a^3*b*e - a^4*f)*x^5)*sqrt(-a*b)*log((b*x^2 - 2*sqrt
(-a*b)*x - a)/(b*x^2 + a)))/(a^5*b^2*x^7 + a^6*b*x^5), -1/30*(15*(7*a*b^4*c
 - 5*a^2*b^3*d + 3*a^3*b^2*e - a^4*b*f)*x^6 + 6*a^4*b*c + 10*(7*a^2*b^3*c -
 5*a^3*b^2*d + 3*a^4*b*e)*x^4 - 2*(7*a^3*b^2*c - 5*a^4*b*d)*x^2 + 15*((7*b^
4*c - 5*a*b^3*d + 3*a^2*b^2*e - a^3*b*f)*x^7 + (7*a*b^3*c - 5*a^2*b^2*d + 3
*a^3*b*e - a^4*f)*x^5)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^5*b^2*x^7 + a^6*
b*x^5)]
```

Sympy [A] (verification not implemented)

Time = 19.29 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.49

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^2}\,dx = -\frac{\sqrt{-\frac{1}{a^9b}}(a^3f-3a^2be+5ab^2d-7b^3c)\log\left(-a^5\sqrt{-\frac{1}{a^9b}}+x\right)}{4}$$
$$+\frac{\sqrt{-\frac{1}{a^9b}}(a^3f-3a^2be+5ab^2d-7b^3c)\log\left(a^5\sqrt{-\frac{1}{a^9b}}+x\right)}{4}$$
$$+\frac{-6a^3c+x^6\cdot(15a^3f-45a^2be+75ab^2d-105b^3c)+x^4(-30a^3e+50a^2bd-70ab^2c)+x^2(-10a^3d+14}{30a^5x^5+30a^4bx^7}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**9*b))*(a**3*f - 3*a**2*b*e + 5*a*b**2*d - 7*b**3*c)*log(-a**5*
sqrt(-1/(a**9*b)) + x)/4 + sqrt(-1/(a**9*b))*(a**3*f - 3*a**2*b*e + 5*a*b**
2*d - 7*b**3*c)*log(a**5*sqrt(-1/(a**9*b)) + x)/4 + (-6*a**3*c + x**6*(15*a
```

```
**3*f - 45*a**2*b*e + 75*a*b**2*d - 105*b**3*c) + x**4*(-30*a**3*e + 50*a**
2*b*d - 70*a*b**2*c) + x**2*(-10*a**3*d + 14*a**2*b*c))/(30*a**5*x**5 + 30*
a**4*b*x**7)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^2}\,dx = -\frac{15\,(7\,b^3c-5\,ab^2d+3\,a^2be-a^3f)x^6+10\,(7\,ab^2c-5\,a^2bd+3\,a^3e)x^4+6\,a^3c-2\,(7\,a^2bc-5\,a^3d)x^2}{30\,(a^4bx^7+a^5x^5)} - \frac{(7\,b^3c-5\,ab^2d+3\,a^2be-a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^4}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/30*(15*(7*b^3*c - 5*a*b^2*d + 3*a^2*b*e - a^3*f)*x^6 + 10*(7*a*b^2*c - 5
*a^2*b*d + 3*a^3*e)*x^4 + 6*a^3*c - 2*(7*a^2*b*c - 5*a^3*d)*x^2)/(a^4*b*x^7
 + a^5*x^5) - 1/2*(7*b^3*c - 5*a*b^2*d + 3*a^2*b*e - a^3*f)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a^4)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^2}\,dx = -\frac{(7\,b^3c-5\,ab^2d+3\,a^2be-a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^4} - \frac{b^3cx-ab^2dx+a^2bex-a^3fx}{2\,(bx^2+a)a^4} - \frac{45\,b^2cx^4-30\,abdx^4+15\,a^2ex^4-10\,abcx^2+5\,a^2dx^2+3\,a^2c}{15\,a^4x^5}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*b^3*c - 5*a*b^2*d + 3*a^2*b*e - a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(
a*b)*a^4) - 1/2*(b^3*c*x - a*b^2*d*x + a^2*b*e*x - a^3*f*x)/((b*x^2 + a)*a^
4) - 1/15*(45*b^2*c*x^4 - 30*a*b*d*x^4 + 15*a^2*e*x^4 - 10*a*b*c*x^2 + 5*a^
2*d*x^2 + 3*a^2*c)/(a^4*x^5)
```

Mupad [B] (verification not implemented)

Time = 6.01 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^2}\,dx$$
$$= -\frac{\frac{c}{5\,a}+\frac{x^6\,\left(-f\,a^3+3\,e\,a^2\,b-5\,d\,a\,b^2+7\,c\,b^3\right)}{2\,a^4}+\frac{x^2\,\left(5\,a\,d-7\,b\,c\right)}{15\,a^2}+\frac{x^4\,\left(3\,e\,a^2-5\,d\,a\,b+7\,c\,b^2\right)}{3\,a^3}}{b\,x^7+a\,x^5}$$
$$-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-f\,a^3+3\,e\,a^2\,b-5\,d\,a\,b^2+7\,c\,b^3\right)}{2\,a^{9/2}\,\sqrt{b}}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^2),x)
```

```
[Out] - (c/(5*a) + (x^6*(7*b^3*c - a^3*f - 5*a*b^2*d + 3*a^2*b*e))/(2*a^4) + (x^2
*(5*a*d - 7*b*c))/(15*a^2) + (x^4*(7*b^2*c + 3*a^2*e - 5*a*b*d))/(3*a^3))/(
a*x^5 + b*x^7) - (atan((b^(1/2)*x)/a^(1/2))*(7*b^3*c - a^3*f - 5*a*b^2*d +
3*a^2*b*e))/(2*a^(9/2)*b^(1/2))
```

3.131 $\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)^2} dx$

Optimal result . . . 784
Rubi [A] (verified) . . . 784
Mathematica [A] (verified) . . . 786
Maple [A] (verified) . . . 786
Fricas [A] (verification not implemented) . . . 787
Sympy [F(-1)] . . . 787
Maxima [A] (verification not implemented) . . . 788
Giac [A] (verification not implemented) . . . 788
Mupad [B] (verification not implemented) . . . 789

Optimal result

Integrand size = 30, antiderivative size = 189

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)^2} dx = -\frac{c}{7a^2x^7}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{3a^4x^3}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+\frac{b(b^3c-ab^2d+a^2be-a^3f)x}{2a^5(a+bx^2)}+\frac{\sqrt{b}(9b^3c-7ab^2d+5a^2be-3a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{11/2}}$$

```
[Out] -1/7*c/a^2/x^7+1/5*(-a*d+2*b*c)/a^3/x^5+1/3*(-a^2*e+2*a*b*d-3*b^2*c)/a^4/x^
3+(-a^3*f+2*a^2*b*e-3*a*b^2*d+4*b^3*c)/a^5/x+1/2*b*(-a^3*f+a^2*b*e-a*b^2*d+
b^3*c)*x/a^5/(b*x^2+a)+1/2*(-3*a^3*f+5*a^2*b*e-7*a*b^2*d+9*b^3*c)*arctan(x*
b^(1/2)/a^(1/2))*b^(1/2)/a^(11/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1819, 1816, 211}

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)^2} dx = \frac{2bc-ad}{5a^3x^5}-\frac{c}{7a^2x^7}-\frac{a^2e-2abd+3b^2c}{3a^4x^3}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(-3a^3f+5a^2be-7ab^2d+9b^3c)}{2a^{11/2}}+\frac{bx(a^3(-f)+a^2be-ab^2d+b^3c)}{2a^5(a+bx^2)}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{a^5x}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^2),x]
```

```
[Out] -1/7*c/(a^2*x^7) + (2*b*c - a*d)/(5*a^3*x^5) - (3*b^2*c - 2*a*b*d + a^2*e)/
(3*a^4*x^3) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(a^5*x) + (b*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*x)/(2*a^5*(a + b*x^2)) + (Sqrt[b]*(9*b^3*c -
7*a*b^2*d + 5*a^2*b*e - 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{2a^5\,(a+bx^2)} \\
&\quad -\frac{\int \frac{-2c+2\left(\frac{bc}{a}-d\right)x^2-\frac{2\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{2\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{a^3}-\frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x^8}{a^4}}{x^8(a+bx^2)}\,dx}{2a} \\
&= \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{2a^5\,(a+bx^2)} \\
&\quad -\frac{\int \left(-\frac{2c}{ax^8}-\frac{2(-2bc+ad)}{a^2x^6}-\frac{2\left(3b^2c-2abd+a^2e\right)}{a^3x^4}-\frac{2\left(-4b^3c+3ab^2d-2a^2be+a^3f\right)}{a^4x^2}+\frac{b\left(-9b^3c+7ab^2d-5a^2be+3a^3f\right)}{a^4(a+bx^2)}\right)dx}{2a} \\
&= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{3a^4x^3}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x} \\
&\quad +\frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{2a^5\,(a+bx^2)}+\frac{(b(9b^3c-7ab^2d+5a^2be-3a^3f))\int\frac{1}{a+bx^2}\,dx}{2a^5}
\end{aligned}$$

$$= -\frac{c}{7a^2x^7} + \frac{2bc - ad}{5a^3x^5} - \frac{3b^2c - 2abd + a^2e}{3a^4x^3} + \frac{4b^3c - 3ab^2d + 2a^2be - a^3f}{a^5x} + \frac{b(b^3c - ab^2d + a^2be - a^3f)\,x}{2a^5\,(a + bx^2)} + \frac{\sqrt{b}(9b^3c - 7ab^2d + 5a^2be - 3a^3f)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{11/2}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.01

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^8\,(a + bx^2)^2}\,dx = -\frac{c}{7a^2x^7} + \frac{2bc - ad}{5a^3x^5} + \frac{-3b^2c + 2abd - a^2e}{3a^4x^3} + \frac{4b^3c - 3ab^2d + 2a^2be - a^3f}{a^5x} - \frac{b(-b^3c + ab^2d - a^2be + a^3f)\,x}{2a^5\,(a + bx^2)} - \frac{\sqrt{b}(-9b^3c + 7ab^2d - 5a^2be + 3a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{11/2}}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^2),x]
```

```
[Out] -1/7*c/(a^2*x^7) + (2*b*c - a*d)/(5*a^3*x^5) + (-3*b^2*c + 2*a*b*d - a^2*e)
/(3*a^4*x^3) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(a^5*x) - (b*(-(b^
3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(2*a^5*(a + b*x^2)) - (Sqrt[b]*(-9*b^3
*c + 7*a*b^2*d - 5*a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(11
/2))
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.92

method	result
default	$-\frac{c}{7a^2x^7} - \frac{ad-2bc}{5a^3x^5} - \frac{a^2e-2abd+3b^2c}{3a^4x^3} - \frac{f\,a^3-2a^2be+3a\,b^2d-4b^3c}{a^5x} - \frac{b\left(\frac{\left(\frac{1}{2}f\,a^3-\frac{1}{2}a^2be+\frac{1}{2}a\,b^2d-\frac{1}{2}b^3c\right)x}{b\,x^2+a} + \frac{\left(3f\,a^3-5a^2be+7a\,b^2d \cdots\right.}{2\sqrt{\cdots}}\right.}{a^5}$
risch	$\frac{-\frac{b\left(3f\,a^3-5a^2be+7a\,b^2d-9b^3c\right)x^8}{2a^5} - \frac{\left(3f\,a^3-5a^2be+7a\,b^2d-9b^3c\right)x^6}{3a^4} - \frac{\left(5a^2e-7abd+9b^2c\right)x^4}{15a^3} - \frac{(7ad-9bc)x^2}{35a^2} - \frac{c}{7a}}{x^7(b\,x^2+a)} + \frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{11}\_Z^2+\cdots\right)} \cdots\right.}{\cdots}$

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*c/a^2/x^7-1/5*(a*d-2*b*c)/a^3/x^5-1/3*(a^2*e-2*a*b*d+3*b^2*c)/a^4/x^3-
(a^3*f-2*a^2*b*e+3*a*b^2*d-4*b^3*c)/a^5/x-1/a^5*b*((1/2*f*a^3-1/2*a^2*b*e+1
```

```
/2*a*b^2*d-1/2*b^3*c)*x/(b*x^2+a)+1/2*(3*a^3*f-5*a^2*b*e+7*a*b^2*d-9*b^3*c)
/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.58

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^2}\,dx$$

$$= \left[\frac{210\,(9\,b^4c-7\,ab^3d+5\,a^2b^2e-3\,a^3bf)x^8+140\,(9\,ab^3c-7\,a^2b^2d+5\,a^3be-3\,a^4f)x^6-60\,a^4c-28\,(9\,a^2}{}\right.$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(210*(9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)*x^8 + 140*(9*a*
b^3*c - 7*a^2*b^2*d + 5*a^3*b*e - 3*a^4*f)*x^6 - 60*a^4*c - 28*(9*a^2*b^2*c
 - 7*a^3*b*d + 5*a^4*e)*x^4 + 12*(9*a^3*b*c - 7*a^4*d)*x^2 - 105*((9*b^4*c
- 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)*x^9 + (9*a*b^3*c - 7*a^2*b^2*d + 5*a
^3*b*e - 3*a^4*f)*x^7)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2
 + a)))/(a^5*b*x^9 + a^6*x^7), 1/210*(105*(9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*
e - 3*a^3*b*f)*x^8 + 70*(9*a*b^3*c - 7*a^2*b^2*d + 5*a^3*b*e - 3*a^4*f)*x^6
 - 30*a^4*c - 14*(9*a^2*b^2*c - 7*a^3*b*d + 5*a^4*e)*x^4 + 6*(9*a^3*b*c - 7
*a^4*d)*x^2 + 105*((9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)*x^9 + (9
*a*b^3*c - 7*a^2*b^2*d + 5*a^3*b*e - 3*a^4*f)*x^7)*sqrt(b/a)*arctan(x*sqrt(
b/a)))/(a^5*b*x^9 + a^6*x^7)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^2}\,dx$$
$$= \frac{105\,(9\,b^4c-7\,ab^3d+5\,a^2b^2e-3\,a^3bf)x^8+70\,(9\,ab^3c-7\,a^2b^2d+5\,a^3be-3\,a^4f)x^6-30\,a^4c-14\,(9\,a^2b^2c-}{210\,(a^5bx^9+a^6x^7)}$$
$$+\frac{(9\,b^4c-7\,ab^3d+5\,a^2b^2e-3\,a^3bf)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^5}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/210*(105*(9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)*x^8 + 70*(9*a*b^
3*c - 7*a^2*b^2*d + 5*a^3*b*e - 3*a^4*f)*x^6 - 30*a^4*c - 14*(9*a^2*b^2*c -
 7*a^3*b*d + 5*a^4*e)*x^4 + 6*(9*a^3*b*c - 7*a^4*d)*x^2)/(a^5*b*x^9 + a^6*x
^7) + 1/2*(9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a^5)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.04

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^2}\,dx$$
$$= \frac{(9\,b^4c-7\,ab^3d+5\,a^2b^2e-3\,a^3bf)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^5}+\frac{b^4cx-ab^3dx+a^2b^2ex-a^3bfx}{2\,(bx^2+a)a^5}$$
$$+\frac{420\,b^3cx^6-315\,ab^2dx^6+210\,a^2bex^6-105\,a^3fx^6-105\,ab^2cx^4+70\,a^2bdx^4-35\,a^3ex^4+42\,a^2bcx^2-21}{105\,a^5x^7}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*a^5) + 1/2*(b^4*c*x - a*b^3*d*x + a^2*b^2*e*x - a^3*b*f*x)/((b*x^
2 + a)*a^5) + 1/105*(420*b^3*c*x^6 - 315*a*b^2*d*x^6 + 210*a^2*b*e*x^6 - 10
5*a^3*f*x^6 - 105*a*b^2*c*x^4 + 70*a^2*b*d*x^4 - 35*a^3*e*x^4 + 42*a^2*b*c*
x^2 - 21*a^3*d*x^2 - 15*a^3*c)/(a^5*x^7)
```

Mupad [B] (verification not implemented)

Time = 5.96 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^2}\,dx=\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-3\,f\,a^3+5\,e\,a^2\,b-7\,d\,a\,b^2+9\,c\,b^3\right)}{2\,a^{11/2}}-\frac{\frac{c}{7\,a}-\frac{x^6\,\left(-3\,f\,a^3+5\,e\,a^2\,b-7\,d\,a\,b^2+9\,c\,b^3\right)}{3\,a^4}+\frac{x^2\,\left(7\,a\,d-9\,b\,c\right)}{35\,a^2}+\frac{x^4\,\left(5\,e\,a^2-7\,d\,a\,b+9\,c\,b^2\right)}{15\,a^3}-\frac{b\,x^8\,\left(-3\,f\,a^3+5\,e\,a^2\,b-7\,d\,a\,b^2+9\,c\,b^3\right)}{2\,a^5}}{b\,x^9+a\,x^7}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^2),x)
```

```
[Out] (b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(9*b^3*c - 3*a^3*f - 7*a*b^2*d + 5*a^2*b
*e))/(2*a^(11/2)) - (c/(7*a) - (x^6*(9*b^3*c - 3*a^3*f - 7*a*b^2*d + 5*a^2*
b*e))/(3*a^4) + (x^2*(7*a*d - 9*b*c))/(35*a^2) + (x^4*(9*b^2*c + 5*a^2*e -
7*a*b*d))/(15*a^3) - (b*x^8*(9*b^3*c - 3*a^3*f - 7*a*b^2*d + 5*a^2*b*e))/(2
*a^5))/(a*x^7 + b*x^9)
```

3.132 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)^2} \, dx$

Optimal result . . . 790
Rubi [A] (verified) . . . 791
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Optimal result

Integrand size = 30, antiderivative size = 230

$$\begin{aligned}\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)^2} \, dx = & -\frac{c}{9a^2x^9}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{5a^4x^5} \\ & +\frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5x^3} \\ & -\frac{b(5b^3c-4ab^2d+3a^2be-2a^3f)}{a^6x} \\ & -\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x}{2a^6(a+bx^2)} \\ & -\frac{b^{3/2}(11b^3c-9ab^2d+7a^2be-5a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{13/2}}\end{aligned}$$

```
[Out] -1/9*c/a^2/x^9+1/7*(-a*d+2*b*c)/a^3/x^7+1/5*(-a^2*e+2*a*b*d-3*b^2*c)/a^4/x^
5+1/3*(-a^3*f+2*a^2*b*e-3*a*b^2*d+4*b^3*c)/a^5/x^3-b*(-2*a^3*f+3*a^2*b*e-4*
a*b^2*d+5*b^3*c)/a^6/x-1/2*b^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^6/(b*x^2+
a)-1/2*b^(3/2)*(-5*a^3*f+7*a^2*b*e-9*a*b^2*d+11*b^3*c)*arctan(x*b^(1/2)/a^(
1/2))/a^(13/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1819, 1816, 211}

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^2}\,dx = \frac{2bc-ad}{7a^3x^7}-\frac{c}{9a^2x^9}-\frac{a^2e-2abd+3b^2c}{5a^4x^5}-\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-5a^3f+7a^2be-9ab^2d+11b^3c\right)}{2a^{13/2}}-\frac{b^2x(a^3(-f)+a^2be-ab^2d+b^3c)}{2a^6\left(a+bx^2\right)}-\frac{b(-2a^3f+3a^2be-4ab^2d+5b^3c)}{a^6x}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{3a^5x^3}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^2),x]
```

```
[Out] -1/9*c/(a^2*x^9) + (2*b*c - a*d)/(7*a^3*x^7) - (3*b^2*c - 2*a*b*d + a^2*e)/
(5*a^4*x^5) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*x^3) - (b*(5
*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/(a^6*x) - (b^2*(b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)*x)/(2*a^6*(a + b*x^2)) - (b^(3/2)*(11*b^3*c - 9*a*b^2*d
+ 7*a^2*b*e - 5*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(13/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x}{2a^6\,(a+bx^2)} - \frac{\int \frac{-2c+2\left(\frac{bc}{a}-d\right)x^2-\frac{2\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{2\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{a^3}-\frac{2b\left(b^3c-ab^2d+a^2be-a^3f\right)x^8}{a^4}+\frac{b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^{10}}{a^5}}{x^{10}(a+bx^2)}\,dx}{2a}$$

$$= -\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x}{2a^6\,(a+bx^2)} - \frac{\int \left(-\frac{2c}{ax^{10}}-\frac{2(-2bc+ad)}{a^2x^8}-\frac{2(3b^2c-2abd+a^2e)}{a^3x^6}-\frac{2(-4b^3c+3ab^2d-2a^2be+a^3f)}{a^4x^4}+\frac{2b(-5b^3c+4ab^2d-3a^2be+2a^3f)}{a^5x^2}-\frac{b^2}{}\right.}{2a}$$

$$= -\frac{c}{9a^2x^9}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5x^3} - \frac{b(5b^3c-4ab^2d+3a^2be-2a^3f)}{a^6x}-\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x}{2a^6\,(a+bx^2)} - \frac{(b^2(11b^3c-9ab^2d+7a^2be-5a^3f))\int\frac{1}{a+bx^2}\,dx}{2a^6}$$

$$= -\frac{c}{9a^2x^9}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5x^3} - \frac{b(5b^3c-4ab^2d+3a^2be-2a^3f)}{a^6x}-\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x}{2a^6\,(a+bx^2)} - \frac{b^{3/2}(11b^3c-9ab^2d+7a^2be-5a^3f)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{13/2}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^2}\,dx = -\frac{c}{9a^2x^9}+\frac{2bc-ad}{7a^3x^7}+\frac{-3b^2c+2abd-a^2e}{5a^4x^5} + \frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5x^3} + \frac{b(-5b^3c+4ab^2d-3a^2be+2a^3f)}{a^6x} + \frac{b^2(-b^3c+ab^2d-a^2be+a^3f)\,x}{2a^6\,(a+bx^2)} + \frac{b^{3/2}(-11b^3c+9ab^2d-7a^2be+5a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{13/2}}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^2),x]
```

```
[Out] -1/9*c/(a^2*x^9) + (2*b*c - a*d)/(7*a^3*x^7) + (-3*b^2*c + 2*a*b*d - a^2*e)
/(5*a^4*x^5) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*x^3) + (b*(
-5*b^3*c + 4*a*b^2*d - 3*a^2*b*e + 2*a^3*f))/(a^6*x) + (b^2*(-(b^3*c) + a*b
^2*d - a^2*b*e + a^3*f)*x)/(2*a^6*(a + b*x^2)) + (b^(3/2)*(-11*b^3*c + 9*a*
b^2*d - 7*a^2*b*e + 5*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(13/2))
```

Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.91

method	result
default	$-\frac{c}{9a^2x^9}-\frac{ad-2bc}{7a^3x^7}-\frac{a^2e-2abd+3b^2c}{5a^4x^5}-\frac{f\,a^3-2a^2be+3a\,b^2d-4b^3c}{3a^5x^3}+\frac{b(2f\,a^3-3a^2be+4a\,b^2d-5b^3c)}{a^6x}+\frac{b^2\left(\frac{\left(\frac{1}{2}f\,a^3-\frac{1}{2}a^2be+\right.}{b\,x^2}\right.}{}$
risch	$\frac{\frac{b^2\left(5f\,a^3-7a^2be+9a\,b^2d-11b^3c\right)x^{10}}{2a^6}+\frac{b\left(5f\,a^3-7a^2be+9a\,b^2d-11b^3c\right)x^8}{3a^5}-\frac{\left(5f\,a^3-7a^2be+9a\,b^2d-11b^3c\right)x^6}{15a^4}-\frac{\left(7a^2e-9abd+11b^2c\right)x^4}{35a^3}-\frac{(9ad-1}{63}}{x^9(b\,x^2+a)}$

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*c/a^2/x^9-1/7*(a*d-2*b*c)/a^3/x^7-1/5*(a^2*e-2*a*b*d+3*b^2*c)/a^4/x^5-
1/3*(a^3*f-2*a^2*b*e+3*a*b^2*d-4*b^3*c)/a^5/x^3+b*(2*a^3*f-3*a^2*b*e+4*a*b^
2*d-5*b^3*c)/a^6/x+b^2/a^6*((1/2*f*a^3-1/2*a^2*b*e+1/2*a*b^2*d-1/2*b^3*c)*x
/(b*x^2+a)+1/2*(5*a^3*f-7*a^2*b*e+9*a*b^2*d-11*b^3*c)/(a*b)^(1/2)*arctan(b*
x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 582, normalized size of antiderivative = 2.53

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^2}\,dx$$

$$=\left[-\frac{630\,(11\,b^5c-9\,ab^4d+7\,a^2b^3e-5\,a^3b^2f)x^{10}+420\,(11\,ab^4c-9\,a^2b^3d+7\,a^3b^2e-5\,a^4bf)x^8-84\,(11\,a^2}{}\right.$$

$$-\frac{315\,(11\,b^5c-9\,ab^4d+7\,a^2b^3e-5\,a^3b^2f)x^{10}+210\,(11\,ab^4c-9\,a^2b^3d+7\,a^3b^2e-5\,a^4bf)x^8-42\,(11\,a^2b}{}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/1260*(630*(11*b^5*c - 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*f)*x^10 + 420
*(11*a*b^4*c - 9*a^2*b^3*d + 7*a^3*b^2*e - 5*a^4*b*f)*x^8 - 84*(11*a^2*b^3*
c - 9*a^3*b^2*d + 7*a^4*b*e - 5*a^5*f)*x^6 + 140*a^5*c + 36*(11*a^3*b^2*c -
 9*a^4*b*d + 7*a^5*e)*x^4 - 20*(11*a^4*b*c - 9*a^5*d)*x^2 + 315*((11*b^5*c
- 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*f)*x^11 + (11*a*b^4*c - 9*a^2*b^3*d +
 7*a^3*b^2*e - 5*a^4*b*f)*x^9)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a
)/(b*x^2 + a)))/(a^6*b*x^11 + a^7*x^9), -1/630*(315*(11*b^5*c - 9*a*b^4*d +
 7*a^2*b^3*e - 5*a^3*b^2*f)*x^10 + 210*(11*a*b^4*c - 9*a^2*b^3*d + 7*a^3*b^
2*e - 5*a^4*b*f)*x^8 - 42*(11*a^2*b^3*c - 9*a^3*b^2*d + 7*a^4*b*e - 5*a^5*f
)*x^6 + 70*a^5*c + 18*(11*a^3*b^2*c - 9*a^4*b*d + 7*a^5*e)*x^4 - 10*(11*a^4
*b*c - 9*a^5*d)*x^2 + 315*((11*b^5*c - 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*
f)*x^11 + (11*a*b^4*c - 9*a^2*b^3*d + 7*a^3*b^2*e - 5*a^4*b*f)*x^9)*sqrt(b/
a)*arctan(x*sqrt(b/a)))/(a^6*b*x^11 + a^7*x^9)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^2}\,dx = -\frac{315\,(11\,b^5c-9\,ab^4d+7\,a^2b^3e-5\,a^3b^2f)x^{10}+210\,(11\,ab^4c-9\,a^2b^3d+7\,a^3b^2e-5\,a^4bf)x^8-42\,(11\,a^2b^3}{630\,(a^6bx^{11}} - \frac{(11\,b^5c-9\,ab^4d+7\,a^2b^3e-5\,a^3b^2f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^6}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/630*(315*(11*b^5*c - 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*f)*x^10 + 210*(
11*a*b^4*c - 9*a^2*b^3*d + 7*a^3*b^2*e - 5*a^4*b*f)*x^8 - 42*(11*a^2*b^3*c
- 9*a^3*b^2*d + 7*a^4*b*e - 5*a^5*f)*x^6 + 70*a^5*c + 18*(11*a^3*b^2*c - 9*
a^4*b*d + 7*a^5*e)*x^4 - 10*(11*a^4*b*c - 9*a^5*d)*x^2)/(a^6*b*x^11 + a^7*x
^9) - 1/2*(11*b^5*c - 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*f)*arctan(b*x/sqr
t(a*b))/(sqrt(a*b)*a^6)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.07

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)^2}\,dx$$
$$= -\frac{(11\,b^5c-9\,ab^4d+7\,a^2b^3e-5\,a^3b^2f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^6} - \frac{b^5cx-ab^4dx+a^2b^3ex-a^3b^2fx}{2\,(bx^2+a)a^6}$$
$$-\frac{1575\,b^4cx^8-1260\,ab^3dx^8+945\,a^2b^2ex^8-630\,a^3bfx^8-420\,ab^3cx^6+315\,a^2b^2dx^6-210\,a^3bex^6+105\,a}{315\,a^6x^9}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(11*b^5*c - 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*f)*arctan(b*x/sqrt(a*b
))/(sqrt(a*b)*a^6) - 1/2*(b^5*c*x - a*b^4*d*x + a^2*b^3*e*x - a^3*b^2*f*x)/
((b*x^2 + a)*a^6) - 1/315*(1575*b^4*c*x^8 - 1260*a*b^3*d*x^8 + 945*a^2*b^2*
e*x^8 - 630*a^3*b*f*x^8 - 420*a*b^3*c*x^6 + 315*a^2*b^2*d*x^6 - 210*a^3*b*e
*x^6 + 105*a^4*f*x^6 + 189*a^2*b^2*c*x^4 - 126*a^3*b*d*x^4 + 63*a^4*e*x^4 -
 90*a^3*b*c*x^2 + 45*a^4*d*x^2 + 35*a^4*c)/(a^6*x^9)
```

Mupad [B] (verification not implemented)

Time = 5.61 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)^2}\,dx =$$
$$-\frac{\frac{c}{9\,a}-\frac{x^6\,(-5\,f\,a^3+7\,e\,a^2\,b-9\,d\,a\,b^2+11\,c\,b^3)}{15\,a^4}+\frac{x^2\,(9\,a\,d-11\,b\,c)}{63\,a^2}+\frac{x^4\,(7\,e\,a^2-9\,d\,a\,b+11\,c\,b^2)}{35\,a^3}+\frac{b\,x^8\,(-5\,f\,a^3+7\,e\,a^2\,b-9\,d\,a\,b^2+11\,c\,b^3}{3\,a^5}}{b\,x^{11}+a\,x^9}$$
$$-\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(-5\,f\,a^3+7\,e\,a^2\,b-9\,d\,a\,b^2+11\,c\,b^3)}{2\,a^{13/2}}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^2),x)
```

```
[Out] - (c/(9*a) - (x^6*(11*b^3*c - 5*a^3*f - 9*a*b^2*d + 7*a^2*b*e))/(15*a^4) +
(x^2*(9*a*d - 11*b*c))/(63*a^2) + (x^4*(11*b^2*c + 7*a^2*e - 9*a*b*d))/(35*
a^3) + (b*x^8*(11*b^3*c - 5*a^3*f - 9*a*b^2*d + 7*a^2*b*e))/(3*a^5) + (b^2*
x^10*(11*b^3*c - 5*a^3*f - 9*a*b^2*d + 7*a^2*b*e))/(2*a^6))/(a*x^9 + b*x^11
) - (b^(3/2)*atan((b^(1/2)*x)/a^(1/2))*(11*b^3*c - 5*a^3*f - 9*a*b^2*d + 7*
a^2*b*e))/(2*a^(13/2))
```

3.133 $\int \frac{x^8(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$

Optimal result 796
Rubi [A] (verified) 797
Mathematica [A] (verified) 799
Maple [A] (verified) 800
Fricas [A] (verification not implemented) 800
Sympy [A] (verification not implemented) 801
Maxima [A] (verification not implemented) 802
Giac [A] (verification not implemented) 802
Mupad [B] (verification not implemented) 803

Optimal result

Integrand size = 30, antiderivative size = 287

$$\begin{aligned}\int \frac{x^8(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = &-\frac{a(15b^3c-27ab^2d+43a^2be-63a^3f)\,x}{4b^7}\\ &+\frac{(5b^3c-9ab^2d+15a^2be-23a^3f)\,x^3}{6b^6}\\ &-\frac{(5b^3c-9ab^2d+17a^2be-29a^3f)\,x^5}{20ab^5}\\ &+\frac{(be-3af)x^7}{7b^4}+\frac{fx^9}{9b^3}+\frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^9}{4a\,(a+bx^2)^2}\\ &-\frac{a^2(5b^3c-9ab^2d+13a^2be-17a^3f)\,x}{8b^7\,(a+bx^2)}\\ &+\frac{a^{3/2}(35b^3c-63ab^2d+99a^2be-143a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{15/2}}\end{aligned}$$

```
[Out] -1/4*a*(-63*a^3*f+43*a^2*b*e-27*a*b^2*d+15*b^3*c)*x/b^7+1/6*(-23*a^3*f+15*a
^2*b*e-9*a*b^2*d+5*b^3*c)*x^3/b^6-1/20*(-29*a^3*f+17*a^2*b*e-9*a*b^2*d+5*b^
3*c)*x^5/a/b^5+1/7*(-3*a*f+b*e)*x^7/b^4+1/9*f*x^9/b^3+1/4*(c-a*(a^2*f-a*b*e
+b^2*d)/b^3)*x^9/a/(b*x^2+a)^2-1/8*a^2*(-17*a^3*f+13*a^2*b*e-9*a*b^2*d+5*b^
3*c)*x/b^7/(b*x^2+a)+1/8*a^(3/2)*(-143*a^3*f+99*a^2*b*e-63*a*b^2*d+35*b^3*c
)*arctan(x*b^(1/2)/a^(1/2))/b^(15/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1818, 1599, 1271, 1824, 211}

$$\int \frac{x^8(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = \frac{x^9\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a(a+bx^2)^2} - \frac{a^2x(-17a^3f+13a^2be-9ab^2d+5b^3c)}{8b^7(a+bx^2)} - \frac{ax(-63a^3f+43a^2be-27ab^2d+15b^3c)}{4b^7} + \frac{x^3(-23a^3f+15a^2be-9ab^2d+5b^3c)}{6b^6} - \frac{x^5(-29a^3f+17a^2be-9ab^2d+5b^3c)}{20ab^5} + \frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(-143a^3f+99a^2be-63ab^2d+35b^3c)}{8b^{15/2}} + \frac{x^7(be-3af)}{7b^4} + \frac{fx^9}{9b^3}$$

```
[In] Int[(x^8*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] -1/4*(a*(15*b^3*c - 27*a*b^2*d + 43*a^2*b*e - 63*a^3*f)*x)/b^7 + ((5*b^3*c
- 9*a*b^2*d + 15*a^2*b*e - 23*a^3*f)*x^3)/(6*b^6) - ((5*b^3*c - 9*a*b^2*d +
 17*a^2*b*e - 29*a^3*f)*x^5)/(20*a*b^5) + ((b*e - 3*a*f)*x^7)/(7*b^4) + (f*
x^9)/(9*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^9)/(4*a*(a + b*x^2)
^2) - (a^2*(5*b^3*c - 9*a*b^2*d + 13*a^2*b*e - 17*a^3*f)*x)/(8*b^7*(a + b*x
^2)) + (a^(3/2)*(35*b^3*c - 63*a*b^2*d + 99*a^2*b*e - 143*a^3*f)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(8*b^(15/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^9}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^7\left(\left(5bc-9ad+\frac{9a^2e}{b}-\frac{9a^3f}{b^2}\right)x-4a\left(e-\frac{af}{b}\right)x^3-4afx^5\right)}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^9}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^8\left(5bc-9ad+\frac{9a^2e}{b}-\frac{9a^3f}{b^2}-4a\left(e-\frac{af}{b}\right)x^2-4afx^4\right)}{\left(a+bx^2\right)^2}\,dx}{4ab}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^9}{4a\left(a+bx^2\right)^2} - \frac{a^2(5b^3c-9ab^2d+13a^2be-17a^3f)\,x}{8b^7\left(a+bx^2\right)} + \frac{\int \frac{a^3\left(5b^3c-9ab^2d+13a^2be-17a^3f\right)-2a^2b\left(5b^3c-9ab^2d+13a^2be-17a^3f\right)x^2+2ab^2\left(5b^3c-9ab^2d+13a^2be-17a^3f\right)x^4-2b^3\left(5b^3c-9ab^2 \right.}{a+bx^2}}{8ab^7}$$

$$= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^9}{4a\left(a+bx^2\right)^2} - \frac{a^2(5b^3c-9ab^2d+13a^2be-17a^3f)\,x}{8b^7\left(a+bx^2\right)} + \frac{\int \left(-2a^2(15b^3c-27ab^2d+43a^2be-63a^3f)+4ab(5b^3c-9ab^2d+15a^2be-23a^3f)\,x^2-2b^2(5b^3 \right.}{8a}$$

$$= -\frac{a(15b^3c - 27ab^2d + 43a^2be - 63a^3f)\,x}{4b^7} + \frac{(5b^3c - 9ab^2d + 15a^2be - 23a^3f)\,x^3}{6b^6} - \frac{(5b^3c - 9ab^2d + 17a^2be - 29a^3f)\,x^5}{20ab^5} + \frac{(be - 3af)x^7}{7b^4} + \frac{fx^9}{9b^3} + \frac{\left(c - \frac{a(b^2d - abe + a^2f)}{b^3}\right)x^9}{4a\,(a + bx^2)^2} - \frac{a^2(5b^3c - 9ab^2d + 13a^2be - 17a^3f)\,x}{8b^7\,(a + bx^2)} + \frac{(a^2(35b^3c - 63ab^2d + 99a^2be - 143a^3f))\int \frac{1}{a+bx^2}\,dx}{8b^7}$$

$$= -\frac{a(15b^3c - 27ab^2d + 43a^2be - 63a^3f)\,x}{4b^7} + \frac{(5b^3c - 9ab^2d + 15a^2be - 23a^3f)\,x^3}{6b^6} - \frac{(5b^3c - 9ab^2d + 17a^2be - 29a^3f)\,x^5}{20ab^5} + \frac{(be - 3af)x^7}{7b^4} + \frac{fx^9}{9b^3} + \frac{\left(c - \frac{a(b^2d - abe + a^2f)}{b^3}\right)x^9}{4a\,(a + bx^2)^2} - \frac{a^2(5b^3c - 9ab^2d + 13a^2be - 17a^3f)\,x}{8b^7\,(a + bx^2)} + \frac{a^{3/2}(35b^3c - 63ab^2d + 99a^2be - 143a^3f)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{15/2}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.95

$$\int \frac{x^8(c + dx^2 + ex^4 + fx^6)}{(a + bx^2)^3}\,dx = \frac{a(-3b^3c + 6ab^2d - 10a^2be + 15a^3f)\,x}{b^7} + \frac{(b^3c - 3ab^2d + 6a^2be - 10a^3f)\,x^3}{3b^6} + \frac{(b^2d - 3abe + 6a^2f)\,x^5}{5b^5} + \frac{(be - 3af)x^7}{7b^4} + \frac{fx^9}{9b^3} + \frac{a^3(b^3c - ab^2d + a^2be - a^3f)\,x}{4b^7\,(a + bx^2)^2} + \frac{a^2(-13b^3c + 17ab^2d - 21a^2be + 25a^3f)\,x}{8b^7\,(a + bx^2)} - \frac{a^{3/2}(-35b^3c + 63ab^2d - 99a^2be + 143a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{15/2}}$$

```
[In] Integrate[(x^8*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] (a*(-3*b^3*c + 6*a*b^2*d - 10*a^2*b*e + 15*a^3*f)*x)/b^7 + ((b^3*c - 3*a*b^
2*d + 6*a^2*b*e - 10*a^3*f)*x^3)/(3*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*f)*x^5
)/(5*b^5) + ((b*e - 3*a*f)*x^7)/(7*b^4) + (f*x^9)/(9*b^3) + (a^3*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f)*x)/(4*b^7*(a + b*x^2)^2) + (a^2*(-13*b^3*c + 17*a
*b^2*d - 21*a^2*b*e + 25*a^3*f)*x)/(8*b^7*(a + b*x^2)) - (a^(3/2)*(-35*b^3*
c + 63*a*b^2*d - 99*a^2*b*e + 143*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^
(15/2))
```

Maple [A] (verified)

Time = 3.62 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.93

method	result
default	$\frac{\frac{1}{9}f\,x^9b^4-\frac{3}{7}a\,b^3f\,x^7+\frac{1}{7}b^4e\,x^7+\frac{6}{5}a^2b^2f\,x^5-\frac{3}{5}a\,b^3e\,x^5+\frac{1}{5}b^4d\,x^5-\frac{10}{3}a^3bf\,x^3+2a^2b^2e\,x^3-a\,b^3d\,x^3+\frac{1}{3}b^4c\,x^3+15a^4fx-10a^3bex+6a^2b^2d}{b^7}$
risch	$\frac{f\,x^9}{9b^3}-\frac{3af\,x^7}{7b^4}+\frac{e\,x^7}{7b^3}+\frac{6a^2f\,x^5}{5b^5}-\frac{3ae\,x^5}{5b^4}+\frac{d\,x^5}{5b^3}-\frac{10a^3f\,x^3}{3b^6}+\frac{2a^2e\,x^3}{b^5}-\frac{ad\,x^3}{b^4}+\frac{c\,x^3}{3b^3}+\frac{15a^4fx}{b^7}-\frac{10a^3ex}{b^6}+\frac{6a^2}{b^5}$

```
[In] int(x^8*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^7*(1/9*f*x^9*b^4-3/7*a*b^3*f*x^7+1/7*b^4*e*x^7+6/5*a^2*b^2*f*x^5-3/5*a*
b^3*e*x^5+1/5*b^4*d*x^5-10/3*a^3*b*f*x^3+2*a^2*b^2*e*x^3-a*b^3*d*x^3+1/3*b^
4*c*x^3+15*a^4*f*x-10*a^3*b*e*x+6*a^2*b^2*d*x-3*a*b^3*c*x)-a^2/b^7*(((-25/8
*a^3*b*f+21/8*a^2*e*b^2-17/8*a*b^3*d+13/8*b^4*c)*x^3-1/8*a*(23*a^3*f-19*a^2
*b*e+15*a*b^2*d-11*b^3*c)*x)/(b*x^2+a)^2+1/8*(143*a^3*f-99*a^2*b*e+63*a*b^2
*d-35*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 762, normalized size of antiderivative = 2.66

$$\int \frac{x^8(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$$

$$=\left[\frac{560\,b^6fx^{13}+80\,(9\,b^6e-13\,ab^5f)x^{11}+16\,(63\,b^6d-99\,ab^5e+143\,a^2b^4f)x^9+48\,(35\,b^6c-63\,ab^5d+99\,a^2b}{}\right.$$

```
[In] integrate(x^8*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/5040*(560*b^6*f*x^13 + 80*(9*b^6*e - 13*a*b^5*f)*x^11 + 16*(63*b^6*d - 9
9*a*b^5*e + 143*a^2*b^4*f)*x^9 + 48*(35*b^6*c - 63*a*b^5*d + 99*a^2*b^4*e -
 143*a^3*b^3*f)*x^7 - 336*(35*a*b^5*c - 63*a^2*b^4*d + 99*a^3*b^3*e - 143*a
^4*b^2*f)*x^5 - 1050*(35*a^2*b^4*c - 63*a^3*b^3*d + 99*a^4*b^2*e - 143*a^5*
b*f)*x^3 - 315*(35*a^3*b^3*c - 63*a^4*b^2*d + 99*a^5*b*e - 143*a^6*f + (35*
a*b^5*c - 63*a^2*b^4*d + 99*a^3*b^3*e - 143*a^4*b^2*f)*x^4 + 2*(35*a^2*b^4*
c - 63*a^3*b^3*d + 99*a^4*b^2*e - 143*a^5*b*f)*x^2)*sqrt(-a/b)*log((b*x^2 -
 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 630*(35*a^3*b^3*c - 63*a^4*b^2*d + 99
*a^5*b*e - 143*a^6*f)*x)/(b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7), 1/2520*(280*b^6
*f*x^13 + 40*(9*b^6*e - 13*a*b^5*f)*x^11 + 8*(63*b^6*d - 99*a*b^5*e + 143*a
```

```
^2*b^4*f)*x^9 + 24*(35*b^6*c - 63*a*b^5*d + 99*a^2*b^4*e - 143*a^3*b^3*f)*x
^7 - 168*(35*a*b^5*c - 63*a^2*b^4*d + 99*a^3*b^3*e - 143*a^4*b^2*f)*x^5 - 5
25*(35*a^2*b^4*c - 63*a^3*b^3*d + 99*a^4*b^2*e - 143*a^5*b*f)*x^3 + 315*(35
*a^3*b^3*c - 63*a^4*b^2*d + 99*a^5*b*e - 143*a^6*f + (35*a*b^5*c - 63*a^2*b
^4*d + 99*a^3*b^3*e - 143*a^4*b^2*f)*x^4 + 2*(35*a^2*b^4*c - 63*a^3*b^3*d +
 99*a^4*b^2*e - 143*a^5*b*f)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 315*(
35*a^3*b^3*c - 63*a^4*b^2*d + 99*a^5*b*e - 143*a^6*f)*x)/(b^9*x^4 + 2*a*b^8
*x^2 + a^2*b^7)]
```

Sympy [A] (verification not implemented)

Time = 19.39 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.75

$$\int \frac{x^8(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = x^7\left(-\frac{3af}{7b^4}+\frac{e}{7b^3}\right)+x^5\cdot\left(\frac{6a^2f}{5b^5}-\frac{3ae}{5b^4}+\frac{d}{5b^3}\right)$$
$$+x^3\left(-\frac{10a^3f}{3b^6}+\frac{2a^2e}{b^5}-\frac{ad}{b^4}+\frac{c}{3b^3}\right)+x\left(\frac{15a^4f}{b^7}-\frac{10a^3e}{b^6}+\frac{6a^2d}{b^5}-\frac{3ac}{b^4}\right)$$
$$+\frac{\sqrt{-\frac{a^3}{b^{15}}}\cdot(143a^3f-99a^2be+63ab^2d-35b^3c)\log\left(-\frac{b^7\sqrt{-\frac{a^3}{b^{15}}}\cdot(143a^3f-99a^2be+63ab^2d-35b^3c)}{143a^4f-99a^3be+63a^2b^2d-35ab^3c}+x\right)}{16}$$
$$-\frac{\sqrt{-\frac{a^3}{b^{15}}}\cdot(143a^3f-99a^2be+63ab^2d-35b^3c)\log\left(\frac{b^7\sqrt{-\frac{a^3}{b^{15}}}\cdot(143a^3f-99a^2be+63ab^2d-35b^3c)}{143a^4f-99a^3be+63a^2b^2d-35ab^3c}+x\right)}{16}$$
$$+\frac{x^3\cdot(25a^5bf-21a^4b^2e+17a^3b^3d-13a^2b^4c)+x(23a^6f-19a^5be+15a^4b^2d-11a^3b^3c)}{8a^2b^7+16ab^8x^2+8b^9x^4}$$
$$+\frac{fx^9}{9b^3}$$

```
[In] integrate(x**8*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] x**7*(-3*a*f/(7*b**4) + e/(7*b**3)) + x**5*(6*a**2*f/(5*b**5) - 3*a*e/(5*b*
*4) + d/(5*b**3)) + x**3*(-10*a**3*f/(3*b**6) + 2*a**2*e/b**5 - a*d/b**4 +
c/(3*b**3)) + x*(15*a**4*f/b**7 - 10*a**3*e/b**6 + 6*a**2*d/b**5 - 3*a*c/b*
*4) + sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*a*b**2*d - 35*b**3*c
)*log(-b**7*sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*a*b**2*d - 35*
b**3*c)/(143*a**4*f - 99*a**3*b*e + 63*a**2*b**2*d - 35*a*b**3*c) + x)/16 -
 sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*a*b**2*d - 35*b**3*c)*log
(b**7*sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*a*b**2*d - 35*b**3*c
)/(143*a**4*f - 99*a**3*b*e + 63*a**2*b**2*d - 35*a*b**3*c) + x)/16 + (x**3
*(25*a**5*b*f - 21*a**4*b**2*e + 17*a**3*b**3*d - 13*a**2*b**4*c) + x*(23*a
**6*f - 19*a**5*b*e + 15*a**4*b**2*d - 11*a**3*b**3*c))/(8*a**2*b**7 + 16*a
*b**8*x**2 + 8*b**9*x**4) + f*x**9/(9*b**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.98

$$\int \frac{x^8(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = -\frac{(13\,a^2b^4c-17\,a^3b^3d+21\,a^4b^2e-25\,a^5bf)x^3+(11\,a^3b^3c-15\,a^4b^2d+19\,a^5be-23\,a^6f)x}{8\,(b^9x^4+2\,ab^8x^2+a^2b^7)} + \frac{(35\,a^2b^3c-63\,a^3b^2d+99\,a^4be-143\,a^5f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^7} + \frac{35\,b^4fx^9+45\,(b^4e-3\,ab^3f)x^7+63\,(b^4d-3\,ab^3e+6\,a^2b^2f)x^5+105\,(b^4c-3\,ab^3d+6\,a^2b^2e-10\,a^3bf)x}{315\,b^7}$$

```
[In] integrate(x^8*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((13*a^2*b^4*c - 17*a^3*b^3*d + 21*a^4*b^2*e - 25*a^5*b*f)*x^3 + (11*a
^3*b^3*c - 15*a^4*b^2*d + 19*a^5*b*e - 23*a^6*f)*x)/(b^9*x^4 + 2*a*b^8*x^2
+ a^2*b^7) + 1/8*(35*a^2*b^3*c - 63*a^3*b^2*d + 99*a^4*b*e - 143*a^5*f)*arc
tan(b*x/sqrt(a*b))/(sqrt(a*b)*b^7) + 1/315*(35*b^4*f*x^9 + 45*(b^4*e - 3*a*
b^3*f)*x^7 + 63*(b^4*d - 3*a*b^3*e + 6*a^2*b^2*f)*x^5 + 105*(b^4*c - 3*a*b^
3*d + 6*a^2*b^2*e - 10*a^3*b*f)*x^3 - 315*(3*a*b^3*c - 6*a^2*b^2*d + 10*a^3
*b*e - 15*a^4*f)*x)/b^7
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.02

$$\int \frac{x^8(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = \frac{(35\,a^2b^3c-63\,a^3b^2d+99\,a^4be-143\,a^5f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^7} - \frac{13\,a^2b^4cx^3-17\,a^3b^3dx^3+21\,a^4b^2ex^3-25\,a^5bfx^3+11\,a^3b^3cx-15\,a^4b^2dx+19\,a^5bex-23\,a^6fx}{8\,(bx^2+a)^2b^7} + \frac{35\,b^{24}fx^9+45\,b^{24}ex^7-135\,ab^{23}fx^7+63\,b^{24}dx^5-189\,ab^{23}ex^5+378\,a^2b^{22}fx^5+105\,b^{24}cx^3-315\,ab^{23}dx}{315\,b^{27}}$$

```
[In] integrate(x^8*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(35*a^2*b^3*c - 63*a^3*b^2*d + 99*a^4*b*e - 143*a^5*f)*arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*b^7) - 1/8*(13*a^2*b^4*c*x^3 - 17*a^3*b^3*d*x^3 + 21*a^4*b
^2*e*x^3 - 25*a^5*b*f*x^3 + 11*a^3*b^3*c*x - 15*a^4*b^2*d*x + 19*a^5*b*e*x
- 23*a^6*f*x)/((b*x^2 + a)^2*b^7) + 1/315*(35*b^24*f*x^9 + 45*b^24*e*x^7 -
```

135*a*b^23*f*x^7 + 63*b^24*d*x^5 - 189*a*b^23*e*x^5 + 378*a^2*b^22*f*x^5 +
105*b^24*c*x^3 - 315*a*b^23*d*x^3 + 630*a^2*b^22*e*x^3 - 1050*a^3*b^21*f*x^
3 - 945*a*b^23*c*x + 1890*a^2*b^22*d*x - 3150*a^3*b^21*e*x + 4725*a^4*b^20*
f*x)/b^27

Mupad [B] (verification not implemented)

Time = 5.59 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.76

$$\int \frac{x^8(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$$

$$= x^7\left(\frac{e}{7\,b^3}-\frac{3\,a\,f}{7\,b^4}\right)+x^3\left(\frac{c}{3\,b^3}-\frac{a^3\,f}{3\,b^6}-\frac{a^2\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^2}+\frac{a\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b}\right) + \frac{x\left(\frac{23\,f\,a^6}{8}-\frac{19\,e\,a^5\,b}{8}+\frac{15\,d\,a^4\,b^2}{8}-\frac{11\,c\,a^3\,b^3}{8}\right)-x^3\left(-\frac{25\,f\,a^5\,b}{8}+\frac{21\,e\,a^4\,b^2}{8}-\frac{17\,d\,a^3\,b^3}{8}+\frac{13\,c\,a^2\,b^4}{8}\right)}{a^2\,b^7+2\,a\,b^8\,x^2+b^9\,x^4} - x\left(\frac{3\,a\left(\frac{c}{b^3}-\frac{a^3\,f}{b^6}-\frac{3\,a^2\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^2}+\frac{3\,a\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b}\right)}{b} - \frac{3\,a^2\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b^2}+\frac{a^3\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^3}\right) - x^5\left(\frac{3\,a^2\,f}{5\,b^5}-\frac{d}{5\,b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{5\,b}\right)+\frac{f\,x^9}{9\,b^3} - \frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,\left(-143\,f\,a^3+99\,e\,a^2\,b-63\,d\,a\,b^2+35\,c\,b^3\right)}{143\,f\,a^5-99\,e\,a^4\,b+63\,d\,a^3\,b^2-35\,c\,a^2\,b^3}\right)\,\left(-143\,f\,a^3+99\,e\,a^2\,b-63\,d\,a\,b^2+35\,c\,b^3\right)}{8\,b^{15/2}}$$

[In] int((x^8*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x)

[Out] x^7*(e/(7*b^3) - (3*a*f)/(7*b^4)) + x^3*(c/(3*b^3) - (a^3*f)/(3*b^6) - (a^2
*(e/b^3 - (3*a*f)/b^4))/b^2 + (a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*

```
a*f)/b^4))/b))/b) + (x*((23*a^6*f)/8 - (11*a^3*b^3*c)/8 + (15*a^4*b^2*d)/8
- (19*a^5*b*e)/8) - x^3*((13*a^2*b^4*c)/8 - (17*a^3*b^3*d)/8 + (21*a^4*b^2*
e)/8 - (25*a^5*b*f)/8))/(a^2*b^7 + b^9*x^4 + 2*a*b^8*x^2) - x*((3*a*(c/b^3
- (a^3*f)/b^6 - (3*a^2*(e/b^3 - (3*a*f)/b^4))/b^2 + (3*a*((3*a^2*f)/b^5 - d
/b^3 + (3*a*(e/b^3 - (3*a*f)/b^4))/b))/b))/b - (3*a^2*((3*a^2*f)/b^5 - d/b^
3 + (3*a*(e/b^3 - (3*a*f)/b^4))/b))/b^2 + (a^3*(e/b^3 - (3*a*f)/b^4))/b^3)
- x^5*((3*a^2*f)/(5*b^5) - d/(5*b^3) + (3*a*(e/b^3 - (3*a*f)/b^4))/(5*b)) +
 (f*x^9)/(9*b^3) - (a^(3/2)*atan((a^(3/2)*b^(1/2)*x*(35*b^3*c - 143*a^3*f -
 63*a*b^2*d + 99*a^2*b*e))/(143*a^5*f - 35*a^2*b^3*c + 63*a^3*b^2*d - 99*a^
4*b*e))*(35*b^3*c - 143*a^3*f - 63*a*b^2*d + 99*a^2*b*e))/(8*b^(15/2))
```

3.134 $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$

Optimal result 805
Rubi [A] (verified) 806
Mathematica [A] (verified) 808
Maple [A] (verified) 809
Fricas [A] (verification not implemented) 809
Sympy [A] (verification not implemented) 810
Maxima [A] (verification not implemented) 810
Giac [A] (verification not implemented) 811
Mupad [B] (verification not implemented) 812

Optimal result

Integrand size = 30, antiderivative size = 247

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = \frac{(3b^3c-7ab^2d+13a^2be-21a^3f)\,x}{2b^6} - \frac{(3b^3c-7ab^2d+15a^2be-27a^3f)\,x^3}{12ab^5} + \frac{(be-3af)x^5}{5b^4} + \frac{fx^7}{7b^3} + \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^7}{4a\,(a+bx^2)^2} + \frac{a(3b^3c-7ab^2d+11a^2be-15a^3f)\,x}{8b^6\,(a+bx^2)} - \frac{\sqrt{a}(15b^3c-35ab^2d+63a^2be-99a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{13/2}}$$

```
[Out] 1/2*(-21*a^3*f+13*a^2*b*e-7*a*b^2*d+3*b^3*c)*x/b^6-1/12*(-27*a^3*f+15*a^2*b
*e-7*a*b^2*d+3*b^3*c)*x^3/a/b^5+1/5*(-3*a*f+b*e)*x^5/b^4+1/7*f*x^7/b^3+1/4*
(c-a*(a^2*f-a*b*e+b^2*d)/b^3)*x^7/a/(b*x^2+a)^2+1/8*a*(-15*a^3*f+11*a^2*b*e
-7*a*b^2*d+3*b^3*c)*x/b^6/(b*x^2+a)-1/8*(-99*a^3*f+63*a^2*b*e-35*a*b^2*d+15
*b^3*c)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(13/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1818, 1599, 1271, 1824, 211}

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = \frac{x^7\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a(a+bx^2)^2} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(-99a^3f+63a^2be-35ab^2d+15b^3c)}{8b^{13/2}} + \frac{ax(-15a^3f+11a^2be-7ab^2d+3b^3c)}{8b^6(a+bx^2)} + \frac{x(-21a^3f+13a^2be-7ab^2d+3b^3c)}{2b^6} - \frac{x^3(-27a^3f+15a^2be-7ab^2d+3b^3c)}{12ab^5} + \frac{x^5(be-3af)}{5b^4} + \frac{fx^7}{7b^3}$$

```
[In] Int[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] ((3*b^3*c - 7*a*b^2*d + 13*a^2*b*e - 21*a^3*f)*x)/(2*b^6) - ((3*b^3*c - 7*a
*b^2*d + 15*a^2*b*e - 27*a^3*f)*x^3)/(12*a*b^5) + ((b*e - 3*a*f)*x^5)/(5*b^
4) + (f*x^7)/(7*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^7)/(4*a*(a
+ b*x^2)^2) + (a*(3*b^3*c - 7*a*b^2*d + 11*a^2*b*e - 15*a^3*f)*x)/(8*b^6*(a
 + b*x^2)) - (Sqrt[a]*(15*b^3*c - 35*a*b^2*d + 63*a^2*b*e - 99*a^3*f)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(13/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^7}{4a\,(a+bx^2)^2}-\frac{\int \frac{x^5\left(\left(3bc-7ad+\frac{7a^2e}{b}-\frac{7a^3f}{b^2}\right)x-4a\left(e-\frac{af}{b}\right)x^3-4afx^5\right)}{(a+bx^2)^2}\,dx}{4ab}$$

$$= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^7}{4a\,(a+bx^2)^2}-\frac{\int \frac{x^6\left(3bc-7ad+\frac{7a^2e}{b}-\frac{7a^3f}{b^2}-4a\left(e-\frac{af}{b}\right)x^2-4afx^4\right)}{(a+bx^2)^2}\,dx}{4ab}$$

$$= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^7}{4a\,(a+bx^2)^2}+\frac{a(3b^3c-7ab^2d+11a^2be-15a^3f)\,x}{8b^6\,(a+bx^2)}$$
$$+\frac{\int \frac{-a^2(3b^3c-7ab^2d+11a^2be-15a^3f)+2ab(3b^3c-7ab^2d+11a^2be-15a^3f)x^2-2b^2(3b^3c-7ab^2d+11a^2be-15a^3f)x^4+8ab^3(be-2af)}{a+bx^2}}{8ab^6}$$

$$= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^7}{4a\,(a+bx^2)^2}+\frac{a(3b^3c-7ab^2d+11a^2be-15a^3f)\,x}{8b^6\,(a+bx^2)}$$
$$+\frac{\int \left(4a(3b^3c-7ab^2d+13a^2be-21a^3f)-2b(3b^3c-7ab^2d+15a^2be-27a^3f)\,x^2+8ab^2(be-3a\right.}{8ab^6}$$

$$= \frac{(3b^3c - 7ab^2d + 13a^2be - 21a^3f)\,x}{2b^6} - \frac{(3b^3c - 7ab^2d + 15a^2be - 27a^3f)\,x^3}{12ab^5}$$
$$+ \frac{(be - 3af)x^5}{5b^4} + \frac{fx^7}{7b^3} + \frac{\left(c - \frac{a(b^2d - abe + a^2f)}{b^3}\right)x^7}{4a\,(a + bx^2)^2}$$
$$+ \frac{a(3b^3c - 7ab^2d + 11a^2be - 15a^3f)\,x}{8b^6\,(a + bx^2)}$$
$$- \frac{(a(15b^3c - 35ab^2d + 63a^2be - 99a^3f))\int \frac{1}{a+bx^2}\,dx}{8b^6}$$
$$= \frac{(3b^3c - 7ab^2d + 13a^2be - 21a^3f)\,x}{2b^6} - \frac{(3b^3c - 7ab^2d + 15a^2be - 27a^3f)\,x^3}{12ab^5}$$
$$+ \frac{(be - 3af)x^5}{5b^4} + \frac{fx^7}{7b^3} + \frac{\left(c - \frac{a(b^2d - abe + a^2f)}{b^3}\right)x^7}{4a\,(a + bx^2)^2}$$
$$+ \frac{a(3b^3c - 7ab^2d + 11a^2be - 15a^3f)\,x}{8b^6\,(a + bx^2)}$$
$$- \frac{\sqrt{a}(15b^3c - 35ab^2d + 63a^2be - 99a^3f)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{13/2}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.94

$$\int \frac{x^6(c + dx^2 + ex^4 + fx^6)}{(a + bx^2)^3}\,dx = \frac{(b^3c - 3ab^2d + 6a^2be - 10a^3f)\,x}{b^6} + \frac{(b^2d - 3abe + 6a^2f)\,x^3}{3b^5}$$
$$+ \frac{(be - 3af)x^5}{5b^4} + \frac{fx^7}{7b^3} + \frac{a^2(-b^3c + ab^2d - a^2be + a^3f)\,x}{4b^6\,(a + bx^2)^2}$$
$$+ \frac{a(9b^3c - 13ab^2d + 17a^2be - 21a^3f)\,x}{8b^6\,(a + bx^2)}$$
$$+ \frac{\sqrt{a}(-15b^3c + 35ab^2d - 63a^2be + 99a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{13/2}}$$

```
[In] Integrate[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x)/b^6 + ((b^2*d - 3*a*b*e + 6*
a^2*f)*x^3)/(3*b^5) + ((b*e - 3*a*f)*x^5)/(5*b^4) + (f*x^7)/(7*b^3) + (a^2*
(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(4*b^6*(a + b*x^2)^2) + (a*(9*b^3
*c - 13*a*b^2*d + 17*a^2*b*e - 21*a^3*f)*x)/(8*b^6*(a + b*x^2)) + (Sqrt[a]*
(-15*b^3*c + 35*a*b^2*d - 63*a^2*b*e + 99*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]
])/(8*b^(13/2))
```

Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.89

method	result
default	$-\frac{-\frac{1}{7} f x^7 b^3+\frac{3}{5} a b^2 f x^5-\frac{1}{5} b^3 e x^5-2 a^2 b f x^3+a b^2 e x^3-\frac{1}{3} b^3 d x^3+10 f a^3 x-6 a^2 b e x+3 a b^2 d x-b^3 c x}{b^6}+\frac{a\left(\frac{\left(-\frac{21}{8} a^3 b f+\frac{17}{8} a^2 e b^2-\frac{13}{8} a \cdots\right.}{\cdots}\right.}{\cdots}$
risch	$\frac{f x^7}{7 b^3}-\frac{3 a f x^5}{5 b^4}+\frac{e x^5}{5 b^3}+\frac{2 a^2 f x^3}{b^5}-\frac{a e x^3}{b^4}+\frac{d x^3}{3 b^3}-\frac{10 f a^3 x}{b^6}+\frac{6 a^2 e x}{b^5}-\frac{3 a d x}{b^4}+\frac{c x}{b^3}+\frac{\left(-\frac{21}{8} a^4 b f+\frac{17}{8} a^3 b^2 e-\frac{13}{8} a^2 b^3 d-\cdots\right.}{\cdots}$

```
[In] int(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^6*(-1/7*f*x^7*b^3+3/5*a*b^2*f*x^5-1/5*b^3*e*x^5-2*a^2*b*f*x^3+a*b^2*e*
x^3-1/3*b^3*d*x^3+10*f*a^3*x-6*a^2*b*e*x+3*a*b^2*d*x-b^3*c*x)+a/b^6*(((-21/
8*a^3*b*f+17/8*a^2*e*b^2-13/8*a*b^3*d+9/8*b^4*c)*x^3-1/8*a*(19*a^3*f-15*a^2
*b*e+11*a*b^2*d-7*b^3*c)*x)/(b*x^2+a)^2+1/8*(99*a^3*f-63*a^2*b*e+35*a*b^2*d
-15*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 668, normalized size of antiderivative = 2.70

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$$

$$= \left[\frac{240\,b^5 f x^{11}+48\,(7\,b^5 e-11\,ab^4 f)x^9+16\,(35\,b^5 d-63\,ab^4 e+99\,a^2 b^3 f)x^7+112\,(15\,b^5 c-35\,ab^4 d+63\,a^2 \cdots}{\cdots}\right.$$

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/1680*(240*b^5*f*x^11 + 48*(7*b^5*e - 11*a*b^4*f)*x^9 + 16*(35*b^5*d - 63
*a*b^4*e + 99*a^2*b^3*f)*x^7 + 112*(15*b^5*c - 35*a*b^4*d + 63*a^2*b^3*e -
99*a^3*b^2*f)*x^5 + 350*(15*a*b^4*c - 35*a^2*b^3*d + 63*a^3*b^2*e - 99*a^4*
b*f)*x^3 - 105*(15*a^2*b^3*c - 35*a^3*b^2*d + 63*a^4*b*e - 99*a^5*f + (15*b
^5*c - 35*a*b^4*d + 63*a^2*b^3*e - 99*a^3*b^2*f)*x^4 + 2*(15*a*b^4*c - 35*a
^2*b^3*d + 63*a^3*b^2*e - 99*a^4*b*f)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sq
rt(-a/b) - a)/(b*x^2 + a)) + 210*(15*a^2*b^3*c - 35*a^3*b^2*d + 63*a^4*b*e
- 99*a^5*f)*x)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6), 1/840*(120*b^5*f*x^11 + 2
4*(7*b^5*e - 11*a*b^4*f)*x^9 + 8*(35*b^5*d - 63*a*b^4*e + 99*a^2*b^3*f)*x^7
 + 56*(15*b^5*c - 35*a*b^4*d + 63*a^2*b^3*e - 99*a^3*b^2*f)*x^5 + 175*(15*a
*b^4*c - 35*a^2*b^3*d + 63*a^3*b^2*e - 99*a^4*b*f)*x^3 - 105*(15*a^2*b^3*c
```

```
- 35*a^3*b^2*d + 63*a^4*b*e - 99*a^5*f + (15*b^5*c - 35*a*b^4*d + 63*a^2*b^
3*e - 99*a^3*b^2*f)*x^4 + 2*(15*a*b^4*c - 35*a^2*b^3*d + 63*a^3*b^2*e - 99*
a^4*b*f)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 105*(15*a^2*b^3*c - 35*a^
3*b^2*d + 63*a^4*b*e - 99*a^5*f)*x)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6)]
```

Sympy [A] (verification not implemented)

Time = 17.92 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.28

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$$
$$= x^5\left(-\frac{3af}{5b^4}+\frac{e}{5b^3}\right)+x^3\cdot\left(\frac{2a^2f}{b^5}-\frac{ae}{b^4}+\frac{d}{3b^3}\right)+x\left(-\frac{10a^3f}{b^6}+\frac{6a^2e}{b^5}-\frac{3ad}{b^4}+\frac{c}{b^3}\right)$$
$$-\frac{\sqrt{-\frac{a}{b^{13}}}\cdot(99a^3f-63a^2be+35ab^2d-15b^3c)\log\left(-b^6\sqrt{-\frac{a}{b^{13}}}+x\right)}{16}$$
$$+\frac{\sqrt{-\frac{a}{b^{13}}}\cdot(99a^3f-63a^2be+35ab^2d-15b^3c)\log\left(b^6\sqrt{-\frac{a}{b^{13}}}+x\right)}{16}$$
$$+\frac{x^3(-21a^4bf+17a^3b^2e-13a^2b^3d+9ab^4c)+x(-19a^5f+15a^4be-11a^3b^2d+7a^2b^3c)}{8a^2b^6+16ab^7x^2+8b^8x^4}$$
$$+\frac{fx^7}{7b^3}$$

```
[In] integrate(x**6*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] x**5*(-3*a*f/(5*b**4) + e/(5*b**3)) + x**3*(2*a**2*f/b**5 - a*e/b**4 + d/(3
*b**3)) + x*(-10*a**3*f/b**6 + 6*a**2*e/b**5 - 3*a*d/b**4 + c/b**3) - sqrt(
-a/b**13)*(99*a**3*f - 63*a**2*b*e + 35*a*b**2*d - 15*b**3*c)*log(-b**6*sqr
t(-a/b**13) + x)/16 + sqrt(-a/b**13)*(99*a**3*f - 63*a**2*b*e + 35*a*b**2*d
 - 15*b**3*c)*log(b**6*sqrt(-a/b**13) + x)/16 + (x**3*(-21*a**4*b*f + 17*a*
*3*b**2*e - 13*a**2*b**3*d + 9*a*b**4*c) + x*(-19*a**5*f + 15*a**4*b*e - 11
*a**3*b**2*d + 7*a**2*b**3*c))/(8*a**2*b**6 + 16*a*b**7*x**2 + 8*b**8*x**4)
 + f*x**7/(7*b**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.96

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$$
$$= \frac{(9\,ab^4c-13\,a^2b^3d+17\,a^3b^2e-21\,a^4bf)x^3+(7\,a^2b^3c-11\,a^3b^2d+15\,a^4be-19\,a^5f)x}{8\,(b^8x^4+2\,ab^7x^2+a^2b^6)}$$
$$-\frac{(15\,ab^3c-35\,a^2b^2d+63\,a^3be-99\,a^4f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^6}$$
$$+\frac{15\,b^3fx^7+21\,(b^3e-3\,ab^2f)x^5+35\,(b^3d-3\,ab^2e+6\,a^2bf)x^3+105\,(b^3c-3\,ab^2d+6\,a^2be-10\,a^3f)x}{105\,b^6}$$

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((9*a*b^4*c - 13*a^2*b^3*d + 17*a^3*b^2*e - 21*a^4*b*f)*x^3 + (7*a^2*b^
3*c - 11*a^3*b^2*d + 15*a^4*b*e - 19*a^5*f)*x)/(b^8*x^4 + 2*a*b^7*x^2 + a^2
*b^6) - 1/8*(15*a*b^3*c - 35*a^2*b^2*d + 63*a^3*b*e - 99*a^4*f)*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*b^6) + 1/105*(15*b^3*f*x^7 + 21*(b^3*e - 3*a*b^2*f)*x
^5 + 35*(b^3*d - 3*a*b^2*e + 6*a^2*b*f)*x^3 + 105*(b^3*c - 3*a*b^2*d + 6*a^
2*b*e - 10*a^3*f)*x)/b^6
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.99

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = -\frac{(15\,ab^3c-35\,a^2b^2d+63\,a^3be-99\,a^4f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^6}$$
$$+\frac{9\,ab^4cx^3-13\,a^2b^3dx^3+17\,a^3b^2ex^3-21\,a^4bfx^3+7\,a^2b^3cx-11\,a^3b^2dx+15\,a^4bex-19\,a^5fx}{8\,(bx^2+a)^2b^6}$$
$$+\frac{15\,b^{18}fx^7+21\,b^{18}ex^5-63\,ab^{17}fx^5+35\,b^{18}dx^3-105\,ab^{17}ex^3+210\,a^2b^{16}fx^3+105\,b^{18}cx-315\,ab^{17}dx}{105\,b^{21}}$$

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*a*b^3*c - 35*a^2*b^2*d + 63*a^3*b*e - 99*a^4*f)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*b^6) + 1/8*(9*a*b^4*c*x^3 - 13*a^2*b^3*d*x^3 + 17*a^3*b^2*e*
x^3 - 21*a^4*b*f*x^3 + 7*a^2*b^3*c*x - 11*a^3*b^2*d*x + 15*a^4*b*e*x - 19*a
^5*f*x)/((b*x^2 + a)^2*b^6) + 1/105*(15*b^18*f*x^7 + 21*b^18*e*x^5 - 63*a*b
^17*f*x^5 + 35*b^18*d*x^3 - 105*a*b^17*e*x^3 + 210*a^2*b^16*f*x^3 + 105*b^1
8*c*x - 315*a*b^17*d*x + 630*a^2*b^16*e*x - 1050*a^3*b^15*f*x)/b^21
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.41

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$$
$$= x^5\left(\frac{e}{5\,b^3}-\frac{3\,a\,f}{5\,b^4}\right)+x\left(\frac{c}{b^3}-\frac{a^3\,f}{b^6}-\frac{3\,a^2\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^2}+\frac{3\,a\,\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b}\right)$$
$$-x^3\left(\frac{a^2\,f}{b^5}-\frac{d}{3\,b^3}+\frac{a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)$$
$$-\frac{\left(\frac{21\,f\,a^4\,b}{8}-\frac{17\,e\,a^3\,b^2}{8}+\frac{13\,d\,a^2\,b^3}{8}-\frac{9\,c\,a\,b^4}{8}\right)\,x^3+\left(\frac{19\,f\,a^5}{8}-\frac{15\,e\,a^4\,b}{8}+\frac{11\,d\,a^3\,b^2}{8}-\frac{7\,c\,a^2\,b^3}{8}\right)\,x}{a^2\,b^6+2\,a\,b^7\,x^2+b^8\,x^4}$$
$$+\frac{f\,x^7}{7\,b^3}$$
$$+\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,\left(-99\,f\,a^3+63\,e\,a^2\,b-35\,d\,a\,b^2+15\,c\,b^3\right)}{99\,f\,a^4-63\,e\,a^3\,b+35\,d\,a^2\,b^2-15\,c\,a\,b^3}\right)\,\left(-99\,f\,a^3+63\,e\,a^2\,b-35\,d\,a\,b^2+15\,c\,b^3\right)}{8\,b^{13/2}}$$

```
[In] int((x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x)
```

```
[Out] x^5*(e/(5*b^3) - (3*a*f)/(5*b^4)) + x*(c/b^3 - (a^3*f)/b^6 - (3*a^2*(e/b^3
- (3*a*f)/b^4))/b^2 + (3*a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a*f)/b
^4))/b))/b) - x^3*((a^2*f)/b^5 - d/(3*b^3) + (a*(e/b^3 - (3*a*f)/b^4))/b) -
 (x*((19*a^5*f)/8 - (7*a^2*b^3*c)/8 + (11*a^3*b^2*d)/8 - (15*a^4*b*e)/8) +
x^3*((13*a^2*b^3*d)/8 - (17*a^3*b^2*e)/8 - (9*a*b^4*c)/8 + (21*a^4*b*f)/8))
/(a^2*b^6 + b^8*x^4 + 2*a*b^7*x^2) + (f*x^7)/(7*b^3) + (a^(1/2)*atan((a^(1/
2)*b^(1/2)*x*(15*b^3*c - 99*a^3*f - 35*a*b^2*d + 63*a^2*b*e))/(99*a^4*f + 3
5*a^2*b^2*d - 15*a*b^3*c - 63*a^3*b*e))*(15*b^3*c - 99*a^3*f - 35*a*b^2*d +
 63*a^2*b*e))/(8*b^(13/2))
```

3.135 $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3} dx$

Optimal result 813
Rubi [A] (verified) 813
Mathematica [A] (verified) 816
Maple [A] (verified) 816
Fricas [A] (verification not implemented) 817
Sympy [A] (verification not implemented) 817
Maxima [A] (verification not implemented) 818
Giac [A] (verification not implemented) 818
Mupad [B] (verification not implemented) 819

Optimal result

Integrand size = 30, antiderivative size = 207

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3} dx = -\frac{(b^3c-5ab^2d+13a^2be-25a^3f)\,x}{4ab^5} + \frac{(be-3af)x^3}{3b^4} + \frac{fx^5}{5b^3} + \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^5}{4a\,(a+bx^2)^2} - \frac{(b^3c-5ab^2d+9a^2be-13a^3f)\,x}{8b^5\,(a+bx^2)} + \frac{(3b^3c-15ab^2d+35a^2be-63a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{11/2}}$$

```
[Out] -1/4*(-25*a^3*f+13*a^2*b*e-5*a*b^2*d+b^3*c)*x/a/b^5+1/3*(-3*a*f+b*e)*x^3/b^
4+1/5*f*x^5/b^3+1/4*(c-a*(a^2*f-a*b*e+b^2*d)/b^3)*x^5/a/(b*x^2+a)^2-1/8*(-1
3*a^3*f+9*a^2*b*e-5*a*b^2*d+b^3*c)*x/b^5/(b*x^2+a)+1/8*(-63*a^3*f+35*a^2*b*
e-15*a*b^2*d+3*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/b^(11/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used

$= \{1818, 1599, 1271, 1824, 211\}$

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = \frac{x^5\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\,(a+bx^2)^2} + \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(-63a^3f+35a^2be-15ab^2d+3b^3c)}{8\sqrt{a}b^{11/2}} - \frac{x(-13a^3f+9a^2be-5ab^2d+b^3c)}{8b^5\,(a+bx^2)} - \frac{x(-25a^3f+13a^2be-5ab^2d+b^3c)}{4ab^5} + \frac{x^3(be-3af)}{3b^4} + \frac{fx^5}{5b^3}$$

```
[In] Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] -1/4*((b^3*c - 5*a*b^2*d + 13*a^2*b*e - 25*a^3*f)*x)/(a*b^5) + ((b*e - 3*a*
f)*x^3)/(3*b^4) + (f*x^5)/(5*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*
x^5)/(4*a*(a + b*x^2)^2) - ((b^3*c - 5*a*b^2*d + 9*a^2*b*e - 13*a^3*f)*x)/(
8*b^5*(a + b*x^2)) + ((3*b^3*c - 15*a*b^2*d + 35*a^2*b*e - 63*a^3*f)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\text{integral} = \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^5}{4a\left(a+bx^2\right)^2}-\frac{\int\frac{x^3\left(\left(bc-5ad+\frac{5a^2e}{b}-\frac{5a^3f}{b^2}\right)x-4a\left(e-\frac{af}{b}\right)x^3-4afx^5\right)}{(a+bx^2)^2}\,dx}{4ab}$$

$$=\frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^5}{4a\left(a+bx^2\right)^2}-\frac{\int\frac{x^4\left(bc-5ad+\frac{5a^2e}{b}-\frac{5a^3f}{b^2}-4a\left(e-\frac{af}{b}\right)x^2-4afx^4\right)}{(a+bx^2)^2}\,dx}{4ab}$$

$$=\frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^5}{4a\left(a+bx^2\right)^2}-\frac{(b^3c-5ab^2d+9a^2be-13a^3f)\,x}{8b^5\left(a+bx^2\right)}+\frac{\int\frac{a(b^3c-5ab^2d+9a^2be-13a^3f)-2b(b^3c-5ab^2d+9a^2be-13a^3f)x^2+8ab^2(be-2af)x^4+8ab^3fx^6}{a+bx^2}\,dx}{8ab^5}$$

$$=\frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^5}{4a\left(a+bx^2\right)^2}-\frac{(b^3c-5ab^2d+9a^2be-13a^3f)\,x}{8b^5\left(a+bx^2\right)}+\frac{\int\left(-2(b^3c-5ab^2d+13a^2be-25a^3f)+8ab(be-3af)x^2+8ab^2fx^4+\frac{3ab^3c-15a^2b^2d+35a^3be-63a^4f}{a+bx^2}\right.}{8ab^5}$$

$$=-\frac{(b^3c-5ab^2d+13a^2be-25a^3f)\,x}{4ab^5}+\frac{(be-3af)x^3}{3b^4}+\frac{fx^5}{5b^3}+\frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^5}{4a\left(a+bx^2\right)^2}-\frac{(b^3c-5ab^2d+9a^2be-13a^3f)\,x}{8b^5\left(a+bx^2\right)}+\frac{(3b^3c-15ab^2d+35a^2be-63a^3f)\int\frac{1}{a+bx^2}\,dx}{8b^5}$$

$$=-\frac{(b^3c-5ab^2d+13a^2be-25a^3f)\,x}{4ab^5}+\frac{(be-3af)x^3}{3b^4}+\frac{fx^5}{5b^3}+\frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^5}{4a\left(a+bx^2\right)^2}-\frac{(b^3c-5ab^2d+9a^2be-13a^3f)\,x}{8b^5\left(a+bx^2\right)}+\frac{(3b^3c-15ab^2d+35a^2be-63a^3f)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{11/2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.85

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$$
$$= \frac{x(945a^4f-525a^3b(e-3fx^2)+a^2b^2(225d-875ex^2+504fx^4)-ab^3(45c-375dx^2+280ex^4+72fx^6)+}{120b^5\,(a+bx^2)^2}$$
$$+\frac{(3b^3c-15ab^2d+35a^2be-63a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{11/2}}$$

```
[In] Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] (x*(945*a^4*f - 525*a^3*b*(e - 3*f*x^2) + a^2*b^2*(225*d - 875*e*x^2 + 504*
f*x^4) - a*b^3*(45*c - 375*d*x^2 + 280*e*x^4 + 72*f*x^6) + b^4*x^2*(-75*c +
 8*(15*d*x^2 + 5*e*x^4 + 3*f*x^6))))/(120*b^5*(a + b*x^2)^2) + ((3*b^3*c -
15*a*b^2*d + 35*a^2*b*e - 63*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]
*b^(11/2))
```

Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.86

method	result
default	$\frac{\frac{1}{5}f\,x^5b^2-abf\,x^3+\frac{1}{3}b^2e\,x^3+6a^2fx-3abex+b^2dx}{b^5}-\frac{\frac{\left(-\frac{17}{8}a^3bf+\frac{13}{8}a^2e\,b^2-\frac{9}{8}a\,b^3d+\frac{5}{8}b^4c\right)x^3-\frac{a\left(15f\,a^3-11a^2be+7a\,b^2d-3b^3c\right)x}{8}}{\left(b\,x^2+a\right)^2}+\frac{(63f\,a^3}{}}{b^5}$
risch	$\frac{f\,x^5}{5b^3}-\frac{af\,x^3}{b^4}+\frac{e\,x^3}{3b^3}+\frac{6a^2fx}{b^5}-\frac{3aex}{b^4}+\frac{dx}{b^3}+\frac{(\frac{17}{8}a^3bf-\frac{13}{8}a^2e\,b^2+\frac{9}{8}a\,b^3d-\frac{5}{8}b^4c)x^3+\frac{a\left(15f\,a^3-11a^2be+7a\,b^2d-3b^3c\right)x}{8}}{b^5(b\,x^2+a)^2}-\frac{63}{}$

```
[In] int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(1/5*f*x^5*b^2-a*b*f*x^3+1/3*b^2*e*x^3+6*a^2*f*x-3*a*b*e*x+b^2*d*x)-1
/b^5*(((-17/8*a^3*b*f+13/8*a^2*e*b^2-9/8*a*b^3*d+5/8*b^4*c)*x^3-1/8*a*(15*a
^3*f-11*a^2*b*e+7*a*b^2*d-3*b^3*c)*x)/(b*x^2+a)^2+1/8*(63*a^3*f-35*a^2*b*e+
15*a*b^2*d-3*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 614, normalized size of antiderivative = 2.97

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$$
$$= \left[\frac{48\,ab^5fx^9+16\,(5\,ab^5e-9\,a^2b^4f)x^7+16\,(15\,ab^5d-35\,a^2b^4e+63\,a^3b^3f)x^5-50\,(3\,ab^5c-15\,a^2b^4d+35\right.$$

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
[Out] [1/240*(48*a*b^5*f*x^9 + 16*(5*a*b^5*e - 9*a^2*b^4*f)*x^7 + 16*(15*a*b^5*d
- 35*a^2*b^4*e + 63*a^3*b^3*f)*x^5 - 50*(3*a*b^5*c - 15*a^2*b^4*d + 35*a^3*
b^3*e - 63*a^4*b^2*f)*x^3 + 15*(3*a^2*b^3*c - 15*a^3*b^2*d + 35*a^4*b*e - 6
3*a^5*f + (3*b^5*c - 15*a*b^4*d + 35*a^2*b^3*e - 63*a^3*b^2*f)*x^4 + 2*(3*a
*b^4*c - 15*a^2*b^3*d + 35*a^3*b^2*e - 63*a^4*b*f)*x^2)*sqrt(-a*b)*log((b*x
^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 30*(3*a^2*b^4*c - 15*a^3*b^3*d + 35
*a^4*b^2*e - 63*a^5*b*f)*x)/(a*b^8*x^4 + 2*a^2*b^7*x^2 + a^3*b^6), 1/120*(2
4*a*b^5*f*x^9 + 8*(5*a*b^5*e - 9*a^2*b^4*f)*x^7 + 8*(15*a*b^5*d - 35*a^2*b^
4*e + 63*a^3*b^3*f)*x^5 - 25*(3*a*b^5*c - 15*a^2*b^4*d + 35*a^3*b^3*e - 63*
a^4*b^2*f)*x^3 + 15*(3*a^2*b^3*c - 15*a^3*b^2*d + 35*a^4*b*e - 63*a^5*f + (
3*b^5*c - 15*a*b^4*d + 35*a^2*b^3*e - 63*a^3*b^2*f)*x^4 + 2*(3*a*b^4*c - 15
*a^2*b^3*d + 35*a^3*b^2*e - 63*a^4*b*f)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a
) - 15*(3*a^2*b^4*c - 15*a^3*b^3*d + 35*a^4*b^2*e - 63*a^5*b*f)*x)/(a*b^8*x
^4 + 2*a^2*b^7*x^2 + a^3*b^6)]
```

Sympy [A] (verification not implemented)

Time = 15.94 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.35

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$$
$$= x^3\left(-\frac{af}{b^4}+\frac{e}{3b^3}\right)+x\left(\frac{6a^2f}{b^5}-\frac{3ae}{b^4}+\frac{d}{b^3}\right)$$
$$+\frac{\sqrt{-\frac{1}{ab^{11}}}\cdot(63a^3f-35a^2be+15ab^2d-3b^3c)\log\left(-ab^5\sqrt{-\frac{1}{ab^{11}}}+x\right)}{16}$$
$$-\frac{\sqrt{-\frac{1}{ab^{11}}}\cdot(63a^3f-35a^2be+15ab^2d-3b^3c)\log\left(ab^5\sqrt{-\frac{1}{ab^{11}}}+x\right)}{16}$$
$$+\frac{x^3\cdot(17a^3bf-13a^2b^2e+9ab^3d-5b^4c)+x(15a^4f-11a^3be+7a^2b^2d-3ab^3c)}{8a^2b^5+16ab^6x^2+8b^7x^4}+\frac{fx^5}{5b^3}$$

```
[In] integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] x**3*(-a*f/b**4 + e/(3*b**3)) + x*(6*a**2*f/b**5 - 3*a*e/b**4 + d/b**3) + s
qrt(-1/(a*b**11))*(63*a**3*f - 35*a**2*b*e + 15*a*b**2*d - 3*b**3*c)*log(-a
*b**5*sqrt(-1/(a*b**11)) + x)/16 - sqrt(-1/(a*b**11))*(63*a**3*f - 35*a**2*
b*e + 15*a*b**2*d - 3*b**3*c)*log(a*b**5*sqrt(-1/(a*b**11)) + x)/16 + (x**3
*(17*a**3*b*f - 13*a**2*b**2*e + 9*a*b**3*d - 5*b**4*c) + x*(15*a**4*f - 11
*a**3*b*e + 7*a**2*b**2*d - 3*a*b**3*c))/(8*a**2*b**5 + 16*a*b**6*x**2 + 8*
b**7*x**4) + f*x**5/(5*b**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.93

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = -\frac{(5\,b^4c-9\,ab^3d+13\,a^2b^2e-17\,a^3bf)x^3+(3\,ab^3c-7\,a^2b^2d+11\,a^3be-15\,a^4f)x}{8\,(b^7x^4+2\,ab^6x^2+a^2b^5)} + \frac{(3\,b^3c-15\,ab^2d+35\,a^2be-63\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^5} + \frac{3\,b^2fx^5+5\,(b^2e-3\,abf)x^3+15\,(b^2d-3\,abe+6\,a^2f)x}{15\,b^5}$$

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*b^4*c - 9*a*b^3*d + 13*a^2*b^2*e - 17*a^3*b*f)*x^3 + (3*a*b^3*c -
7*a^2*b^2*d + 11*a^3*b*e - 15*a^4*f)*x)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5) +
 1/8*(3*b^3*c - 15*a*b^2*d + 35*a^2*b*e - 63*a^3*f)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*b^5) + 1/15*(3*b^2*f*x^5 + 5*(b^2*e - 3*a*b*f)*x^3 + 15*(b^2*d -
3*a*b*e + 6*a^2*f)*x)/b^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.94

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = \frac{(3\,b^3c-15\,ab^2d+35\,a^2be-63\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^5} - \frac{5\,b^4cx^3-9\,ab^3dx^3+13\,a^2b^2ex^3-17\,a^3bfx^3+3\,ab^3cx-7\,a^2b^2dx+11\,a^3bex-15\,a^4fx}{8\,(bx^2+a)^2b^5} + \frac{3\,b^{12}fx^5+5\,b^{12}ex^3-15\,ab^{11}fx^3+15\,b^{12}dx-45\,ab^{11}ex+90\,a^2b^{10}fx}{15\,b^{15}}$$

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^3*c - 15*a*b^2*d + 35*a^2*b*e - 63*a^3*f)*arctan(b*x/sqrt(a*b))/(s
qrt(a*b)*b^5) - 1/8*(5*b^4*c*x^3 - 9*a*b^3*d*x^3 + 13*a^2*b^2*e*x^3 - 17*a^
3*b*f*x^3 + 3*a*b^3*c*x - 7*a^2*b^2*d*x + 11*a^3*b*e*x - 15*a^4*f*x)/((b*x^
2 + a)^2*b^5) + 1/15*(3*b^12*f*x^5 + 5*b^12*e*x^3 - 15*a*b^11*f*x^3 + 15*b^
12*d*x - 45*a*b^11*e*x + 90*a^2*b^10*f*x)/b^15
```

Mupad [B] (verification not implemented)

Time = 5.62 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = x^3\left(\frac{e}{3\,b^3}-\frac{a\,f}{b^4}\right) - x\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right) - \frac{x^3\left(-\frac{17\,f\,a^3\,b}{8}+\frac{13\,e\,a^2\,b^2}{8}-\frac{9\,d\,a\,b^3}{8}+\frac{5\,c\,b^4}{8}\right)-x\left(\frac{15\,f\,a^4}{8}-\frac{11\,e\,a^3\,b}{8}+\frac{7\,d\,a^2\,b^2}{8}-\frac{3\,c\,a\,b^3}{8}\right)}{a^2\,b^5+2\,a\,b^6\,x^2+b^7\,x^4} + \frac{f\,x^5}{5\,b^3} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-63\,f\,a^3+35\,e\,a^2\,b-15\,d\,a\,b^2+3\,c\,b^3\right)}{8\,\sqrt{a}\,b^{11/2}}$$

```
[In] int((x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x)
```

```
[Out] x^3*(e/(3*b^3) - (a*f)/b^4) - x*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a
*f)/b^4))/b) - (x^3*((5*b^4*c)/8 + (13*a^2*b^2*e)/8 - (9*a*b^3*d)/8 - (17*a
^3*b*f)/8) - x*((15*a^4*f)/8 + (7*a^2*b^2*d)/8 - (3*a*b^3*c)/8 - (11*a^3*b*
e)/8))/(a^2*b^5 + b^7*x^4 + 2*a*b^6*x^2) + (f*x^5)/(5*b^3) + (atan((b^(1/2)
*x)/a^(1/2))*(3*b^3*c - 63*a^3*f - 15*a*b^2*d + 35*a^2*b*e))/(8*a^(1/2)*b^(
11/2))
```

3.136 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$

Optimal result 820
Rubi [A] (verified) 820
Mathematica [A] (verified) 822
Maple [A] (verified) 823
Fricas [A] (verification not implemented) 823
Sympy [A] (verification not implemented) 824
Maxima [A] (verification not implemented) 824
Giac [A] (verification not implemented) 825
Mupad [B] (verification not implemented) 825

Optimal result

Integrand size = 30, antiderivative size = 167

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = \frac{(be-3af)x}{b^4}+\frac{fx^3}{3b^3}+\frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^3}{4a\,(a+bx^2)^2} - \frac{(b^3c+3ab^2d-7a^2be+11a^3f)\,x}{8ab^4\,(a+bx^2)} + \frac{(b^3c+3ab^2d-15a^2be+35a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{9/2}}$$

```
[Out] (-3*a*f+b*e)*x/b^4+1/3*f*x^3/b^3+1/4*(c-a*(a^2*f-a*b*e+b^2*d)/b^3)*x^3/a/(b
*x^2+a)^2-1/8*(11*a^3*f-7*a^2*b*e+3*a*b^2*d+b^3*c)*x/a/b^4/(b*x^2+a)+1/8*(3
5*a^3*f-15*a^2*b*e+3*a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(9/
2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1818, 1599, 1271, 1167, 211}

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = \frac{x^3\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\,(a+bx^2)^2} - \frac{x(11a^3f-7a^2be+3ab^2d+b^3c)}{8ab^4\,(a+bx^2)} + \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(35a^3f-15a^2be+3ab^2d+b^3c)}{8a^{3/2}b^{9/2}} + \frac{x(be-3af)}{b^4}+\frac{fx^3}{3b^3}$$

```
[In] Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] ((b*e - 3*a*f)*x)/b^4 + (f*x^3)/(3*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))
/b^3)*x^3)/(4*a*(a + b*x^2)^2) - ((b^3*c + 3*a*b^2*d - 7*a^2*b*e + 11*a^3*f
)*x)/(8*a*b^4*(a + b*x^2)) + ((b^3*c + 3*a*b^2*d - 15*a^2*b*e + 35*a^3*f)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^3}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x\left(-\left(\left(bc+3ad-\frac{3a^2e}{b}+\frac{3a^3f}{b^2}\right)x\right)-4a\left(e-\frac{af}{b}\right)x^3-4afx^5\right)}{(a+bx^2)^2}\,dx}{4ab}$$

$$= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^3}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^2\left(-bc-3ad+\frac{3a^2e}{b}-\frac{3a^3f}{b^2}-4a\left(e-\frac{af}{b}\right)x^2-4afx^4\right)}{(a+bx^2)^2}\,dx}{4ab}$$

$$= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^3}{4a\left(a+bx^2\right)^2} - \frac{\left(b^3c+3ab^2d-7a^2be+11a^3f\right)x}{8ab^4\left(a+bx^2\right)} + \frac{\int \frac{b^3c+3ab^2d-7a^2be+11a^3f+8ab(be-2af)x^2+8ab^2fx^4}{a+bx^2}\,dx}{8ab^4}$$

$$= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^3}{4a\left(a+bx^2\right)^2} - \frac{\left(b^3c+3ab^2d-7a^2be+11a^3f\right)x}{8ab^4\left(a+bx^2\right)} + \frac{\int \left(8a(be-3af)+8abfx^2+\frac{b^3c+3ab^2d-15a^2be+35a^3f}{a+bx^2}\right)dx}{8ab^4}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^3}{3b^3} + \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^3}{4a\left(a+bx^2\right)^2} - \frac{\left(b^3c+3ab^2d-7a^2be+11a^3f\right)x}{8ab^4\left(a+bx^2\right)} + \frac{\left(b^3c+3ab^2d-15a^2be+35a^3f\right)\int \frac{1}{a+bx^2}\,dx}{8ab^4}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^3}{3b^3} + \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^3}{4a\left(a+bx^2\right)^2} - \frac{\left(b^3c+3ab^2d-7a^2be+11a^3f\right)x}{8ab^4\left(a+bx^2\right)} + \frac{\left(b^3c+3ab^2d-15a^2be+35a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{9/2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.93

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{\left(a+bx^2\right)^3}\,dx$$
$$= \frac{x(-105a^4f+3b^4cx^2+5a^3b(9e-35fx^2)+a^2b^2(-9d+75ex^2-56fx^4)+ab^3(-3c-15dx^2+24ex^4+8f}{24ab^4\left(a+bx^2\right)^2} + \frac{\left(b^3c+3ab^2d-15a^2be+35a^3f\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{9/2}}$$

```
[In] Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] (x*(-105*a^4*f + 3*b^4*c*x^2 + 5*a^3*b*(9*e - 35*f*x^2) + a^2*b^2*(-9*d + 7
5*e*x^2 - 56*f*x^4) + a*b^3*(-3*c - 15*d*x^2 + 24*e*x^4 + 8*f*x^6)))/(24*a*
b^4*(a + b*x^2)^2) + ((b^3*c + 3*a*b^2*d - 15*a^2*b*e + 35*a^3*f)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(9/2))
```

Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

method	result
default	$-\frac{-\frac{1}{3} f\, x^{3} b+3 a f x-b e x}{b^{4}}+\frac{\frac{-\frac{b\left(13 f\, a^{3}-9 a^{2} b e+5 a\, b^{2} d-b^{3} c\right) x^{3}}{8 a}+\left(-\frac{11}{8} f\, a^{3}+\frac{7}{8} a^{2} b e-\frac{3}{8} a\, b^{2} d-\frac{1}{8} b^{3} c\right) x}{\left(b\, x^{2}+a\right)^{2}}+\frac{\left(35 f\, a^{3}-15 a^{2} b e+3 a\, b^{2} d+b^{3} c\right) \arctan}{8 a \sqrt{a b}}}{b^{4}}$
risch	$\frac{f\, x^{3}}{3 b^{3}}-\frac{3 a f x}{b^{4}}+\frac{e x}{b^{3}}+\frac{-\frac{b\left(13 f\, a^{3}-9 a^{2} b e+5 a\, b^{2} d-b^{3} c\right) x^{3}}{8 a}+\left(-\frac{11}{8} f\, a^{3}+\frac{7}{8} a^{2} b e-\frac{3}{8} a\, b^{2} d-\frac{1}{8} b^{3} c\right) x}{b^{4}\left(b\, x^{2}+a\right)^{2}}-\frac{35 a^{2} \ln\left(b x+\sqrt{-a b}\right) f}{16 b^{4} \sqrt{-a b}}+\frac{15 a \ln}{16}$

```
[In] int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(-1/3*f*x^3*b+3*a*f*x-b*e*x)+1/b^4*((-1/8*b*(13*a^3*f-9*a^2*b*e+5*a*
b^2*d-b^3*c)/a*x^3+(-11/8*f*a^3+7/8*a^2*b*e-3/8*a*b^2*d-1/8*b^3*c)*x)/(b*x^
2+a)^2+1/8*(35*a^3*f-15*a^2*b*e+3*a*b^2*d+b^3*c)/a/(a*b)^(1/2)*arctan(b*x/(
a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 555, normalized size of antiderivative = 3.32

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$$
$$=\left[\frac{16\,a^2b^4fx^7+16\,(3\,a^2b^4e-7\,a^3b^3f)x^5+2\,(3\,ab^5c-15\,a^2b^4d+75\,a^3b^3e-175\,a^4b^2f)x^3-3\,(a^2b^3c+3\,a^3}{}\right.$$

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*a^2*b^4*f*x^7 + 16*(3*a^2*b^4*e - 7*a^3*b^3*f)*x^5 + 2*(3*a*b^5*c
 - 15*a^2*b^4*d + 75*a^3*b^3*e - 175*a^4*b^2*f)*x^3 - 3*(a^2*b^3*c + 3*a^3*
b^2*d - 15*a^4*b*e + 35*a^5*f + (b^5*c + 3*a*b^4*d - 15*a^2*b^3*e + 35*a^3*
b^2*f)*x^4 + 2*(a*b^4*c + 3*a^2*b^3*d - 15*a^3*b^2*e + 35*a^4*b*f)*x^2)*sqr
t(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 6*(a^2*b^4*c + 3*a^
3*b^3*d - 15*a^4*b^2*e + 35*a^5*b*f)*x)/(a^2*b^7*x^4 + 2*a^3*b^6*x^2 + a^4*
b^5), 1/24*(8*a^2*b^4*f*x^7 + 8*(3*a^2*b^4*e - 7*a^3*b^3*f)*x^5 + (3*a*b^5*
```

```
c - 15*a^2*b^4*d + 75*a^3*b^3*e - 175*a^4*b^2*f)*x^3 + 3*(a^2*b^3*c + 3*a^3
*b^2*d - 15*a^4*b*e + 35*a^5*f + (b^5*c + 3*a*b^4*d - 15*a^2*b^3*e + 35*a^3
*b^2*f)*x^4 + 2*(a*b^4*c + 3*a^2*b^3*d - 15*a^3*b^2*e + 35*a^4*b*f)*x^2)*sq
rt(a*b)*arctan(sqrt(a*b)*x/a) - 3*(a^2*b^4*c + 3*a^3*b^3*d - 15*a^4*b^2*e +
 35*a^5*b*f)*x)/(a^2*b^7*x^4 + 2*a^3*b^6*x^2 + a^4*b^5)]
```

Sympy [A] (verification not implemented)

Time = 5.71 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.56

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$$
$$= x\left(-\frac{3af}{b^4}+\frac{e}{b^3}\right)-\frac{\sqrt{-\frac{1}{a^3b^9}}\cdot(35a^3f-15a^2be+3ab^2d+b^3c)\log\left(-a^2b^4\sqrt{-\frac{1}{a^3b^9}}+x\right)}{16}$$
$$+\frac{\sqrt{-\frac{1}{a^3b^9}}\cdot(35a^3f-15a^2be+3ab^2d+b^3c)\log\left(a^2b^4\sqrt{-\frac{1}{a^3b^9}}+x\right)}{16}$$
$$+\frac{x^3(-13a^3bf+9a^2b^2e-5ab^3d+b^4c)+x(-11a^4f+7a^3be-3a^2b^2d-ab^3c)}{8a^3b^4+16a^2b^5x^2+8ab^6x^4}+\frac{fx^3}{3b^3}$$

```
[In] integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] x*(-3*a*f/b**4 + e/b**3) - sqrt(-1/(a**3*b**9))*(35*a**3*f - 15*a**2*b*e +
3*a*b**2*d + b**3*c)*log(-a**2*b**4*sqrt(-1/(a**3*b**9)) + x)/16 + sqrt(-1/
(a**3*b**9))*(35*a**3*f - 15*a**2*b*e + 3*a*b**2*d + b**3*c)*log(a**2*b**4*
sqrt(-1/(a**3*b**9)) + x)/16 + (x**3*(-13*a**3*b*f + 9*a**2*b**2*e - 5*a*b*
*3*d + b**4*c) + x*(-11*a**4*f + 7*a**3*b*e - 3*a**2*b**2*d - a*b**3*c))/(8
*a**3*b**4 + 16*a**2*b**5*x**2 + 8*a*b**6*x**4) + f*x**3/(3*b**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.01

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$$
$$=\frac{(b^4c-5\,ab^3d+9\,a^2b^2e-13\,a^3bf)x^3-(ab^3c+3\,a^2b^2d-7\,a^3be+11\,a^4f)x}{8\,(ab^6x^4+2\,a^2b^5x^2+a^3b^4)}$$
$$+\frac{bfx^3+3\,(be-3\,af)x}{3\,b^4}+\frac{(b^3c+3\,ab^2d-15\,a^2be+35\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^4}$$

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((b^4*c - 5*a*b^3*d + 9*a^2*b^2*e - 13*a^3*b*f)*x^3 - (a*b^3*c + 3*a^2*
b^2*d - 7*a^3*b*e + 11*a^4*f)*x)/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4) + 1/
3*(b*f*x^3 + 3*(b*e - 3*a*f)*x)/b^4 + 1/8*(b^3*c + 3*a*b^2*d - 15*a^2*b*e +
 35*a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^4)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.01

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = \frac{(b^3c+3\,ab^2d-15\,a^2be+35\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^4} + \frac{b^4cx^3-5\,ab^3dx^3+9\,a^2b^2ex^3-13\,a^3bfx^3-ab^3cx-3\,a^2b^2dx+7\,a^3bex-11\,a^4fx}{8\,(bx^2+a)^2ab^4} + \frac{b^6fx^3+3\,b^6ex-9\,ab^5fx}{3\,b^9}$$

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(b^3*c + 3*a*b^2*d - 15*a^2*b*e + 35*a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*a*b^4) + 1/8*(b^4*c*x^3 - 5*a*b^3*d*x^3 + 9*a^2*b^2*e*x^3 - 13*a^3*b*
f*x^3 - a*b^3*c*x - 3*a^2*b^2*d*x + 7*a^3*b*e*x - 11*a^4*f*x)/((b*x^2 + a)^
2*a*b^4) + 1/3*(b^6*f*x^3 + 3*b^6*e*x - 9*a*b^5*f*x)/b^9
```

Mupad [B] (verification not implemented)

Time = 5.51 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.98

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = x\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right) - \frac{x\left(\frac{11\,f\,a^3}{8}-\frac{7\,e\,a^2\,b}{8}+\frac{3\,d\,a\,b^2}{8}+\frac{c\,b^3}{8}\right)-\frac{x^3\,(-13\,f\,a^3\,b+9\,e\,a^2\,b^2-5\,d\,a\,b^3+c\,b^4)}{8\,a}}{a^2\,b^4+2\,a\,b^5\,x^2+b^6\,x^4} + \frac{f\,x^3}{3\,b^3} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(35\,f\,a^3-15\,e\,a^2\,b+3\,d\,a\,b^2+c\,b^3)}{8\,a^{3/2}\,b^{9/2}}$$

```
[In] int((x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x)
```

```
[Out] x*(e/b^3 - (3*a*f)/b^4) - (x*((b^3*c)/8 + (11*a^3*f)/8 + (3*a*b^2*d)/8 - (7
*a^2*b*e)/8) - (x^3*(b^4*c + 9*a^2*b^2*e - 5*a*b^3*d - 13*a^3*b*f))/(8*a))/
(a^2*b^4 + b^6*x^4 + 2*a*b^5*x^2) + (f*x^3)/(3*b^3) + (atan((b^(1/2)*x)/a^(
1/2))*(b^3*c + 35*a^3*f + 3*a*b^2*d - 15*a^2*b*e))/(8*a^(3/2)*b^(9/2))
```

3.137 $\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^3}\,dx$

Optimal result 826
Rubi [A] (verified) 826
Mathematica [A] (verified) 828
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Optimal result

Integrand size = 27, antiderivative size = 147

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^3}\,dx=\frac{fx}{b^3}+\frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x}{4a(a+bx^2)^2}+\frac{(3b^3c+ab^2d-5a^2be+9a^3f)x}{8a^2b^3(a+bx^2)}+\frac{(3b^3c+ab^2d+3a^2be-15a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}$$

```
[Out] f*x/b^3+1/4*(c-a*(a^2*f-a*b*e+b^2*d)/b^3)*x/a/(b*x^2+a)^2+1/8*(9*a^3*f-5*a^
2*b*e+a*b^2*d+3*b^3*c)*x/a^2/b^3/(b*x^2+a)+1/8*(-15*a^3*f+3*a^2*b*e+a*b^2*d
+3*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(7/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1828, 1171, 396, 211}

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^3}\,dx=\frac{x\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a(a+bx^2)^2}+\frac{x(9a^3f-5a^2be+ab^2d+3b^3c)}{8a^2b^3(a+bx^2)}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(-15a^3f+3a^2be+ab^2d+3b^3c)}{8a^{5/2}b^{7/2}}+\frac{fx}{b^3}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^3,x]
```

```
[Out] (f*x)/b^3 + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x)/(4*a*(a + b*x^2)^2) +
 ((3*b^3*c + a*b^2*d - 5*a^2*b*e + 9*a^3*f)*x)/(8*a^2*b^3*(a + b*x^2)) + ((
```

```
3*b^3*c + a*b^2*d + 3*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a
^(5/2)*b^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{4a\left(a+bx^2\right)^2}-\frac{\int \frac{-\frac{3b^3c+ab^2d-a^2be+a^3f}{b^3}-\frac{4a(be-af)x^2}{b^2}-\frac{4afx^4}{b}}{\left(a+bx^2\right)^2}\,dx}{4a}\\
&= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{4a\left(a+bx^2\right)^2}+\frac{\left(3b^3c+ab^2d-5a^2be+9a^3f\right)x}{8a^2b^3\left(a+bx^2\right)}+\frac{\int \frac{\frac{3b^3c+ab^2d+3a^2be-7a^3f}{b^3}+\frac{8a^2fx^2}{b^2}}{a+bx^2}\,dx}{8a^2}\\
&= \frac{fx}{b^3}+\frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{4a\left(a+bx^2\right)^2}+\frac{\left(3b^3c+ab^2d-5a^2be+9a^3f\right)x}{8a^2b^3\left(a+bx^2\right)}\\
&\quad+\frac{\left(3b^3c+ab^2d+3a^2be-15a^3f\right)\int \frac{1}{a+bx^2}\,dx}{8a^2b^3}
\end{aligned}
$$

$$= \frac{fx}{b^3} + \frac{\left(c - \frac{a(b^2d - abe + a^2f)}{b^3}\right)x}{4a\left(a + bx^2\right)^2} + \frac{(3b^3c + ab^2d - 5a^2be + 9a^3f)\,x}{8a^2b^3\left(a + bx^2\right)} + \frac{(3b^3c + ab^2d + 3a^2be - 15a^3f)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.96

$$\int \frac{c + dx^2 + ex^4 + fx^6}{\left(a + bx^2\right)^3}\,dx = \frac{x(15a^4f + 3b^4cx^2 + ab^3(5c + dx^2) + a^3b(-3e + 25fx^2) - a^2b^2(d + 5ex^2 - 8fx^4))}{8a^2b^3\left(a + bx^2\right)^2} + \frac{(3b^3c + ab^2d + 3a^2be - 15a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^3,x]
```

```
[Out] (x*(15*a^4*f + 3*b^4*c*x^2 + a*b^3*(5*c + d*x^2) + a^3*b*(-3*e + 25*f*x^2)
- a^2*b^2*(d + 5*e*x^2 - 8*f*x^4)))/(8*a^2*b^3*(a + b*x^2)^2) + ((3*b^3*c +
 a*b^2*d + 3*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^
(7/2))
```

Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.95

method	result
default	$\frac{fx}{b^3} - \frac{\frac{-\frac{b\left(9f\,a^3-5a^2be+a\,b^2d+3b^3c\right)x^3}{8a^2}-\frac{\left(7f\,a^3-3a^2be-a\,b^2d+5b^3c\right)x}{8a}}{\left(b\,x^2+a\right)^2}+\frac{\left(15f\,a^3-3a^2be-a\,b^2d-3b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}}{b^3}$
risch	$\frac{fx}{b^3} + \frac{\frac{b\left(9f\,a^3-5a^2be+a\,b^2d+3b^3c\right)x^3}{8a^2}+\frac{\left(7f\,a^3-3a^2be-a\,b^2d+5b^3c\right)x}{8a}}{b^3\left(b\,x^2+a\right)^2} - \frac{15a\ln\left(bx-\sqrt{-ab}\right)f}{16b^3\sqrt{-ab}} + \frac{3\ln\left(bx-\sqrt{-ab}\right)e}{16b^2\sqrt{-ab}} + \frac{\ln\left(bx-\sqrt{-ab}\right)d}{16b\sqrt{-ab}\,a}$

```
[In] int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] f*x/b^3-1/b^3*((-1/8*b*(9*a^3*f-5*a^2*b*e+a*b^2*d+3*b^3*c)/a^2*x^3-1/8*(7*a
^3*f-3*a^2*b*e-a*b^2*d+5*b^3*c)/a*x)/(b*x^2+a)^2+1/8*(15*a^3*f-3*a^2*b*e-a*
b^2*d-3*b^3*c)/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 504, normalized size of antiderivative = 3.43

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^3}\,dx$$
$$= \left[\frac{16\,a^3b^3fx^5+2\,(3\,ab^5c+a^2b^4d-5\,a^3b^3e+25\,a^4b^2f)x^3+(3\,a^2b^3c+a^3b^2d+3\,a^4be-15\,a^5f+(3\,b^5c+a\right.$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*a^3*b^3*f*x^5 + 2*(3*a*b^5*c + a^2*b^4*d - 5*a^3*b^3*e + 25*a^4*b
^2*f)*x^3 + (3*a^2*b^3*c + a^3*b^2*d + 3*a^4*b*e - 15*a^5*f + (3*b^5*c + a*
b^4*d + 3*a^2*b^3*e - 15*a^3*b^2*f)*x^4 + 2*(3*a*b^4*c + a^2*b^3*d + 3*a^3*
b^2*e - 15*a^4*b*f)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2
 + a)) + 2*(5*a^2*b^4*c - a^3*b^3*d - 3*a^4*b^2*e + 15*a^5*b*f)*x)/(a^3*b^6
*x^4 + 2*a^4*b^5*x^2 + a^5*b^4), 1/8*(8*a^3*b^3*f*x^5 + (3*a*b^5*c + a^2*b^
4*d - 5*a^3*b^3*e + 25*a^4*b^2*f)*x^3 + (3*a^2*b^3*c + a^3*b^2*d + 3*a^4*b*
e - 15*a^5*f + (3*b^5*c + a*b^4*d + 3*a^2*b^3*e - 15*a^3*b^2*f)*x^4 + 2*(3*
a*b^4*c + a^2*b^3*d + 3*a^3*b^2*e - 15*a^4*b*f)*x^2)*sqrt(a*b)*arctan(sqrt(
a*b)*x/a) + (5*a^2*b^4*c - a^3*b^3*d - 3*a^4*b^2*e + 15*a^5*b*f)*x)/(a^3*b^
6*x^4 + 2*a^4*b^5*x^2 + a^5*b^4)]
```

Sympy [A] (verification not implemented)

Time = 3.40 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.65

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^3}\,dx$$
$$= \frac{\sqrt{-\frac{1}{a^5b^7}}\cdot(15a^3f-3a^2be-ab^2d-3b^3c)\log\left(-a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x\right)}{16}$$
$$-\frac{\sqrt{-\frac{1}{a^5b^7}}\cdot(15a^3f-3a^2be-ab^2d-3b^3c)\log\left(a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x\right)}{16}$$
$$+\frac{x^3\cdot(9a^3bf-5a^2b^2e+ab^3d+3b^4c)+x(7a^4f-3a^3be-a^2b^2d+5ab^3c)}{8a^4b^3+16a^3b^4x^2+8a^2b^5x^4}+\frac{fx}{b^3}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] sqrt(-1/(a**5*b**7))*(15*a**3*f - 3*a**2*b*e - a*b**2*d - 3*b**3*c)*log(-a*
*3*b**3*sqrt(-1/(a**5*b**7)) + x)/16 - sqrt(-1/(a**5*b**7))*(15*a**3*f - 3*
a**2*b*e - a*b**2*d - 3*b**3*c)*log(a**3*b**3*sqrt(-1/(a**5*b**7)) + x)/16
```

```
+ (x**3*(9*a**3*b*f - 5*a**2*b**2*e + a*b**3*d + 3*b**4*c) + x*(7*a**4*f -
3*a**3*b*e - a**2*b**2*d + 5*a*b**3*c))/(8*a**4*b**3 + 16*a**3*b**4*x**2 +
8*a**2*b**5*x**4) + f*x/b**3
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^3}\,dx = \frac{(3\,b^4c+ab^3d-5\,a^2b^2e+9\,a^3bf)x^3+(5\,ab^3c-a^2b^2d-3\,a^3be+7\,a^4f)x}{8\,(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3)} + \frac{fx}{b^3} + \frac{(3\,b^3c+ab^2d+3\,a^2be-15\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^3}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*b^4*c + a*b^3*d - 5*a^2*b^2*e + 9*a^3*b*f)*x^3 + (5*a*b^3*c - a^2*b
^2*d - 3*a^3*b*e + 7*a^4*f)*x)/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3) + f*
x/b^3 + 1/8*(3*b^3*c + a*b^2*d + 3*a^2*b*e - 15*a^3*f)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*a^2*b^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^3}\,dx = \frac{fx}{b^3} + \frac{(3\,b^3c+ab^2d+3\,a^2be-15\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^3} + \frac{3\,b^4cx^3+ab^3dx^3-5\,a^2b^2ex^3+9\,a^3bfx^3+5\,ab^3cx-a^2b^2dx-3\,a^3bex+7\,a^4fx}{8\,(bx^2+a)^2a^2b^3}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] f*x/b^3 + 1/8*(3*b^3*c + a*b^2*d + 3*a^2*b*e - 15*a^3*f)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*a^2*b^3) + 1/8*(3*b^4*c*x^3 + a*b^3*d*x^3 - 5*a^2*b^2*e*x^3
+ 9*a^3*b*f*x^3 + 5*a*b^3*c*x - a^2*b^2*d*x - 3*a^3*b*e*x + 7*a^4*f*x)/((b*
x^2 + a)^2*a^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 5.54 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^3}\,dx=\frac{\frac{x\,(7\,f\,a^3-3\,e\,a^2\,b-d\,a\,b^2+5\,c\,b^3)}{8\,a}+\frac{x^3\,(9\,f\,a^3\,b-5\,e\,a^2\,b^2+d\,a\,b^3+3\,c\,b^4)}{8\,a^2}}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4}+\frac{f\,x}{b^3}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(-15\,f\,a^3+3\,e\,a^2\,b+d\,a\,b^2+3\,c\,b^3)}{8\,a^{5/2}\,b^{7/2}}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^3,x)
```

```
[Out] ((x*(5*b^3*c + 7*a^3*f - a*b^2*d - 3*a^2*b*e))/(8*a) + (x^3*(3*b^4*c - 5*a^
2*b^2*e + a*b^3*d + 9*a^3*b*f))/(8*a^2))/(a^2*b^3 + b^5*x^4 + 2*a*b^4*x^2)
+ (f*x)/b^3 + (atan((b^(1/2)*x)/a^(1/2))*(3*b^3*c - 15*a^3*f + a*b^2*d + 3*
a^2*b*e))/(8*a^(5/2)*b^(7/2))
```

3.138 $\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)^3} dx$

Optimal result 832
Rubi [A] (verified) 832
Mathematica [A] (verified) 834
Maple [A] (verified) 834
Fricas [A] (verification not implemented) 835
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Maxima [A] (verification not implemented) 836
Giac [A] (verification not implemented) 837
Mupad [B] (verification not implemented) 837

Optimal result

Integrand size = 30, antiderivative size = 153

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)^3} dx = -\frac{c}{a^3x} - \frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{4a(a+bx^2)^2} - \frac{(7b^3c-3ab^2d-a^2be+5a^3f)x}{8a^3b^2(a+bx^2)} - \frac{(15b^3c-3ab^2d-a^2be-3a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}b^{5/2}}$$

```
[Out] -c/a^3/x-1/4*(b*c/a-d+a*e/b-a^2*f/b^2)*x/a/(b*x^2+a)^2-1/8*(5*a^3*f-a^2*b*e
-3*a*b^2*d+7*b^3*c)*x/a^3/b^2/(b*x^2+a)-1/8*(-3*a^3*f-a^2*b*e-3*a*b^2*d+15*
b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(5/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1819, 1273, 464, 211}

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)^3} dx = -\frac{c}{a^3x} - \frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{4a(a+bx^2)^2} - \frac{x(5a^3f-a^2be-3ab^2d+7b^3c)}{8a^3b^2(a+bx^2)} - \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(-3a^3f-a^2be-3ab^2d+15b^3c)}{8a^{7/2}b^{5/2}}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(c/(a^3*x)) - (((b*c)/a - d + (a*e)/b - (a^2*f)/b^2)*x)/(4*a*(a + b*x^2)^2
) - ((7*b^3*c - 3*a*b^2*d - a^2*b*e + 5*a^3*f)*x)/(8*a^3*b^2*(a + b*x^2)) -
 ((15*b^3*c - 3*a*b^2*d - a^2*b*e - 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
8*a^(7/2)*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{4a\left(a+bx^2\right)^2}-\frac{\int\frac{-4c+\left(\frac{3bc}{a}-3d-\frac{ae}{b}+\frac{a^2f}{b^2}\right)x^2-\frac{4afx^4}{b}}{x^2\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= -\frac{\left(\frac{bc}{a} - d + \frac{ae}{b} - \frac{a^2f}{b^2}\right)x}{4a(a+bx^2)^2} - \frac{(7b^3c - 3ab^2d - a^2be + 5a^3f)x}{8a^3b^2(a+bx^2)} + \frac{\int \frac{8ab^2c - (7b^3c - 3ab^2d - a^2be - 3a^3f)x^2}{x^2(a+bx^2)}dx}{8a^3b^2}$$

$$= -\frac{c}{a^3x} - \frac{\left(\frac{bc}{a} - d + \frac{ae}{b} - \frac{a^2f}{b^2}\right)x}{4a(a+bx^2)^2} - \frac{(7b^3c - 3ab^2d - a^2be + 5a^3f)x}{8a^3b^2(a+bx^2)} - \frac{(15b^3c - 3ab^2d - a^2be - 3a^3f)\int \frac{1}{a+bx^2}dx}{8a^3b^2}$$

$$= -\frac{c}{a^3x} - \frac{\left(\frac{bc}{a} - d + \frac{ae}{b} - \frac{a^2f}{b^2}\right)x}{4a(a+bx^2)^2} - \frac{(7b^3c - 3ab^2d - a^2be + 5a^3f)x}{8a^3b^2(a+bx^2)} - \frac{(15b^3c - 3ab^2d - a^2be - 3a^3f)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}b^{5/2}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.01

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^2(a+bx^2)^3}dx = -\frac{c}{a^3x} + \frac{(-b^3c + ab^2d - a^2be + a^3f)x}{4a^2b^2(a+bx^2)^2} - \frac{(7b^3c - 3ab^2d - a^2be + 5a^3f)x}{8a^3b^2(a+bx^2)} + \frac{(-15b^3c + 3ab^2d + a^2be + 3a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}b^{5/2}}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(c/(a^3*x)) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(4*a^2*b^2*(a + b
*x^2)^2) - ((7*b^3*c - 3*a*b^2*d - a^2*b*e + 5*a^3*f)*x)/(8*a^3*b^2*(a + b*
x^2)) + ((-15*b^3*c + 3*a*b^2*d + a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(8*a^(7/2)*b^(5/2))
```

Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.92

method	result
default	$-\frac{c}{a^{3}x}+\frac{\frac{-\frac{\left(5f\,a^{3}-a^{2}be-3a\,b^{2}d+7b^{3}c\right)x^{3}}{8b}-\frac{a\left(3f\,a^{3}+a^{2}be-5a\,b^{2}d+9b^{3}c\right)x}{8b^{2}}}{\left(b\,x^{2}+a\right)^{2}}+\frac{\left(3f\,a^{3}+a^{2}be+3a\,b^{2}d-15b^{3}c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8b^{2}\sqrt{ab}}}{a^{3}}$
risch	$\frac{-\frac{\left(5f\,a^{3}-a^{2}be-3a\,b^{2}d+15b^{3}c\right)x^{4}}{8a^{3}b}-\frac{\left(3f\,a^{3}+a^{2}be-5a\,b^{2}d+25b^{3}c\right)x^{2}}{8a^{2}b^{2}}-\frac{c}{a}}{x\left(b\,x^{2}+a\right)^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{7}b^{5}\_Z^{2}+9a^{6}f^{2}+6a^{5}bef+18a^{4}b^{2}df+a^{4}b^{2}e^{2}-90a\ldots\right.}\right.}{}$

[In] int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)

[Out] -c/a^3/x+1/a^3*((-1/8*(5*a^3*f-a^2*b*e-3*a*b^2*d+7*b^3*c)/b*x^3-1/8*a*(3*a^3*f+a^2*b*e-5*a*b^2*d+9*b^3*c)/b^2*x)/(b*x^2+a)^2+1/8*(3*a^3*f+a^2*b*e+3*a*b^2*d-15*b^3*c)/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.38

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^3}\,dx$$

$$=\left[-\frac{16\,a^{3}b^{3}c+2\,(15\,ab^{5}c-3\,a^{2}b^{4}d-a^{3}b^{3}e+5\,a^{4}b^{2}f)x^{4}+2\,(25\,a^{2}b^{4}c-5\,a^{3}b^{3}d+a^{4}b^{2}e+3\,a^{5}bf)x^{2}-((1\ldots}{\ldots}\right.$$

$$-\frac{8\,a^{3}b^{3}c+(15\,ab^{5}c-3\,a^{2}b^{4}d-a^{3}b^{3}e+5\,a^{4}b^{2}f)x^{4}+(25\,a^{2}b^{4}c-5\,a^{3}b^{3}d+a^{4}b^{2}e+3\,a^{5}bf)x^{2}+((15\,b^{5}c\ldots}{\ldots}$$

[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^3,x, algorithm="fricas")

[Out] [-1/16*(16*a^3*b^3*c + 2*(15*a*b^5*c - 3*a^2*b^4*d - a^3*b^3*e + 5*a^4*b^2*f)*x^4 + 2*(25*a^2*b^4*c - 5*a^3*b^3*d + a^4*b^2*e + 3*a^5*b*f)*x^2 - ((15*b^5*c - 3*a*b^4*d - a^2*b^3*e - 3*a^3*b^2*f)*x^5 + 2*(15*a*b^4*c - 3*a^2*b^3*d - a^3*b^2*e - 3*a^4*b*f)*x^3 + (15*a^2*b^3*c - 3*a^3*b^2*d - a^4*b*e - 3*a^5*f)*x)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*b^5*x^5 + 2*a^5*b^4*x^3 + a^6*b^3*x), -1/8*(8*a^3*b^3*c + (15*a*b^5*c - 3*a^2*b^4*d - a^3*b^3*e + 5*a^4*b^2*f)*x^4 + (25*a^2*b^4*c - 5*a^3*b^3*d + a^4*b^2*e + 3*a^5*b*f)*x^2 + ((15*b^5*c - 3*a*b^4*d - a^2*b^3*e - 3*a^3*b^2*f)*x^5 + 2*(15*a*b^4*c - 3*a^2*b^3*d - a^3*b^2*e - 3*a^4*b*f)*x^3 + (15*a^2*b^3*c - 3*a^3*b^2*d - a^4*b*e - 3*a^5*f)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b^5*x^5 + 2*a^5*b^4*x^3 + a^6*b^3*x)]

Sympy [A] (verification not implemented)

Time = 11.08 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.63

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^3}\,dx$$
$$= -\frac{\sqrt{-\frac{1}{a^7b^5}}\cdot(3a^3f+a^2be+3ab^2d-15b^3c)\log\left(-a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x\right)}{16}$$
$$+\frac{\sqrt{-\frac{1}{a^7b^5}}\cdot(3a^3f+a^2be+3ab^2d-15b^3c)\log\left(a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x\right)}{16}$$
$$+\frac{-8a^2b^2c+x^4(-5a^3bf+a^2b^2e+3ab^3d-15b^4c)+x^2(-3a^4f-a^3be+5a^2b^2d-25ab^3c)}{8a^5b^2x+16a^4b^3x^3+8a^3b^4x^5}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**7*b**5))*(3*a**3*f + a**2*b*e + 3*a*b**2*d - 15*b**3*c)*log(-a
**4*b**2*sqrt(-1/(a**7*b**5)) + x)/16 + sqrt(-1/(a**7*b**5))*(3*a**3*f + a*
*2*b*e + 3*a*b**2*d - 15*b**3*c)*log(a**4*b**2*sqrt(-1/(a**7*b**5)) + x)/16
 + (-8*a**2*b**2*c + x**4*(-5*a**3*b*f + a**2*b**2*e + 3*a*b**3*d - 15*b**4
*c) + x**2*(-3*a**4*f - a**3*b*e + 5*a**2*b**2*d - 25*a*b**3*c))/(8*a**5*b*
*2*x + 16*a**4*b**3*x**3 + 8*a**3*b**4*x**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^3}\,dx$$
$$= -\frac{8\,a^2b^2c+(15\,b^4c-3\,ab^3d-a^2b^2e+5\,a^3bf)x^4+(25\,ab^3c-5\,a^2b^2d+a^3be+3\,a^4f)x^2}{8\,(a^3b^4x^5+2\,a^4b^3x^3+a^5b^2x)}$$
$$-\frac{(15\,b^3c-3\,ab^2d-a^2be-3\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3b^2}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(8*a^2*b^2*c + (15*b^4*c - 3*a*b^3*d - a^2*b^2*e + 5*a^3*b*f)*x^4 + (2
5*a*b^3*c - 5*a^2*b^2*d + a^3*b*e + 3*a^4*f)*x^2)/(a^3*b^4*x^5 + 2*a^4*b^3*
x^3 + a^5*b^2*x) - 1/8*(15*b^3*c - 3*a*b^2*d - a^2*b*e - 3*a^3*f)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^3*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^3}\,dx = -\frac{c}{a^3x} - \frac{\left(15\,b^3c-3\,ab^2d-a^2be-3\,a^3f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3b^2} - \frac{7\,b^4cx^3-3\,ab^3dx^3-a^2b^2ex^3+5\,a^3bfx^3+9\,ab^3cx-5\,a^2b^2dx+a^3bex+3\,a^4fx}{8\,\left(bx^2+a\right)^2a^3b^2}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -c/(a^3*x) - 1/8*(15*b^3*c - 3*a*b^2*d - a^2*b*e - 3*a^3*f)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a^3*b^2) - 1/8*(7*b^4*c*x^3 - 3*a*b^3*d*x^3 - a^2*b^2*e*x
^3 + 5*a^3*b*f*x^3 + 9*a*b^3*c*x - 5*a^2*b^2*d*x + a^3*b*e*x + 3*a^4*f*x)/(
(b*x^2 + a)^2*a^3*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^3}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(3\,f\,a^3+e\,a^2\,b+3\,d\,a\,b^2-15\,c\,b^3\right)}{8\,a^{7/2}\,b^{5/2}} - \frac{\frac{c}{a}+\frac{x^4\,\left(5\,f\,a^3-e\,a^2\,b-3\,d\,a\,b^2+15\,c\,b^3\right)}{8\,a^3\,b}+\frac{x^2\,\left(3\,f\,a^3+e\,a^2\,b-5\,d\,a\,b^2+25\,c\,b^3\right)}{8\,a^2\,b^2}}{a^2\,x+2\,a\,b\,x^3+b^2\,x^5}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^3),x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(3*a^3*f - 15*b^3*c + 3*a*b^2*d + a^2*b*e))/(8*a
^(7/2)*b^(5/2)) - (c/a + (x^4*(15*b^3*c + 5*a^3*f - 3*a*b^2*d - a^2*b*e))/(
8*a^3*b) + (x^2*(25*b^3*c + 3*a^3*f - 5*a*b^2*d + a^2*b*e))/(8*a^2*b^2))/(a
^2*x + b^2*x^5 + 2*a*b*x^3)
```

3.139 $\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)^3}\,dx$

Optimal result . . . 838
Rubi [A] (verified) . . . 838
Mathematica [A] (verified) . . . 840
Maple [A] (verified) . . . 841
Fricas [A] (verification not implemented) . . . 841
Sympy [F(-1)] . . . 842
Maxima [A] (verification not implemented) . . . 842
Giac [A] (verification not implemented) . . . 842
Mupad [B] (verification not implemented) . . . 843

Optimal result

Integrand size = 30, antiderivative size = 168

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)^3}\,dx = -\frac{c}{3a^3x^3}+\frac{3bc-ad}{a^4x}+\frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{4a(a+bx^2)^2}+\frac{(11b^3c-7ab^2d+3a^2be+a^3f)\,x}{8a^4b(a+bx^2)}+\frac{(35b^3c-15ab^2d+3a^2be+a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}b^{3/2}}$$

```
[Out] -1/3*c/a^3/x^3+(-a*d+3*b*c)/a^4/x+1/4*(b^2*c/a^2-b*d/a+e-a*f/b)*x/a/(b*x^2+
a)^2+1/8*(a^3*f+3*a^2*b*e-7*a*b^2*d+11*b^3*c)*x/a^4/b/(b*x^2+a)+1/8*(a^3*f+
3*a^2*b*e-15*a*b^2*d+35*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(9/2)/b^(3/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1819, 1273, 1275, 211}

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)^3}\,dx = \frac{3bc-ad}{a^4x}-\frac{c}{3a^3x^3}+\frac{x\left(\frac{b^2c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{4a(a+bx^2)^2}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(a^3f+3a^2be-15ab^2d+35b^3c)}{8a^{9/2}b^{3/2}}+\frac{x(a^3f+3a^2be-7ab^2d+11b^3c)}{8a^4b(a+bx^2)}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^3),x]
```

```
[Out] -1/3*c/(a^3*x^3) + (3*b*c - a*d)/(a^4*x) + (((b^2*c)/a^2 - (b*d)/a + e - (a
*f)/b)*x)/(4*a*(a + b*x^2)^2) + ((11*b^3*c - 7*a*b^2*d + 3*a^2*b*e + a^3*f)
*x)/(8*a^4*b*(a + b*x^2)) + ((35*b^3*c - 15*a*b^2*d + 3*a^2*b*e + a^3*f)*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2}-\frac{\int \frac{-4c+4\left(\frac{bc}{a}-d\right)x^2+\left(-\frac{3b^2c}{a^2}+\frac{3bd}{a}-3e-\frac{af}{b}\right)x^4}{x^4\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= \frac{\left(\frac{b^2c}{a^2} - \frac{bd}{a} + e - \frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2} + \frac{\left(11b^3c - 7ab^2d + 3a^2be + a^3f\right)x}{8a^4b\left(a+bx^2\right)} - \frac{\int \frac{-8a^2b^2c + 8ab^2(2bc-ad)x^2 - b\left(11b^3c - 7ab^2d + 3a^2be + a^3f\right)x^4}{x^4(a+bx^2)}\,dx}{8a^4b^2}$$

$$= \frac{\left(\frac{b^2c}{a^2} - \frac{bd}{a} + e - \frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2} + \frac{\left(11b^3c - 7ab^2d + 3a^2be + a^3f\right)x}{8a^4b\left(a+bx^2\right)} - \frac{\int \left(-\frac{8ab^2c}{x^4} + \frac{8b^2(3bc-ad)}{x^2} - \frac{b\left(35b^3c - 15ab^2d + 3a^2be + a^3f\right)}{a+bx^2}\right)dx}{8a^4b^2}$$

$$= -\frac{c}{3a^3x^3} + \frac{3bc-ad}{a^4x} + \frac{\left(\frac{b^2c}{a^2} - \frac{bd}{a} + e - \frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2} + \frac{\left(11b^3c - 7ab^2d + 3a^2be + a^3f\right)x}{8a^4b\left(a+bx^2\right)} + \frac{\left(35b^3c - 15ab^2d + 3a^2be + a^3f\right)\int \frac{1}{a+bx^2}\,dx}{8a^4b}$$

$$= -\frac{c}{3a^3x^3} + \frac{3bc-ad}{a^4x} + \frac{\left(\frac{b^2c}{a^2} - \frac{bd}{a} + e - \frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2} + \frac{\left(11b^3c - 7ab^2d + 3a^2be + a^3f\right)x}{8a^4b\left(a+bx^2\right)} + \frac{\left(35b^3c - 15ab^2d + 3a^2be + a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}b^{3/2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^3}\,dx = \frac{-3a^4fx^4 + 105b^4cx^6 + 5ab^3x^4(35c-9dx^2) + a^2b^2x^2(56c-75dx^2+9ex^4) + a^3b(-8c+3x^2(-8d+5ex^2+\ldots}{24a^4bx^3\left(a+bx^2\right)^2} + \frac{\left(35b^3c - 15ab^2d + 3a^2be + a^3f\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}b^{3/2}}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^3),x]
```

```
[Out] (-3*a^4*f*x^4 + 105*b^4*c*x^6 + 5*a*b^3*x^4*(35*c - 9*d*x^2) + a^2*b^2*x^2*
(56*c - 75*d*x^2 + 9*e*x^4) + a^3*b*(-8*c + 3*x^2*(-8*d + 5*e*x^2 + f*x^4))
)/(24*a^4*b*x^3*(a + b*x^2)^2) + ((35*b^3*c - 15*a*b^2*d + 3*a^2*b*e + a^3*
f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2)*b^(3/2))
```

Maple [A] (verified)

Time = 3.51 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.90

method	result
default	$-\frac{c}{3a^3x^3}-\frac{ad-3bc}{a^4x}+\frac{\frac{\left(\frac{1}{8}fa^3+\frac{3}{8}a^2be-\frac{7}{8}ab^2d+\frac{11}{8}b^3c\right)x^3-\frac{a\left(fa^3-5a^2be+9ab^2d-13b^3c\right)x}{8b}}{\left(bx^2+a\right)^2}+\frac{\left(fa^3+3a^2be-15ab^2d+35b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right.}{8b\sqrt{ab}}}{a^4}$
risch	$\frac{\frac{\left(fa^3+3a^2be-15ab^2d+35b^3c\right)x^6}{8a^4}-\frac{\left(3fa^3-15a^2be+75ab^2d-175b^3c\right)x^4}{24a^3b}-\frac{(3ad-7bc)x^2}{3a^2}-\frac{c}{3a}}{x^3\left(bx^2+a\right)^2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^9b^3\_Z^2+a^6f^2+6a^5bef-30a\right.}\right.}{}$

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*c/a^3/x^3-(a*d-3*b*c)/a^4/x+1/a^4*(((1/8*f*a^3+3/8*a^2*b*e-7/8*a*b^2*d
+11/8*b^3*c)*x^3-1/8*a*(a^3*f-5*a^2*b*e+9*a*b^2*d-13*b^3*c)/b*x)/(b*x^2+a)^
2+1/8*(a^3*f+3*a^2*b*e-15*a*b^2*d+35*b^3*c)/b/(a*b)^(1/2)*arctan(b*x/(a*b)^
(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 570, normalized size of antiderivative = 3.39

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^3}\,dx$$
$$=\left[-\frac{16\,a^4b^2c-6\,(35\,ab^5c-15\,a^2b^4d+3\,a^3b^3e+a^4b^2f)x^6-2\,(175\,a^2b^4c-75\,a^3b^3d+15\,a^4b^2e-3\,a^5bf)x^4}{}\right.$$
$$-\frac{8\,a^4b^2c-3\,(35\,ab^5c-15\,a^2b^4d+3\,a^3b^3e+a^4b^2f)x^6-(175\,a^2b^4c-75\,a^3b^3d+15\,a^4b^2e-3\,a^5bf)x^4-}{}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(16*a^4*b^2*c - 6*(35*a*b^5*c - 15*a^2*b^4*d + 3*a^3*b^3*e + a^4*b^2
*f)*x^6 - 2*(175*a^2*b^4*c - 75*a^3*b^3*d + 15*a^4*b^2*e - 3*a^5*b*f)*x^4 -
 16*(7*a^3*b^3*c - 3*a^4*b^2*d)*x^2 + 3*((35*b^5*c - 15*a*b^4*d + 3*a^2*b^3
*e + a^3*b^2*f)*x^7 + 2*(35*a*b^4*c - 15*a^2*b^3*d + 3*a^3*b^2*e + a^4*b*f)
*x^5 + (35*a^2*b^3*c - 15*a^3*b^2*d + 3*a^4*b*e + a^5*f)*x^3)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^5*b^4*x^7 + 2*a^6*b^3*x^5 +
 a^7*b^2*x^3), -1/24*(8*a^4*b^2*c - 3*(35*a*b^5*c - 15*a^2*b^4*d + 3*a^3*b^
3*e + a^4*b^2*f)*x^6 - (175*a^2*b^4*c - 75*a^3*b^3*d + 15*a^4*b^2*e - 3*a^5
*b*f)*x^4 - 8*(7*a^3*b^3*c - 3*a^4*b^2*d)*x^2 - 3*((35*b^5*c - 15*a*b^4*d +
```

```
 3*a^2*b^3*e + a^3*b^2*f)*x^7 + 2*(35*a*b^4*c - 15*a^2*b^3*d + 3*a^3*b^2*e
+ a^4*b*f)*x^5 + (35*a^2*b^3*c - 15*a^3*b^2*d + 3*a^4*b*e + a^5*f)*x^3)*sqr
t(a*b)*arctan(sqrt(a*b)*x/a))/(a^5*b^4*x^7 + 2*a^6*b^3*x^5 + a^7*b^2*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.08

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^3}\,dx = \frac{3\,(35\,b^4c-15\,ab^3d+3\,a^2b^2e+a^3bf)x^6-8\,a^3bc+(175\,ab^3c-75\,a^2b^2d+15\,a^3be-3\,a^4f)x^4+8\,(7\,a^2b^2c-}{24\,(a^4b^3x^7+2\,a^5b^2x^5+a^6bx^3)} + \frac{(35\,b^3c-15\,ab^2d+3\,a^2be+a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^4b}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(3*(35*b^4*c - 15*a*b^3*d + 3*a^2*b^2*e + a^3*b*f)*x^6 - 8*a^3*b*c + (
175*a*b^3*c - 75*a^2*b^2*d + 15*a^3*b*e - 3*a^4*f)*x^4 + 8*(7*a^2*b^2*c - 3
*a^3*b*d)*x^2)/(a^4*b^3*x^7 + 2*a^5*b^2*x^5 + a^6*b*x^3) + 1/8*(35*b^3*c -
15*a*b^2*d + 3*a^2*b*e + a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*b)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^3}\,dx = \frac{\left(35\,b^3c-15\,ab^2d+3\,a^2be+a^3f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^4b} + \frac{11\,b^4cx^3-7\,ab^3dx^3+3\,a^2b^2ex^3+a^3bfx^3+13\,ab^3cx-9\,a^2b^2dx+5\,a^3bex-a^4fx}{8\,\left(bx^2+a\right)^2a^4b} + \frac{9\,bcx^2-3\,adx^2-ac}{3\,a^4x^3}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(35*b^3*c - 15*a*b^2*d + 3*a^2*b*e + a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*a^4*b) + 1/8*(11*b^4*c*x^3 - 7*a*b^3*d*x^3 + 3*a^2*b^2*e*x^3 + a^3*b*
f*x^3 + 13*a*b^3*c*x - 9*a^2*b^2*d*x + 5*a^3*b*e*x - a^4*f*x)/((b*x^2 + a)^
2*a^4*b) + 1/3*(9*b*c*x^2 - 3*a*d*x^2 - a*c)/(a^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 5.72 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^3}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(f\,a^3+3\,e\,a^2\,b-15\,d\,a\,b^2+35\,c\,b^3\right)}{8\,a^{9/2}\,b^{3/2}} - \frac{\frac{c}{3\,a}-\frac{x^6\,\left(f\,a^3+3\,e\,a^2\,b-15\,d\,a\,b^2+35\,c\,b^3\right)}{8\,a^4}+\frac{x^2\,\left(3\,a\,d-7\,b\,c\right)}{3\,a^2}-\frac{x^4\,\left(-3\,f\,a^3+15\,e\,a^2\,b-75\,d\,a\,b^2+175\,c\,b^3\right)}{24\,a^3\,b}}{a^2\,x^3+2\,a\,b\,x^5+b^2\,x^7}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^3),x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(35*b^3*c + a^3*f - 15*a*b^2*d + 3*a^2*b*e))/(8*
a^(9/2)*b^(3/2)) - (c/(3*a) - (x^6*(35*b^3*c + a^3*f - 15*a*b^2*d + 3*a^2*b
*e))/(8*a^4) + (x^2*(3*a*d - 7*b*c))/(3*a^2) - (x^4*(175*b^3*c - 3*a^3*f -
75*a*b^2*d + 15*a^2*b*e))/(24*a^3*b))/(a^2*x^3 + b^2*x^7 + 2*a*b*x^5)
```

3.140 $\int \frac{c+dx^2+ex^4+fx^6}{x^6(a+bx^2)^3} dx$

Optimal result . . . 844
Rubi [A] (verified) . . . 844
Mathematica [A] (verified) . . . 846
Maple [A] (verified) . . . 847
Fricas [A] (verification not implemented) . . . 847
Sympy [F(-1)] . . . 848
Maxima [A] (verification not implemented) . . . 848
Giac [A] (verification not implemented) . . . 849
Mupad [B] (verification not implemented) . . . 849

Optimal result

Integrand size = 30, antiderivative size = 196

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6(a+bx^2)^3} dx = -\frac{c}{5a^3x^5} + \frac{3bc-ad}{3a^4x^3} - \frac{6b^2c-3abd+a^2e}{a^5x} - \frac{(b^3c-ab^2d+a^2be-a^3f)x}{4a^4(a+bx^2)^2} - \frac{(15b^3c-11ab^2d+7a^2be-3a^3f)x}{8a^5(a+bx^2)} - \frac{(63b^3c-35ab^2d+15a^2be-3a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{11/2}\sqrt{b}}$$

```
[Out] -1/5*c/a^3/x^5+1/3*(-a*d+3*b*c)/a^4/x^3+(-a^2*e+3*a*b*d-6*b^2*c)/a^5/x-1/4*
(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^4/(b*x^2+a)^2-1/8*(-3*a^3*f+7*a^2*b*e-11
*a*b^2*d+15*b^3*c)*x/a^5/(b*x^2+a)-1/8*(-3*a^3*f+15*a^2*b*e-35*a*b^2*d+63*b
^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(11/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used

$= \{1819, 1816, 211\}$

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^3}\,dx = \frac{3bc-ad}{3a^4x^3}-\frac{c}{5a^3x^5}-\frac{a^2e-3abd+6b^2c}{a^5x} - \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-3a^3f+15a^2be-35ab^2d+63b^3c\right)}{8a^{11/2}\sqrt{b}} - \frac{x(-3a^3f+7a^2be-11ab^2d+15b^3c)}{8a^5\left(a+bx^2\right)} - \frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{4a^4\left(a+bx^2\right)^2}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^3),x]
```

```
[Out] -1/5*c/(a^3*x^5) + (3*b*c - a*d)/(3*a^4*x^3) - (6*b^2*c - 3*a*b*d + a^2*e)/
(a^5*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^4*(a + b*x^2)^2) - (
(15*b^3*c - 11*a*b^2*d + 7*a^2*b*e - 3*a^3*f)*x)/(8*a^5*(a + b*x^2)) - ((63
*b^3*c - 35*a*b^2*d + 15*a^2*b*e - 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8
*a^(11/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^4\left(a+bx^2\right)^2} - \frac{\int \frac{-4c+4\left(\frac{bc}{a}-d\right)x^2-\frac{4\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{3\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{a^3}}{x^6\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= -\frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{4a^4(a+bx^2)^2} - \frac{(15b^3c - 11ab^2d + 7a^2be - 3a^3f)\,x}{8a^5(a+bx^2)} + \frac{\int \frac{8c - 8\left(\frac{2bc}{a} - d\right)x^2 + 8\left(\frac{3b^2c}{a^2} - \frac{2bd}{a} + e\right)x^4 - \frac{(15b^3c - 11ab^2d + 7a^2be - 3a^3f)x^6}{a^3}}{x^6(a+bx^2)}\,dx}{8a^2}$$

$$= -\frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{4a^4(a+bx^2)^2} - \frac{(15b^3c - 11ab^2d + 7a^2be - 3a^3f)\,x}{8a^5(a+bx^2)} + \frac{\int \left(\frac{8c}{ax^6} + \frac{8(-3bc+ad)}{a^2x^4} + \frac{8(6b^2c - 3abd + a^2e)}{a^3x^2} + \frac{-63b^3c + 35ab^2d - 15a^2be + 3a^3f}{a^3(a+bx^2)}\right)dx}{8a^2}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc - ad}{3a^4x^3} - \frac{6b^2c - 3abd + a^2e}{a^5x} - \frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{4a^4(a+bx^2)^2} - \frac{(15b^3c - 11ab^2d + 7a^2be - 3a^3f)\,x}{8a^5(a+bx^2)} - \frac{(63b^3c - 35ab^2d + 15a^2be - 3a^3f)\int \frac{1}{a+bx^2}\,dx}{8a^5}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc - ad}{3a^4x^3} - \frac{6b^2c - 3abd + a^2e}{a^5x} - \frac{(b^3c - ab^2d + a^2be - a^3f)\,x}{4a^4(a+bx^2)^2} - \frac{(15b^3c - 11ab^2d + 7a^2be - 3a^3f)\,x}{8a^5(a+bx^2)} - \frac{(63b^3c - 35ab^2d + 15a^2be - 3a^3f)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{11/2}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^6(a+bx^2)^3}\,dx = -\frac{c}{5a^3x^5} + \frac{3bc - ad}{3a^4x^3} + \frac{-6b^2c + 3abd - a^2e}{a^5x} + \frac{(-b^3c + ab^2d - a^2be + a^3f)\,x}{4a^4(a+bx^2)^2} + \frac{(-15b^3c + 11ab^2d - 7a^2be + 3a^3f)\,x}{8a^5(a+bx^2)} + \frac{(-63b^3c + 35ab^2d - 15a^2be + 3a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{11/2}\sqrt{b}}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^3),x]
[Out] -1/5*c/(a^3*x^5) + (3*b*c - a*d)/(3*a^4*x^3) + (-6*b^2*c + 3*a*b*d - a^2*e)
/(a^5*x) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(4*a^4*(a + b*x^2)^2)
 + ((-15*b^3*c + 11*a*b^2*d - 7*a^2*b*e + 3*a^3*f)*x)/(8*a^5*(a + b*x^2)) +
 ((-63*b^3*c + 35*a*b^2*d - 15*a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(8*a^(11/2)*Sqrt[b])
```

Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.90

method	result
default	$-\frac{c}{5a^3x^5}-\frac{ad-3bc}{3a^4x^3}-\frac{a^2e-3abd+6b^2c}{a^5x}+\frac{\frac{\left(\frac{3}{8}a^3bf-\frac{7}{8}a^2eb^2+\frac{11}{8}ab^3d-\frac{15}{8}b^4c\right)x^3+\frac{a\left(5fa^3-9a^2be+13ab^2d-17b^3c\right)x}{8}}{\left(bx^2+a\right)^2}+\frac{\left(3fa^3-15a^2be\right.}{}}{a^5}$
risch	$\frac{\frac{b\left(3fa^3-15a^2be+35ab^2d-63b^3c\right)x^8}{8a^5}+\frac{5\left(3fa^3-15a^2be+35ab^2d-63b^3c\right)x^6}{24a^4}-\frac{\left(15a^2e-35abd+63b^2c\right)x^4}{15a^3}-\frac{(5ad-9bc)x^2}{15a^2}-\frac{c}{5a}}{x^5\left(bx^2+a\right)^2}-\frac{3\ln\left(-\sqrt{-ab}\,x\right.}{16\sqrt{-ab}\,}$

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*c/a^3/x^5-1/3*(a*d-3*b*c)/a^4/x^3-(a^2*e-3*a*b*d+6*b^2*c)/a^5/x+1/a^5*
(((3/8*a^3*b*f-7/8*a^2*e*b^2+11/8*a*b^3*d-15/8*b^4*c)*x^3+1/8*a*(5*a^3*f-9*
a^2*b*e+13*a*b^2*d-17*b^3*c)*x)/(b*x^2+a)^2+1/8*(3*a^3*f-15*a^2*b*e+35*a*b^
2*d-63*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 628, normalized size of antiderivative = 3.20

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^3}\,dx$$

$$=\left[-\frac{30\,(63\,ab^5c-35\,a^2b^4d+15\,a^3b^3e-3\,a^4b^2f)x^8+48\,a^5bc+50\,(63\,a^2b^4c-35\,a^3b^3d+15\,a^4b^2e-3\,a^5bf}{}\right.$$

$$-\frac{15\,(63\,ab^5c-35\,a^2b^4d+15\,a^3b^3e-3\,a^4b^2f)x^8+24\,a^5bc+25\,(63\,a^2b^4c-35\,a^3b^3d+15\,a^4b^2e-3\,a^5bf)}{}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/240*(30*(63*a*b^5*c - 35*a^2*b^4*d + 15*a^3*b^3*e - 3*a^4*b^2*f)*x^8 +
48*a^5*b*c + 50*(63*a^2*b^4*c - 35*a^3*b^3*d + 15*a^4*b^2*e - 3*a^5*b*f)*x^
6 + 16*(63*a^3*b^3*c - 35*a^4*b^2*d + 15*a^5*b*e)*x^4 - 16*(9*a^4*b^2*c - 5
*a^5*b*d)*x^2 - 15*((63*b^5*c - 35*a*b^4*d + 15*a^2*b^3*e - 3*a^3*b^2*f)*x^
9 + 2*(63*a*b^4*c - 35*a^2*b^3*d + 15*a^3*b^2*e - 3*a^4*b*f)*x^7 + (63*a^2*
b^3*c - 35*a^3*b^2*d + 15*a^4*b*e - 3*a^5*f)*x^5)*sqrt(-a*b)*log((b*x^2 - 2
*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^6*b^3*x^9 + 2*a^7*b^2*x^7 + a^8*b*x^5),
 -1/120*(15*(63*a*b^5*c - 35*a^2*b^4*d + 15*a^3*b^3*e - 3*a^4*b^2*f)*x^8 +
24*a^5*b*c + 25*(63*a^2*b^4*c - 35*a^3*b^3*d + 15*a^4*b^2*e - 3*a^5*b*f)*x^
6 + 8*(63*a^3*b^3*c - 35*a^4*b^2*d + 15*a^5*b*e)*x^4 - 8*(9*a^4*b^2*c - 5*a
```

```
^5*b*d)*x^2 + 15*((63*b^5*c - 35*a*b^4*d + 15*a^2*b^3*e - 3*a^3*b^2*f)*x^9
+ 2*(63*a*b^4*c - 35*a^2*b^3*d + 15*a^3*b^2*e - 3*a^4*b*f)*x^7 + (63*a^2*b^
3*c - 35*a^3*b^2*d + 15*a^4*b*e - 3*a^5*f)*x^5)*sqrt(a*b)*arctan(sqrt(a*b)*
x/a))/(a^6*b^3*x^9 + 2*a^7*b^2*x^7 + a^8*b*x^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^3}\,dx = -\frac{15\,(63\,b^4c-35\,ab^3d+15\,a^2b^2e-3\,a^3bf)x^8+25\,(63\,ab^3c-35\,a^2b^2d+15\,a^3be-3\,a^4f)x^6+24\,a^4c+8\,(6}{120\,(a^5b^2x^9+2\,a^6bx^7+a^7x^5)} - \frac{(63\,b^3c-35\,ab^2d+15\,a^2be-3\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^5}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/120*(15*(63*b^4*c - 35*a*b^3*d + 15*a^2*b^2*e - 3*a^3*b*f)*x^8 + 25*(63*
a*b^3*c - 35*a^2*b^2*d + 15*a^3*b*e - 3*a^4*f)*x^6 + 24*a^4*c + 8*(63*a^2*b
^2*c - 35*a^3*b*d + 15*a^4*e)*x^4 - 8*(9*a^3*b*c - 5*a^4*d)*x^2)/(a^5*b^2*x
^9 + 2*a^6*b*x^7 + a^7*x^5) - 1/8*(63*b^3*c - 35*a*b^2*d + 15*a^2*b*e - 3*a
^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^3}\,dx = -\frac{\left(63\,b^3c-35\,ab^2d+15\,a^2be-3\,a^3f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^5} - \frac{15\,b^4cx^3-11\,ab^3dx^3+7\,a^2b^2ex^3-3\,a^3bfx^3+17\,ab^3cx-13\,a^2b^2dx+9\,a^3bex-5\,a^4fx}{8\,\left(bx^2+a\right)^2a^5} - \frac{90\,b^2cx^4-45\,abdx^4+15\,a^2ex^4-15\,abcx^2+5\,a^2dx^2+3\,a^2c}{15\,a^5x^5}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(63*b^3*c - 35*a*b^2*d + 15*a^2*b*e - 3*a^3*f)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*a^5) - 1/8*(15*b^4*c*x^3 - 11*a*b^3*d*x^3 + 7*a^2*b^2*e*x^3 - 3*a
^3*b*f*x^3 + 17*a*b^3*c*x - 13*a^2*b^2*d*x + 9*a^3*b*e*x - 5*a^4*f*x)/((b*x
^2 + a)^2*a^5) - 1/15*(90*b^2*c*x^4 - 45*a*b*d*x^4 + 15*a^2*e*x^4 - 15*a*b*
c*x^2 + 5*a^2*d*x^2 + 3*a^2*c)/(a^5*x^5)
```

Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^3}\,dx = -\frac{\frac{c}{5\,a}+\frac{5\,x^6\,\left(-3\,f\,a^3+15\,e\,a^2\,b-35\,d\,a\,b^2+63\,c\,b^3\right)}{24\,a^4}+\frac{x^2\,\left(5\,a\,d-9\,b\,c\right)}{15\,a^2}+\frac{x^4\,\left(15\,e\,a^2-35\,d\,a\,b+63\,c\,b^2\right)}{15\,a^3}+\frac{b\,x^8\,\left(-3\,f\,a^3+15\,e\,a^2\,b-35\,d\,a\,b^2+\right.}{8\,a^5}}{a^2\,x^5+2\,a\,b\,x^7+b^2\,x^9} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-3\,f\,a^3+15\,e\,a^2\,b-35\,d\,a\,b^2+63\,c\,b^3\right)}{8\,a^{11/2}\,\sqrt{b}}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^3),x)
```

```
[Out] - (c/(5*a) + (5*x^6*(63*b^3*c - 3*a^3*f - 35*a*b^2*d + 15*a^2*b*e))/(24*a^4
) + (x^2*(5*a*d - 9*b*c))/(15*a^2) + (x^4*(63*b^2*c + 15*a^2*e - 35*a*b*d))
/(15*a^3) + (b*x^8*(63*b^3*c - 3*a^3*f - 35*a*b^2*d + 15*a^2*b*e))/(8*a^5))
/(a^2*x^5 + b^2*x^9 + 2*a*b*x^7) - (atan((b^(1/2)*x)/a^(1/2))*(63*b^3*c - 3
*a^3*f - 35*a*b^2*d + 15*a^2*b*e))/(8*a^(11/2)*b^(1/2))
```

3.141 $\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)^3} dx$

Optimal result 850
Rubi [A] (verified) 850
Mathematica [A] (verified) 852
Maple [A] (verified) 853
Fricas [A] (verification not implemented) 853
Sympy [F(-1)] 854
Maxima [A] (verification not implemented) 854
Giac [A] (verification not implemented) 855
Mupad [B] (verification not implemented) 855

Optimal result

Integrand size = 30, antiderivative size = 234

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)^3} dx = -\frac{c}{7a^3x^7} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{3a^5x^3} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{4a^5(a+bx^2)^2} + \frac{b(19b^3c-15ab^2d+11a^2be-7a^3f)\,x}{8a^6(a+bx^2)} + \frac{\sqrt{b}(99b^3c-63ab^2d+35a^2be-15a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{13/2}}$$

```
[Out] -1/7*c/a^3/x^7+1/5*(-a*d+3*b*c)/a^4/x^5+1/3*(-a^2*e+3*a*b*d-6*b^2*c)/a^5/x^
3+(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)/a^6/x+1/4*b*(-a^3*f+a^2*b*e-a*b^2*d
+b^3*c)*x/a^5/(b*x^2+a)^2+1/8*b*(-7*a^3*f+11*a^2*b*e-15*a*b^2*d+19*b^3*c)*x
/a^6/(b*x^2+a)+1/8*(-15*a^3*f+35*a^2*b*e-63*a*b^2*d+99*b^3*c)*arctan(x*b^(1
/2)/a^(1/2))*b^(1/2)/a^(13/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used

$= \{1819, 1816, 211\}$

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)^3}\,dx = \frac{3bc-ad}{5a^4x^5} - \frac{c}{7a^3x^7} - \frac{a^2e-3abd+6b^2c}{3a^5x^3} + \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(-15a^3f+35a^2be-63ab^2d+99b^3c)}{8a^{13/2}} + \frac{bx(-7a^3f+11a^2be-15ab^2d+19b^3c)}{8a^6(a+bx^2)} + \frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{a^6x} + \frac{bx(a^3(-f)+a^2be-ab^2d+b^3c)}{4a^5(a+bx^2)^2}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^3),x]
```

```
[Out] -1/7*c/(a^3*x^7) + (3*b*c - a*d)/(5*a^4*x^5) - (6*b^2*c - 3*a*b*d + a^2*e)/
(3*a^5*x^3) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(a^6*x) + (b*(b^3*
c - a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^5*(a + b*x^2)^2) + (b*(19*b^3*c - 15
*a*b^2*d + 11*a^2*b*e - 7*a^3*f)*x)/(8*a^6*(a + b*x^2)) + (Sqrt[b]*(99*b^3*
c - 63*a*b^2*d + 35*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(
13/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{4a^5\,(a+bx^2)^2} - \frac{\int \frac{-4c+4\left(\frac{bc}{a}-d\right)x^2-\frac{4\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{4\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{a^3}-\frac{3b\left(b^3c-ab^2d+a^2be-a^3f\right)x^8}{a^4}}{x^8(a+bx^2)^2}\,dx}{4a} \\
&= \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{4a^5\,(a+bx^2)^2} + \frac{b(19b^3c-15ab^2d+11a^2be-7a^3f)\,x}{8a^6\,(a+bx^2)} \\
&\quad + \frac{\int \frac{8c-8\left(\frac{2bc}{a}-d\right)x^2+8\left(\frac{3b^2c}{a^2}-\frac{2bd}{a}+e\right)x^4-\frac{8\left(4b^3c-3ab^2d+2a^2be-a^3f\right)x^6}{a^3}+\frac{b\left(19b^3c-15ab^2d+11a^2be-7a^3f\right)x^8}{a^4}}{x^8(a+bx^2)}\,dx}{8a^2} \\
&= \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{4a^5\,(a+bx^2)^2} + \frac{b(19b^3c-15ab^2d+11a^2be-7a^3f)\,x}{8a^6\,(a+bx^2)} \\
&\quad + \frac{\int \left(\frac{8c}{ax^8}+\frac{8(-3bc+ad)}{a^2x^6}+\frac{8\left(6b^2c-3abd+a^2e\right)}{a^3x^4}+\frac{8\left(-10b^3c+6ab^2d-3a^2be+a^3f\right)}{a^4x^2}-\frac{b\left(-99b^3c+63ab^2d-35a^2be+15a^3f\right)}{a^4(a+bx^2)}\right)dx}{8a^2} \\
&= -\frac{c}{7a^3x^7} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{3a^5x^3} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} \\
&\quad + \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{4a^5\,(a+bx^2)^2} + \frac{b(19b^3c-15ab^2d+11a^2be-7a^3f)\,x}{8a^6\,(a+bx^2)} \\
&\quad + \frac{(b(99b^3c-63ab^2d+35a^2be-15a^3f))\int \frac{1}{a+bx^2}\,dx}{8a^6} \\
&= -\frac{c}{7a^3x^7} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{3a^5x^3} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} \\
&\quad + \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{4a^5\,(a+bx^2)^2} + \frac{b(19b^3c-15ab^2d+11a^2be-7a^3f)\,x}{8a^6\,(a+bx^2)} \\
&\quad + \frac{\sqrt{b}(99b^3c-63ab^2d+35a^2be-15a^3f)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{13/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00

$$
\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{x^8\,(a+bx^2)^3}\,dx &= -\frac{c}{7a^3x^7} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{3a^5x^3} \\
&\quad + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{4a^5\,(a+bx^2)^2} \\
&\quad + \frac{b(19b^3c-15ab^2d+11a^2be-7a^3f)\,x}{8a^6\,(a+bx^2)} \\
&\quad + \frac{\sqrt{b}(99b^3c-63ab^2d+35a^2be-15a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{13/2}}
\end{aligned}
$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^3),x]
```

```
[Out] -1/7*c/(a^3*x^7) + (3*b*c - a*d)/(5*a^4*x^5) - (6*b^2*c - 3*a*b*d + a^2*e)/
(3*a^5*x^3) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(a^6*x) + (b*(b^3*
c - a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^5*(a + b*x^2)^2) + (b*(19*b^3*c - 15
*a*b^2*d + 11*a^2*b*e - 7*a^3*f)*x)/(8*a^6*(a + b*x^2)) + (Sqrt[b]*(99*b^3*
c - 63*a*b^2*d + 35*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(
13/2))
```

Maple [A] (verified)

Time = 3.64 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.91

method	result
default	$-\frac{c}{7a^3x^7}-\frac{ad-3bc}{5a^4x^5}-\frac{a^2e-3abd+6b^2c}{3a^5x^3}-\frac{f\,a^3-3a^2be+6a\,b^2d-10b^3c}{a^6x}-\frac{b\left(\frac{\left(\frac{7}{8}a^3bf-\frac{11}{8}a^2e\,b^2+\frac{15}{8}a\,b^3d-\frac{19}{8}b^4c\right)x^3+\frac{a\left(9f\,a^3-13a\right.}{}}{\left(b\,x^2+a\right)^2}\right.}{}$
risch	$\frac{-\frac{b^2\left(15f\,a^3-35a^2be+63a\,b^2d-99b^3c\right)x^{10}}{8a^6}-\frac{5b\left(15f\,a^3-35a^2be+63a\,b^2d-99b^3c\right)x^8}{24a^5}-\frac{\left(15f\,a^3-35a^2be+63a\,b^2d-99b^3c\right)x^6}{15a^4}-\frac{\left(35a^2e-63abd+99b\right.}{105a^3}}{x^7\left(b\,x^2+a\right)^2}$

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*c/a^3/x^7-1/5*(a*d-3*b*c)/a^4/x^5-1/3*(a^2*e-3*a*b*d+6*b^2*c)/a^5/x^3-
(a^3*f-3*a^2*b*e+6*a*b^2*d-10*b^3*c)/a^6/x-b/a^6*(((7/8*a^3*b*f-11/8*a^2*e*
b^2+15/8*a*b^3*d-19/8*b^4*c)*x^3+1/8*a*(9*a^3*f-13*a^2*b*e+17*a*b^2*d-21*b^
3*c)*x)/(b*x^2+a)^2+1/8*(15*a^3*f-35*a^2*b*e+63*a*b^2*d-99*b^3*c)/(a*b)^(1/
2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 678, normalized size of antiderivative = 2.90

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^3}\,dx$$

$$= \left[\frac{210\,(99\,b^5c-63\,ab^4d+35\,a^2b^3e-15\,a^3b^2f)x^{10}+350\,(99\,ab^4c-63\,a^2b^3d+35\,a^3b^2e-15\,a^4bf)x^8+112}{}\right.$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/1680*(210*(99*b^5*c - 63*a*b^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f)*x^10 + 3
50*(99*a*b^4*c - 63*a^2*b^3*d + 35*a^3*b^2*e - 15*a^4*b*f)*x^8 + 112*(99*a^
```

```
2*b^3*c - 63*a^3*b^2*d + 35*a^4*b*e - 15*a^5*f)*x^6 - 240*a^5*c - 16*(99*a^
3*b^2*c - 63*a^4*b*d + 35*a^5*e)*x^4 + 48*(11*a^4*b*c - 7*a^5*d)*x^2 - 105*
((99*b^5*c - 63*a*b^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f)*x^11 + 2*(99*a*b^4*c
 - 63*a^2*b^3*d + 35*a^3*b^2*e - 15*a^4*b*f)*x^9 + (99*a^2*b^3*c - 63*a^3*b
^2*d + 35*a^4*b*e - 15*a^5*f)*x^7)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a)
 - a)/(b*x^2 + a)))/(a^6*b^2*x^11 + 2*a^7*b*x^9 + a^8*x^7), 1/840*(105*(99*
b^5*c - 63*a*b^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f)*x^10 + 175*(99*a*b^4*c -
63*a^2*b^3*d + 35*a^3*b^2*e - 15*a^4*b*f)*x^8 + 56*(99*a^2*b^3*c - 63*a^3*b
^2*d + 35*a^4*b*e - 15*a^5*f)*x^6 - 120*a^5*c - 8*(99*a^3*b^2*c - 63*a^4*b*
d + 35*a^5*e)*x^4 + 24*(11*a^4*b*c - 7*a^5*d)*x^2 + 105*((99*b^5*c - 63*a*b
^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f)*x^11 + 2*(99*a*b^4*c - 63*a^2*b^3*d + 3
5*a^3*b^2*e - 15*a^4*b*f)*x^9 + (99*a^2*b^3*c - 63*a^3*b^2*d + 35*a^4*b*e -
 15*a^5*f)*x^7)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^6*b^2*x^11 + 2*a^7*b*x^9
+ a^8*x^7)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.06

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^3}\,dx$$
$$=\frac{105\,(99\,b^5c-63\,ab^4d+35\,a^2b^3e-15\,a^3b^2f)x^{10}+175\,(99\,ab^4c-63\,a^2b^3d+35\,a^3b^2e-15\,a^4bf)x^8+56\,(99}{840\,(a^6b^2x^{11}+}$$
$$+\frac{(99\,b^4c-63\,ab^3d+35\,a^2b^2e-15\,a^3bf)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^6}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/840*(105*(99*b^5*c - 63*a*b^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f)*x^10 + 175
*(99*a*b^4*c - 63*a^2*b^3*d + 35*a^3*b^2*e - 15*a^4*b*f)*x^8 + 56*(99*a^2*b
^3*c - 63*a^3*b^2*d + 35*a^4*b*e - 15*a^5*f)*x^6 - 120*a^5*c - 8*(99*a^3*b^
2*c - 63*a^4*b*d + 35*a^5*e)*x^4 + 24*(11*a^4*b*c - 7*a^5*d)*x^2)/(a^6*b^2*
x^11 + 2*a^7*b*x^9 + a^8*x^7) + 1/8*(99*b^4*c - 63*a*b^3*d + 35*a^2*b^2*e -
 15*a^3*b*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.05

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^3}\,dx=\frac{\left(99\,b^4c-63\,ab^3d+35\,a^2b^2e-15\,a^3bf\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^6}+\frac{19\,b^5cx^3-15\,ab^4dx^3+11\,a^2b^3ex^3-7\,a^3b^2fx^3+21\,ab^4cx-17\,a^2b^3dx+13\,a^3b^2ex-9\,a^4bfx}{8\,\left(bx^2+a\right)^2a^6}+\frac{1050\,b^3cx^6-630\,ab^2dx^6+315\,a^2bex^6-105\,a^3fx^6-210\,ab^2cx^4+105\,a^2bdx^4-35\,a^3ex^4+63\,a^2bcx^2-}{105\,a^6x^7}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(99*b^4*c - 63*a*b^3*d + 35*a^2*b^2*e - 15*a^3*b*f)*arctan(b*x/sqrt(a*b
))/(sqrt(a*b)*a^6) + 1/8*(19*b^5*c*x^3 - 15*a*b^4*d*x^3 + 11*a^2*b^3*e*x^3
- 7*a^3*b^2*f*x^3 + 21*a*b^4*c*x - 17*a^2*b^3*d*x + 13*a^3*b^2*e*x - 9*a^4*
b*f*x)/((b*x^2 + a)^2*a^6) + 1/105*(1050*b^3*c*x^6 - 630*a*b^2*d*x^6 + 315*
a^2*b*e*x^6 - 105*a^3*f*x^6 - 210*a*b^2*c*x^4 + 105*a^2*b*d*x^4 - 35*a^3*e*
x^4 + 63*a^2*b*c*x^2 - 21*a^3*d*x^2 - 15*a^3*c)/(a^6*x^7)
```

Mupad [B] (verification not implemented)

Time = 5.61 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^3}\,dx=\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-15\,f\,a^3+35\,e\,a^2\,b-63\,d\,a\,b^2+99\,c\,b^3\right)}{8\,a^{13/2}}-\frac{\frac{c}{7\,a}-\frac{x^6\,\left(-15\,f\,a^3+35\,e\,a^2\,b-63\,d\,a\,b^2+99\,c\,b^3\right)}{15\,a^4}+\frac{x^2\,\left(7\,a\,d-11\,b\,c\right)}{35\,a^2}+\frac{x^4\,\left(35\,e\,a^2-63\,d\,a\,b+99\,c\,b^2\right)}{105\,a^3}-\frac{5\,b\,x^8\,\left(-15\,f\,a^3+35\,e\,a^2\,b-63\,d\,a\right.}{24\,a^5}}{a^2\,x^7+2\,a\,b\,x^9+b^2\,x^{11}}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^3),x)
```

```
[Out] (b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(99*b^3*c - 15*a^3*f - 63*a*b^2*d + 35*a
^2*b*e))/(8*a^(13/2)) - (c/(7*a) - (x^6*(99*b^3*c - 15*a^3*f - 63*a*b^2*d +
 35*a^2*b*e))/(15*a^4) + (x^2*(7*a*d - 11*b*c))/(35*a^2) + (x^4*(99*b^2*c +
 35*a^2*e - 63*a*b*d))/(105*a^3) - (5*b*x^8*(99*b^3*c - 15*a^3*f - 63*a*b^2
*d + 35*a^2*b*e))/(24*a^5) - (b^2*x^10*(99*b^3*c - 15*a^3*f - 63*a*b^2*d +
35*a^2*b*e))/(8*a^6))/(a^2*x^7 + b^2*x^11 + 2*a*b*x^9)
```

3.142 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)^3}\,dx$

Optimal result 856
Rubi [A] (verified) 857
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Optimal result

Integrand size = 30, antiderivative size = 277

$$\begin{aligned}\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)^3}\,dx = &-\frac{c}{9a^3x^9}+\frac{3bc-ad}{7a^4x^7}-\frac{6b^2c-3abd+a^2e}{5a^5x^5} \\ &+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{3a^6x^3} \\ &-\frac{b(15b^3c-10ab^2d+6a^2be-3a^3f)}{a^7x} \\ &-\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x}{4a^6(a+bx^2)^2} \\ &-\frac{b^2(23b^3c-19ab^2d+15a^2be-11a^3f)\,x}{8a^7(a+bx^2)} \\ &-\frac{b^{3/2}(143b^3c-99ab^2d+63a^2be-35a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{15/2}}\end{aligned}$$

```
[Out] -1/9*c/a^3/x^9+1/7*(-a*d+3*b*c)/a^4/x^7+1/5*(-a^2*e+3*a*b*d-6*b^2*c)/a^5/x^
5+1/3*(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)/a^6/x^3-b*(-3*a^3*f+6*a^2*b*e-1
0*a*b^2*d+15*b^3*c)/a^7/x-1/4*b^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^6/(b*x
^2+a)^2-1/8*b^2*(-11*a^3*f+15*a^2*b*e-19*a*b^2*d+23*b^3*c)*x/a^7/(b*x^2+a)-
1/8*b^(3/2)*(-35*a^3*f+63*a^2*b*e-99*a*b^2*d+143*b^3*c)*arctan(x*b^(1/2)/a^
(1/2))/a^(15/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1819, 1816, 211}

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^3}\,dx=\frac{3bc-ad}{7a^4x^7}-\frac{c}{9a^3x^9}-\frac{a^2e-3abd+6b^2c}{5a^5x^5}-\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-35a^3f+63a^2be-99ab^2d+143b^3c\right)}{8a^{15/2}}-\frac{b^2x(-11a^3f+15a^2be-19ab^2d+23b^3c)}{8a^7\left(a+bx^2\right)}-\frac{b(-3a^3f+6a^2be-10ab^2d+15b^3c)}{a^7x}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{3a^6x^3}-\frac{b^2x(a^3(-f)+a^2be-ab^2d+b^3c)}{4a^6\left(a+bx^2\right)^2}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^3),x]
```

```
[Out] -1/9*c/(a^3*x^9) + (3*b*c - a*d)/(7*a^4*x^7) - (6*b^2*c - 3*a*b*d + a^2*e)/
(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(3*a^6*x^3) - (b*(
15*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f))/(a^7*x) - (b^2*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*x)/(4*a^6*(a + b*x^2)^2) - (b^2*(23*b^3*c - 19*a*b^2*
d + 15*a^2*b*e - 11*a^3*f)*x)/(8*a^7*(a + b*x^2)) - (b^(3/2)*(143*b^3*c - 9
9*a*b^2*d + 63*a^2*b*e - 35*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(15/2)
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
```

```
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x}{4a^6\,(a+bx^2)^2} - \frac{\int \frac{-4c+4\left(\frac{bc}{a}-d\right)x^2-\frac{4\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{4\left(b^3c-ab^2d+a^2be-a^3f\right)x^6}{a^3}-\frac{4b\left(b^3c-ab^2d+a^2be-a^3f\right)x^8}{a^4}+\frac{3b^2\left(b^3c-ab^2d+a^2be-a^3f\right)x^{10}}{a^5}}{x^{10}(a+bx^2)^2}\,dx}{4a}$$

$$= -\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x}{4a^6\,(a+bx^2)^2} - \frac{b^2(23b^3c-19ab^2d+15a^2be-11a^3f)\,x}{8a^7\,(a+bx^2)} + \frac{\int \frac{8c-8\left(\frac{2bc}{a}-d\right)x^2+8\left(\frac{3b^2c}{a^2}-\frac{2bd}{a}+e\right)x^4-\frac{8\left(4b^3c-3ab^2d+2a^2be-a^3f\right)x^6}{a^3}+\frac{8b\left(5b^3c-4ab^2d+3a^2be-2a^3f\right)x^8}{a^4}-\frac{b^2\left(23b^3c-19ab^2d+15a^2be-\right.}{a^5}}{x^{10}(a+bx^2)}}{8a^2}$$

$$= -\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x}{4a^6\,(a+bx^2)^2} - \frac{b^2(23b^3c-19ab^2d+15a^2be-11a^3f)\,x}{8a^7\,(a+bx^2)} + \frac{\int \left(\frac{8c}{ax^{10}}+\frac{8(-3bc+ad)}{a^2x^8}+\frac{8(6b^2c-3abd+a^2e)}{a^3x^6}+\frac{8(-10b^3c+6ab^2d-3a^2be+a^3f)}{a^4x^4}-\frac{8b(-15b^3c+10ab^2d-6a^2be+3a^3f)}{a^5x^2}+\frac{b^2}{}\right.}{8a^2}$$

$$= -\frac{c}{9a^3x^9} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{3a^6x^3} - \frac{b(15b^3c-10ab^2d+6a^2be-3a^3f)}{a^7x} - \frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x}{4a^6\,(a+bx^2)^2} - \frac{b^2(23b^3c-19ab^2d+15a^2be-11a^3f)\,x}{8a^7\,(a+bx^2)} - \frac{(b^2(143b^3c-99ab^2d+63a^2be-35a^3f))\int \frac{1}{a+bx^2}\,dx}{8a^7}$$

$$= -\frac{c}{9a^3x^9} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{3a^6x^3} - \frac{b(15b^3c-10ab^2d+6a^2be-3a^3f)}{a^7x} - \frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x}{4a^6\,(a+bx^2)^2} - \frac{b^2(23b^3c-19ab^2d+15a^2be-11a^3f)\,x}{8a^7\,(a+bx^2)} - \frac{b^{3/2}(143b^3c-99ab^2d+63a^2be-35a^3f)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{15/2}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^3}\,dx = -\frac{c}{9a^3x^9}+\frac{3bc-ad}{7a^4x^7}-\frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{3a^6x^3} + \frac{b(-15b^3c+10ab^2d-6a^2be+3a^3f)}{a^7x} + \frac{b^2(-b^3c+ab^2d-a^2be+a^3f)\,x}{4a^6\left(a+bx^2\right)^2} + \frac{b^2(-23b^3c+19ab^2d-15a^2be+11a^3f)\,x}{8a^7\left(a+bx^2\right)} + \frac{b^{3/2}(-143b^3c+99ab^2d-63a^2be+35a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{15/2}}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^3),x]
```

```
[Out] -1/9*c/(a^3*x^9) + (3*b*c - a*d)/(7*a^4*x^7) - (6*b^2*c - 3*a*b*d + a^2*e)/
(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(3*a^6*x^3) + (b*(
-15*b^3*c + 10*a*b^2*d - 6*a^2*b*e + 3*a^3*f))/(a^7*x) + (b^2*(-(b^3*c) + a
*b^2*d - a^2*b*e + a^3*f)*x)/(4*a^6*(a + b*x^2)^2) + (b^2*(-23*b^3*c + 19*a
*b^2*d - 15*a^2*b*e + 11*a^3*f)*x)/(8*a^7*(a + b*x^2)) + (b^(3/2)*(-143*b^3
*c + 99*a*b^2*d - 63*a^2*b*e + 35*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^
(15/2))
```

Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.90

method	result
default	$-\frac{c}{9a^3x^9}-\frac{ad-3bc}{7a^4x^7}-\frac{a^2e-3abd+6b^2c}{5a^5x^5}-\frac{f\,a^3-3a^2be+6a\,b^2d-10b^3c}{3a^6x^3}+\frac{b\left(3f\,a^3-6a^2be+10a\,b^2d-15b^3c\right)}{a^7x}+\frac{b^2\left(\frac{\left(\frac{11}{8}a^3bf-\frac{15}{8}\right.}{}\right.}{}$
risch	$\frac{\frac{b^3\left(35f\,a^3-63a^2be+99a\,b^2d-143b^3c\right)x^{12}}{8a^7}+\frac{5b^2\left(35f\,a^3-63a^2be+99a\,b^2d-143b^3c\right)x^{10}}{24a^6}+\frac{b\left(35f\,a^3-63a^2be+99a\,b^2d-143b^3c\right)x^8}{15a^5}-\frac{\left(35f\,a^3-63a^2b\right.}{}}{x^9\left(b\,x^2+a\right)^2}$

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*c/a^3/x^9-1/7*(a*d-3*b*c)/a^4/x^7-1/5*(a^2*e-3*a*b*d+6*b^2*c)/a^5/x^5-
1/3*(a^3*f-3*a^2*b*e+6*a*b^2*d-10*b^3*c)/a^6/x^3+b*(3*a^3*f-6*a^2*b*e+10*a*
b^2*d-15*b^3*c)/a^7/x+b^2/a^7*(((11/8*a^3*b*f-15/8*a^2*e*b^2+19/8*a*b^3*d-2
```

```
3/8*b^4*c)*x^3+1/8*a*(13*a^3*f-17*a^2*b*e+21*a*b^2*d-25*b^3*c)*x)/(b*x^2+a)
^2+1/8*(35*a^3*f-63*a^2*b*e+99*a*b^2*d-143*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a
*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 772, normalized size of antiderivative = 2.79

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^3}\,dx$$

$$= \left[-\frac{630\,(143\,b^6c-99\,ab^5d+63\,a^2b^4e-35\,a^3b^3f)x^{12}+1050\,(143\,ab^5c-99\,a^2b^4d+63\,a^3b^3e-35\,a^4b^2f)x^{10}}{}\right.$$

$$-\frac{315\,(143\,b^6c-99\,ab^5d+63\,a^2b^4e-35\,a^3b^3f)x^{12}+525\,(143\,ab^5c-99\,a^2b^4d+63\,a^3b^3e-35\,a^4b^2f)x^{10}+}{}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/5040*(630*(143*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)*x^12 +
 1050*(143*a*b^5*c - 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)*x^10 + 336
*(143*a^2*b^4*c - 99*a^3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^8 + 560*a^6*c
 - 48*(143*a^3*b^3*c - 99*a^4*b^2*d + 63*a^5*b*e - 35*a^6*f)*x^6 + 16*(143*
a^4*b^2*c - 99*a^5*b*d + 63*a^6*e)*x^4 - 80*(13*a^5*b*c - 9*a^6*d)*x^2 + 31
5*((143*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)*x^13 + 2*(143*a*b
^5*c - 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)*x^11 + (143*a^2*b^4*c -
99*a^3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^9)*sqrt(-b/a)*log((b*x^2 + 2*a*
x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^7*b^2*x^13 + 2*a^8*b*x^11 + a^9*x^9), -1
/2520*(315*(143*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)*x^12 + 52
5*(143*a*b^5*c - 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)*x^10 + 168*(14
3*a^2*b^4*c - 99*a^3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^8 + 280*a^6*c - 2
4*(143*a^3*b^3*c - 99*a^4*b^2*d + 63*a^5*b*e - 35*a^6*f)*x^6 + 8*(143*a^4*b
^2*c - 99*a^5*b*d + 63*a^6*e)*x^4 - 40*(13*a^5*b*c - 9*a^6*d)*x^2 + 315*((1
43*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)*x^13 + 2*(143*a*b^5*c
- 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)*x^11 + (143*a^2*b^4*c - 99*a^
3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^9)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a
^7*b^2*x^13 + 2*a^8*b*x^11 + a^9*x^9)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^3}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a)**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.05

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^3}\,dx = -\frac{315\,(143\,b^6c-99\,ab^5d+63\,a^2b^4e-35\,a^3b^3f)x^{12}+525\,(143\,ab^5c-99\,a^2b^4d+63\,a^3b^3e-35\,a^4b^2f)x^{10}+\cdots}{\cdots} - \frac{(143\,b^5c-99\,ab^4d+63\,a^2b^3e-35\,a^3b^2f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^7}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^3,x, algorithm="maxima")
[Out] -1/2520*(315*(143*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)*x^12 +
525*(143*a*b^5*c - 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)*x^10 + 168*(
143*a^2*b^4*c - 99*a^3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^8 + 280*a^6*c -
 24*(143*a^3*b^3*c - 99*a^4*b^2*d + 63*a^5*b*e - 35*a^6*f)*x^6 + 8*(143*a^4
*b^2*c - 99*a^5*b*d + 63*a^6*e)*x^4 - 40*(13*a^5*b*c - 9*a^6*d)*x^2)/(a^7*b
^2*x^13 + 2*a^8*b*x^11 + a^9*x^9) - 1/8*(143*b^5*c - 99*a*b^4*d + 63*a^2*b^
3*e - 35*a^3*b^2*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^7)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.06

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^3}\,dx = -\frac{(143\,b^5c-99\,ab^4d+63\,a^2b^3e-35\,a^3b^2f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^7} - \frac{23\,b^6cx^3-19\,ab^5dx^3+15\,a^2b^4ex^3-11\,a^3b^3fx^3+25\,ab^5cx-21\,a^2b^4dx+17\,a^3b^3ex-13\,a^4b^2fx}{8\,(bx^2+a)^2a^7} - \frac{4725\,b^4cx^8-3150\,ab^3dx^8+1890\,a^2b^2ex^8-945\,a^3bfx^8-1050\,ab^3cx^6+630\,a^2b^2dx^6-315\,a^3bex^6+\cdots}{315\,a^7x^9}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(143*b^5*c - 99*a*b^4*d + 63*a^2*b^3*e - 35*a^3*b^2*f)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a^7) - 1/8*(23*b^6*c*x^3 - 19*a*b^5*d*x^3 + 15*a^2*b^4*e*
x^3 - 11*a^3*b^3*f*x^3 + 25*a*b^5*c*x - 21*a^2*b^4*d*x + 17*a^3*b^3*e*x - 1
3*a^4*b^2*f*x)/((b*x^2 + a)^2*a^7) - 1/315*(4725*b^4*c*x^8 - 3150*a*b^3*d*x
^8 + 1890*a^2*b^2*e*x^8 - 945*a^3*b*f*x^8 - 1050*a*b^3*c*x^6 + 630*a^2*b^2*
d*x^6 - 315*a^3*b*e*x^6 + 105*a^4*f*x^6 + 378*a^2*b^2*c*x^4 - 189*a^3*b*d*x
^4 + 63*a^4*e*x^4 - 135*a^3*b*c*x^2 + 45*a^4*d*x^2 + 35*a^4*c)/(a^7*x^9)
```

Mupad [B] (verification not implemented)

Time = 5.65 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^3}\,dx = -\frac{\frac{c}{9\,a}-\frac{x^6\,\left(-35\,f\,a^3+63\,e\,a^2\,b-99\,d\,a\,b^2+143\,c\,b^3\right)}{105\,a^4}+\frac{x^2\,\left(9\,a\,d-13\,b\,c\right)}{63\,a^2}+\frac{x^4\,\left(63\,e\,a^2-99\,d\,a\,b+143\,c\,b^2\right)}{315\,a^3}+\frac{b\,x^8\,\left(-35\,f\,a^3+63\,e\,a^2\,b-99\,d\,a\,b}{15\,a^5}}{a^2\,x^9+2\,a\,b\,x^{11}+b^2\,x} - \frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-35\,f\,a^3+63\,e\,a^2\,b-99\,d\,a\,b^2+143\,c\,b^3\right)}{8\,a^{15/2}}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^3),x)
```

```
[Out] - (c/(9*a) - (x^6*(143*b^3*c - 35*a^3*f - 99*a*b^2*d + 63*a^2*b*e))/(105*a^
4) + (x^2*(9*a*d - 13*b*c))/(63*a^2) + (x^4*(143*b^2*c + 63*a^2*e - 99*a*b*
d))/(315*a^3) + (b*x^8*(143*b^3*c - 35*a^3*f - 99*a*b^2*d + 63*a^2*b*e))/(1
5*a^5) + (5*b^2*x^10*(143*b^3*c - 35*a^3*f - 99*a*b^2*d + 63*a^2*b*e))/(24*
a^6) + (b^3*x^12*(143*b^3*c - 35*a^3*f - 99*a*b^2*d + 63*a^2*b*e))/(8*a^7))
/(a^2*x^9 + b^2*x^13 + 2*a*b*x^11) - (b^(3/2)*atan((b^(1/2)*x)/a^(1/2))*(14
3*b^3*c - 35*a^3*f - 99*a*b^2*d + 63*a^2*b*e))/(8*a^(15/2))
```

3.143 $\int \frac{x^5(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

Optimal result 863
Rubi [A] (verified) 863
Mathematica [A] (verified) 865
Maple [A] (verified) 865
Fricas [A] (verification not implemented) 866
Sympy [B] (verification not implemented) 867
Maxima [A] (verification not implemented) 868
Giac [A] (verification not implemented) 868
Mupad [B] (verification not implemented) 869

Optimal result

Integrand size = 32, antiderivative size = 214

$$\begin{aligned}\int \frac{x^5(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx &= \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^6}\\ &\quad-\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)\left(a+bx^2\right)^{3/2}}{3b^6}\\ &\quad+\frac{(b^3c-3ab^2d+6a^2be-10a^3f)\left(a+bx^2\right)^{5/2}}{5b^6}\\ &\quad+\frac{(b^2d-4abe+10a^2f)\left(a+bx^2\right)^{7/2}}{7b^6}\\ &\quad+\frac{(be-5af)\left(a+bx^2\right)^{9/2}}{9b^6}+\frac{f\left(a+bx^2\right)^{11/2}}{11b^6}\end{aligned}$$

```
[Out] -1/3*a*(-5*a^3*f+4*a^2*b*e-3*a*b^2*d+2*b^3*c)*(b*x^2+a)^(3/2)/b^6+1/5*(-10*
a^3*f+6*a^2*b*e-3*a*b^2*d+b^3*c)*(b*x^2+a)^(5/2)/b^6+1/7*(10*a^2*f-4*a*b*e+
b^2*d)*(b*x^2+a)^(7/2)/b^6+1/9*(-5*a*f+b*e)*(b*x^2+a)^(9/2)/b^6+1/11*f*(b*x
^2+a)^(11/2)/b^6+a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x^2+a)^(1/2)/b^6
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used

$= \{1813, 1634\}$

$$\int \frac{x^5(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{(a+bx^2)^{7/2}(10a^2f-4abe+b^2d)}{7b^6} + \frac{(a+bx^2)^{5/2}(-10a^3f+6a^2be-3ab^2d+b^3c)}{5b^6} - \frac{a(a+bx^2)^{3/2}(-5a^3f+4a^2be-3ab^2d+2b^3c)}{3b^6} + \frac{a^2\sqrt{a+bx^2}(a^3(-f)+a^2be-ab^2d+b^3c)}{b^6} + \frac{(a+bx^2)^{9/2}(be-5af)}{9b^6} + \frac{f(a+bx^2)^{11/2}}{11b^6}$$

```
[In] Int[(x^5*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^6 - (a*(2*b^3*c
 - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*(a + b*x^2)^(3/2))/(3*b^6) + ((b^3*c -
3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*(a + b*x^2)^(5/2))/(5*b^6) + ((b^2*d - 4*
a*b*e + 10*a^2*f)*(a + b*x^2)^(7/2))/(7*b^6) + ((b*e - 5*a*f)*(a + b*x^2)^(
9/2))/(9*b^6) + (f*(a + b*x^2)^(11/2))/(11*b^6)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^2(-b^3c+ab^2d-a^2be+a^3f)}{b^5\sqrt{a+bx}} + \frac{a(-2b^3c+3ab^2d-4a^2be+5a^3f)\sqrt{a+bx}}{b^5} + \frac{(b^3c-3ab^2d+6a^2be-10a^3f)(a+bx)^{3/2}}{b^5} + \frac{(b^2d-4abe+10a^2f)(a+bx)^{5/2}}{b^5} + \frac{(be-5af)(a+bx)^{7/2}}{b^5} + \frac{f(a+bx)^{9/2}}{b^5}\right)dx, x, x^2\right)$$

$$
\begin{aligned}
&= \frac{a^2(b^3c - ab^2d + a^2be - a^3f)\sqrt{a+bx^2}}{b^6} \\
&\quad - \frac{a(2b^3c - 3ab^2d + 4a^2be - 5a^3f)\,(a+bx^2)^{3/2}}{3b^6} \\
&\quad + \frac{(b^3c - 3ab^2d + 6a^2be - 10a^3f)\,(a+bx^2)^{5/2}}{5b^6} \\
&\quad + \frac{(b^2d - 4abe + 10a^2f)\,(a+bx^2)^{7/2}}{7b^6} + \frac{(be - 5af)\,(a+bx^2)^{9/2}}{9b^6} + \frac{f(a+bx^2)^{11/2}}{11b^6}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.74

$$
\int \frac{x^5(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx
$$

$$
= \frac{\sqrt{a+bx^2}(-1280a^5f + 128a^4b(11e+5fx^2) - 16a^3b^2(99d+44ex^2+30fx^4) + 8a^2b^3(231c+99dx^2+66e}{3465}
$$

```
[In] Integrate[(x^5*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-1280*a^5*f + 128*a^4*b*(11*e + 5*f*x^2) - 16*a^3*b^2*(99
*d + 44*e*x^2 + 30*f*x^4) + 8*a^2*b^3*(231*c + 99*d*x^2 + 66*e*x^4 + 50*f*x
^6) - 2*a*b^4*x^2*(462*c + 297*d*x^2 + 220*e*x^4 + 175*f*x^6) + b^5*x^4*(69
3*c + 5*(99*d*x^2 + 77*e*x^4 + 63*f*x^6))))/(3465*b^6)
```

Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$-\frac{256\left(-\frac{693\left(\frac{5}{11}f\,x^{6}+\frac{5}{9}e\,x^{4}+\frac{5}{7}d\,x^{2}+c\right)x^{4}b^{5}}{1280}+\frac{231\left(\frac{25}{66}f\,x^{6}+\frac{10}{21}e\,x^{4}+\frac{9}{14}d\,x^{2}+c\right)x^{2}a\,b^{4}}{320}-\frac{231\left(\frac{50}{231}f\,x^{6}+\frac{2}{7}e\,x^{4}+\frac{3}{7}d\,x^{2}+c\right)a^{2}b^{3}}{160}+\frac{99\left(\frac{10}{33}\right.}{}\right.}{693b^{6}}$
gosper	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-315f\,x^{10}b^{5}+350a\,b^{4}f\,x^{8}-385b^{5}e\,x^{8}-400a^{2}b^{3}f\,x^{6}+440a\,b^{4}e\,x^{6}-495b^{5}d\,x^{6}+480a^{3}b^{2}f\,x^{4}-528a^{2}b^{3}e\,x^{4}+594a\,b\right.}{3465b}$
trager	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-315f\,x^{10}b^{5}+350a\,b^{4}f\,x^{8}-385b^{5}e\,x^{8}-400a^{2}b^{3}f\,x^{6}+440a\,b^{4}e\,x^{6}-495b^{5}d\,x^{6}+480a^{3}b^{2}f\,x^{4}-528a^{2}b^{3}e\,x^{4}+594a\,b\right.}{3465b}$
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-315f\,x^{10}b^{5}+350a\,b^{4}f\,x^{8}-385b^{5}e\,x^{8}-400a^{2}b^{3}f\,x^{6}+440a\,b^{4}e\,x^{6}-495b^{5}d\,x^{6}+480a^{3}b^{2}f\,x^{4}-528a^{2}b^{3}e\,x^{4}+594a\,b\right.}{3465b}$
default	$e\left(\frac{x^{8}\sqrt{b\,x^{2}+a}}{9b}-\frac{8a\left(\frac{x^{6}\sqrt{b\,x^{2}+a}}{7b}-\frac{6a\left(\frac{x^{4}\sqrt{b\,x^{2}+a}}{5b}-\frac{4a\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+a}}{3b^{2}}\right)}{5b}\right)}{7b}\right)}{9b}\right)+d\left(\frac{x^{6}\sqrt{b\,x^{2}+a}}{7b}-\frac{6a\left(\frac{x^{4}\sqrt{}}{}\right.}{}\right.$

[In] int(x^5*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] -256/693*(-693/1280*(5/11*f*x^6+5/9*e*x^4+5/7*d*x^2+c)*x^4*b^5+231/320*(25/66*f*x^6+10/21*e*x^4+9/14*d*x^2+c)*x^2*a*b^4-231/160*(50/231*f*x^6+2/7*e*x^4+3/7*d*x^2+c)*a^2*b^3+99/80*(10/33*f*x^4+4/9*e*x^2+d)*a^3*b^2-11/10*(5/11*f*x^2+e)*a^4*b+f*a^5)*(b*x^2+a)^(1/2)/b^6

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.83

$$\int \frac{x^5(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$$
$$=\frac{(315\,b^5fx^{10}+35\,(11\,b^5e-10\,ab^4f)x^8+5\,(99\,b^5d-88\,ab^4e+80\,a^2b^3f)x^6+1848\,a^2b^3c-1584\,a^3b^2d+140}{}$$

[In] integrate(x^5*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")

```
[Out] 1/3465*(315*b^5*f*x^10 + 35*(11*b^5*e - 10*a*b^4*f)*x^8 + 5*(99*b^5*d - 88*
a*b^4*e + 80*a^2*b^3*f)*x^6 + 1848*a^2*b^3*c - 1584*a^3*b^2*d + 1408*a^4*b*
e - 1280*a^5*f + 3*(231*b^5*c - 198*a*b^4*d + 176*a^2*b^3*e - 160*a^3*b^2*f
)*x^4 - 4*(231*a*b^4*c - 198*a^2*b^3*d + 176*a^3*b^2*e - 160*a^4*b*f)*x^2)*
sqrt(b*x^2 + a)/b^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(214) = 428$.

Time = 0.46 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.07

$$\int \frac{x^5(c + dx^2 + ex^4 + fx^6)}{\sqrt{a + bx^2}}\,dx$$
$$= \begin{cases} -\frac{256a^5f\sqrt{a+bx^2}}{693b^6} + \frac{128a^4e\sqrt{a+bx^2}}{315b^5} + \frac{128a^4fx^2\sqrt{a+bx^2}}{693b^5} - \frac{16a^3d\sqrt{a+bx^2}}{35b^4} - \frac{64a^3ex^2\sqrt{a+bx^2}}{315b^4} - \frac{32a^3fx^4\sqrt{a+bx^2}}{231b^4} + \frac{8a^2c\sqrt{a+bx^2}}{15b^3} \\ \frac{\frac{cx^6}{6} + \frac{dx^8}{8} + \frac{ex^{10}}{10} + \frac{fx^{12}}{12}}{\sqrt{a}} \end{cases}$$

```
[In] integrate(x**5*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-256*a**5*f*sqrt(a + b*x**2)/(693*b**6) + 128*a**4*e*sqrt(a + b*
x**2)/(315*b**5) + 128*a**4*f*x**2*sqrt(a + b*x**2)/(693*b**5) - 16*a**3*d*
sqrt(a + b*x**2)/(35*b**4) - 64*a**3*e*x**2*sqrt(a + b*x**2)/(315*b**4) - 3
2*a**3*f*x**4*sqrt(a + b*x**2)/(231*b**4) + 8*a**2*c*sqrt(a + b*x**2)/(15*b
**3) + 8*a**2*d*x**2*sqrt(a + b*x**2)/(35*b**3) + 16*a**2*e*x**4*sqrt(a + b
*x**2)/(105*b**3) + 80*a**2*f*x**6*sqrt(a + b*x**2)/(693*b**3) - 4*a*c*x**2
*sqrt(a + b*x**2)/(15*b**2) - 6*a*d*x**4*sqrt(a + b*x**2)/(35*b**2) - 8*a*e
*x**6*sqrt(a + b*x**2)/(63*b**2) - 10*a*f*x**8*sqrt(a + b*x**2)/(99*b**2) +
 c*x**4*sqrt(a + b*x**2)/(5*b) + d*x**6*sqrt(a + b*x**2)/(7*b) + e*x**8*sqr
t(a + b*x**2)/(9*b) + f*x**10*sqrt(a + b*x**2)/(11*b), Ne(b, 0)), ((c*x**6/
6 + d*x**8/8 + e*x**10/10 + f*x**12/12)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.62

$$\int \frac{x^5(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}fx^{10}}{11\,b} + \frac{\sqrt{bx^2+a}ex^8}{9\,b} - \frac{10\,\sqrt{bx^2+a}afx^8}{99\,b^2} + \frac{\sqrt{bx^2+a}dx^6}{7\,b} - \frac{8\,\sqrt{bx^2+a}aex^6}{63\,b^2} + \frac{80\,\sqrt{bx^2+a}a^2fx^6}{693\,b^3} + \frac{\sqrt{bx^2+a}cx^4}{5\,b} - \frac{6\,\sqrt{bx^2+a}adx^4}{35\,b^2} + \frac{16\,\sqrt{bx^2+a}a^2ex^4}{105\,b^3} - \frac{32\,\sqrt{bx^2+a}a^3fx^4}{231\,b^4} - \frac{4\,\sqrt{bx^2+a}acx^2}{15\,b^2} + \frac{8\,\sqrt{bx^2+a}a^2dx^2}{35\,b^3} - \frac{64\,\sqrt{bx^2+a}a^3ex^2}{315\,b^4} + \frac{128\,\sqrt{bx^2+a}a^4fx^2}{693\,b^5} + \frac{8\,\sqrt{bx^2+a}a^2c}{15\,b^3} - \frac{16\,\sqrt{bx^2+a}a^3d}{35\,b^4} + \frac{128\,\sqrt{bx^2+a}a^4e}{315\,b^5} - \frac{256\,\sqrt{bx^2+a}a^5f}{693\,b^6}$$

```
[In] integrate(x^5*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/11*sqrt(b*x^2 + a)*f*x^10/b + 1/9*sqrt(b*x^2 + a)*e*x^8/b - 10/99*sqrt(b*
x^2 + a)*a*f*x^8/b^2 + 1/7*sqrt(b*x^2 + a)*d*x^6/b - 8/63*sqrt(b*x^2 + a)*a
*e*x^6/b^2 + 80/693*sqrt(b*x^2 + a)*a^2*f*x^6/b^3 + 1/5*sqrt(b*x^2 + a)*c*x
^4/b - 6/35*sqrt(b*x^2 + a)*a*d*x^4/b^2 + 16/105*sqrt(b*x^2 + a)*a^2*e*x^4/
b^3 - 32/231*sqrt(b*x^2 + a)*a^3*f*x^4/b^4 - 4/15*sqrt(b*x^2 + a)*a*c*x^2/b
^2 + 8/35*sqrt(b*x^2 + a)*a^2*d*x^2/b^3 - 64/315*sqrt(b*x^2 + a)*a^3*e*x^2/
b^4 + 128/693*sqrt(b*x^2 + a)*a^4*f*x^2/b^5 + 8/15*sqrt(b*x^2 + a)*a^2*c/b^
3 - 16/35*sqrt(b*x^2 + a)*a^3*d/b^4 + 128/315*sqrt(b*x^2 + a)*a^4*e/b^5 - 2
56/693*sqrt(b*x^2 + a)*a^5*f/b^6
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.21

$$\int \frac{x^5(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{(a^2b^3c-a^3b^2d+a^4be-a^5f)\sqrt{bx^2+a}}{b^6} + \frac{693\,(bx^2+a)^{\frac{5}{2}}b^3c-2310\,(bx^2+a)^{\frac{3}{2}}ab^3c+495\,(bx^2+a)^{\frac{7}{2}}b^2d-2079\,(bx^2+a)^{\frac{5}{2}}ab^2d+3465\,(bx^2+a)^{\frac{3}{2}}a}{}$$

```
[In] integrate(x^5*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] (a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*sqrt(b*x^2 + a)/b^6 + 1/3465*(693
*(b*x^2 + a)^(5/2)*b^3*c - 2310*(b*x^2 + a)^(3/2)*a*b^3*c + 495*(b*x^2 + a)
^(7/2)*b^2*d - 2079*(b*x^2 + a)^(5/2)*a*b^2*d + 3465*(b*x^2 + a)^(3/2)*a^2*
b^2*d + 385*(b*x^2 + a)^(9/2)*b*e - 1980*(b*x^2 + a)^(7/2)*a*b*e + 4158*(b*
x^2 + a)^(5/2)*a^2*b*e - 4620*(b*x^2 + a)^(3/2)*a^3*b*e + 315*(b*x^2 + a)^(
11/2)*f - 1925*(b*x^2 + a)^(9/2)*a*f + 4950*(b*x^2 + a)^(7/2)*a^2*f - 6930*
(b*x^2 + a)^(5/2)*a^3*f + 5775*(b*x^2 + a)^(3/2)*a^4*f)/b^6
```

Mupad [B] (verification not implemented)

Time = 5.82 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.87

$$\int \frac{x^5(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \sqrt{b\,x^2+a}\,\left(\frac{x^6\,\left(400\,f\,a^2\,b^3-440\,e\,a\,b^4+495\,d\,b^5\right)}{3465\,b^6} - \frac{1280\,f\,a^5-1408\,e\,a^4\,b+1584\,d\,a^3\,b^2-1848\,c\,a^2\,b^3}{3465\,b^6} + \frac{x^4\,\left(-480\,f\,a^3\,b^2+528\,e\,a^2\,b^3-594\,d\,a\,b^4+693\,c\,b^5\right)}{3465\,b^6} + \frac{f\,x^{10}}{11\,b} + \frac{x^8\,\left(385\,b^5\,e-350\,a\,b^4\,f\right)}{3465\,b^6} - \frac{4\,a\,x^2\,\left(-160\,f\,a^3+176\,e\,a^2\,b-198\,d\,a\,b^2+231\,c\,b^3\right)}{3465\,b^5}\right)$$

```
[In] int((x^5*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((x^6*(495*b^5*d + 400*a^2*b^3*f - 440*a*b^4*e))/(3465*b^
6) - (1280*a^5*f - 1848*a^2*b^3*c + 1584*a^3*b^2*d - 1408*a^4*b*e)/(3465*b^
6) + (x^4*(693*b^5*c + 528*a^2*b^3*e - 480*a^3*b^2*f - 594*a*b^4*d))/(3465*
b^6) + (f*x^10)/(11*b) + (x^8*(385*b^5*e - 350*a*b^4*f))/(3465*b^6) - (4*a*
x^2*(231*b^3*c - 160*a^3*f - 198*a*b^2*d + 176*a^2*b*e))/(3465*b^5))
```

3.144 $\int \frac{x^3(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

Optimal result 870
Rubi [A] (verified) 871
Mathematica [A] (verified) 872
Maple [A] (verified) 872
Fricas [A] (verification not implemented) 873
Sympy [B] (verification not implemented) 873
Maxima [A] (verification not implemented) 874
Giac [A] (verification not implemented) 874
Mupad [B] (verification not implemented) 875

Optimal result

Integrand size = 32, antiderivative size = 167

$$\int \frac{x^3(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = -\frac{a(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^5} + \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,(a+bx^2)^{3/2}}{3b^5} + \frac{(b^2d-3abe+6a^2f)\,(a+bx^2)^{5/2}}{5b^5} + \frac{(be-4af)\,(a+bx^2)^{7/2}}{7b^5} + \frac{f(a+bx^2)^{9/2}}{9b^5}$$

```
[Out] 1/3*(-4*a^3*f+3*a^2*b*e-2*a*b^2*d+b^3*c)*(b*x^2+a)^(3/2)/b^5+1/5*(6*a^2*f-3
*a*b*e+b^2*d)*(b*x^2+a)^(5/2)/b^5+1/7*(-4*a*f+b*e)*(b*x^2+a)^(7/2)/b^5+1/9*
f*(b*x^2+a)^(9/2)/b^5-a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x^2+a)^(1/2)/b^5
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1813, 1634}

$$\int \frac{x^3(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{(a+bx^2)^{5/2}\,(6a^2f-3abe+b^2d)}{5b^5} + \frac{(a+bx^2)^{3/2}\,(-4a^3f+3a^2be-2ab^2d+b^3c)}{3b^5} - \frac{a\sqrt{a+bx^2}(a^3(-f)+a^2be-ab^2d+b^3c)}{b^5} + \frac{(a+bx^2)^{7/2}\,(be-4af)}{7b^5} + \frac{f(a+bx^2)^{9/2}}{9b^5}$$

```
[In] Int[(x^3*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^5) + ((b^3*c -
2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*(a + b*x^2)^(3/2))/(3*b^5) + ((b^2*d - 3*a
*b*e + 6*a^2*f)*(a + b*x^2)^(5/2))/(5*b^5) + ((b*e - 4*a*f)*(a + b*x^2)^(7/
2))/(7*b^5) + (f*(a + b*x^2)^(9/2))/(9*b^5)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a(-b^3c+ab^2d-a^2be+a^3f)}{b^4\sqrt{a+bx}} + \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\sqrt{a+bx}}{b^4} + \frac{(b^2d-3abe+6a^2f)(a+bx)^{3/2}}{b^4} + \frac{(be-4af)(a+bx)^{5/2}}{b^4} + \frac{f(a+bx)^{7/2}}{b^4}\right) dx, x, x^2\right)$$

$$= -\frac{a(b^3c - ab^2d + a^2be - a^3f)\sqrt{a+bx^2}}{b^5} + \frac{(b^3c - 2ab^2d + 3a^2be - 4a^3f)(a+bx^2)^{3/2}}{3b^5} + \frac{(b^2d - 3abe + 6a^2f)(a+bx^2)^{5/2}}{5b^5} + \frac{(be - 4af)(a+bx^2)^{7/2}}{7b^5} + \frac{f(a+bx^2)^{9/2}}{9b^5}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

$$\int \frac{x^3(c + dx^2 + ex^4 + fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(128a^4f - 16a^3b(9e + 4fx^2) + 24a^2b^2(7d + 3ex^2 + 2fx^4) - 2ab^3(105c + 42dx^2 + 27ex^4 + 20fx^6)}{315b^5}$$

```
[In] Integrate[(x^3*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(128*a^4*f - 16*a^3*b*(9*e + 4*f*x^2) + 24*a^2*b^2*(7*d +
3*e*x^2 + 2*f*x^4) - 2*a*b^3*(105*c + 42*d*x^2 + 27*e*x^4 + 20*f*x^6) + b^4
*x^2*(105*c + 63*d*x^2 + 45*e*x^4 + 35*f*x^6)))/(315*b^5)
```

Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{128\left(\frac{105\left(\frac{1}{3}f\,x^{6}+\frac{3}{7}e\,x^{4}+\frac{3}{5}d\,x^{2}+c\right)x^{2}b^{4}}{128}-\frac{105\left(\frac{4}{21}f\,x^{6}+\frac{9}{35}e\,x^{4}+\frac{2}{5}d\,x^{2}+c\right)a\,b^{3}}{64}+\frac{21\left(\frac{2}{7}f\,x^{4}+\frac{3}{7}e\,x^{2}+d\right)a^{2}b^{2}}{16}-\frac{9\left(\frac{4f\,x^{2}}{9}+e\right)a^{3}b}{8}+a^{4}f\right)}{315b^{5}}$
gosper	$\frac{\sqrt{b\,x^{2}+a}\,(35f\,x^{8}b^{4}-40a\,b^{3}f\,x^{6}+45b^{4}e\,x^{6}+48a^{2}b^{2}f\,x^{4}-54a\,b^{3}e\,x^{4}+63b^{4}d\,x^{4}-64a^{3}bf\,x^{2}+72a^{2}b^{2}e\,x^{2}-84a\,b^{3}d\,x^{2}+105b^{4}c\,x^{2}}{315b^{5}}$
trager	$\frac{\sqrt{b\,x^{2}+a}\,(35f\,x^{8}b^{4}-40a\,b^{3}f\,x^{6}+45b^{4}e\,x^{6}+48a^{2}b^{2}f\,x^{4}-54a\,b^{3}e\,x^{4}+63b^{4}d\,x^{4}-64a^{3}bf\,x^{2}+72a^{2}b^{2}e\,x^{2}-84a\,b^{3}d\,x^{2}+105b^{4}c\,x^{2}}{315b^{5}}$
risch	$\frac{\sqrt{b\,x^{2}+a}\,(35f\,x^{8}b^{4}-40a\,b^{3}f\,x^{6}+45b^{4}e\,x^{6}+48a^{2}b^{2}f\,x^{4}-54a\,b^{3}e\,x^{4}+63b^{4}d\,x^{4}-64a^{3}bf\,x^{2}+72a^{2}b^{2}e\,x^{2}-84a\,b^{3}d\,x^{2}+105b^{4}c\,x^{2}}{315b^{5}}$
default	$f\left(\frac{x^{8}\sqrt{b\,x^{2}+a}}{9b}-\frac{8a\left(\frac{x^{6}\sqrt{b\,x^{2}+a}}{7b}-\frac{6a\left(\frac{x^{4}\sqrt{b\,x^{2}+a}}{5b}-\frac{4a\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+a}}{3b^{2}}\right)}{5b}\right)}{7b}\right)}{9b}\right)+e\left(\frac{x^{6}\sqrt{b\,x^{2}+a}}{7b}-\frac{6a\left(\frac{x^{4}\sqrt{}}{}\right)}{}\right)$

```
[In] int(x^3*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 128/315*(105/128*(1/3*f*x^6+3/7*e*x^4+3/5*d*x^2+c)*x^2*b^4-105/64*(4/21*f*x
^6+9/35*e*x^4+2/5*d*x^2+c)*a*b^3+21/16*(2/7*f*x^4+3/7*e*x^2+d)*a^2*b^2-9/8*
(4/9*f*x^2+e)*a^3*b+a^4*f)*(b*x^2+a)^(1/2)/b^5
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.80

$$\int \frac{x^3(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{(35\,b^4fx^8+5\,(9\,b^4e-8\,ab^3f)x^6-210\,ab^3c+168\,a^2b^2d-144\,a^3be+128\,a^4f+3\,(21\,b^4d-18\,ab^3e+16\,a}{315\,b^5}$$

```
[In] integrate(x^3*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^4*f*x^8 + 5*(9*b^4*e - 8*a*b^3*f)*x^6 - 210*a*b^3*c + 168*a^2*b
^2*d - 144*a^3*b*e + 128*a^4*f + 3*(21*b^4*d - 18*a*b^3*e + 16*a^2*b^2*f)*x
^4 + (105*b^4*c - 84*a*b^3*d + 72*a^2*b^2*e - 64*a^3*b*f)*x^2)*sqrt(b*x^2 +
 a)/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(163) = 326$.

Time = 0.38 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.04

$$\int \frac{x^3(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{128a^4f\sqrt{a+bx^2}}{315b^5} - \frac{16a^3e\sqrt{a+bx^2}}{35b^4} - \frac{64a^3fx^2\sqrt{a+bx^2}}{315b^4} + \frac{8a^2d\sqrt{a+bx^2}}{15b^3} + \frac{8a^2ex^2\sqrt{a+bx^2}}{35b^3} + \frac{16a^2fx^4\sqrt{a+bx^2}}{105b^3} - \frac{2ac\sqrt{a+bx^2}}{3b^2} - \frac{4ad \\ \frac{\frac{cx^4}{4}+\frac{dx^6}{6}+\frac{ex^8}{8}+\frac{fx^{10}}{10}}{\sqrt{a}} \end{cases}$$

```
[In] integrate(x**3*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((128*a**4*f*sqrt(a + b*x**2)/(315*b**5) - 16*a**3*e*sqrt(a + b*x*
*2)/(35*b**4) - 64*a**3*f*x**2*sqrt(a + b*x**2)/(315*b**4) + 8*a**2*d*sqrt(
a + b*x**2)/(15*b**3) + 8*a**2*e*x**2*sqrt(a + b*x**2)/(35*b**3) + 16*a**2*
f*x**4*sqrt(a + b*x**2)/(105*b**3) - 2*a*c*sqrt(a + b*x**2)/(3*b**2) - 4*a*
d*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*e*x**4*sqrt(a + b*x**2)/(35*b**2) -
 8*a*f*x**6*sqrt(a + b*x**2)/(63*b**2) + c*x**2*sqrt(a + b*x**2)/(3*b) + d*
x**4*sqrt(a + b*x**2)/(5*b) + e*x**6*sqrt(a + b*x**2)/(7*b) + f*x**8*sqrt(a
 + b*x**2)/(9*b), Ne(b, 0)), ((c*x**4/4 + d*x**6/6 + e*x**8/8 + f*x**10/10)
/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.57

$$\int \frac{x^3(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}fx^8}{9\,b} + \frac{\sqrt{bx^2+a}ex^6}{7\,b} - \frac{8\,\sqrt{bx^2+a}afx^6}{63\,b^2} + \frac{\sqrt{bx^2+a}dx^4}{5\,b} - \frac{6\,\sqrt{bx^2+a}aex^4}{35\,b^2} + \frac{16\,\sqrt{bx^2+a}a^2fx^4}{105\,b^3} + \frac{\sqrt{bx^2+a}cx^2}{3\,b} - \frac{4\,\sqrt{bx^2+a}adx^2}{15\,b^2} + \frac{8\,\sqrt{bx^2+a}a^2ex^2}{35\,b^3} - \frac{64\,\sqrt{bx^2+a}a^3fx^2}{315\,b^4} - \frac{2\,\sqrt{bx^2+a}ac}{3\,b^2} + \frac{8\,\sqrt{bx^2+a}a^2d}{15\,b^3} - \frac{16\,\sqrt{bx^2+a}a^3e}{35\,b^4} + \frac{128\,\sqrt{bx^2+a}a^4f}{315\,b^5}$$

```
[In] integrate(x^3*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(b*x^2 + a)*f*x^8/b + 1/7*sqrt(b*x^2 + a)*e*x^6/b - 8/63*sqrt(b*x^2
 + a)*a*f*x^6/b^2 + 1/5*sqrt(b*x^2 + a)*d*x^4/b - 6/35*sqrt(b*x^2 + a)*a*e*
x^4/b^2 + 16/105*sqrt(b*x^2 + a)*a^2*f*x^4/b^3 + 1/3*sqrt(b*x^2 + a)*c*x^2/
b - 4/15*sqrt(b*x^2 + a)*a*d*x^2/b^2 + 8/35*sqrt(b*x^2 + a)*a^2*e*x^2/b^3 -
 64/315*sqrt(b*x^2 + a)*a^3*f*x^2/b^4 - 2/3*sqrt(b*x^2 + a)*a*c/b^2 + 8/15*
sqrt(b*x^2 + a)*a^2*d/b^3 - 16/35*sqrt(b*x^2 + a)*a^3*e/b^4 + 128/315*sqrt(
b*x^2 + a)*a^4*f/b^5
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.16

$$\int \frac{x^3(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = -\frac{(ab^3c-a^2b^2d+a^3be-a^4f)\sqrt{bx^2+a}}{b^5} + \frac{105\,(bx^2+a)^{\frac{3}{2}}b^3c+63\,(bx^2+a)^{\frac{5}{2}}b^2d-210\,(bx^2+a)^{\frac{3}{2}}ab^2d+45\,(bx^2+a)^{\frac{7}{2}}be-189\,(bx^2+a)^{\frac{5}{2}}abe+315}{315\,b^5}$$

```
[In] integrate(x^3*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*sqrt(b*x^2 + a)/b^5 + 1/315*(105*(
b*x^2 + a)^(3/2)*b^3*c + 63*(b*x^2 + a)^(5/2)*b^2*d - 210*(b*x^2 + a)^(3/2)
*a*b^2*d + 45*(b*x^2 + a)^(7/2)*b*e - 189*(b*x^2 + a)^(5/2)*a*b*e + 315*(b*
x^2 + a)^(3/2)*a^2*b*e + 35*(b*x^2 + a)^(9/2)*f - 180*(b*x^2 + a)^(7/2)*a*f
 + 378*(b*x^2 + a)^(5/2)*a^2*f - 420*(b*x^2 + a)^(3/2)*a^3*f)/b^5
```

Mupad [B] (verification not implemented)

Time = 5.78 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.87

$$\int \frac{x^3(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx=\sqrt{b\,x^2+a}\,\left(\frac{128\,f\,a^4-144\,e\,a^3\,b+168\,d\,a^2\,b^2-210\,c\,a\,b^3}{315\,b^5}+\frac{x^4\,\left(48\,f\,a^2\,b^2-54\,e\,a\,b^3+63\,d\,b^4\right)}{315\,b^5}+\frac{f\,x^8}{9\,b}+\frac{x^6\,\left(45\,b^4\,e-40\,a\,b^3\,f\right)}{315\,b^5}+\frac{x^2\,\left(-64\,f\,a^3\,b+72\,e\,a^2\,b^2-84\,d\,a\,b^3+105\,c\,b^4\right)}{315\,b^5}\right)$$

```
[In] int((x^3*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((128*a^4*f + 168*a^2*b^2*d - 210*a*b^3*c - 144*a^3*b*e)/
(315*b^5) + (x^4*(63*b^4*d + 48*a^2*b^2*f - 54*a*b^3*e))/(315*b^5) + (f*x^8
)/(9*b) + (x^6*(45*b^4*e - 40*a*b^3*f))/(315*b^5) + (x^2*(105*b^4*c + 72*a^
2*b^2*e - 84*a*b^3*d - 64*a^3*b*f))/(315*b^5))
```

3.145 $\int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

Optimal result 876
Rubi [A] (verified) 876
Mathematica [A] (verified) 877
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Fricas [A] (verification not implemented) 878
Sympy [B] (verification not implemented) 879
Maxima [A] (verification not implemented) 879
Giac [A] (verification not implemented) 880
Mupad [B] (verification not implemented) 880

Optimal result

Integrand size = 30, antiderivative size = 121

$$\int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^4} + \frac{(b^2d-2abe+3a^2f)(a+bx^2)^{3/2}}{3b^4} + \frac{(be-3af)(a+bx^2)^{5/2}}{5b^4} + \frac{f(a+bx^2)^{7/2}}{7b^4}$$

```
[Out] 1/3*(3*a^2*f-2*a*b*e+b^2*d)*(b*x^2+a)^(3/2)/b^4+1/5*(-3*a*f+b*e)*(b*x^2+a)^
(5/2)/b^4+1/7*f*(b*x^2+a)^(7/2)/b^4+(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x^2+a
)^(1/2)/b^4
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1813, 1864}

$$\int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{(a+bx^2)^{3/2}(3a^2f-2abe+b^2d)}{3b^4} + \frac{\sqrt{a+bx^2}(a^3(-f)+a^2be-ab^2d+b^3c)}{b^4} + \frac{(a+bx^2)^{5/2}(be-3af)}{5b^4} + \frac{f(a+bx^2)^{7/2}}{7b^4}$$

```
[In] Int[(x*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^4 + ((b^2*d - 2*a*b
*e + 3*a^2*f)*(a + b*x^2)^(3/2))/(3*b^4) + ((b*e - 3*a*f)*(a + b*x^2)^(5/2)
)/(5*b^4) + (f*(a + b*x^2)^(7/2))/(7*b^4)
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b^3c-ab^2d+a^2be-a^3f}{b^3\sqrt{a+bx}}+\frac{(b^2d-2abe+3a^2f)\sqrt{a+bx}}{b^3}\right.\right.\\
&\qquad\left.\left.+\frac{(be-3af)(a+bx)^{3/2}}{b^3}+\frac{f(a+bx)^{5/2}}{b^3}\right)\,dx, x, x^2\right)\\
&= \frac{(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^4}+\frac{(b^2d-2abe+3a^2f)\left(a+bx^2\right)^{3/2}}{3b^4}\\
&\quad+\frac{(be-3af)\left(a+bx^2\right)^{5/2}}{5b^4}+\frac{f\left(a+bx^2\right)^{7/2}}{7b^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(-48a^3f+8a^2b(7e+3fx^2)-2ab^2(35d+14ex^2+9fx^4)+b^3(105c+35dx^2+21ex^4+15fx^6))}{105b^4}$$

```
[In] Integrate[(x*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-48*a^3*f + 8*a^2*b*(7*e + 3*f*x^2) - 2*a*b^2*(35*d + 14*
e*x^2 + 9*f*x^4) + b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6)))/(105*b^4)
```

Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$-\frac{16\left(\frac{\left(-5 f\,x^{6}-7 e\,x^{4}-\frac{35}{3} d\,x^{2}-35 c \right) b^{3}}{16}+\frac{35\left(\frac{9}{35} f\,x^{4}+\frac{2}{5} e\,x^{2}+d \right) a\,b^{2}}{24}-\frac{7\left(\frac{3 f\,x^{2}}{7}+e \right) a^{2} b}{6}+f\,a^{3}\right) \sqrt{b\,x^{2}+a}}{35 b^{4}}$
gosper	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-15 f\,x^{6} b^{3}+18 a\,b^{2} f\,x^{4}-21 b^{3} e\,x^{4}-24 a^{2} b f\,x^{2}+28 a\,b^{2} e\,x^{2}-35 b^{3} d\,x^{2}+48 f\,a^{3}-56 a^{2} b e+70 a\,b^{2} d-105 b^{3} c \right)}{105 b^{4}}$
trager	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-15 f\,x^{6} b^{3}+18 a\,b^{2} f\,x^{4}-21 b^{3} e\,x^{4}-24 a^{2} b f\,x^{2}+28 a\,b^{2} e\,x^{2}-35 b^{3} d\,x^{2}+48 f\,a^{3}-56 a^{2} b e+70 a\,b^{2} d-105 b^{3} c \right)}{105 b^{4}}$
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-15 f\,x^{6} b^{3}+18 a\,b^{2} f\,x^{4}-21 b^{3} e\,x^{4}-24 a^{2} b f\,x^{2}+28 a\,b^{2} e\,x^{2}-35 b^{3} d\,x^{2}+48 f\,a^{3}-56 a^{2} b e+70 a\,b^{2} d-105 b^{3} c \right)}{105 b^{4}}$
default	$f\left(\frac{x^{6}\sqrt{b\,x^{2}+a}}{7 b}-\frac{6 a\left(\frac{x^{4}\sqrt{b\,x^{2}+a}}{5 b}-\frac{4 a\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3 b}-\frac{2 a\sqrt{b\,x^{2}+a}}{3 b^{2}}\right)}{5 b}\right)}{7 b}\right)+e\left(\frac{x^{4}\sqrt{b\,x^{2}+a}}{5 b}-\frac{4 a\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3 b}-\frac{2 a\sqrt{b\,x^{2}+a}}{3 b^{2}}\right.}{5 b}\right.$

```
[In] int(x*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -16/35*(1/16*(-5*f*x^6-7*e*x^4-35/3*d*x^2-35*c)*b^3+35/24*(9/35*f*x^4+2/5*e
*x^2+d)*a*b^2-7/6*(3/7*f*x^2+e)*a^2*b+f*a^3)*(b*x^2+a)^(1/2)/b^4
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.78

$$\int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$$
$$=\frac{(15\,b^3fx^6+3\,(7\,b^3e-6\,ab^2f)x^4+105\,b^3c-70\,ab^2d+56\,a^2be-48\,a^3f+(35\,b^3d-28\,ab^2e+24\,a^2bf)x^2)\sqrt{}}{105\,b^4}$$

```
[In] integrate(x*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^3*f*x^6 + 3*(7*b^3*e - 6*a*b^2*f)*x^4 + 105*b^3*c - 70*a*b^2*d
+ 56*a^2*b*e - 48*a^3*f + (35*b^3*d - 28*a*b^2*e + 24*a^2*b*f)*x^2)*sqrt(b*
x^2 + a)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(112) = 224$.

Time = 0.29 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.97

$$\int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \begin{cases} -\frac{16a^3f\sqrt{a+bx^2}}{35b^4} + \frac{8a^2e\sqrt{a+bx^2}}{15b^3} + \frac{8a^2fx^2\sqrt{a+bx^2}}{35b^3} - \frac{2ad\sqrt{a+bx^2}}{3b^2} - \frac{4aex^2\sqrt{a+bx^2}}{15b^2} - \frac{6afx^4\sqrt{a+bx^2}}{35b^2} + \frac{c\sqrt{a+bx^2}}{b} + \frac{dx^2\sqrt{a+bx^2}}{3b} \\ \frac{\frac{cx^2}{2}+\frac{dx^4}{4}+\frac{ex^6}{6}+\frac{fx^8}{8}}{\sqrt{a}} \end{cases}$$

```
[In] integrate(x*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-16*a**3*f*sqrt(a + b*x**2)/(35*b**4) + 8*a**2*e*sqrt(a + b*x**2
)/(15*b**3) + 8*a**2*f*x**2*sqrt(a + b*x**2)/(35*b**3) - 2*a*d*sqrt(a + b*x
**2)/(3*b**2) - 4*a*e*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*f*x**4*sqrt(a +
 b*x**2)/(35*b**2) + c*sqrt(a + b*x**2)/b + d*x**2*sqrt(a + b*x**2)/(3*b) +
 e*x**4*sqrt(a + b*x**2)/(5*b) + f*x**6*sqrt(a + b*x**2)/(7*b), Ne(b, 0)),
((c*x**2/2 + d*x**4/4 + e*x**6/6 + f*x**8/8)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.49

$$\begin{aligned}\int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx &= \frac{\sqrt{bx^2+a}fx^6}{7\,b} + \frac{\sqrt{bx^2+a}ex^4}{5\,b} - \frac{6\,\sqrt{bx^2+a}afx^4}{35\,b^2} \\ &\quad + \frac{\sqrt{bx^2+a}dx^2}{3\,b} - \frac{4\,\sqrt{bx^2+a}aex^2}{15\,b^2} \\ &\quad + \frac{8\,\sqrt{bx^2+a}a^2fx^2}{35\,b^3} + \frac{\sqrt{bx^2+a}c}{b} - \frac{2\,\sqrt{bx^2+a}ad}{3\,b^2} \\ &\quad + \frac{8\,\sqrt{bx^2+a}a^2e}{15\,b^3} - \frac{16\,\sqrt{bx^2+a}a^3f}{35\,b^4}\end{aligned}$$

```
[In] integrate(x*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*sqrt(b*x^2 + a)*f*x^6/b + 1/5*sqrt(b*x^2 + a)*e*x^4/b - 6/35*sqrt(b*x^2
 + a)*a*f*x^4/b^2 + 1/3*sqrt(b*x^2 + a)*d*x^2/b - 4/15*sqrt(b*x^2 + a)*a*e*
x^2/b^2 + 8/35*sqrt(b*x^2 + a)*a^2*f*x^2/b^3 + sqrt(b*x^2 + a)*c/b - 2/3*sq
rt(b*x^2 + a)*a*d/b^2 + 8/15*sqrt(b*x^2 + a)*a^2*e/b^3 - 16/35*sqrt(b*x^2 +
 a)*a^3*f/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.05

$$\int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{(b^3c-ab^2d+a^2be-a^3f)\sqrt{bx^2+a}}{b^4} + \frac{35\,(bx^2+a)^{\frac{3}{2}}b^2d+21\,(bx^2+a)^{\frac{5}{2}}be-70\,(bx^2+a)^{\frac{3}{2}}abe+15\,(bx^2+a)^{\frac{7}{2}}f-63\,(bx^2+a)^{\frac{5}{2}}af+105\,(bx^2+}{105\,b^4}$$

```
[In] integrate(x*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] (b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(b*x^2 + a)/b^4 + 1/105*(35*(b*x^2
+ a)^(3/2)*b^2*d + 21*(b*x^2 + a)^(5/2)*b*e - 70*(b*x^2 + a)^(3/2)*a*b*e +
15*(b*x^2 + a)^(7/2)*f - 63*(b*x^2 + a)^(5/2)*a*f + 105*(b*x^2 + a)^(3/2)*a
^2*f)/b^4
```

Mupad [B] (verification not implemented)

Time = 5.72 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.85

$$\int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \sqrt{b\,x^2+a}\left(\frac{-48\,f\,a^3+56\,e\,a^2\,b-70\,d\,a\,b^2+105\,c\,b^3}{105\,b^4} + \frac{f\,x^6}{7\,b} + \frac{x^2\,(24\,f\,a^2\,b-28\,e\,a\,b^2+35\,d\,b^3)}{105\,b^4} + \frac{x^4\,(21\,b^3\,e-18\,a\,b^2\,f)}{105\,b^4}\right)$$

```
[In] int((x*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((105*b^3*c - 48*a^3*f - 70*a*b^2*d + 56*a^2*b*e)/(105*b^
4) + (f*x^6)/(7*b) + (x^2*(35*b^3*d - 28*a*b^2*e + 24*a^2*b*f))/(105*b^4) +
 (x^4*(21*b^3*e - 18*a*b^2*f))/(105*b^4))
```

3.146 $\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx$

Optimal result 881
Rubi [A] (verified) 881
Mathematica [A] (verified) 883
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Optimal result

Integrand size = 32, antiderivative size = 103

$$\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx = \frac{(b^2d-abe+a^2f)\sqrt{a+bx^2}}{b^3}+\frac{(be-2af)\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{f\left(a+bx^2\right)^{5/2}}{5b^3}-\frac{c\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] 1/3*(-2*a*f+b*e)*(b*x^2+a)^(3/2)/b^3+1/5*f*(b*x^2+a)^(5/2)/b^3-c*arctanh((b
*x^2+a)^(1/2)/a^(1/2))/a^(1/2)+(a^2*f-a*b*e+b^2*d)*(b*x^2+a)^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1813, 1634, 65, 214}

$$\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(a^2f-abe+b^2d)}{b^3}-\frac{c\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\left(a+bx^2\right)^{3/2}(be-2af)}{3b^3}+\frac{f\left(a+bx^2\right)^{5/2}}{5b^3}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] ((b^2*d - a*b*e + a^2*f)*Sqrt[a + b*x^2])/b^3 + ((b*e - 2*a*f)*(a + b*x^2)^
(3/2))/(3*b^3) + (f*(a + b*x^2)^(5/2))/(5*b^3) - (c*ArcTanh[Sqrt[a + b*x^2]
/Sqrt[a]])/Sqrt[a]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx}}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b^2d-abe+a^2f}{b^2\sqrt{a+bx}}+\frac{c}{x\sqrt{a+bx}}+\frac{(be-2af)\sqrt{a+bx}}{b^2}+\frac{f(a+bx)^{3/2}}{b^2}\right)\,dx, x, x^2\right) \\
&= \frac{(b^2d-abe+a^2f)\sqrt{a+bx^2}}{b^3}+\frac{(be-2af)(a+bx^2)^{3/2}}{3b^3} \\
&\quad +\frac{f(a+bx^2)^{5/2}}{5b^3}+\frac{1}{2}c\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right) \\
&= \frac{(b^2d-abe+a^2f)\sqrt{a+bx^2}}{b^3}+\frac{(be-2af)(a+bx^2)^{3/2}}{3b^3} \\
&\quad +\frac{f(a+bx^2)^{5/2}}{5b^3}+\frac{c\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b} \\
&= \frac{(b^2d-abe+a^2f)\sqrt{a+bx^2}}{b^3}+\frac{(be-2af)(a+bx^2)^{3/2}}{3b^3}+\frac{f(a+bx^2)^{5/2}}{5b^3}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(8a^2f-2ab(5e+2fx^2)+b^2(15d+5ex^2+3fx^4))}{15b^3} - \frac{c\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x*Sqrt[a + b*x^2]),x]
[Out] (Sqrt[a + b*x^2]*(8*a^2*f - 2*a*b*(5*e + 2*f*x^2) + b^2*(15*d + 5*e*x^2 + 3
*f*x^4)))/(15*b^3) - (c*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$-\frac{2\left(\frac{3b^3c\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2}+\sqrt{b\,x^2+a}\left(b\left(\frac{2f\,x^2}{5}+e\right)a^{\frac{3}{2}}-\frac{4a^{\frac{5}{2}}f}{5}-\frac{3\sqrt{a}\,b^2\left(\frac{1}{5}f\,x^4+\frac{1}{3}e\,x^2+d\right)}{2}\right)\right)}{3\sqrt{a}\,b^3}$
default	$f\left(\frac{x^4\sqrt{b\,x^2+a}}{5b}-\frac{4a\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)}{5b}\right)+e\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)+\frac{d\sqrt{b\,x^2+a}}{b}-\frac{c\ln\left(\frac{2a+2\sqrt{a}}{}\right.}{\sqrt{a}}$

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(3/2*b^3*c*arctanh((b*x^2+a)^(1/2)/a^(1/2))+(b*x^2+a)^(1/2)*(b*(2/5*f*
x^2+e)*a^(3/2)-4/5*a^(5/2)*f-3/2*a^(1/2)*b^2*(1/5*f*x^4+1/3*e*x^2+d)))/a^(1
/2)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.99

$$\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx = \left[\frac{15\,\sqrt{a}b^3c\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(3\,ab^2fx^4+15\,ab^2d-10\,a^2be+8\,a^3f+(5\,ab^2e-4\,a^2bf)x^2)\sqrt{b}}{30\,ab^3}\right.$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(a)*b^3*c*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)
 + 2*(3*a*b^2*f*x^4 + 15*a*b^2*d - 10*a^2*b*e + 8*a^3*f + (5*a*b^2*e - 4*a^
2*b*f)*x^2)*sqrt(b*x^2 + a))/(a*b^3), 1/15*(15*sqrt(-a)*b^3*c*arctan(sqrt(-
a)/sqrt(b*x^2 + a)) + (3*a*b^2*f*x^4 + 15*a*b^2*d - 10*a^2*b*e + 8*a^3*f +
(5*a*b^2*e - 4*a^2*b*f)*x^2)*sqrt(b*x^2 + a))/(a*b^3)]
```

Sympy [A] (verification not implemented)

Time = 4.48 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.30

$$\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx = \frac{\begin{cases} \frac{2c\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2f(a+bx^2)^{\frac{5}{2}}}{5b^3} + \frac{2(a+bx^2)^{\frac{3}{2}}(-2af+be)}{3b^3} + \frac{2\sqrt{a+bx^2}(a^2f-abe+b^2d)}{b^3} & \text{for}\ b \neq 0 \\ \frac{c\log(x^2)+dx^2+\frac{ex^4}{2}+\frac{fx^6}{3}}{\sqrt{a}} & \text{otherwise} \end{cases}}{2}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((2*c*atan(sqrt(a + b*x**2)/sqrt(-a))/sqrt(-a) + 2*f*(a + b*x**2)*
*(5/2)/(5*b**3) + 2*(a + b*x**2)**(3/2)*(-2*a*f + b*e)/(3*b**3) + 2*sqrt(a
+ b*x**2)*(a**2*f - a*b*e + b**2*d)/b**3, Ne(b, 0)), ((c*log(x**2) + d*x**2
 + e*x**4/2 + f*x**6/3)/sqrt(a), True))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.18

$$\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}fx^4}{5\,b} + \frac{\sqrt{bx^2+a}ex^2}{3\,b} - \frac{4\,\sqrt{bx^2+a}afx^2}{15\,b^2} - \frac{c\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} + \frac{\sqrt{bx^2+a}d}{b} - \frac{2\,\sqrt{bx^2+a}ae}{3\,b^2} + \frac{8\,\sqrt{bx^2+a}a^2f}{15\,b^3}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(b*x^2 + a)*f*x^4/b + 1/3*sqrt(b*x^2 + a)*e*x^2/b - 4/15*sqrt(b*x^2
 + a)*a*f*x^2/b^2 - c*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + sqrt(b*x^2 +
a)*d/b - 2/3*sqrt(b*x^2 + a)*a*e/b^2 + 8/15*sqrt(b*x^2 + a)*a^2*f/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.21

$$\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx = \frac{c\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{15\sqrt{bx^2+a}ab^{14}d+5\left(bx^2+a\right)^{\frac{3}{2}}b^{13}e-15\sqrt{bx^2+a}ab^{13}e+3\left(bx^2+a\right)^{\frac{5}{2}}b^{12}f-10\left(bx^2+a\right)^{\frac{3}{2}}ab^{12}f+15}{15\,b^{15}}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] c*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/15*(15*sqrt(b*x^2 + a)*b^14
*d + 5*(b*x^2 + a)^(3/2)*b^13*e - 15*sqrt(b*x^2 + a)*a*b^13*e + 3*(b*x^2 +
a)^(5/2)*b^12*f - 10*(b*x^2 + a)^(3/2)*a*b^12*f + 15*sqrt(b*x^2 + a)*a^2*b^
12*f)/b^15
```

Mupad [B] (verification not implemented)

Time = 6.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx = \sqrt{b\,x^2+a}\left(\frac{8\,a^2\,f}{15\,b^3}+\frac{f\,x^4}{5\,b}-\frac{4\,a\,f\,x^2}{15\,b^2}\right)+\frac{d\,\sqrt{b\,x^2+a}}{b} - \frac{c\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{e\,\sqrt{b\,x^2+a}\,\left(2\,a-b\,x^2\right)}{3\,b^2}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x*(a + b*x^2)^(1/2)),x)
```

```
[Out] (a + b*x^2)^(1/2)*((8*a^2*f)/(15*b^3) + (f*x^4)/(5*b) - (4*a*f*x^2)/(15*b^2
)) + (d*(a + b*x^2)^(1/2))/b - (c*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2)
 - (e*(a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2)
```

3.147 $\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx$

Optimal result 886
Rubi [A] (verified) 886
Mathematica [A] (verified) 888
Maple [A] (verified) 888
Fricas [A] (verification not implemented) 889
Sympy [A] (verification not implemented) 889
Maxima [A] (verification not implemented) 890
Giac [A] (verification not implemented) 890
Mupad [B] (verification not implemented) 891

Optimal result

Integrand size = 32, antiderivative size = 100

$$\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx = \frac{(be-af)\sqrt{a+bx^2}}{b^2}-\frac{c\sqrt{a+bx^2}}{2ax^2}+\frac{f(a+bx^2)^{3/2}}{3b^2}+\frac{(bc-2ad)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

```
[Out] 1/3*f*(b*x^2+a)^(3/2)/b^2+1/2*(-2*a*d+b*c)*arctanh((b*x^2+a)^(1/2)/a^(1/2))
/a^(3/2)+(-a*f+b*e)*(b*x^2+a)^(1/2)/b^2-1/2*c*(b*x^2+a)^(1/2)/a/x^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1813, 1635, 911, 1167, 214}

$$\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(bc-2ad)}{2a^{3/2}}+\frac{\sqrt{a+bx^2}(be-af)}{b^2}+\frac{f(a+bx^2)^{3/2}}{3b^2}-\frac{c\sqrt{a+bx^2}}{2ax^2}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] ((b*e - a*f)*Sqrt[a + b*x^2])/b^2 - (c*Sqrt[a + b*x^2])/(2*a*x^2) + (f*(a +
 b*x^2)^(3/2))/(3*b^2) + ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(
2*a^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1635

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= -\frac{c\sqrt{a+bx^2}}{2ax^2} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-2ad)-aex-afx^2}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{2a}$$

$$= -\frac{c\sqrt{a+bx^2}}{2ax^2} - \frac{\text{Subst}\left(\int \frac{\frac{\frac{1}{2}b^2(bc-2ad)+a^2be-a^3f}{b^2} - \frac{\left(abe-2a^2f\right)x^2}{b^2} - \frac{afx^4}{b^2}}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{ab}$$

$$= -\frac{c\sqrt{a+bx^2}}{2ax^2} - \frac{\text{Subst}\left(\int \left(-a\left(e-\frac{af}{b}\right) - \frac{afx^2}{b} + \frac{bc-2ad}{2\left(-\frac{a}{b}+\frac{x^2}{b}\right)}\right) dx, x, \sqrt{a+bx^2}\right)}{ab}$$

$$= \frac{(be-af)\sqrt{a+bx^2}}{b^2} - \frac{c\sqrt{a+bx^2}}{2ax^2} + \frac{f(a+bx^2)^{3/2}}{3b^2} - \frac{(bc-2ad)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+bx^2}\right)}{2ab}$$

$$= \frac{(be-af)\sqrt{a+bx^2}}{b^2} - \frac{c\sqrt{a+bx^2}}{2ax^2} + \frac{f(a+bx^2)^{3/2}}{3b^2} + \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}} dx = \frac{\sqrt{a+bx^2}(-3b^2c+6abex^2-4a^2fx^2+2abfx^4)}{6ab^2x^2} + \frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^3*Sqrt[a + b*x^2]),x]
[Out] (Sqrt[a + b*x^2]*(-3*b^2*c + 6*a*b*e*x^2 - 4*a^2*f*x^2 + 2*a*b*f*x^4))/(6*a
*b^2*x^2) + ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2))
```

Maple [A] (verified)

Time = 3.54 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$-\frac{b^2x^2\left(ad-\frac{bc}{2}\right)\text{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)+\frac{\sqrt{b\,x^2+a}\left(-2\left(\frac{f\,x^2}{3}+e\right)b\,x^2a^{\frac{3}{2}}+\sqrt{a}\,b^2c+\frac{4a^{\frac{5}{2}}f\,x^2}{3}\right)}{2}}{a^{\frac{3}{2}}b^2x^2}$
risch	$-\frac{c\sqrt{b\,x^2+a}}{2a\,x^2}+\frac{2af\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)+\frac{2ae\sqrt{b\,x^2+a}}{b}-\frac{(2ad-bc)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{\sqrt{a}}}{2a}$
default	$f\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)+\frac{e\sqrt{b\,x^2+a}}{b}-\frac{d\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{\sqrt{a}}+c\left(-\frac{\sqrt{b\,x^2+a}}{2a\,x^2}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{2a^{\frac{3}{2}}}\right.$

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b^2*x^2*(a*d-1/2*b*c)*arctanh((b*x^2+a)^(1/2)/a^(1/2))+1/2*(b*x^2+a)^(1/2
)*(-2*(1/3*f*x^2+e)*b*x^2*a^(3/2)+a^(1/2)*b^2*c+4/3*a^(5/2)*f*x^2))/a^(3/2)
/b^2/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.10

$$\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx = \left[-\frac{3\,(b^3c-2\,ab^2d)\sqrt{a}x^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(2\,a^2bfx^4-3\,ab^2c+2\,(3\,a^2be-2\,a^3f)x^2)\sqrt{bx^2+a}}{12\,a^2b^2x^2}, -\frac{3\,(b^3c-2\,ab^2d)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-(2\,a^2bfx^4-3\,ab^2c+2\,(3\,a^2be-2\,a^3f)x^2)\sqrt{bx^2+a}}{6\,a^2b^2x^2}\right]$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(b^3*c - 2*a*b^2*d)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) - 2*(2*a^2*b*f*x^4 - 3*a*b^2*c + 2*(3*a^2*b*e - 2*a^3*f)
*x^2)*sqrt(b*x^2 + a))/(a^2*b^2*x^2), -1/6*(3*(b^3*c - 2*a*b^2*d)*sqrt(-a)*
x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (2*a^2*b*f*x^4 - 3*a*b^2*c + 2*(3*a^
2*b*e - 2*a^3*f)*x^2)*sqrt(b*x^2 + a))/(a^2*b^2*x^2)]
```

Sympy [A] (verification not implemented)

Time = 12.83 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.38

$$\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx = e\left(\begin{cases}\frac{\sqrt{a+bx^2}}{b} & \text{for}\ b\neq 0\\ \frac{x^2}{2\sqrt{a}} & \text{otherwise}\end{cases}\right) + f\left(\begin{cases}-\frac{2a\sqrt{a+bx^2}}{3b^2}+\frac{x^2\sqrt{a+bx^2}}{3b} & \text{for}\ b\neq 0\\ \frac{x^4}{4\sqrt{a}} & \text{otherwise}\end{cases}\right) - \frac{\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{2ax} - \frac{d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}} + \frac{bc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] e*Piecewise((sqrt(a + b*x**2)/b, Ne(b, 0)), (x**2/(2*sqrt(a)), True)) + f*P
iecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)/(3*b), Ne(
```

```
b, 0)), (x**4/(4*sqrt(a)), True)) - sqrt(b)*c*sqrt(a/(b*x**2) + 1)/(2*a*x)
- d*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + b*c*asinh(sqrt(a)/(sqrt(b)*x))/(2*
a**(3/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}fx^2}{3\,b} + \frac{bc\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{d\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} + \frac{\sqrt{bx^2+a}e}{b} - \frac{2\sqrt{bx^2+a}af}{3\,b^2} - \frac{\sqrt{bx^2+a}c}{2\,ax^2}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(b*x^2 + a)*f*x^2/b + 1/2*b*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2)
 - d*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + sqrt(b*x^2 + a)*e/b - 2/3*sqrt
(b*x^2 + a)*a*f/b^2 - 1/2*sqrt(b*x^2 + a)*c/(a*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.13

$$\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx = -\frac{\frac{3\,(b^2c-2\,abd)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{3\sqrt{bx^2+a}bc}{ax^2} - \frac{2\left(3\sqrt{bx^2+a}b^3e+(bx^2+a)^{\frac{3}{2}}b^2f-3\sqrt{bx^2+a}ab^2f\right)}{b^3}}{6\,b}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*(3*(b^2*c - 2*a*b*d)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + 3
*sqrt(b*x^2 + a)*b*c/(a*x^2) - 2*(3*sqrt(b*x^2 + a)*b^3*e + (b*x^2 + a)^(3/
2)*b^2*f - 3*sqrt(b*x^2 + a)*a*b^2*f)/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 6.42 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx=\frac{e\,\sqrt{b\,x^2+a}}{b}-\frac{d\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{c\,\sqrt{b\,x^2+a}}{2\,a\,x^2}+\frac{b\,c\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{f\,\sqrt{b\,x^2+a}\,\left(2\,a-b\,x^2\right)}{3\,b^2}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^3*(a + b*x^2)^(1/2)),x)
[Out] (e*(a + b*x^2)^(1/2))/b - (d*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2) - (c
*(a + b*x^2)^(1/2))/(2*a*x^2) + (b*c*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a
^(3/2)) - (f*(a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2)
```

3.148 $\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx$

Optimal result . . . 892
Rubi [A] (verified) . . . 892
Mathematica [A] (verified) . . . 894
Maple [A] (verified) . . . 895
Fricas [A] (verification not implemented) . . . 895
Sympy [A] (verification not implemented) . . . 896
Maxima [A] (verification not implemented) . . . 896
Giac [A] (verification not implemented) . . . 897
Mupad [B] (verification not implemented) . . . 897

Optimal result

Integrand size = 32, antiderivative size = 114

$$\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx = \frac{f\sqrt{a+bx^2}}{b}-\frac{c\sqrt{a+bx^2}}{4ax^4}+\frac{(3bc-4ad)\sqrt{a+bx^2}}{8a^2x^2}-\frac{(3b^2c-4abd+8a^2e)\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

```
[Out] -1/8*(8*a^2*e-4*a*b*d+3*b^2*c)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)+f*(
b*x^2+a)^(1/2)/b-1/4*c*(b*x^2+a)^(1/2)/a/x^4+1/8*(-4*a*d+3*b*c)*(b*x^2+a)^(
1/2)/a^2/x^2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1813, 1635, 911, 1171, 396, 214}

$$\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(3bc-4ad)}{8a^2x^2}-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(8a^2e-4abd+3b^2c)}{8a^{5/2}}-\frac{c\sqrt{a+bx^2}}{4ax^4}+\frac{f\sqrt{a+bx^2}}{b}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] (f*Sqrt[a + b*x^2])/b - (c*Sqrt[a + b*x^2])/(4*a*x^4) + ((3*b*c - 4*a*d)*Sq
rt[a + b*x^2])/(8*a^2*x^2) - ((3*b^2*c - 4*a*b*d + 8*a^2*e)*ArcTanh[Sqrt[a
+ b*x^2]/Sqrt[a]])/(8*a^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1635

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
```

```
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= -\frac{c\sqrt{a+bx^2}}{4ax^4} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(3bc-4ad)-2aex-2afx^2}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)}{4a}$$

$$= -\frac{c\sqrt{a+bx^2}}{4ax^4} - \frac{\text{Subst}\left(\int \frac{\frac{\frac{1}{2}b^2(3bc-4ad)+2a^2be-2a^3f}{b^2} - \frac{(2abe-4a^2f)x^2}{b^2} - \frac{2afx^4}{b^2}}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^2}\,dx, x, \sqrt{a+bx^2}\right)}{2ab}$$

$$= -\frac{c\sqrt{a+bx^2}}{4ax^4} + \frac{(3bc-4ad)\sqrt{a+bx^2}}{8a^2x^2} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}\left(-3bc+4ad-\frac{8a^2e}{b}+\frac{8a^3f}{b^2}\right)-\frac{4a^2fx^2}{b^2}}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{4a^2}$$

$$= \frac{f\sqrt{a+bx^2}}{b} - \frac{c\sqrt{a+bx^2}}{4ax^4} + \frac{(3bc-4ad)\sqrt{a+bx^2}}{8a^2x^2} + \frac{\left(3bc-4ad+\frac{8a^2e}{b}\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{8a^2}$$

$$= \frac{f\sqrt{a+bx^2}}{b} - \frac{c\sqrt{a+bx^2}}{4ax^4} + \frac{(3bc-4ad)\sqrt{a+bx^2}}{8a^2x^2} - \frac{(3b^2c-4abd+8a^2e)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(-2abc+3b^2cx^2-4abdx^2+8a^2fx^4)}{8a^2bx^4} + \frac{(-3b^2c+4abd-8a^2e)\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^5*Sqrt[a + b*x^2]),x]
[Out] (Sqrt[a + b*x^2]*(-2*a*b*c + 3*b^2*c*x^2 - 4*a*b*d*x^2 + 8*a^2*f*x^4))/(8*a
^2*b*x^4) + ((-3*b^2*c + 4*a*b*d - 8*a^2*e)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
])/(8*a^(5/2))
```

Maple [A] (verified)

Time = 3.56 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{-\left(a^{2} e-\frac{1}{2} a b d+\frac{3}{8} b^{2} c\right) b\,x^{4} a^{2} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)+\sqrt{b\,x^{2}+a}\,\left(a^{2} f\,x^{4}-\frac{b\left(2 d\,x^{2}+c\right) a}{4}+\frac{3 b^{2} c\,x^{2}}{8}\right) a^{\frac{5}{2}}}{b\,a^{\frac{9}{2}} x^{4}}$
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(4 a d\,x^{2}-3 c b\,x^{2}+2 a c\right)}{8 a^{2} x^{4}}+\frac{\frac{8 a^{2} f \sqrt{b\,x^{2}+a}}{b}-\frac{\left(8 a^{2} e-4 a b d+3 b^{2} c\right) \ln\left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{\sqrt{a}}}{8 a^{2}}$
default	$\frac{f\sqrt{b\,x^{2}+a}}{b}-\frac{e\ln\left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{\sqrt{a}}+d\left(-\frac{\sqrt{b\,x^{2}+a}}{2 a\,x^{2}}+\frac{b\ln\left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2 a^{\frac{3}{2}}}\right)+c\left(-\frac{\sqrt{b\,x^{2}+a}}{4 a\,x^{4}}-\frac{3 b\left(-\sqrt{\right.}}{}\right.$

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^5/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(a^2*e-1/2*a*b*d+3/8*b^2*c)*b*x^4*a^2*arctanh((b*x^2+a)^(1/2)/a^(1/2))+(b
*x^2+a)^(1/2)*(a^2*f*x^4-1/4*b*(2*d*x^2+c)*a+3/8*b^2*c*x^2)*a^(5/2))/b/a^(9
/2)/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.94

$$\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx$$
$$=\left[\frac{(3\,b^3c-4\,ab^2d+8\,a^2be)\sqrt{a}x^4\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(8\,a^3fx^4-2\,a^2bc+(3\,ab^2c-4\,a^2bd)x^2)\sqrt{bx}}{16\,a^3bx^4}\right.$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^5/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*b^3*c - 4*a*b^2*d + 8*a^2*b*e)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b
*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(8*a^3*f*x^4 - 2*a^2*b*c + (3*a*b^2*c - 4
*a^2*b*d)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^4), 1/8*((3*b^3*c - 4*a*b^2*d + 8*
a^2*b*e)*sqrt(-a)*x^4*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (8*a^3*f*x^4 - 2*a
^2*b*c + (3*a*b^2*c - 4*a^2*b*d)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^4)]
```

Sympy [A] (verification not implemented)

Time = 33.69 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.70

$$\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx = f\left(\begin{cases}\frac{\sqrt{a+bx^2}}{b} & \text{for } b\neq 0\\ \frac{x^2}{2\sqrt{a}} & \text{otherwise}\end{cases}\right) - \frac{c}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{\sqrt{b}c}{8ax^3\sqrt{\frac{a}{bx^2}+1}} - \frac{\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{2ax} + \frac{3b^{\frac{3}{2}}c}{8a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}} + \frac{bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}} - \frac{3b^2c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{5}{2}}}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**5/(b*x**2+a)**(1/2),x)
```

```
[Out] f*Piecewise((sqrt(a + b*x**2)/b, Ne(b, 0)), (x**2/(2*sqrt(a)), True)) - c/(
4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + sqrt(b)*c/(8*a*x**3*sqrt(a/(b*x**2)
+ 1)) - sqrt(b)*d*sqrt(a/(b*x**2) + 1)/(2*a*x) + 3*b**(3/2)*c/(8*a**2*x*sqr
t(a/(b*x**2) + 1)) - e*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + b*d*asinh(sqrt(
a)/(sqrt(b)*x))/(2*a**(3/2)) - 3*b**2*c*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(5
/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.12

$$\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx = -\frac{3\,b^2c\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{5}{2}}} + \frac{bd\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{e\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} + \frac{\sqrt{bx^2+a}f}{b} + \frac{3\sqrt{bx^2+a}bc}{8\,a^2x^2} - \frac{\sqrt{bx^2+a}d}{2\,ax^2} - \frac{\sqrt{bx^2+a}c}{4\,ax^4}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^5/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/8*b^2*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/2*b*d*arcsinh(a/(sqrt(
a*b)*abs(x)))/a^(3/2) - e*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + sqrt(b*x^
2 + a)*f/b + 3/8*sqrt(b*x^2 + a)*b*c/(a^2*x^2) - 1/2*sqrt(b*x^2 + a)*d/(a*x
^2) - 1/4*sqrt(b*x^2 + a)*c/(a*x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.23

$$\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx = \frac{8\sqrt{bx^2+a}f+\frac{(3b^3c-4ab^2d+8a^2be)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{3(bx^2+a)^{\frac{3}{2}}b^3c-5\sqrt{bx^2+a}ab^3c-4(bx^2+a)^{\frac{3}{2}}ab^2d+4\sqrt{bx^2+a}a^2b^2d}{a^2b^2x^4}}{8b}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^5/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(8*sqrt(b*x^2 + a)*f + (3*b^3*c - 4*a*b^2*d + 8*a^2*b*e)*arctan(sqrt(b*
x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^2 + a)^(3/2)*b^3*c - 5*sqrt(b*x
^2 + a)*a*b^3*c - 4*(b*x^2 + a)^(3/2)*a*b^2*d + 4*sqrt(b*x^2 + a)*a^2*b^2*d
)/(a^2*b^2*x^4))/b
```

Mupad [B] (verification not implemented)

Time = 6.99 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.17

$$\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx = \frac{f\sqrt{b\,x^2+a}}{b}-\frac{e\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{5\,c\sqrt{b\,x^2+a}}{8\,a\,x^4}+\frac{3\,c\,(b\,x^2+a)^{3/2}}{8\,a^2\,x^4}-\frac{d\sqrt{b\,x^2+a}}{2\,a\,x^2}+\frac{b\,d\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{3\,b^2\,c\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{5/2}}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^5*(a + b*x^2)^(1/2)),x)
```

```
[Out] (f*(a + b*x^2)^(1/2))/b - (e*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2) - (5
*c*(a + b*x^2)^(1/2))/(8*a*x^4) + (3*c*(a + b*x^2)^(3/2))/(8*a^2*x^4) - (d*
(a + b*x^2)^(1/2))/(2*a*x^2) + (b*d*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^
(3/2)) - (3*b^2*c*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(5/2))
```

3.149 $\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx$

Optimal result 898
Rubi [A] (verified) 898
Mathematica [A] (verified) 901
Maple [A] (verified) 901
Fricas [A] (verification not implemented) 902
Sympy [B] (verification not implemented) 902
Maxima [A] (verification not implemented) 903
Giac [A] (verification not implemented) 903
Mupad [B] (verification not implemented) 904

Optimal result

Integrand size = 32, antiderivative size = 146

$$\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx = -\frac{c\sqrt{a+bx^2}}{6ax^6}+\frac{(5bc-6ad)\sqrt{a+bx^2}}{24a^2x^4} - \frac{(5b^2c-6abd+8a^2e)\sqrt{a+bx^2}}{16a^3x^2} + \frac{(5b^3c-6ab^2d+8a^2be-16a^3f)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{7/2}}$$

```
[Out] 1/16*(-16*a^3*f+8*a^2*b*e-6*a*b^2*d+5*b^3*c)*arctanh((b*x^2+a)^(1/2)/a^(1/2
))/a^(7/2)-1/6*c*(b*x^2+a)^(1/2)/a/x^6+1/24*(-6*a*d+5*b*c)*(b*x^2+a)^(1/2)/
a^2/x^4-1/16*(8*a^2*e-6*a*b*d+5*b^2*c)*(b*x^2+a)^(1/2)/a^3/x^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1813, 1635, 911, 1171, 393, 214}

$$\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(5bc-6ad)}{24a^2x^4} - \frac{\sqrt{a+bx^2}(8a^2e-6abd+5b^2c)}{16a^3x^2} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(-16a^3f+8a^2be-6ab^2d+5b^3c)}{16a^{7/2}} - \frac{c\sqrt{a+bx^2}}{6ax^6}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^7*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/6*(c*Sqrt[a + b*x^2])/(a*x^6) + ((5*b*c - 6*a*d)*Sqrt[a + b*x^2])/(24*a^
2*x^4) - ((5*b^2*c - 6*a*b*d + 8*a^2*e)*Sqrt[a + b*x^2])/(16*a^3*x^2) + ((5
*b^3*c - 6*a*b^2*d + 8*a^2*b*e - 16*a^3*f)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
)/(16*a^(7/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1635

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
```

```
2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= -\frac{c\sqrt{a+bx^2}}{6ax^6} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(5bc-6ad)-3aex-3afx^2}{x^3\sqrt{a+bx}}\,dx, x, x^2\right)}{6a}$$

$$= -\frac{c\sqrt{a+bx^2}}{6ax^6} - \frac{\text{Subst}\left(\int \frac{\frac{\frac{1}{2}b^2(5bc-6ad)+3a^2be-3a^3f}{b^2} - \frac{\left(3abe-6a^2f\right)x^2}{b^2} - \frac{3afx^4}{b^2}}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^3}\,dx, x, \sqrt{a+bx^2}\right)}{3ab}$$

$$= -\frac{c\sqrt{a+bx^2}}{6ax^6} + \frac{(5bc-6ad)\sqrt{a+bx^2}}{24a^2x^4} - \frac{\text{Subst}\left(\int \frac{-\frac{3}{2}\left(5bc-6ad+\frac{8a^2e}{b}-\frac{8a^3f}{b^2}\right)-\frac{12a^2fx^2}{b^2}}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^2}\,dx, x, \sqrt{a+bx^2}\right)}{12a^2}$$

$$= -\frac{c\sqrt{a+bx^2}}{6ax^6} + \frac{(5bc-6ad)\sqrt{a+bx^2}}{24a^2x^4} - \frac{(5b^2c-6abd+8a^2e)\sqrt{a+bx^2}}{16a^3x^2} + \frac{\left(b^2\left(\frac{12a^3f}{b^3} - \frac{3\left(5bc-6ad+\frac{8a^2e}{b}-\frac{8a^3f}{b^2}\right)}{2b}\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{24a^3}$$

$$= -\frac{c\sqrt{a+bx^2}}{6ax^6} + \frac{(5bc-6ad)\sqrt{a+bx^2}}{24a^2x^4} - \frac{(5b^2c-6abd+8a^2e)\sqrt{a+bx^2}}{16a^3x^2} + \frac{(5b^3c-6ab^2d+8a^2be-16a^3f)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{7/2}}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86

$$\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx$$
$$=\frac{\sqrt{a+bx^2}(-8a^2c+10abcx^2-12a^2dx^2-15b^2cx^4+18abdx^4-24a^2ex^4)}{48a^3x^6}$$
$$+\frac{(5b^3c-6ab^2d+8a^2be-16a^3f)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{7/2}}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^7*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^2*c + 10*a*b*c*x^2 - 12*a^2*d*x^2 - 15*b^2*c*x^4 + 1
8*a*b*d*x^4 - 24*a^2*e*x^4))/(48*a^3*x^6) + ((5*b^3*c - 6*a*b^2*d + 8*a^2*b
*e - 16*a^3*f)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(7/2))
```

Maple [A] (verified)

Time = 3.57 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$-\frac{x^6\left(f a^3-\frac{1}{2}a^2be+\frac{3}{8}a b^2d-\frac{5}{16}b^3c\right)\operatorname{arctanh}\left(\frac{\sqrt{b x^2+a}}{\sqrt{a}}\right)+\frac{5\left(\frac{4\left(2e x^4+d x^2+\frac{2}{3}c\right)a^{\frac{5}{2}}}{5}+b\left(2\left(-\frac{3d x^2}{5}-\frac{c}{3}\right)a^{\frac{3}{2}}+bc x^2\sqrt{a}\right)x^2\right)\sqrt{b x^2 \cdots}}{16}}{a^{\frac{7}{2}}x^6}$
risch	$-\frac{\sqrt{b x^2+a}\,(24a^2e x^4-18abd x^4+15b^2c x^4+12a^2d x^2-10abc x^2+8a^2c)}{48a^3x^6}-\frac{(16f a^3-8a^2be+6a b^2d-5b^3c)\ln\left(\frac{2a+2\sqrt{a}\sqrt{b x^{\cdots}}}{x}\cdots\right.}{16a^{\frac{7}{2}}}$
default	$-\frac{f\ln\left(\frac{2a+2\sqrt{a}\sqrt{b x^2+a}}{x}\right)}{\sqrt{a}}+c\left(-\frac{\sqrt{b x^2+a}}{6a x^6}-\frac{5b\left(-\frac{\sqrt{b x^2+a}}{4a x^4}-\frac{3b\left(-\frac{\sqrt{b x^2+a}}{2a x^2}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\sqrt{b x^2+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)}{6a}\right)+e\left(\cdots\right.$

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^7/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(x^6*(f*a^3-1/2*a^2*b*e+3/8*a*b^2*d-5/16*b^3*c)*arctanh((b*x^2+a)^(1/2)/a^
(1/2))+5/16*(4/5*(2*e*x^4+d*x^2+2/3*c)*a^(5/2)+b*(2*(-3/5*d*x^2-1/3*c)*a^(3
/2)+b*c*x^2*a^(1/2))*x^2)*(b*x^2+a)^(1/2))/a^(7/2)/x^6
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.79

$$\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx = \left[-\frac{3\,(5\,b^3c-6\,ab^2d+8\,a^2be-16\,a^3f)\sqrt{a}x^6\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(3\,(5\,ab^2c-6\,a^2bd+8\,a^3e)x^4+ \right.}{96\,a^4x^6}$$

$$-\frac{3\,(5\,b^3c-6\,ab^2d+8\,a^2be-16\,a^3f)\sqrt{-a}x^6\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(3\,(5\,ab^2c-6\,a^2bd+8\,a^3e)x^4+8\,a^3c-2}{48\,a^4x^6}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^7/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*b^3*c - 6*a*b^2*d + 8*a^2*b*e - 16*a^3*f)*sqrt(a)*x^6*log(-(b*
x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*(5*a*b^2*c - 6*a^2*b*d +
 8*a^3*e)*x^4 + 8*a^3*c - 2*(5*a^2*b*c - 6*a^3*d)*x^2)*sqrt(b*x^2 + a))/(a^
4*x^6), -1/48*(3*(5*b^3*c - 6*a*b^2*d + 8*a^2*b*e - 16*a^3*f)*sqrt(-a)*x^6*
arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(5*a*b^2*c - 6*a^2*b*d + 8*a^3*e)*x^4
 + 8*a^3*c - 2*(5*a^2*b*c - 6*a^3*d)*x^2)*sqrt(b*x^2 + a))/(a^4*x^6)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(141) = 282.

Time = 40.70 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.08

$$\begin{aligned}\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx = &-\frac{c}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{d}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{\sqrt{b}c}{24ax^5\sqrt{\frac{a}{bx^2}+1}}\\ &+\frac{\sqrt{b}d}{8ax^3\sqrt{\frac{a}{bx^2}+1}}-\frac{\sqrt{b}e\sqrt{\frac{a}{bx^2}+1}}{2ax}-\frac{5b^{\frac{3}{2}}c}{48a^2x^3\sqrt{\frac{a}{bx^2}+1}}\\ &+\frac{3b^{\frac{3}{2}}d}{8a^2x\sqrt{\frac{a}{bx^2}+1}}-\frac{5b^{\frac{5}{2}}c}{16a^3x\sqrt{\frac{a}{bx^2}+1}}-\frac{f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}\\ &+\frac{be\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}-\frac{3b^2d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{5}{2}}}+\frac{5b^3c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{7}{2}}}\end{aligned}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**7/(b*x**2+a)**(1/2),x)
```

```
[Out] -c/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - d/(4*sqrt(b)*x**5*sqrt(a/(b*x**2
) + 1)) + sqrt(b)*c/(24*a*x**5*sqrt(a/(b*x**2) + 1)) + sqrt(b)*d/(8*a*x**3*
```

```
sqrt(a/(b*x**2) + 1)) - sqrt(b)*e*sqrt(a/(b*x**2) + 1)/(2*a*x) - 5*b**(3/2)
*c/(48*a**2*x**3*sqrt(a/(b*x**2) + 1)) + 3*b**(3/2)*d/(8*a**2*x*sqrt(a/(b*x
**2) + 1)) - 5*b**(5/2)*c/(16*a**3*x*sqrt(a/(b*x**2) + 1)) - f*asinh(sqrt(a
)/(sqrt(b)*x))/sqrt(a) + b*e*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/2)) - 3*b*
*2*d*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(5/2)) + 5*b**3*c*asinh(sqrt(a)/(sqrt
(b)*x))/(16*a**(7/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.32

$$\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx = \frac{5\,b^3c\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{7}{2}}} - \frac{3\,b^2d\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{5}{2}}} + \frac{be\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{f\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} - \frac{5\,\sqrt{bx^2+a}b^2c}{16\,a^3x^2} + \frac{3\,\sqrt{bx^2+a}bd}{8\,a^2x^2} - \frac{\sqrt{bx^2+a}e}{2\,ax^2} + \frac{5\,\sqrt{bx^2+a}bc}{24\,a^2x^4} - \frac{\sqrt{bx^2+a}d}{4\,ax^4} - \frac{\sqrt{bx^2+a}c}{6\,ax^6}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^7/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/16*b^3*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) - 3/8*b^2*d*arcsinh(a/(sqr
t(a*b)*abs(x)))/a^(5/2) + 1/2*b*e*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - f
*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) - 5/16*sqrt(b*x^2 + a)*b^2*c/(a^3*x^
2) + 3/8*sqrt(b*x^2 + a)*b*d/(a^2*x^2) - 1/2*sqrt(b*x^2 + a)*e/(a*x^2) + 5/
24*sqrt(b*x^2 + a)*b*c/(a^2*x^4) - 1/4*sqrt(b*x^2 + a)*d/(a*x^4) - 1/6*sqrt
(b*x^2 + a)*c/(a*x^6)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.56

$$\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx = -\frac{\frac{3\,(5\,b^4c-6\,ab^3d+8\,a^2b^2e-16\,a^3bf)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3} + \frac{15\,(bx^2+a)^{\frac{5}{2}}b^4c-40\,(bx^2+a)^{\frac{3}{2}}ab^4c+33\,\sqrt{bx^2+a}a^2b^4c-18\,(bx^2+a)^{\frac{5}{2}}ab^3d+48\,(bx}{}}{48\,b}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^7/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/48*(3*(5*b^4*c - 6*a*b^3*d + 8*a^2*b^2*e - 16*a^3*b*f)*arctan(sqrt(b*x^2
 + a)/sqrt(-a))/(sqrt(-a)*a^3) + (15*(b*x^2 + a)^(5/2)*b^4*c - 40*(b*x^2 +
```

```
a)^(3/2)*a*b^4*c + 33*sqrt(b*x^2 + a)*a^2*b^4*c - 18*(b*x^2 + a)^(5/2)*a*b^
3*d + 48*(b*x^2 + a)^(3/2)*a^2*b^3*d - 30*sqrt(b*x^2 + a)*a^3*b^3*d + 24*(b
*x^2 + a)^(5/2)*a^2*b^2*e - 48*(b*x^2 + a)^(3/2)*a^3*b^2*e + 24*sqrt(b*x^2
+ a)*a^4*b^2*e)/(a^3*b^3*x^6))/b
```

Mupad [B] (verification not implemented)

Time = 7.62 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.36

$$\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx = \frac{5\,c\,{\left(b\,x^2+a\right)}^{3/2}}{6\,a^2\,x^6}-\frac{11\,c\,\sqrt{b\,x^2+a}}{16\,a\,x^6}-\frac{f\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{5\,c\,{\left(b\,x^2+a\right)}^{5/2}}{16\,a^3\,x^6}-\frac{5\,d\,\sqrt{b\,x^2+a}}{8\,a\,x^4}+\frac{3\,d\,{\left(b\,x^2+a\right)}^{3/2}}{8\,a^2\,x^4}-\frac{e\,\sqrt{b\,x^2+a}}{2\,a\,x^2}+\frac{b\,e\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{3\,b^2\,d\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{5/2}}-\frac{b^3\,c\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{16\,a^{7/2}}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^7*(a + b*x^2)^(1/2)),x)
```

```
[Out] (5*c*(a + b*x^2)^(3/2))/(6*a^2*x^6) - (11*c*(a + b*x^2)^(1/2))/(16*a*x^6) -
 (f*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2) - (5*c*(a + b*x^2)^(5/2))/(16
*a^3*x^6) - (5*d*(a + b*x^2)^(1/2))/(8*a*x^4) + (3*d*(a + b*x^2)^(3/2))/(8*
a^2*x^4) - (e*(a + b*x^2)^(1/2))/(2*a*x^2) + (b*e*atanh((a + b*x^2)^(1/2)/a
^(1/2)))/(2*a^(3/2)) - (b^3*c*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(16*
a^(7/2)) - (3*b^2*d*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(5/2))
```

3.150 $\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx$

Optimal result 905
Rubi [A] (verified) 905
Mathematica [A] (verified) 908
Maple [A] (verified) 908
Fricas [A] (verification not implemented) 909
Sympy [B] (verification not implemented) 910
Maxima [A] (verification not implemented) 911
Giac [B] (verification not implemented) 911
Mupad [B] (verification not implemented) 912

Optimal result

Integrand size = 32, antiderivative size = 195

$$\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx = -\frac{c\sqrt{a+bx^2}}{8ax^8}+\frac{(7bc-8ad)\sqrt{a+bx^2}}{48a^2x^6} - \frac{(35b^2c-40abd+48a^2e)\sqrt{a+bx^2}}{192a^3x^4} + \frac{(35b^3c-40ab^2d+48a^2be-64a^3f)\sqrt{a+bx^2}}{128a^4x^2} - \frac{b(35b^3c-40ab^2d+48a^2be-64a^3f)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{9/2}}$$

```
[Out] -1/128*b*(-64*a^3*f+48*a^2*b*e-40*a*b^2*d+35*b^3*c)*arctanh((b*x^2+a)^(1/2)
/a^(1/2))/a^(9/2)-1/8*c*(b*x^2+a)^(1/2)/a/x^8+1/48*(-8*a*d+7*b*c)*(b*x^2+a)
^(1/2)/a^2/x^6-1/192*(48*a^2*e-40*a*b*d+35*b^2*c)*(b*x^2+a)^(1/2)/a^3/x^4+1
/128*(-64*a^3*f+48*a^2*b*e-40*a*b^2*d+35*b^3*c)*(b*x^2+a)^(1/2)/a^4/x^2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1813, 1635, 911, 1171, 393, 205, 214}

$$\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(7bc-8ad)}{48a^2x^6}-\frac{\sqrt{a+bx^2}(48a^2e-40abd+35b^2c)}{192a^3x^4} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(-64a^3f+48a^2be-40ab^2d+35b^3c)}{128a^{9/2}} + \frac{\sqrt{a+bx^2}(-64a^3f+48a^2be-40ab^2d+35b^3c)}{128a^4x^2}-\frac{c\sqrt{a+bx^2}}{8ax^8}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^9*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/8*(c*Sqrt[a + b*x^2])/(a*x^8) + ((7*b*c - 8*a*d)*Sqrt[a + b*x^2])/(48*a^
2*x^6) - ((35*b^2*c - 40*a*b*d + 48*a^2*e)*Sqrt[a + b*x^2])/(192*a^3*x^4) +
 ((35*b^3*c - 40*a*b^2*d + 48*a^2*b*e - 64*a^3*f)*Sqrt[a + b*x^2])/(128*a^4
*x^2) - (b*(35*b^3*c - 40*a*b^2*d + 48*a^2*b*e - 64*a^3*f)*ArcTanh[Sqrt[a +
 b*x^2]/Sqrt[a]])/(128*a^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
```

```
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1635

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx}}\,dx, x, x^2\right)\\
&= -\frac{c\sqrt{a+bx^2}}{8ax^8} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(7bc-8ad)-4aex-4afx^2}{x^4\sqrt{a+bx}}\,dx, x, x^2\right)}{8a}\\
&= -\frac{c\sqrt{a+bx^2}}{8ax^8} - \frac{\text{Subst}\left(\int \frac{\frac{\frac{1}{2}b^2(7bc-8ad)+4a^2be-4a^3f}{b^2}-\frac{\left(4abe-8a^2f\right)x^2}{b^2}-\frac{4afx^4}{b^2}}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^4}\,dx, x, \sqrt{a+bx^2}\right)}{4ab}\\
&= -\frac{c\sqrt{a+bx^2}}{8ax^8} + \frac{(7bc-8ad)\sqrt{a+bx^2}}{48a^2x^6}\\
&\quad - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}\left(-35bc+40ad-\frac{48a^2e}{b}+\frac{48a^3f}{b^2}\right)-\frac{24a^2fx^2}{b^2}}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^3}\,dx, x, \sqrt{a+bx^2}\right)}{24a^2}\\
&= -\frac{c\sqrt{a+bx^2}}{8ax^8} + \frac{(7bc-8ad)\sqrt{a+bx^2}}{48a^2x^6} - \frac{(35b^2c-40abd+48a^2e)\sqrt{a+bx^2}}{192a^3x^4}\\
&\quad + \frac{\left(b^2\left(\frac{24a^3f}{b^3}+\frac{3\left(-35bc+40ad-\frac{48a^2e}{b}+\frac{48a^3f}{b^2}\right)}{2b}\right)\right)\text{Subst}\left(\int \frac{1}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^2}\,dx, x, \sqrt{a+bx^2}\right)}{96a^3}
\end{aligned}$$

$$= -\frac{c\sqrt{a+bx^2}}{8ax^8} + \frac{(7bc-8ad)\sqrt{a+bx^2}}{48a^2x^6} - \frac{(35b^2c-40abd+48a^2e)\sqrt{a+bx^2}}{192a^3x^4} + \frac{(35b^3c-40ab^2d+48a^2be-64a^3f)\sqrt{a+bx^2}}{128a^4x^2} + \frac{(35b^3c-40ab^2d+48a^2be-64a^3f)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{128a^4}$$

$$= -\frac{c\sqrt{a+bx^2}}{8ax^8} + \frac{(7bc-8ad)\sqrt{a+bx^2}}{48a^2x^6} - \frac{(35b^2c-40abd+48a^2e)\sqrt{a+bx^2}}{192a^3x^4} + \frac{(35b^3c-40ab^2d+48a^2be-64a^3f)\sqrt{a+bx^2}}{128a^4x^2} - \frac{b(35b^3c-40ab^2d+48a^2be-64a^3f)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{9/2}}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.82

$$\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx = \frac{\frac{\sqrt{a}\sqrt{a+bx^2}(105b^3cx^6-10ab^2x^4(7c+12dx^2)+8a^2bx^2(7c+10dx^2+18ex^4)-16a^3(3c+4dx^2+6ex^4+12fx^6))}{x^8} - 3b(35b^3c-40ab^2d+48a^2}{384a^{9/2}}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^9*Sqrt[a + b*x^2]),x]
```

```
[Out] ((Sqrt[a]*Sqrt[a + b*x^2]*(105*b^3*c*x^6 - 10*a*b^2*x^4*(7*c + 12*d*x^2) +
8*a^2*b*x^2*(7*c + 10*d*x^2 + 18*e*x^4) - 16*a^3*(3*c + 4*d*x^2 + 6*e*x^4 +
 12*f*x^6)))/x^8 - 3*b*(35*b^3*c - 40*a*b^2*d + 48*a^2*b*e - 64*a^3*f)*ArcT
anh[Sqrt[a + b*x^2]/Sqrt[a]])/(384*a^(9/2))
```

Maple [A] (verified)

Time = 3.57 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{b\,x^{8}\left(f\,a^{3}-\frac{3}{4}a^{2}be+\frac{5}{8}a\,b^{2}d-\frac{35}{64}b^{3}c\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)+\frac{35\left(\left(-\frac{64}{35}f\,x^{6}-\frac{32}{35}e\,x^{4}-\frac{64}{105}d\,x^{2}-\frac{16}{35}c\right)a^{\frac{7}{2}}+b\,x^{2}\left(\left(\frac{48}{35}e\,x^{4}+\frac{16}{21}d\,x^{2}+\frac{8}{15}c\right.\right.\right.}{64}}{2a^{\frac{9}{2}}x^{8}}$
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(192a^{3}f\,x^{6}-144a^{2}be\,x^{6}+120a\,b^{2}d\,x^{6}-105b^{3}c\,x^{6}+96a^{3}e\,x^{4}-80a^{2}bd\,x^{4}+70a\,b^{2}c\,x^{4}+64a^{3}d\,x^{2}-56a^{2}bc\,x^{2}+48c\,a\right.}{384a^{4}x^{8}}$
default	$d\left(-\frac{\sqrt{b\,x^{2}+a}}{6a\,x^{6}}-\frac{5b\left(-\frac{\sqrt{b\,x^{2}+a}}{4a\,x^{4}}-\frac{3b\left(-\frac{\sqrt{b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)}{6a}\right)+c\left(-\frac{\sqrt{b\,x^{2}+a}}{8a\,x^{8}}-\frac{7b\left(-\frac{\sqrt{b\,x^{2}+a}}{6a\,x^{6}}\right.}{}\right.$

[In] int((f*x^6+e*x^4+d*x^2+c)/x^9/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*(b*x^8*(f*a^3-3/4*a^2*b*e+5/8*a*b^2*d-35/64*b^3*c)*arctanh((b*x^2+a)^(1/2)/a^(1/2))+35/64*((-64/35*f*x^6-32/35*e*x^4-64/105*d*x^2-16/35*c)*a^(7/2)+b*x^2*((48/35*e*x^4+16/21*d*x^2+8/15*c)*a^(5/2)+((-8/7*d*x^2-2/3*c)*a^(3/2)+b*c*x^2*a^(1/2))*b*x^2))*(b*x^2+a)^(1/2))/a^(9/2)/x^8

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.75

$$\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx$$
$$=\left[-\frac{3\left(35\,b^4c-40\,ab^3d+48\,a^2b^2e-64\,a^3bf\right)\sqrt{a}x^8\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\left(3\left(35\,ab^3c-40\,a^2b^2d+4\right.\right.}{768\,a^5x}\right.$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^9/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(35*b^4*c - 40*a*b^3*d + 48*a^2*b^2*e - 64*a^3*b*f)*sqrt(a)*x^8*
log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(3*(35*a*b^3*c - 40
*a^2*b^2*d + 48*a^3*b*e - 64*a^4*f)*x^6 - 48*a^4*c - 2*(35*a^2*b^2*c - 40*a
^3*b*d + 48*a^4*e)*x^4 + 8*(7*a^3*b*c - 8*a^4*d)*x^2)*sqrt(b*x^2 + a))/(a^5
*x^8), 1/384*(3*(35*b^4*c - 40*a*b^3*d + 48*a^2*b^2*e - 64*a^3*b*f)*sqrt(-a
)*x^8*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(35*a*b^3*c - 40*a^2*b^2*d + 48
*a^3*b*e - 64*a^4*f)*x^6 - 48*a^4*c - 2*(35*a^2*b^2*c - 40*a^3*b*d + 48*a^4
*e)*x^4 + 8*(7*a^3*b*c - 8*a^4*d)*x^2)*sqrt(b*x^2 + a))/(a^5*x^8)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(196) = 392$.

Time = 103.42 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.28

$$\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx = -\frac{c}{8\sqrt{b}x^9\sqrt{\frac{a}{bx^2}+1}} - \frac{d}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{e}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{\sqrt{b}c}{48ax^7\sqrt{\frac{a}{bx^2}+1}} + \frac{\sqrt{b}d}{24ax^5\sqrt{\frac{a}{bx^2}+1}} + \frac{\sqrt{b}e}{8ax^3\sqrt{\frac{a}{bx^2}+1}} - \frac{\sqrt{b}f\sqrt{\frac{a}{bx^2}+1}}{2ax} - \frac{7b^{\frac{3}{2}}c}{192a^2x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{5b^{\frac{3}{2}}d}{48a^2x^3\sqrt{\frac{a}{bx^2}+1}} + \frac{3b^{\frac{3}{2}}e}{8a^2x\sqrt{\frac{a}{bx^2}+1}} + \frac{35b^{\frac{5}{2}}c}{384a^3x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{5b^{\frac{5}{2}}d}{16a^3x\sqrt{\frac{a}{bx^2}+1}} + \frac{35b^{\frac{7}{2}}c}{128a^4x\sqrt{\frac{a}{bx^2}+1}} + \frac{bf\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}} - \frac{3b^2e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{5}{2}}} + \frac{5b^3d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{7}{2}}} - \frac{35b^4c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{128a^{\frac{9}{2}}}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**9/(b*x**2+a)**(1/2),x)
```

```
[Out] -c/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - d/(6*sqrt(b)*x**7*sqrt(a/(b*x**2
) + 1)) - e/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + sqrt(b)*c/(48*a*x**7*sq
rt(a/(b*x**2) + 1)) + sqrt(b)*d/(24*a*x**5*sqrt(a/(b*x**2) + 1)) + sqrt(b)*
e/(8*a*x**3*sqrt(a/(b*x**2) + 1)) - sqrt(b)*f*sqrt(a/(b*x**2) + 1)/(2*a*x)
- 7*b**(3/2)*c/(192*a**2*x**5*sqrt(a/(b*x**2) + 1)) - 5*b**(3/2)*d/(48*a**2
*x**3*sqrt(a/(b*x**2) + 1)) + 3*b**(3/2)*e/(8*a**2*x*sqrt(a/(b*x**2) + 1))
+ 35*b**(5/2)*c/(384*a**3*x**3*sqrt(a/(b*x**2) + 1)) - 5*b**(5/2)*d/(16*a**
3*x*sqrt(a/(b*x**2) + 1)) + 35*b**(7/2)*c/(128*a**4*x*sqrt(a/(b*x**2) + 1))
 + b*f*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/2)) - 3*b**2*e*asinh(sqrt(a)/(sq
rt(b)*x))/(8*a**(5/2)) + 5*b**3*d*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(7/2))
- 35*b**4*c*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(9/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.41

$$\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx = -\frac{35\,b^4c\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{128\,a^{\frac{9}{2}}}+\frac{5\,b^3d\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{7}{2}}} - \frac{3\,b^2e\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{5}{2}}}+\frac{bf\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} + \frac{35\,\sqrt{bx^2+a}b^3c}{128\,a^4x^2}-\frac{5\,\sqrt{bx^2+a}b^2d}{16\,a^3x^2}+\frac{3\,\sqrt{bx^2+a}be}{8\,a^2x^2} - \frac{\sqrt{bx^2+a}f}{2\,ax^2}-\frac{35\,\sqrt{bx^2+a}b^2c}{192\,a^3x^4}+\frac{5\,\sqrt{bx^2+a}bd}{24\,a^2x^4} - \frac{\sqrt{bx^2+a}e}{4\,ax^4}+\frac{7\,\sqrt{bx^2+a}bc}{48\,a^2x^6}-\frac{\sqrt{bx^2+a}d}{6\,ax^6}-\frac{\sqrt{bx^2+a}c}{8\,ax^8}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^9/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -35/128*b^4*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(9/2) + 5/16*b^3*d*arcsinh(a/
(sqrt(a*b)*abs(x)))/a^(7/2) - 3/8*b^2*e*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/
2) + 1/2*b*f*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + 35/128*sqrt(b*x^2 + a)
*b^3*c/(a^4*x^2) - 5/16*sqrt(b*x^2 + a)*b^2*d/(a^3*x^2) + 3/8*sqrt(b*x^2 +
a)*b*e/(a^2*x^2) - 1/2*sqrt(b*x^2 + a)*f/(a*x^2) - 35/192*sqrt(b*x^2 + a)*b
^2*c/(a^3*x^4) + 5/24*sqrt(b*x^2 + a)*b*d/(a^2*x^4) - 1/4*sqrt(b*x^2 + a)*e
/(a*x^4) + 7/48*sqrt(b*x^2 + a)*b*c/(a^2*x^6) - 1/6*sqrt(b*x^2 + a)*d/(a*x^
6) - 1/8*sqrt(b*x^2 + a)*c/(a*x^8)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(171) = 342.

Time = 0.29 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.83

$$\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx = \frac{\frac{3\left(35\,b^5c-40\,ab^4d+48\,a^2b^3e-64\,a^3b^2f\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4}+\frac{105\,\left(bx^2+a\right)^{\frac{7}{2}}b^5c-385\,\left(bx^2+a\right)^{\frac{5}{2}}ab^5c+511\,\left(bx^2+a\right)^{\frac{3}{2}}a^2b^5c-279\,\sqrt{bx^2+a}a^3b^5c-}{}}{}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^9/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(3*(35*b^5*c - 40*a*b^4*d + 48*a^2*b^3*e - 64*a^3*b^2*f)*arctan(sqrt(
b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4) + (105*(b*x^2 + a)^(7/2)*b^5*c - 385*(b
*x^2 + a)^(5/2)*a*b^5*c + 511*(b*x^2 + a)^(3/2)*a^2*b^5*c - 279*sqrt(b*x^2
```

```
+ a)*a^3*b^5*c - 120*(b*x^2 + a)^(7/2)*a*b^4*d + 440*(b*x^2 + a)^(5/2)*a^2*
b^4*d - 584*(b*x^2 + a)^(3/2)*a^3*b^4*d + 264*sqrt(b*x^2 + a)*a^4*b^4*d + 1
44*(b*x^2 + a)^(7/2)*a^2*b^3*e - 528*(b*x^2 + a)^(5/2)*a^3*b^3*e + 624*(b*x
^2 + a)^(3/2)*a^4*b^3*e - 240*sqrt(b*x^2 + a)*a^5*b^3*e - 192*(b*x^2 + a)^(
7/2)*a^3*b^2*f + 576*(b*x^2 + a)^(5/2)*a^4*b^2*f - 576*(b*x^2 + a)^(3/2)*a^
5*b^2*f + 192*sqrt(b*x^2 + a)*a^6*b^2*f)/(a^4*b^4*x^8))/b
```

Mupad [B] (verification not implemented)

Time = 7.85 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.42

$$\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx = \frac{511\,c\,(b\,x^2+a)^{3/2}}{384\,a^2\,x^8} - \frac{93\,c\,\sqrt{b\,x^2+a}}{128\,a\,x^8} - \frac{385\,c\,(b\,x^2+a)^{5/2}}{384\,a^3\,x^8} + \frac{35\,c\,(b\,x^2+a)^{7/2}}{128\,a^4\,x^8} - \frac{11\,d\,\sqrt{b\,x^2+a}}{16\,a\,x^6} + \frac{5\,d\,(b\,x^2+a)^{3/2}}{6\,a^2\,x^6} - \frac{5\,d\,(b\,x^2+a)^{5/2}}{16\,a^3\,x^6} - \frac{5\,e\,\sqrt{b\,x^2+a}}{8\,a\,x^4} + \frac{3\,e\,(b\,x^2+a)^{3/2}}{8\,a^2\,x^4} - \frac{f\,\sqrt{b\,x^2+a}}{2\,a\,x^2} + \frac{b\,f\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}} - \frac{3\,b^2\,e\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{5/2}} + \frac{b^4\,c\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,35\mathrm{i}}{128\,a^{9/2}} - \frac{b^3\,d\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{16\,a^{7/2}}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^9*(a + b*x^2)^(1/2)),x)
```

```
[Out] (511*c*(a + b*x^2)^(3/2))/(384*a^2*x^8) - (93*c*(a + b*x^2)^(1/2))/(128*a*x
^8) - (385*c*(a + b*x^2)^(5/2))/(384*a^3*x^8) + (35*c*(a + b*x^2)^(7/2))/(1
28*a^4*x^8) - (11*d*(a + b*x^2)^(1/2))/(16*a*x^6) + (5*d*(a + b*x^2)^(3/2))
/(6*a^2*x^6) - (5*d*(a + b*x^2)^(5/2))/(16*a^3*x^6) - (5*e*(a + b*x^2)^(1/2
))/(8*a*x^4) + (3*e*(a + b*x^2)^(3/2))/(8*a^2*x^4) - (f*(a + b*x^2)^(1/2))/
(2*a*x^2) + (b*f*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2)) + (b^4*c*ata
n(((a + b*x^2)^(1/2)*1i)/a^(1/2))*35i)/(128*a^(9/2)) - (b^3*d*atan(((a + b*
x^2)^(1/2)*1i)/a^(1/2))*5i)/(16*a^(7/2)) - (3*b^2*e*atanh((a + b*x^2)^(1/2)
/a^(1/2)))/(8*a^(5/2))
```

3.151 $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

Optimal result 913
Rubi [A] (verified) 914
Mathematica [A] (verified) 916
Maple [A] (verified) 917
Fricas [A] (verification not implemented) 918
Sympy [A] (verification not implemented) 918
Maxima [A] (verification not implemented) 919
Giac [A] (verification not implemented) 920
Mupad [F(-1)] 920

Optimal result

Integrand size = 32, antiderivative size = 245

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = -\frac{a(96b^3c-80ab^2d+70a^2be-63a^3f)\,x\sqrt{a+bx^2}}{256b^5} + \frac{(96b^3c-80ab^2d+70a^2be-63a^3f)\,x^3\sqrt{a+bx^2}}{384b^4} + \frac{(80b^2d-70abe+63a^2f)\,x^5\sqrt{a+bx^2}}{480b^3} + \frac{(10be-9af)x^7\sqrt{a+bx^2}}{80b^2} + \frac{fx^9\sqrt{a+bx^2}}{10b} + \frac{a^2(96b^3c-80ab^2d+70a^2be-63a^3f)\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{11/2}}$$

```
[Out] 1/256*a^2*(-63*a^3*f+70*a^2*b*e-80*a*b^2*d+96*b^3*c)*arctanh(x*b^(1/2)/(b*x
^2+a)^(1/2))/b^(11/2)-1/256*a*(-63*a^3*f+70*a^2*b*e-80*a*b^2*d+96*b^3*c)*x*
(b*x^2+a)^(1/2)/b^5+1/384*(-63*a^3*f+70*a^2*b*e-80*a*b^2*d+96*b^3*c)*x^3*(b
*x^2+a)^(1/2)/b^4+1/480*(63*a^2*f-70*a*b*e+80*b^2*d)*x^5*(b*x^2+a)^(1/2)/b^
3+1/80*(-9*a*f+10*b*e)*x^7*(b*x^2+a)^(1/2)/b^2+1/10*f*x^9*(b*x^2+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1823, 1281, 470, 327, 223, 212}

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{x^5\sqrt{a+bx^2}(63a^2f-70abe+80b^2d)}{480b^3} + \frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-63a^3f+70a^2be-80ab^2d+96b^3c)}{256b^{11/2}} - \frac{ax\sqrt{a+bx^2}(-63a^3f+70a^2be-80ab^2d+96b^3c)}{256b^5} + \frac{x^3\sqrt{a+bx^2}(-63a^3f+70a^2be-80ab^2d+96b^3c)}{384b^4} + \frac{x^7\sqrt{a+bx^2}(10be-9af)}{80b^2} + \frac{fx^9\sqrt{a+bx^2}}{10b}$$

```
[In] Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] -1/256*(a*(96*b^3*c - 80*a*b^2*d + 70*a^2*b*e - 63*a^3*f)*x*Sqrt[a + b*x^2]
)/b^5 + ((96*b^3*c - 80*a*b^2*d + 70*a^2*b*e - 63*a^3*f)*x^3*Sqrt[a + b*x^2
])/(384*b^4) + ((80*b^2*d - 70*a*b*e + 63*a^2*f)*x^5*Sqrt[a + b*x^2])/(480*
b^3) + ((10*b*e - 9*a*f)*x^7*Sqrt[a + b*x^2])/(80*b^2) + (f*x^9*Sqrt[a + b*
x^2])/(10*b) + (a^2*(96*b^3*c - 80*a*b^2*d + 70*a^2*b*e - 63*a^3*f)*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{fx^9\sqrt{a+bx^2}}{10b}+\frac{\int \frac{x^4\left(10bc+10bdx^2+(10be-9af)x^4\right)}{\sqrt{a+bx^2}}\,dx}{10b}\\
&= \frac{(10be-9af)x^7\sqrt{a+bx^2}}{80b^2}+\frac{fx^9\sqrt{a+bx^2}}{10b}+\frac{\int \frac{x^4\left(80b^2c+\left(80b^2d-70abe+63a^2f\right)x^2\right)}{\sqrt{a+bx^2}}\,dx}{80b^2}\\
&= \frac{\left(80b^2d-70abe+63a^2f\right)x^5\sqrt{a+bx^2}}{480b^3}+\frac{(10be-9af)x^7\sqrt{a+bx^2}}{80b^2}\\
&\quad+\frac{fx^9\sqrt{a+bx^2}}{10b}+\frac{\left(96b^3c-80ab^2d+70a^2be-63a^3f\right)\int \frac{x^4}{\sqrt{a+bx^2}}\,dx}{96b^3}\\
&= \frac{\left(96b^3c-80ab^2d+70a^2be-63a^3f\right)x^3\sqrt{a+bx^2}}{384b^4}\\
&\quad+\frac{\left(80b^2d-70abe+63a^2f\right)x^5\sqrt{a+bx^2}}{480b^3}+\frac{(10be-9af)x^7\sqrt{a+bx^2}}{80b^2}\\
&\quad+\frac{fx^9\sqrt{a+bx^2}}{10b}-\frac{\left(a\left(96b^3c-80ab^2d+70a^2be-63a^3f\right)\right)\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{128b^4}
\end{aligned}$$

$$= -\frac{a(96b^3c - 80ab^2d + 70a^2be - 63a^3f)\,x\sqrt{a+bx^2}}{256b^5} + \frac{(96b^3c - 80ab^2d + 70a^2be - 63a^3f)\,x^3\sqrt{a+bx^2}}{384b^4} + \frac{(80b^2d - 70abe + 63a^2f)\,x^5\sqrt{a+bx^2}}{480b^3} + \frac{(10be - 9af)x^7\sqrt{a+bx^2}}{80b^2} + \frac{fx^9\sqrt{a+bx^2}}{10b} + \frac{(a^2(96b^3c - 80ab^2d + 70a^2be - 63a^3f))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{256b^5}$$

$$= -\frac{a(96b^3c - 80ab^2d + 70a^2be - 63a^3f)\,x\sqrt{a+bx^2}}{256b^5} + \frac{(96b^3c - 80ab^2d + 70a^2be - 63a^3f)\,x^3\sqrt{a+bx^2}}{384b^4} + \frac{(80b^2d - 70abe + 63a^2f)\,x^5\sqrt{a+bx^2}}{480b^3} + \frac{(10be - 9af)x^7\sqrt{a+bx^2}}{80b^2} + \frac{fx^9\sqrt{a+bx^2}}{10b} + \frac{(a^2(96b^3c - 80ab^2d + 70a^2be - 63a^3f))\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{256b^5}$$

$$= -\frac{a(96b^3c - 80ab^2d + 70a^2be - 63a^3f)\,x\sqrt{a+bx^2}}{256b^5} + \frac{(96b^3c - 80ab^2d + 70a^2be - 63a^3f)\,x^3\sqrt{a+bx^2}}{384b^4} + \frac{(80b^2d - 70abe + 63a^2f)\,x^5\sqrt{a+bx^2}}{480b^3} + \frac{(10be - 9af)x^7\sqrt{a+bx^2}}{80b^2} + \frac{fx^9\sqrt{a+bx^2}}{10b} + \frac{a^2(96b^3c - 80ab^2d + 70a^2be - 63a^3f)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{11/2}}$$

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.89

$$\int \frac{x^4(c + dx^2 + ex^4 + fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{x\sqrt{a+bx^2}(-1440ab^3c + 1200a^2b^2d - 1050a^3be + 945a^4f + 960b^4cx^2 - 800ab^3dx^2 + 700a^2b^2ex^2 - 630a^3b\ldots}{3840b^5} - \frac{a^2(-96b^3c + 80ab^2d - 70a^2be + 63a^3f)\,\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{128b^{11/2}}$$

```
[In] Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(-1440*a*b^3*c + 1200*a^2*b^2*d - 1050*a^3*b*e + 945*a^4
*f + 960*b^4*c*x^2 - 800*a*b^3*d*x^2 + 700*a^2*b^2*e*x^2 - 630*a^3*b*f*x^2
+ 640*b^4*d*x^4 - 560*a*b^3*e*x^4 + 504*a^2*b^2*f*x^4 + 480*b^4*e*x^6 - 432
```

```
*a*b^3*f*x^6 + 384*b^4*f*x^8))/(3840*b^5) - (a^2*(-96*b^3*c + 80*a*b^2*d -
70*a^2*b*e + 63*a^3*f)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(
128*b^(11/2))
```

Maple [A] (verified)

Time = 3.70 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$-\frac{63\left(a^2\left(f\,a^3-\frac{10}{9}a^2be+\frac{80}{63}a\,b^2d-\frac{32}{21}b^3c\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)-\left(-\frac{32\left(\frac{3}{10}f\,x^6+\frac{7}{18}e\,x^4+\frac{5}{9}d\,x^2+c\right)a\,b^{\frac{7}{2}}}{21}+\frac{64x^2\left(\frac{2}{5}f\,x^6+\frac{1}{2}e\,x^4+\right.}{63}\right.}{256b^{\frac{11}{2}}}$
risch	$\frac{x\left(384f\,x^8b^4-432a\,b^3f\,x^6+480b^4e\,x^6+504a^2b^2f\,x^4-560a\,b^3e\,x^4+640b^4d\,x^4-630a^3bf\,x^2+700a^2b^2e\,x^2-800a\,b^3d\,x^2+960b^4\right.}{3840b^5}$
default	$e\left(\frac{x^7\sqrt{b\,x^2+a}}{8b}-\frac{7a\left(\frac{x^5\sqrt{b\,x^2+a}}{6b}-\frac{5a\left(\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)}{6b}\right)}{8b}\right)+d\left(\frac{x^5\sqrt{b\,x^2+a}}{6b}-\right.$

```
[In] int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -63/256/b^(11/2)*(a^2*(f*a^3-10/9*a^2*b*e+80/63*a*b^2*d-32/21*b^3*c)*arctan
h((b*x^2+a)^(1/2)/x/b^(1/2))-(-32/21*(3/10*f*x^6+7/18*e*x^4+5/9*d*x^2+c)*a*
b^(7/2)+64/63*x^2*(2/5*f*x^6+1/2*e*x^4+2/3*d*x^2+c)*b^(9/2)+((8/15*f*x^4+20
```

```
/27*e*x^2+80/63*d)*b^(5/2)+((-2/3*f*x^2-10/9*e)*b^(3/2)+a*f*b^(1/2))*a)*a^2
)*(b*x^2+a)^(1/2)*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.69

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$$
$$= \left[-\frac{15\,(96\,a^2b^3c-80\,a^3b^2d+70\,a^4be-63\,a^5f)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(384\,b^5fx^9+48\,(\right.$$
$$-\frac{15\,(96\,a^2b^3c-80\,a^3b^2d+70\,a^4be-63\,a^5f)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(384\,b^5fx^9+48\,(10\,b^5e-9\,ab^4f)x^7}{}$$

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] [-1/7680*(15*(96*a^2*b^3*c - 80*a^3*b^2*d + 70*a^4*b*e - 63*a^5*f)*sqrt(b)*
log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(384*b^5*f*x^9 + 48*(10
*b^5*e - 9*a*b^4*f)*x^7 + 8*(80*b^5*d - 70*a*b^4*e + 63*a^2*b^3*f)*x^5 + 10
*(96*b^5*c - 80*a*b^4*d + 70*a^2*b^3*e - 63*a^3*b^2*f)*x^3 - 15*(96*a*b^4*c
 - 80*a^2*b^3*d + 70*a^3*b^2*e - 63*a^4*b*f)*x)*sqrt(b*x^2 + a))/b^6, -1/38
40*(15*(96*a^2*b^3*c - 80*a^3*b^2*d + 70*a^4*b*e - 63*a^5*f)*sqrt(-b)*arcta
n(sqrt(-b)*x/sqrt(b*x^2 + a)) - (384*b^5*f*x^9 + 48*(10*b^5*e - 9*a*b^4*f)*
x^7 + 8*(80*b^5*d - 70*a*b^4*e + 63*a^2*b^3*f)*x^5 + 10*(96*b^5*c - 80*a*b^
4*d + 70*a^2*b^3*e - 63*a^3*b^2*f)*x^3 - 15*(96*a*b^4*c - 80*a^2*b^3*d + 70
*a^3*b^2*e - 63*a^4*b*f)*x)*sqrt(b*x^2 + a))/b^6]
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.99

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} \frac{3a^2\left(-\frac{5a\left(-\frac{7a\left(-\frac{9af}{10b}+e\right)}{8b}+d\right)}{6b}+c\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b^2}+\sqrt{a+bx^2}\left(-\frac{3ax\left(-\frac{5a\left(-\frac{7a\left(-\frac{9af}{10b}+e\right)}{8b}+d\right)}{6b}+c\right)}{8b^2}\right. \\ \frac{\frac{cx^5}{5}+\frac{dx^7}{7}+\frac{ex^9}{9}+\frac{fx^{11}}{11}}{\sqrt{a}} \end{cases}$$

```
[In] integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((3*a**2*(-5*a*(-7*a*(-9*a*f/(10*b) + e)/(8*b) + d)/(6*b) + c)*Pie
cewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(
x)/sqrt(b*x**2), True))/(8*b**2) + sqrt(a + b*x**2)*(-3*a*x*(-5*a*(-7*a*(-9
*a*f/(10*b) + e)/(8*b) + d)/(6*b) + c)/(8*b**2) + f*x**9/(10*b) + x**7*(-9*
a*f/(10*b) + e)/(8*b) + x**5*(-7*a*(-9*a*f/(10*b) + e)/(8*b) + d)/(6*b) + x
**3*(-5*a*(-7*a*(-9*a*f/(10*b) + e)/(8*b) + d)/(6*b) + c)/(4*b)), Ne(b, 0))
, ((c*x**5/5 + d*x**7/7 + e*x**9/9 + f*x**11/11)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.38

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}fx^9}{10\,b} + \frac{\sqrt{bx^2+a}ex^7}{8\,b} - \frac{9\,\sqrt{bx^2+a}afx^7}{80\,b^2} + \frac{\sqrt{bx^2+a}dx^5}{6\,b} - \frac{7\,\sqrt{bx^2+a}aex^5}{48\,b^2} + \frac{21\,\sqrt{bx^2+a}a^2fx^5}{160\,b^3} + \frac{\sqrt{bx^2+a}cx^3}{4\,b} - \frac{5\,\sqrt{bx^2+a}adx^3}{24\,b^2} + \frac{35\,\sqrt{bx^2+a}a^2ex^3}{192\,b^3} - \frac{21\,\sqrt{bx^2+a}a^3fx^3}{128\,b^4} - \frac{3\,\sqrt{bx^2+a}acx}{8\,b^2} + \frac{5\,\sqrt{bx^2+a}a^2dx}{16\,b^3} - \frac{35\,\sqrt{bx^2+a}a^3ex}{128\,b^4} + \frac{63\,\sqrt{bx^2+a}a^4fx}{256\,b^5} + \frac{3\,a^2c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} - \frac{5\,a^3d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}} + \frac{35\,a^4e\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{9}{2}}} - \frac{63\,a^5f\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{11}{2}}}$$

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(b*x^2 + a)*f*x^9/b + 1/8*sqrt(b*x^2 + a)*e*x^7/b - 9/80*sqrt(b*x^
2 + a)*a*f*x^7/b^2 + 1/6*sqrt(b*x^2 + a)*d*x^5/b - 7/48*sqrt(b*x^2 + a)*a*e
*x^5/b^2 + 21/160*sqrt(b*x^2 + a)*a^2*f*x^5/b^3 + 1/4*sqrt(b*x^2 + a)*c*x^3
/b - 5/24*sqrt(b*x^2 + a)*a*d*x^3/b^2 + 35/192*sqrt(b*x^2 + a)*a^2*e*x^3/b^
3 - 21/128*sqrt(b*x^2 + a)*a^3*f*x^3/b^4 - 3/8*sqrt(b*x^2 + a)*a*c*x/b^2 +
5/16*sqrt(b*x^2 + a)*a^2*d*x/b^3 - 35/128*sqrt(b*x^2 + a)*a^3*e*x/b^4 + 63/
256*sqrt(b*x^2 + a)*a^4*f*x/b^5 + 3/8*a^2*c*arcsinh(b*x/sqrt(a*b))/b^(5/2)
- 5/16*a^3*d*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 35/128*a^4*e*arcsinh(b*x/sqrt
(a*b))/b^(9/2) - 63/256*a^5*f*arcsinh(b*x/sqrt(a*b))/b^(11/2)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.89

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$$
$$=\frac{1}{3840}\left(2\left(4\left(6\left(\frac{8\,fx^2}{b}+\frac{10\,b^8e-9\,ab^7f}{b^9}\right)x^2+\frac{80\,b^8d-70\,ab^7e+63\,a^2b^6f}{b^9}\right)x^2+\frac{5\,(96\,b^8c-80\,ab^7d+70}{b^9}\right.\right.$$
$$-\frac{(96\,a^2b^3c-80\,a^3b^2d+70\,a^4be-63\,a^5f)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{11}{2}}}$$

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*f*x^2/b + (10*b^8*e - 9*a*b^7*f)/b^9)*x^2 + (80*b^8*d -
70*a*b^7*e + 63*a^2*b^6*f)/b^9)*x^2 + 5*(96*b^8*c - 80*a*b^7*d + 70*a^2*b^6
*e - 63*a^3*b^5*f)/b^9)*x^2 - 15*(96*a*b^7*c - 80*a^2*b^6*d + 70*a^3*b^5*e
- 63*a^4*b^4*f)/b^9)*sqrt(b*x^2 + a)*x - 1/256*(96*a^2*b^3*c - 80*a^3*b^2*d
 + 70*a^4*b*e - 63*a^5*f)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx=\int \frac{x^4\,(f\,x^6+e\,x^4+d\,x^2+c)}{\sqrt{b\,x^2+a}}\,dx$$

```
[In] int((x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2),x)
```

```
[Out] int((x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2), x)
```

3.152 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}} \, dx$

Optimal result 921
Rubi [A] (verified) 921
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Optimal result

Integrand size = 32, antiderivative size = 194

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}} \, dx = \frac{(64b^3c-48ab^2d+40a^2be-35a^3f)\,x\sqrt{a+bx^2}}{128b^4} + \frac{(48b^2d-40abe+35a^2f)\,x^3\sqrt{a+bx^2}}{192b^3} + \frac{(8be-7af)x^5\sqrt{a+bx^2}}{48b^2} + \frac{fx^7\sqrt{a+bx^2}}{8b} - \frac{a(64b^3c-48ab^2d+40a^2be-35a^3f)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{9/2}}$$

```
[Out] -1/128*a*(-35*a^3*f+40*a^2*b*e-48*a*b^2*d+64*b^3*c)*arctanh(x*b^(1/2)/(b*x^
2+a)^(1/2))/b^(9/2)+1/128*(-35*a^3*f+40*a^2*b*e-48*a*b^2*d+64*b^3*c)*x*(b*x
^2+a)^(1/2)/b^4+1/192*(35*a^2*f-40*a*b*e+48*b^2*d)*x^3*(b*x^2+a)^(1/2)/b^3+
1/48*(-7*a*f+8*b*e)*x^5*(b*x^2+a)^(1/2)/b^2+1/8*f*x^7*(b*x^2+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used

$= \{1823, 1281, 470, 327, 223, 212\}$

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{x^3\sqrt{a+bx^2}(35a^2f-40abe+48b^2d)}{192b^3} - \frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-35a^3f+40a^2be-48ab^2d+64b^3c)}{128b^{9/2}} + \frac{x\sqrt{a+bx^2}(-35a^3f+40a^2be-48ab^2d+64b^3c)}{128b^4} + \frac{x^5\sqrt{a+bx^2}(8be-7af)}{48b^2} + \frac{fx^7\sqrt{a+bx^2}}{8b}$$

```
[In] Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] ((64*b^3*c - 48*a*b^2*d + 40*a^2*b*e - 35*a^3*f)*x*Sqrt[a + b*x^2])/(128*b^
4) + ((48*b^2*d - 40*a*b*e + 35*a^2*f)*x^3*Sqrt[a + b*x^2])/(192*b^3) + ((8
*b*e - 7*a*f)*x^5*Sqrt[a + b*x^2])/(48*b^2) + (f*x^7*Sqrt[a + b*x^2])/(8*b)
 - (a*(64*b^3*c - 48*a*b^2*d + 40*a^2*b*e - 35*a^3*f)*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/(128*b^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{fx^7\sqrt{a+bx^2}}{8b}+\frac{\int \frac{x^2(8bc+8bdx^2+(8be-7af)x^4)}{\sqrt{a+bx^2}}\,dx}{8b}\\
&= \frac{(8be-7af)x^5\sqrt{a+bx^2}}{48b^2}+\frac{fx^7\sqrt{a+bx^2}}{8b}+\frac{\int \frac{x^2(48b^2c+(48b^2d-40abe+35a^2f)x^2)}{\sqrt{a+bx^2}}\,dx}{48b^2}\\
&= \frac{(48b^2d-40abe+35a^2f)\,x^3\sqrt{a+bx^2}}{192b^3}+\frac{(8be-7af)x^5\sqrt{a+bx^2}}{48b^2}\\
&\quad+\frac{fx^7\sqrt{a+bx^2}}{8b}-\frac{1}{64}\left(-64c+\frac{a(48b^2d-40abe+35a^2f)}{b^3}\right)\int \frac{x^2}{\sqrt{a+bx^2}}\,dx\\
&= \frac{\left(64c-\frac{a(48b^2d-40abe+35a^2f)}{b^3}\right)x\sqrt{a+bx^2}}{128b}+\frac{(48b^2d-40abe+35a^2f)\,x^3\sqrt{a+bx^2}}{192b^3}\\
&\quad+\frac{(8be-7af)x^5\sqrt{a+bx^2}}{48b^2}+\frac{fx^7\sqrt{a+bx^2}}{8b}\\
&\quad-\frac{(a(64b^3c-48ab^2d+40a^2be-35a^3f))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{128b^4}\\
&= \frac{\left(64c-\frac{a(48b^2d-40abe+35a^2f)}{b^3}\right)x\sqrt{a+bx^2}}{128b}+\frac{(48b^2d-40abe+35a^2f)\,x^3\sqrt{a+bx^2}}{192b^3}\\
&\quad+\frac{(8be-7af)x^5\sqrt{a+bx^2}}{48b^2}+\frac{fx^7\sqrt{a+bx^2}}{8b}\\
&\quad-\frac{(a(64b^3c-48ab^2d+40a^2be-35a^3f))\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{128b^4}
\end{aligned}
$$

$$= \frac{\left(64c - \frac{a(48b^2d-40abe+35a^2f)}{b^3}\right)x\sqrt{a+bx^2}}{128b} + \frac{(48b^2d - 40abe + 35a^2f)\,x^3\sqrt{a+bx^2}}{192b^3} + \frac{(8be-7af)x^5\sqrt{a+bx^2}}{48b^2} + \frac{fx^7\sqrt{a+bx^2}}{8b} - \frac{a(64b^3c - 48ab^2d + 40a^2be - 35a^3f)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{9/2}}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.82

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{b}x\sqrt{a+bx^2}(-105a^3f+10a^2b(12e+7fx^2)-8ab^2(18d+10ex^2+7fx^4)+16b^3(12c+6dx^2+4ex^4+3f}{384b^{9/2}}$$

```
[In] Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(-105*a^3*f + 10*a^2*b*(12*e + 7*f*x^2) - 8*a*b^
2*(18*d + 10*e*x^2 + 7*f*x^4) + 16*b^3*(12*c + 6*d*x^2 + 4*e*x^4 + 3*f*x^6)
) + 6*a*(-64*b^3*c + 48*a*b^2*d - 40*a^2*b*e + 35*a^3*f)*ArcTanh[(Sqrt[b]*x
)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(384*b^(9/2))
```

Maple [A] (verified)

Time = 3.65 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$\frac{\frac{35a\left(fa^3-\frac{8}{7}a^2be+\frac{48}{35}ab^2d-\frac{64}{35}b^3c\right)\operatorname{arctanh}\left(\frac{\sqrt{bx^2+a}}{x\sqrt{b}}\right)}{128}-\frac{35\left(\left(-\frac{16}{35}fx^6-\frac{64}{105}ex^4-\frac{32}{35}dx^2-\frac{64}{35}c\right)b^{\frac{7}{2}}+\left(\left(\frac{8}{15}fx^4+\frac{16}{21}ex^2+\frac{48}{35}d\right)b^{\frac{5}{2}}+\cdots\right.\right.}{128}}{b^{\frac{9}{2}}}$
risch	$-\frac{x\left(-48fx^6b^3+56ab^2fx^4-64b^3ex^4-70a^2bfx^2+80ab^2ex^2-96b^3dx^2+105fa^3-120a^2be+144ab^2d-192b^3c\right)\sqrt{bx^2+a}}{384b^4}+\cdots$
default	$f\left(\frac{x^7\sqrt{bx^2+a}}{8b}-\frac{7a\left(\frac{x^5\sqrt{bx^2+a}}{6b}-\frac{5a\left(\frac{x^3\sqrt{bx^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{bx^2+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)}{6b}\right)}{8b}\right)+e\left(\frac{x^5\sqrt{bx^2+a}}{6b}-\cdots\right.$

[In] int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 35/128*(a*(f*a^3-8/7*a^2*b*e+48/35*a*b^2*d-64/35*b^3*c)*arctanh((b*x^2+a)^(
1/2)/x/b^(1/2))-((-16/35*f*x^6-64/105*e*x^4-32/35*d*x^2-64/35*c)*b^(7/2)+((
8/15*f*x^4+16/21*e*x^2+48/35*d)*b^(5/2)+a*((-2/3*f*x^2-8/7*e)*b^(3/2)+a*f*b
^(1/2)))*a)*x*(b*x^2+a)^(1/2))/b^(9/2)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.70

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$$
$$=\left[-\frac{3\,(64\,ab^3c-48\,a^2b^2d+40\,a^3be-35\,a^4f)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(48\,b^4fx^7+8\,(8\,b\cdots}{7\cdots}\right.$$

[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] [-1/768*(3*(64*a*b^3*c - 48*a^2*b^2*d + 40*a^3*b*e - 35*a^4*f)*sqrt(b)*log(
-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(48*b^4*f*x^7 + 8*(8*b^4*e
- 7*a*b^3*f)*x^5 + 2*(48*b^4*d - 40*a*b^3*e + 35*a^2*b^2*f)*x^3 + 3*(64*b^4
*c - 48*a*b^3*d + 40*a^2*b^2*e - 35*a^3*b*f)*x)*sqrt(b*x^2 + a))/b^5, 1/384
(3(64*a*b^3*c - 48*a^2*b^2*d + 40*a^3*b*e - 35*a^4*f)*sqrt(-b)*arctan(sqr

```
t(-b)*x/sqrt(b*x^2 + a)) + (48*b^4*f*x^7 + 8*(8*b^4*e - 7*a*b^3*f)*x^5 + 2*
(48*b^4*d - 40*a*b^3*e + 35*a^2*b^2*f)*x^3 + 3*(64*b^4*c - 48*a*b^3*d + 40*
a^2*b^2*e - 35*a^3*b*f)*x)*sqrt(b*x^2 + a))/b^5]
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.03

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \begin{cases} -\frac{a\left(-\frac{3a\left(-\frac{5a\left(-\frac{7af}{8b}+e\right)}{6b}+d\right)}{4b}+c\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b} + \sqrt{a+bx^2}\left(\frac{fx^7}{8b}+\frac{x^5\left(-\frac{7af}{8b}+e\right)}{6b}+\frac{x^3\left(-\frac{5a\left(-\right.}{}\right.}{4}\right. \\ \frac{\frac{cx^3}{3}+\frac{dx^5}{5}+\frac{ex^7}{7}+\frac{fx^9}{9}}{\sqrt{a}} \end{cases}$$

```
[In] integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-a*(-3*a*(-5*a*(-7*a*f/(8*b) + e)/(6*b) + d)/(4*b) + c)*Piecewis
e((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sq
rt(b*x**2), True))/(2*b) + sqrt(a + b*x**2)*(f*x**7/(8*b) + x**5*(-7*a*f/(8
*b) + e)/(6*b) + x**3*(-5*a*(-7*a*f/(8*b) + e)/(6*b) + d)/(4*b) + x*(-3*a*(
-5*a*(-7*a*f/(8*b) + e)/(6*b) + d)/(4*b) + c)/(2*b)), Ne(b, 0)), ((c*x**3/3
 + d*x**5/5 + e*x**7/7 + f*x**9/9)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.31

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}fx^7}{8\,b}+\frac{\sqrt{bx^2+a}ex^5}{6\,b}-\frac{7\,\sqrt{bx^2+a}afx^5}{48\,b^2}+\frac{\sqrt{bx^2+a}dx^3}{4\,b}-\frac{5\,\sqrt{bx^2+a}aex^3}{24\,b^2}+\frac{35\,\sqrt{bx^2+a}a^2fx^3}{192\,b^3}+\frac{\sqrt{bx^2+a}cx}{2\,b}-\frac{3\,\sqrt{bx^2+a}adx}{8\,b^2}+\frac{5\,\sqrt{bx^2+a}a^2ex}{16\,b^3}-\frac{35\,\sqrt{bx^2+a}a^3fx}{128\,b^4}-\frac{ac\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{3\,a^2d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}-\frac{5\,a^3e\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}}+\frac{35\,a^4f\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{9}{2}}}$$

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*f*x^7/b + 1/6*sqrt(b*x^2 + a)*e*x^5/b - 7/48*sqrt(b*x^2
 + a)*a*f*x^5/b^2 + 1/4*sqrt(b*x^2 + a)*d*x^3/b - 5/24*sqrt(b*x^2 + a)*a*e*
x^3/b^2 + 35/192*sqrt(b*x^2 + a)*a^2*f*x^3/b^3 + 1/2*sqrt(b*x^2 + a)*c*x/b
- 3/8*sqrt(b*x^2 + a)*a*d*x/b^2 + 5/16*sqrt(b*x^2 + a)*a^2*e*x/b^3 - 35/128
*sqrt(b*x^2 + a)*a^3*f*x/b^4 - 1/2*a*c*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3/8
*a^2*d*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 5/16*a^3*e*arcsinh(b*x/sqrt(a*b))/b
^(7/2) + 35/128*a^4*f*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.88

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{1}{384}\left(2\left(4\left(\frac{6\,fx^2}{b}+\frac{8\,b^6e-7\,ab^5f}{b^7}\right)x^2+\frac{48\,b^6d-40\,ab^5e+35\,a^2b^4f}{b^7}\right)x^2+\frac{3\,(64\,b^6c-48\,ab^5d+40\,a^2b^{\cdots}}{b^7}\right. + \frac{(64\,ab^3c-48\,a^2b^2d+40\,a^3be-35\,a^4f)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{9}{2}}}$$

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*f*x^2/b + (8*b^6*e - 7*a*b^5*f)/b^7)*x^2 + (48*b^6*d - 40*a*
b^5*e + 35*a^2*b^4*f)/b^7)*x^2 + 3*(64*b^6*c - 48*a*b^5*d + 40*a^2*b^4*e -
35*a^3*b^3*f)/b^7)*sqrt(b*x^2 + a)*x + 1/128*(64*a*b^3*c - 48*a^2*b^2*d + 4
0*a^3*b*e - 35*a^4*f)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \int \frac{x^2\,(f\,x^6+e\,x^4+d\,x^2+c)}{\sqrt{b\,x^2+a}}\,dx$$

```
[In] int((x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2),x)
```

```
[Out] int((x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2), x)
```

3.153 $\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}} \, dx$

Optimal result . . . 929
Rubi [A] (verified) . . . 929
Mathematica [A] (verified) . . . 931
Maple [A] (verified) . . . 931
Fricas [A] (verification not implemented) . . . 932
Sympy [A] (verification not implemented) . . . 933
Maxima [A] (verification not implemented) . . . 933
Giac [A] (verification not implemented) . . . 934
Mupad [F(-1)] . . . 934

Optimal result

Integrand size = 29, antiderivative size = 145

$$\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}} \, dx = \frac{(8b^2d-6abe+5a^2f)\,x\sqrt{a+bx^2}}{16b^3} + \frac{(6be-5af)x^3\sqrt{a+bx^2}}{24b^2} + \frac{fx^5\sqrt{a+bx^2}}{6b} + \frac{(16b^3c-8ab^2d+6a^2be-5a^3f)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}$$

```
[Out] 1/16*(-5*a^3*f+6*a^2*b*e-8*a*b^2*d+16*b^3*c)*arctanh(x*b^(1/2)/(b*x^2+a)^(1
/2))/b^(7/2)+1/16*(5*a^2*f-6*a*b*e+8*b^2*d)*x*(b*x^2+a)^(1/2)/b^3+1/24*(-5*
a*f+6*b*e)*x^3*(b*x^2+a)^(1/2)/b^2+1/6*f*x^5*(b*x^2+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1829, 1173, 396, 223, 212}

$$\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}} \, dx = \frac{x\sqrt{a+bx^2}(5a^2f-6abe+8b^2d)}{16b^3} + \frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-5a^3f+6a^2be-8ab^2d+16b^3c)}{16b^{7/2}} + \frac{x^3\sqrt{a+bx^2}(6be-5af)}{24b^2} + \frac{fx^5\sqrt{a+bx^2}}{6b}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/Sqrt[a + b*x^2],x]
```

```
[Out] ((8*b^2*d - 6*a*b*e + 5*a^2*f)*x*Sqrt[a + b*x^2])/(16*b^3) + ((6*b*e - 5*a*
f)*x^3*Sqrt[a + b*x^2])/(24*b^2) + (f*x^5*Sqrt[a + b*x^2])/(6*b) + ((16*b^3
*c - 8*a*b^2*d + 6*a^2*b*e - 5*a^3*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])
/(16*b^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1173

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{fx^5\sqrt{a+bx^2}}{6b}+\frac{\int \frac{6bc+6bdx^2+(6be-5af)x^4}{\sqrt{a+bx^2}}\,dx}{6b} \\
&= \frac{(6be-5af)x^3\sqrt{a+bx^2}}{24b^2}+\frac{fx^5\sqrt{a+bx^2}}{6b}+\frac{\int \frac{24b^2c+3\left(8b^2d-6abe+5a^2f\right)x^2}{\sqrt{a+bx^2}}\,dx}{24b^2}
\end{aligned}$$

$$= \frac{(8b^2d - 6abe + 5a^2f)\,x\sqrt{a+bx^2}}{16b^3} + \frac{(6be - 5af)x^3\sqrt{a+bx^2}}{24b^2} + \frac{fx^5\sqrt{a+bx^2}}{6b} - \frac{1}{16}\left(-16c + \frac{a(8b^2d - 6abe + 5a^2f)}{b^3}\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx$$

$$= \frac{(8b^2d - 6abe + 5a^2f)\,x\sqrt{a+bx^2}}{16b^3} + \frac{(6be - 5af)x^3\sqrt{a+bx^2}}{24b^2} + \frac{fx^5\sqrt{a+bx^2}}{6b} - \frac{1}{16}\left(-16c + \frac{a(8b^2d - 6abe + 5a^2f)}{b^3}\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)$$

$$= \frac{(8b^2d - 6abe + 5a^2f)\,x\sqrt{a+bx^2}}{16b^3} + \frac{(6be - 5af)x^3\sqrt{a+bx^2}}{24b^2} + \frac{fx^5\sqrt{a+bx^2}}{6b} + \frac{\left(16c - \frac{a(8b^2d-6abe+5a^2f)}{b^3}\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.89

$$\int \frac{c + dx^2 + ex^4 + fx^6}{\sqrt{a+bx^2}}\,dx = \frac{x\sqrt{a+bx^2}(24b^2d - 18abe + 15a^2f + 12b^2ex^2 - 10abfx^2 + 8b^2fx^4)}{48b^3} + \frac{(16b^3c - 8ab^2d + 6a^2be - 5a^3f)\,\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{8b^{7/2}}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/Sqrt[a + b*x^2],x]
[Out] (x*Sqrt[a + b*x^2]*(24*b^2*d - 18*a*b*e + 15*a^2*f + 12*b^2*e*x^2 - 10*a*b*
f*x^2 + 8*b^2*f*x^4))/(48*b^3) + ((16*b^3*c - 8*a*b^2*d + 6*a^2*b*e - 5*a^3
*f)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(8*b^(7/2))
```

Maple [A] (verified)

Time = 3.68 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$-\frac{5\left(\left(f\,a^{3}-\frac{6}{5}a^{2}be+\frac{8}{5}a\,b^{2}d-\frac{16}{5}b^{3}c\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)-\left(\frac{4\left(\frac{2}{3}f\,x^{4}+e\,x^{2}+2d\right)b^{\frac{5}{2}}}{5}+\left(2\left(-\frac{f\,x^{2}}{3}-\frac{3e}{5}\right)b^{\frac{3}{2}}+af\sqrt{b}\right)a\right)x\sqrt{b\,x^{2}+}\right.}{16b^{\frac{7}{2}}}$
risch	$\frac{x\left(8b^{2}f\,x^{4}-10abf\,x^{2}+12b^{2}e\,x^{2}+15a^{2}f-18aeb+24b^{2}d\right)\sqrt{b\,x^{2}+a}}{48b^{3}}-\frac{\left(5f\,a^{3}-6a^{2}be+8a\,b^{2}d-16b^{3}c\right)\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{16b^{\frac{7}{2}}}$
default	$\frac{c\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+f\left(\frac{x^{5}\sqrt{b\,x^{2}+a}}{6b}-\frac{5a\left(\frac{x^{3}\sqrt{b\,x^{2}+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)}{6b}\right)+e\left(\frac{x^{3}\sqrt{b\,x^{2}+}}{4b}\right.$

[In] int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] -5/16*((f*a^3-6/5*a^2*b*e+8/5*a*b^2*d-16/5*b^3*c)*arctanh((b*x^2+a)^(1/2)/x
/b^(1/2))-(4/5*(2/3*f*x^4+e*x^2+2*d)*b^(5/2)+(2*(-1/3*f*x^2-3/5*e)*b^(3/2)+
a*f*b^(1/2))*a)*x*(b*x^2+a)^(1/2))/b^(7/2)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.72

$$\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}}\,dx$$
$$=\left[-\frac{3\,(16\,b^3c-8\,ab^2d+6\,a^2be-5\,a^3f)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(8\,b^3fx^5+2\,(6\,b^3e-5\,ab\right.}{96\,b^4}$$
$$-\frac{3\,(16\,b^3c-8\,ab^2d+6\,a^2be-5\,a^3f)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(8\,b^3fx^5+2\,(6\,b^3e-5\,ab^2f)x^3+3\,(8\,b^3d-(}{48\,b^4}$$

[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] [-1/96*(3*(16*b^3*c - 8*a*b^2*d + 6*a^2*b*e - 5*a^3*f)*sqrt(b)*log(-2*b*x^2
 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(8*b^3*f*x^5 + 2*(6*b^3*e - 5*a*b^2
*f)*x^3 + 3*(8*b^3*d - 6*a*b^2*e + 5*a^2*b*f)*x)*sqrt(b*x^2 + a))/b^4, -1/4
8*(3*(16*b^3*c - 8*a*b^2*d + 6*a^2*b*e - 5*a^3*f)*sqrt(-b)*arctan(sqrt(-b)*
x/sqrt(b*x^2 + a)) - (8*b^3*f*x^5 + 2*(6*b^3*e - 5*a*b^2*f)*x^3 + 3*(8*b^3*
d - 6*a*b^2*e + 5*a^2*b*f)*x)*sqrt(b*x^2 + a))/b^4]

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.07

$$\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}}\,dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{fx^5}{6b}+\frac{x^3\left(-\frac{5af}{6b}+e\right)}{4b}+\frac{x\left(-\frac{3a\left(-\frac{5af}{6b}+e\right)}{4b}+d\right)}{2b}\right)+\left(-\frac{a\left(-\frac{3a\left(-\frac{5af}{6b}+e\right)}{4b}+d\right)}{2b}+c\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} \\ \frac{x\log(x)}{\sqrt{bx^2}}\end{cases}\right. \\ \frac{cx+\frac{dx^3}{3}+\frac{ex^5}{5}+\frac{fx^7}{7}}{\sqrt{a}} \end{cases}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + b*x**2)*(f*x**5/(6*b) + x**3*(-5*a*f/(6*b) + e)/(4*b) +
 x*(-3*a*(-5*a*f/(6*b) + e)/(4*b) + d)/(2*b)) + (-a*(-3*a*(-5*a*f/(6*b) + e
)/(4*b) + d)/(2*b) + c)*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/
sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), ((c*x + d*x*
*3/3 + e*x**5/5 + f*x**7/7)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.20

$$\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}fx^5}{6\,b}+\frac{\sqrt{bx^2+a}ex^3}{4\,b}-\frac{5\,\sqrt{bx^2+a}afx^3}{24\,b^2}+\frac{\sqrt{bx^2+a}dx}{2\,b} - \frac{3\,\sqrt{bx^2+a}aex}{8\,b^2}+\frac{5\,\sqrt{bx^2+a}a^2fx}{16\,b^3}+\frac{c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{ad\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{3\,a^2e\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}-\frac{5\,a^3f\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(b*x^2 + a)*f*x^5/b + 1/4*sqrt(b*x^2 + a)*e*x^3/b - 5/24*sqrt(b*x^2
 + a)*a*f*x^3/b^2 + 1/2*sqrt(b*x^2 + a)*d*x/b - 3/8*sqrt(b*x^2 + a)*a*e*x/b
^2 + 5/16*sqrt(b*x^2 + a)*a^2*f*x/b^3 + c*arcsinh(b*x/sqrt(a*b))/sqrt(b) -
1/2*a*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3/8*a^2*e*arcsinh(b*x/sqrt(a*b))/b
^(5/2) - 5/16*a^3*f*arcsinh(b*x/sqrt(a*b))/b^(7/2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.86

$$\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}}\,dx$$
$$=\frac{1}{48}\left(2\left(\frac{4\,fx^2}{b}+\frac{6\,b^4e-5\,ab^3f}{b^5}\right)x^2+\frac{3\,(8\,b^4d-6\,ab^3e+5\,a^2b^2f)}{b^5}\right)\sqrt{bx^2+ax}$$
$$-\frac{(16\,b^3c-8\,ab^2d+6\,a^2be-5\,a^3f)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{7}{2}}}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*f*x^2/b + (6*b^4*e - 5*a*b^3*f)/b^5)*x^2 + 3*(8*b^4*d - 6*a*b^3*
e + 5*a^2*b^2*f)/b^5)*sqrt(b*x^2 + a)*x - 1/16*(16*b^3*c - 8*a*b^2*d + 6*a^
2*b*e - 5*a^3*f)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}}\,dx=\int \frac{f\,x^6+e\,x^4+d\,x^2+c}{\sqrt{b\,x^2+a}}\,dx$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^(1/2),x)
```

```
[Out] int((c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^(1/2), x)
```

3.154 $\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx$

Optimal result 935
Rubi [A] (verified) 935
Mathematica [A] (verified) 937
Maple [A] (verified) 938
Fricas [A] (verification not implemented) 938
Sympy [A] (verification not implemented) 939
Maxima [A] (verification not implemented) 940
Giac [A] (verification not implemented) 940
Mupad [F(-1)] 941

Optimal result

Integrand size = 32, antiderivative size = 117

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx = -\frac{c\sqrt{a+bx^2}}{ax}+\frac{(4be-3af)x\sqrt{a+bx^2}}{8b^2}+\frac{fx^3\sqrt{a+bx^2}}{4b}+\frac{(8b^2d-4abe+3a^2f)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

```
[Out] 1/8*(3*a^2*f-4*a*b*e+8*b^2*d)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)-c*
(b*x^2+a)^(1/2)/a/x+1/8*(-3*a*f+4*b*e)*x*(b*x^2+a)^(1/2)/b^2+1/4*f*x^3*(b*x
^2+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1821, 1599, 1173, 396, 223, 212}

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3a^2f-4abe+8b^2d)}{8b^{5/2}}+\frac{x\sqrt{a+bx^2}(4be-3af)}{8b^2}-\frac{c\sqrt{a+bx^2}}{ax}+\frac{fx^3\sqrt{a+bx^2}}{4b}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((c*Sqrt[a + b*x^2])/(a*x)) + ((4*b*e - 3*a*f)*x*Sqrt[a + b*x^2])/(8*b^2)
+ (f*x^3*Sqrt[a + b*x^2])/(4*b) + ((8*b^2*d - 4*a*b*e + 3*a^2*f)*ArcTanh[(S
qrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1173

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = -\frac{c\sqrt{a+bx^2}}{ax} - \frac{\int \frac{-adx-aex^3-afx^5}{x\sqrt{a+bx^2}}\,dx}{a}$$

$$
\begin{aligned}
&= -\frac{c\sqrt{a+bx^2}}{ax} - \frac{\int \frac{-ad-aex^2-afx^4}{\sqrt{a+bx^2}}\,dx}{a} \\
&= -\frac{c\sqrt{a+bx^2}}{ax} + \frac{fx^3\sqrt{a+bx^2}}{4b} - \frac{\int \frac{-4abd-a(4be-3af)x^2}{\sqrt{a+bx^2}}\,dx}{4ab} \\
&= -\frac{c\sqrt{a+bx^2}}{ax} + \frac{(4be-3af)x\sqrt{a+bx^2}}{8b^2} + \frac{fx^3\sqrt{a+bx^2}}{4b} \\
&\quad + \frac{(8ab^2d-a^2(4be-3af))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8ab^2} \\
&= -\frac{c\sqrt{a+bx^2}}{ax} + \frac{(4be-3af)x\sqrt{a+bx^2}}{8b^2} + \frac{fx^3\sqrt{a+bx^2}}{4b} \\
&\quad + \frac{(8ab^2d-a^2(4be-3af))\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{8ab^2} \\
&= -\frac{c\sqrt{a+bx^2}}{ax} + \frac{(4be-3af)x\sqrt{a+bx^2}}{8b^2} + \frac{fx^3\sqrt{a+bx^2}}{4b} \\
&\quad + \frac{(8b^2d-4abe+3a^2f)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.90

$$
\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx &= \frac{\sqrt{a+bx^2}(-8b^2c+4abex^2-3a^2fx^2+2abfx^4)}{8ab^2x} \\
&\quad + \frac{(-8b^2d+4abe-3a^2f)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{5/2}}
\end{aligned}
$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*b^2*c + 4*a*b*e*x^2 - 3*a^2*f*x^2 + 2*a*b*f*x^4))/(8*a
*b^2*x) + ((-8*b^2*d + 4*a*b*e - 3*a^2*f)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2
]])/(8*b^(5/2))
```

Maple [A] (verified)

Time = 3.61 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{\sqrt{b x^2+a}\,(-2abf x^4+3a^2 f x^2-4abe x^2+8b^2c)}{8b^2ax}+\frac{(3a^2f-4aeb+8b^2d)\ln\left(x\sqrt{b}+\sqrt{b x^2+a}\right)}{8b^{\frac{5}{2}}}$
pseudoelliptic	$\frac{3b^2x(a^2f-\frac{4}{3}aeb+\frac{8}{3}b^2d)a\,\text{arctanh}\left(\frac{\sqrt{bx^2+a}}{x\sqrt{b}}\right)-8\sqrt{bx^2+a}\left(b^2c-\frac{x^2a\left(\frac{fx^2}{2}+e\right)b}{2}+\frac{3a^2fx^2}{8}\right)b^{\frac{5}{2}}}{8xb^{\frac{9}{2}}a}$
default	$\frac{d\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{\sqrt{b}}+f\left(\frac{x^3\sqrt{bx^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{bx^2+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)+e\left(\frac{x\sqrt{bx^2+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b}\right.}{2b^{\frac{3}{2}}}\right.$

[In] int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/8*(b*x^2+a)^(1/2)*(-2*a*b*f*x^4+3*a^2*f*x^2-4*a*b*e*x^2+8*b^2*c)/b^2/a/x +1/8*(3*a^2*f-4*a*b*e+8*b^2*d)/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.85

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx$$
$$=\left[\frac{(8\,ab^2d-4\,a^2be+3\,a^3f)\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(2\,ab^2fx^4-8\,b^3c+(4\,ab^2e-3\,a^2bf}{16\,ab^3x},\right.$$
$$\left.-\frac{(8\,ab^2d-4\,a^2be+3\,a^3f)\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,ab^2fx^4-8\,b^3c+(4\,ab^2e-3\,a^2bf)x^2)\sqrt{bx^2+a}}{8\,ab^3x}\right]$$

[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] [1/16*((8*a*b^2*d - 4*a^2*b*e + 3*a^3*f)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*a*b^2*f*x^4 - 8*b^3*c + (4*a*b^2*e - 3*a^2*b*f)*x^2)*sqrt(b*x^2 + a))/(a*b^3*x), -1/8*((8*a*b^2*d - 4*a^2*b*e + 3*a^3*f)*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*a*b^2*f*x^4 - 8*b^3*c + (4*a*b^2*e - 3*a^2*b*f)*x^2)*sqrt(b*x^2 + a))/(a*b^3*x)]

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.10

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx = d\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\wedge b\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for}\ b\neq 0\\ \frac{x}{\sqrt{a}} & \text{otherwise}\end{cases}\right) + e\left(\begin{cases}-\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b}+\frac{x\sqrt{a+bx^2}}{2b} & \text{for}\ b\neq 0\\ \frac{x^3}{3\sqrt{a}} & \text{otherwise}\end{cases}\right) + f\left(\begin{cases}\frac{3a^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b^2}-\frac{3ax\sqrt{a+bx^2}}{8b^2}+\frac{x^3\sqrt{a+bx^2}}{4b} & \text{for}\ b\neq 0\\ \frac{x^5}{5\sqrt{a}} & \text{otherwise}\end{cases}\right) - \frac{\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{a}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] d*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0) & Ne
(b, 0)), (x*log(x)/sqrt(b*x**2), Ne(b, 0)), (x/sqrt(a), True)) + e*Piecewis
e((-a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0))
, (x*log(x)/sqrt(b*x**2), True))/(2*b) + x*sqrt(a + b*x**2)/(2*b), Ne(b, 0)
), (x**3/(3*sqrt(a)), True)) + f*Piecewise((3*a**2*Piecewise((log(2*sqrt(b)
*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True
))/(8*b**2) - 3*a*x*sqrt(a + b*x**2)/(8*b**2) + x**3*sqrt(a + b*x**2)/(4*b)
, Ne(b, 0)), (x**5/(5*sqrt(a)), True)) - sqrt(b)*c*sqrt(a/(b*x**2) + 1)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}fx^3}{4\,b}+\frac{\sqrt{bx^2+a}ex}{2\,b}-\frac{3\,\sqrt{bx^2+a}afx}{8\,b^2}+\frac{d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{ae\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{3\,a^2f\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}-\frac{\sqrt{bx^2+a}c}{ax}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(b*x^2 + a)*f*x^3/b + 1/2*sqrt(b*x^2 + a)*e*x/b - 3/8*sqrt(b*x^2 +
a)*a*f*x/b^2 + d*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/2*a*e*arcsinh(b*x/sqrt(
a*b))/b^(3/2) + 3/8*a^2*f*arcsinh(b*x/sqrt(a*b))/b^(5/2) - sqrt(b*x^2 + a)*
c/(a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx = \frac{1}{8}\,\sqrt{bx^2+a}\left(\frac{2\,fx^2}{b}+\frac{4\,b^2e-3\,abf}{b^3}\right)x+\frac{2\,\sqrt{b}c}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}-\frac{(8\,b^2d-4\,abe+3\,a^2f)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{16\,b^{\frac{5}{2}}}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*f*x^2/b + (4*b^2*e - 3*a*b*f)/b^3)*x + 2*sqrt(b)*c/(
(sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) - 1/16*(8*b^2*d - 4*a*b*e + 3*a^2*f)*l
og((sqrt(b)*x - sqrt(b*x^2 + a))^2)/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx=\int \frac{f\,x^6+e\,x^4+d\,x^2+c}{x^2\,\sqrt{b\,x^2+a}}\,dx$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^(1/2)),x)
[Out] int((c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^(1/2)), x)
```

3.155 $\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx$

Optimal result 942
Rubi [A] (verified) 942
Mathematica [A] (verified) 944
Maple [A] (verified) 945
Fricas [A] (verification not implemented) 945
Sympy [A] (verification not implemented) 946
Maxima [A] (verification not implemented) 946
Giac [A] (verification not implemented) 947
Mupad [B] (verification not implemented) 947

Optimal result

Integrand size = 32, antiderivative size = 110

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx = -\frac{c\sqrt{a+bx^2}}{3ax^3}+\frac{(2bc-3ad)\sqrt{a+bx^2}}{3a^2x}+\frac{fx\sqrt{a+bx^2}}{2b}+\frac{(2be-af)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

```
[Out] 1/2*(-a*f+2*b*e)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)-1/3*c*(b*x^2+a)
^(1/2)/a/x^3+1/3*(-3*a*d+2*b*c)*(b*x^2+a)^(1/2)/a^2/x+1/2*f*x*(b*x^2+a)^(1/
2)/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1821, 1599, 1279, 396, 223, 212}

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(2bc-3ad)}{3a^2x}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2be-af)}{2b^{3/2}}-\frac{c\sqrt{a+bx^2}}{3ax^3}+\frac{fx\sqrt{a+bx^2}}{2b}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/3*(c*Sqrt[a + b*x^2])/(a*x^3) + ((2*b*c - 3*a*d)*Sqrt[a + b*x^2])/(3*a^2
*x) + (f*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*e - a*f)*ArcTanh[(Sqrt[b]*x)/Sqrt
[a + b*x^2]])/(2*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\text{integral} = -\frac{c\sqrt{a+bx^2}}{3ax^3} - \frac{\int \frac{(2bc-3ad)x-3aex^3-3afx^5}{x^3\sqrt{a+bx^2}}\,dx}{3a}$$

$$= -\frac{c\sqrt{a+bx^2}}{3ax^3} - \frac{\int \frac{2bc-3ad-3aex^2-3afx^4}{x^2\sqrt{a+bx^2}}\,dx}{3a}$$

$$= -\frac{c\sqrt{a+bx^2}}{3ax^3} + \frac{(2bc-3ad)\sqrt{a+bx^2}}{3a^2x} + \frac{\int \frac{3a^2e+3a^2fx^2}{\sqrt{a+bx^2}}\,dx}{3a^2}$$

$$= -\frac{c\sqrt{a+bx^2}}{3ax^3} + \frac{(2bc-3ad)\sqrt{a+bx^2}}{3a^2x} + \frac{fx\sqrt{a+bx^2}}{2b} + \frac{(2be-af)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}$$

$$= -\frac{c\sqrt{a+bx^2}}{3ax^3} + \frac{(2bc-3ad)\sqrt{a+bx^2}}{3a^2x} + \frac{fx\sqrt{a+bx^2}}{2b} + \frac{(2be-af)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b}$$

$$= -\frac{c\sqrt{a+bx^2}}{3ax^3} + \frac{(2bc-3ad)\sqrt{a+bx^2}}{3a^2x} + \frac{fx\sqrt{a+bx^2}}{2b} + \frac{(2be-af)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(-2abc+4b^2cx^2-6abdx^2+3a^2fx^4)}{6a^2bx^3} + \frac{(-2be+af)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{3/2}}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-2*a*b*c + 4*b^2*c*x^2 - 6*a*b*d*x^2 + 3*a^2*f*x^4))/(6*a
^2*b*x^3) + ((-2*b*e + a*f)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(3/2)
)
```

Maple [A] (verified)

Time = 3.60 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.75

method	result
risch	$\frac{\sqrt{b\,x^{2}+a}\,\left(3 a^{2} f\,x^{4}-6 x^{2} a b d+4 b^{2} c\,x^{2}-2 a b c\right)}{6 b\,a^{2} x^{3}}-\frac{\left(a f -2 b e\right) \ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2 b^{\frac{3}{2}}}$
pseudoelliptic	$-\frac{x^{3} a^{2} \left(a f -2 b e\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x \sqrt{b}}\right)-\left(-\frac{2 a \left(3 d\,x^{2}+c \right) b^{\frac{3}{2}}}{3}+x^{2} \left(\sqrt{b}\,a^{2} f\,x^{2}+\frac{4 b^{\frac{5}{2}} c}{3}\right)\right) \sqrt{b\,x^{2}+a}}{2 b^{\frac{3}{2}} x^{3} a^{2}}$
default	$\frac{e \ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+f \left(\frac{x \sqrt{b\,x^{2}+a}}{2 b}-\frac{a \ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2 b^{\frac{3}{2}}}\right)+c \left(-\frac{\sqrt{b\,x^{2}+a}}{3 a\,x^{3}}+\frac{2 b \sqrt{b\,x^{2}+a}}{3 a^{2} x}\right)-\frac{d \sqrt{b\,x^{2}+a}}{a x}$

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(b*x^2+a)^(1/2)*(3*a^2*f*x^4-6*a*b*d*x^2+4*b^2*c*x^2-2*a*b*c)/b/a^2/x^3
-1/2*(a*f-2*b*e)/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.91

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx$$
$$= \left[-\frac{3\,(2\,a^2be-a^3f)\sqrt{b}x^3\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(3\,a^2bfx^4-2\,ab^2c+2\,(2\,b^3c-3\,ab^2d)x^2}{12\,a^2b^2x^3}\right.$$
$$\left.-\frac{3\,(2\,a^2be-a^3f)\sqrt{-b}x^3\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(3\,a^2bfx^4-2\,ab^2c+2\,(2\,b^3c-3\,ab^2d)x^2)\sqrt{bx^2+a}}{6\,a^2b^2x^3}\right]$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(2*a^2*b*e - a^3*f)*sqrt(b)*x^3*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*
sqrt(b)*x - a) - 2*(3*a^2*b*f*x^4 - 2*a*b^2*c + 2*(2*b^3*c - 3*a*b^2*d)*x^2
)*sqrt(b*x^2 + a))/(a^2*b^2*x^3), -1/6*(3*(2*a^2*b*e - a^3*f)*sqrt(-b)*x^3*
arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (3*a^2*b*f*x^4 - 2*a*b^2*c + 2*(2*b^3*
c - 3*a*b^2*d)*x^2)*sqrt(b*x^2 + a))/(a^2*b^2*x^3)]
```

Sympy [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.78

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx$$
$$= e\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \wedge b\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for } b\neq 0 \\ \frac{x}{\sqrt{a}} & \text{otherwise} \end{cases}\right)$$
$$+ f\left(\begin{cases} -\frac{a\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2b} + \frac{x\sqrt{a+bx^2}}{2b} & \text{for } b\neq 0 \\ \frac{x^3}{3\sqrt{a}} & \text{otherwise} \end{cases}\right)$$
$$- \frac{\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{3ax^2} - \frac{\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{a} + \frac{2b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^2}+1}}{3a^2}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] e*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0) & Ne
(b, 0)), (x*log(x)/sqrt(b*x**2), Ne(b, 0)), (x/sqrt(a), True)) + f*Piecewis
e((-a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0))
, (x*log(x)/sqrt(b*x**2), True))/(2*b) + x*sqrt(a + b*x**2)/(2*b), Ne(b, 0)
), (x**3/(3*sqrt(a)), True)) - sqrt(b)*c*sqrt(a/(b*x**2) + 1)/(3*a*x**2) -
sqrt(b)*d*sqrt(a/(b*x**2) + 1)/a + 2*b**(3/2)*c*sqrt(a/(b*x**2) + 1)/(3*a**
2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}fx}{2\,b} + \frac{e\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{af\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} + \frac{2\sqrt{bx^2+a}bc}{3\,a^2x} - \frac{\sqrt{bx^2+a}d}{ax} - \frac{\sqrt{bx^2+a}c}{3\,ax^3}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*f*x/b + e*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/2*a*f*arcs
inh(b*x/sqrt(a*b))/b^(3/2) + 2/3*sqrt(b*x^2 + a)*b*c/(a^2*x) - sqrt(b*x^2 +
 a)*d/(a*x) - 1/3*sqrt(b*x^2 + a)*c/(a*x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.55

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^4\sqrt{a + bx^2}}\, dx = \frac{\sqrt{bx^2 + a}fx}{2\,b} - \frac{(2\,be - af)\log\left(\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2\right)}{4\,b^{\frac{3}{2}}} + \frac{2\left(3\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^4\sqrt{b}d + 6\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 b^{\frac{3}{2}}c - 6\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 a\sqrt{b}d - 2\,ab^{\frac{3}{2}}c + 3\,a\right.}{3\left(\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 - a\right)^3}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*f*x/b - 1/4*(2*b*e - a*f)*log((sqrt(b)*x - sqrt(b*x^2 +
 a))^2)/b^(3/2) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*sqrt(b)*d + 6*(sqr
t(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a
*sqrt(b)*d - 2*a*b^(3/2)*c + 3*a^2*sqrt(b)*d)/((sqrt(b)*x - sqrt(b*x^2 + a)
)^2 - a)^3
```

Mupad [B] (verification not implemented)

Time = 7.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.30

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^4\sqrt{a + bx^2}}\, dx = \begin{cases} -\frac{-f\,x^6 - 3\,e\,x^4 + 3\,d\,x^2 + c}{3\,\sqrt{a}\,x^3} & \text{if } b = 0 \\ \frac{e\,\ln\left(\sqrt{b}\,x + \sqrt{b\,x^2 + a}\right)}{\sqrt{b}} - \frac{d\,\sqrt{b\,x^2 + a}}{a\,x} - \frac{a\,f\,\ln\left(2\,\sqrt{b}\,x + 2\,\sqrt{b\,x^2 + a}\right)}{2\,b^{3/2}} + \frac{f\,x\,\sqrt{b\,x^2 + a}}{2\,b} - \frac{c\,\sqrt{b\,x^2 + a}\,(a - 2\,b\,x^2)}{3\,a^2\,x^3} & \text{if } b \neq 0 \end{cases}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^(1/2)),x)
```

```
[Out] piecewise(b == 0, -(c + 3*d*x^2 - 3*e*x^4 - f*x^6)/(3*a^(1/2)*x^3), b ~= 0,
 (e*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2) - (d*(a + b*x^2)^(1/2))/(a*
x) - (a*f*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2)) + (f*x*(a + b
*x^2)^(1/2))/(2*b) - (c*(a + b*x^2)^(1/2)*(a - 2*b*x^2))/(3*a^2*x^3))
```

3.156 $\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx$

Optimal result 948
Rubi [A] (verified) 948
Mathematica [A] (verified) 950
Maple [A] (verified) 951
Fricas [A] (verification not implemented) 951
Sympy [A] (verification not implemented) 952
Maxima [A] (verification not implemented) 953
Giac [B] (verification not implemented) 953
Mupad [B] (verification not implemented) 954

Optimal result

Integrand size = 32, antiderivative size = 118

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx = -\frac{c\sqrt{a+bx^2}}{5ax^5}+\frac{(4bc-5ad)\sqrt{a+bx^2}}{15a^2x^3} -\frac{(8b^2c-10abd+15a^2e)\sqrt{a+bx^2}}{15a^3x}+\frac{f\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] f*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)-1/5*c*(b*x^2+a)^(1/2)/a/x^5+1/
15*(-5*a*d+4*b*c)*(b*x^2+a)^(1/2)/a^2/x^3-1/15*(15*a^2*e-10*a*b*d+8*b^2*c)*
(b*x^2+a)^(1/2)/a^3/x
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1821, 1599, 1279, 462, 223, 212}

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(4bc-5ad)}{15a^2x^3}-\frac{\sqrt{a+bx^2}(15a^2e-10abd+8b^2c)}{15a^3x} +\frac{f\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{c\sqrt{a+bx^2}}{5ax^5}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/5*(c*Sqrt[a + b*x^2])/(a*x^5) + ((4*b*c - 5*a*d)*Sqrt[a + b*x^2])/(15*a^
2*x^3) - ((8*b^2*c - 10*a*b*d + 15*a^2*e)*Sqrt[a + b*x^2])/(15*a^3*x) + (f*
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{c\sqrt{a+bx^2}}{5ax^5} - \frac{\int \frac{(4bc-5ad)x-5aex^3-5afx^5}{x^5\sqrt{a+bx^2}}\,dx}{5a} \\
&= -\frac{c\sqrt{a+bx^2}}{5ax^5} - \frac{\int \frac{4bc-5ad-5aex^2-5afx^4}{x^4\sqrt{a+bx^2}}\,dx}{5a} \\
&= -\frac{c\sqrt{a+bx^2}}{5ax^5} + \frac{(4bc-5ad)\sqrt{a+bx^2}}{15a^2x^3} + \frac{\int \frac{8b^2c-10abd+15a^2e+15a^2fx^2}{x^2\sqrt{a+bx^2}}\,dx}{15a^2} \\
&= -\frac{c\sqrt{a+bx^2}}{5ax^5} + \frac{(4bc-5ad)\sqrt{a+bx^2}}{15a^2x^3} - \frac{(8b^2c-10abd+15a^2e)\sqrt{a+bx^2}}{15a^3x} + f\int \frac{1}{\sqrt{a+bx^2}}\,dx \\
&= -\frac{c\sqrt{a+bx^2}}{5ax^5} + \frac{(4bc-5ad)\sqrt{a+bx^2}}{15a^2x^3} - \frac{(8b^2c-10abd+15a^2e)\sqrt{a+bx^2}}{15a^3x} \\
&\quad + f\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right) \\
&= -\frac{c\sqrt{a+bx^2}}{5ax^5} + \frac{(4bc-5ad)\sqrt{a+bx^2}}{15a^2x^3} \\
&\quad - \frac{(8b^2c-10abd+15a^2e)\sqrt{a+bx^2}}{15a^3x} + \frac{f\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.83

$$
\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx &= -\frac{\sqrt{a+bx^2}(8b^2cx^4-2abx^2(2c+5dx^2)+a^2(3c+5dx^2+15ex^4))}{15a^3x^5} \\
&\quad - \frac{f\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}
\end{aligned}
$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*Sqrt[a + b*x^2]),x]
[Out] -1/15*(Sqrt[a + b*x^2]*(8*b^2*c*x^4 - 2*a*b*x^2*(2*c + 5*d*x^2) + a^2*(3*c
 + 5*d*x^2 + 15*e*x^4)))/(a^3*x^5) - (f*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])
/Sqrt[b]
```

Maple [A] (verified)

Time = 3.56 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(15a^{2}e\,x^{4}-10abd\,x^{4}+8b^{2}c\,x^{4}+5a^{2}d\,x^{2}-4abc\,x^{2}+3a^{2}c\right)}{15a^{3}x^{5}}+\frac{f\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}$
pseudoelliptic	$\frac{f\,a^{3}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)x^{5}-\frac{\sqrt{b\,x^{2}+a}\,\left(-\frac{4x^{2}\left(\frac{5d\,x^{2}}{2}+c\right)a\,b^{\frac{3}{2}}}{3}+\frac{8b^{\frac{5}{2}}c\,x^{4}}{3}+a^{2}\sqrt{b}\,\left(5e\,x^{4}+\frac{5}{3}d\,x^{2}+c\right)\right)}{5}}{\sqrt{b}\,x^{5}a^{3}}$
default	$\frac{f\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+d\left(-\frac{\sqrt{b\,x^{2}+a}}{3a\,x^{3}}+\frac{2b\sqrt{b\,x^{2}+a}}{3a^{2}x}\right)-\frac{e\sqrt{b\,x^{2}+a}}{ax}+c\left(-\frac{\sqrt{b\,x^{2}+a}}{5a\,x^{5}}-\frac{4b\left(-\frac{\sqrt{b\,x^{2}+a}}{3a\,x^{3}}+\frac{2b\sqrt{b\,x^{2}}}{3a^{2}x}\right.}{5a}\right.$

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(b*x^2+a)^(1/2)*(15*a^2*e*x^4-10*a*b*d*x^4+8*b^2*c*x^4+5*a^2*d*x^2-4*
a*b*c*x^2+3*a^2*c)/a^3/x^5+f*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.87

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx$$
$$=\left[\frac{15\,a^3\sqrt{b}fx^5\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,((8\,b^3c-10\,ab^2d+15\,a^2be)x^4+3\,a^2bc-(4\,ab^2c-}{30\,a^3bx^5}\right.$$
$$-\frac{15\,a^3\sqrt{-b}fx^5\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+((8\,b^3c-10\,ab^2d+15\,a^2be)x^4+3\,a^2bc-(4\,ab^2c-5\,a^2bd)x^2)\sqrt{bx^2+}}{15\,a^3bx^5}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*a^3*sqrt(b)*f*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 - 2*((8*b^3*c - 10*a*b^2*d + 15*a^2*b*e)*x^4 + 3*a^2*b*c - (4*a*b^2*c - 5*
a^2*b*d)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^5), -1/15*(15*a^3*sqrt(-b)*f*x^5*ar
ctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + ((8*b^3*c - 10*a*b^2*d + 15*a^2*b*e)*x^4
 + 3*a^2*b*c - (4*a*b^2*c - 5*a^2*b*d)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^5)]
```

Sympy [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 427, normalized size of antiderivative = 3.62

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx = -\frac{3a^4b^{\frac{9}{2}}c\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{2a^3b^{\frac{11}{2}}cx^2\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{3a^2b^{\frac{13}{2}}cx^4\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{12ab^{\frac{15}{2}}cx^6\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{8b^{\frac{17}{2}}cx^8\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} + f\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \wedge b\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for } b\neq 0 \\ \frac{x}{\sqrt{a}} & \text{otherwise} \end{cases}\right) - \frac{\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{3ax^2} - \frac{\sqrt{b}e\sqrt{\frac{a}{bx^2}+1}}{a} + \frac{2b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^2}+1}}{3a^2}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*a**4*b**(9/2)*c*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x
**6 + 15*a**3*b**6*x**8) - 2*a**3*b**(11/2)*c*x**2*sqrt(a/(b*x**2) + 1)/(15
*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 3*a**2*b**(13/2)
*c*x**4*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a*
*3*b**6*x**8) - 12*a*b**(15/2)*c*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x*
*4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 8*b**(17/2)*c*x**8*sqrt(a/(b*
x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) + f*
Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0) & Ne(b
, 0)), (x*log(x)/sqrt(b*x**2), Ne(b, 0)), (x/sqrt(a), True)) - sqrt(b)*d*sq
rt(a/(b*x**2) + 1)/(3*a*x**2) - sqrt(b)*e*sqrt(a/(b*x**2) + 1)/a + 2*b**(3/
2)*d*sqrt(a/(b*x**2) + 1)/(3*a**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx = \frac{f\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{8\sqrt{bx^2+a}b^2c}{15\,a^3x} + \frac{2\sqrt{bx^2+a}bd}{3\,a^2x} - \frac{\sqrt{bx^2+a}e}{ax} + \frac{4\sqrt{bx^2+a}bc}{15\,a^2x^3} - \frac{\sqrt{bx^2+a}d}{3\,ax^3} - \frac{\sqrt{bx^2+a}c}{5\,ax^5}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] f*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 8/15*sqrt(b*x^2 + a)*b^2*c/(a^3*x) + 2/3
*sqrt(b*x^2 + a)*b*d/(a^2*x) - sqrt(b*x^2 + a)*e/(a*x) + 4/15*sqrt(b*x^2 +
a)*b*c/(a^2*x^3) - 1/3*sqrt(b*x^2 + a)*d/(a*x^3) - 1/5*sqrt(b*x^2 + a)*c/(a
*x^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(100) = 200.

Time = 0.33 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.70

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx = -\frac{f\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{2\sqrt{b}} + \frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8\sqrt{b}e+30\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6b^{\frac{3}{2}}d-60\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6a\sqrt{b}e+80\left(\sqrt{b}\cdots\right.\right.}{\cdots}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*f*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/sqrt(b) + 2/15*(15*(sqrt(b)*x -
 sqrt(b*x^2 + a))^8*sqrt(b)*e + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(3/2)*
d - 60*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*sqrt(b)*e + 80*(sqrt(b)*x - sqrt(b
*x^2 + a))^4*b^(5/2)*c - 70*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(3/2)*d + 9
0*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*sqrt(b)*e - 40*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*a*b^(5/2)*c + 50*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(3/2)*d -
60*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*sqrt(b)*e + 8*a^2*b^(5/2)*c - 10*a^3
*b^(3/2)*d + 15*a^4*sqrt(b)*e)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

Mupad [B] (verification not implemented)

Time = 6.65 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.89

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx = \frac{f\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}} - \frac{e\,\sqrt{b\,x^2+a}}{a\,x} - \frac{d\,\sqrt{b\,x^2+a}\,(a-2\,b\,x^2)}{3\,a^2\,x^3} - \frac{c\,\sqrt{b\,x^2+a}\,(3\,a^2-4\,a\,b\,x^2+8\,b^2\,x^4)}{15\,a^3\,x^5}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^(1/2)),x)
```

```
[Out] (f*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2) - (e*(a + b*x^2)^(1/2))/(a*x
) - (d*(a + b*x^2)^(1/2)*(a - 2*b*x^2))/(3*a^2*x^3) - (c*(a + b*x^2)^(1/2)*
(3*a^2 + 8*b^2*x^4 - 4*a*b*x^2))/(15*a^3*x^5)
```

3.157 $\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx$

Optimal result 955
Rubi [A] (verified) 955
Mathematica [A] (verified) 957
Maple [A] (verified) 957
Fricas [A] (verification not implemented) 958
Sympy [B] (verification not implemented) 958
Maxima [A] (verification not implemented) 960
Giac [B] (verification not implemented) 960
Mupad [B] (verification not implemented) 961

Optimal result

Integrand size = 32, antiderivative size = 140

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx=-\frac{c\sqrt{a+bx^2}}{7ax^7}+\frac{(6bc-7ad)\sqrt{a+bx^2}}{35a^2x^5}-\frac{(24b^2c-28abd+35a^2e)\sqrt{a+bx^2}}{105a^3x^3}+\frac{(48b^3c-56ab^2d+70a^2be-105a^3f)\sqrt{a+bx^2}}{105a^4x}$$

```
[Out] -1/7*c*(b*x^2+a)^(1/2)/a/x^7+1/35*(-7*a*d+6*b*c)*(b*x^2+a)^(1/2)/a^2/x^5-1/
105*(35*a^2*e-28*a*b*d+24*b^2*c)*(b*x^2+a)^(1/2)/a^3/x^3+1/105*(-105*a^3*f+
70*a^2*b*e-56*a*b^2*d+48*b^3*c)*(b*x^2+a)^(1/2)/a^4/x
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1817, 12, 270}

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx=\frac{\sqrt{a+bx^2}(6bc-7ad)}{35a^2x^5}-\frac{\sqrt{a+bx^2}(35a^2e-28abd+24b^2c)}{105a^3x^3}+\frac{\sqrt{a+bx^2}(-105a^3f+70a^2be-56ab^2d+48b^3c)}{105a^4x}-\frac{c\sqrt{a+bx^2}}{7ax^7}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/7*(c*Sqrt[a + b*x^2])/(a*x^7) + ((6*b*c - 7*a*d)*Sqrt[a + b*x^2])/(35*a^
2*x^5) - ((24*b^2*c - 28*a*b*d + 35*a^2*e)*Sqrt[a + b*x^2])/(105*a^3*x^3) +
 ((48*b^3*c - 56*a*b^2*d + 70*a^2*b*e - 105*a^3*f)*Sqrt[a + b*x^2])/(105*a^
4*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{c\sqrt{a+bx^2}}{7ax^7} - \frac{\int \frac{6bc-7a(d+ex^2+fx^4)}{x^6\sqrt{a+bx^2}}\,dx}{7a} \\
&= -\frac{c\sqrt{a+bx^2}}{7ax^7} + \frac{(6bc-7ad)\sqrt{a+bx^2}}{35a^2x^5} + \frac{\int \frac{4b(6bc-7ad)-5a(-7ae-7afx^2)}{x^4\sqrt{a+bx^2}}\,dx}{35a^2} \\
&= -\frac{c\sqrt{a+bx^2}}{7ax^7} + \frac{(6bc-7ad)\sqrt{a+bx^2}}{35a^2x^5} \\
&\quad - \frac{(24b^2c-28abd+35a^2e)\sqrt{a+bx^2}}{105a^3x^3} - \frac{\int \frac{2b(24b^2c-28abd+35a^2e)-105a^3f}{x^2\sqrt{a+bx^2}}\,dx}{105a^3} \\
&= -\frac{c\sqrt{a+bx^2}}{7ax^7} + \frac{(6bc-7ad)\sqrt{a+bx^2}}{35a^2x^5} - \frac{(24b^2c-28abd+35a^2e)\sqrt{a+bx^2}}{105a^3x^3} \\
&\quad - \frac{(48b^3c-56ab^2d+70a^2be-105a^3f)\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx}{105a^3} \\
&= -\frac{c\sqrt{a+bx^2}}{7ax^7} + \frac{(6bc-7ad)\sqrt{a+bx^2}}{35a^2x^5} - \frac{(24b^2c-28abd+35a^2e)\sqrt{a+bx^2}}{105a^3x^3} \\
&\quad + \frac{(48b^3c-56ab^2d+70a^2be-105a^3f)\sqrt{a+bx^2}}{105a^4x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.74

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a+bx^2}(48b^3cx^6-8ab^2x^4(3c+7dx^2)+2a^2bx^2(9c+14dx^2+35ex^4)-a^3(15c+21dx^2+35x^4(e+3fx}{105a^4x^7}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(48*b^3*c*x^6 - 8*a*b^2*x^4*(3*c + 7*d*x^2) + 2*a^2*b*x^2*
(9*c + 14*d*x^2 + 35*e*x^4) - a^3*(15*c + 21*d*x^2 + 35*x^4*(e + 3*f*x^2)))
)/(105*a^4*x^7)
```

Maple [A] (verified)

Time = 3.58 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$-\frac{\left(\left(7f\,x^6+\frac{7}{3}e\,x^4+\frac{7}{5}d\,x^2+c\right)a^3-\frac{6b\,x^2\left(\frac{35}{9}e\,x^4+\frac{14}{9}d\,x^2+c\right)a^2}{5}+\frac{8b^2\left(\frac{7d\,x^2}{3}+c\right)x^4a}{5}-\frac{16b^3c\,x^6}{5}\right)\sqrt{b\,x^2+a}}{7x^7a^4}$
gosper	$-\frac{\sqrt{b\,x^2+a}\,(105a^3f\,x^6-70a^2be\,x^6+56a\,b^2d\,x^6-48b^3c\,x^6+35a^3e\,x^4-28a^2bd\,x^4+24a\,b^2c\,x^4+21a^3d\,x^2-18a^2bc\,x^2+15c\,a^3)}{105x^7a^4}$
trager	$-\frac{\sqrt{b\,x^2+a}\,(105a^3f\,x^6-70a^2be\,x^6+56a\,b^2d\,x^6-48b^3c\,x^6+35a^3e\,x^4-28a^2bd\,x^4+24a\,b^2c\,x^4+21a^3d\,x^2-18a^2bc\,x^2+15c\,a^3)}{105x^7a^4}$
risch	$-\frac{\sqrt{b\,x^2+a}\,(105a^3f\,x^6-70a^2be\,x^6+56a\,b^2d\,x^6-48b^3c\,x^6+35a^3e\,x^4-28a^2bd\,x^4+24a\,b^2c\,x^4+21a^3d\,x^2-18a^2bc\,x^2+15c\,a^3)}{105x^7a^4}$
default	$c\left(-\frac{\sqrt{b\,x^2+a}}{7a\,x^7}-\frac{6b\left(-\frac{\sqrt{b\,x^2+a}}{5a\,x^5}-\frac{4b\left(-\frac{\sqrt{b\,x^2+a}}{3a\,x^3}+\frac{2b\sqrt{b\,x^2+a}}{3a^2x}\right)}{5a}\right)}{7a}\right)+e\left(-\frac{\sqrt{b\,x^2+a}}{3a\,x^3}+\frac{2b\sqrt{b\,x^2+a}}{3a^2x}\right)-\frac{f\sqrt{b\,x^2+a}}{ax}$

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*((7*f*x^6+7/3*e*x^4+7/5*d*x^2+c)*a^3-6/5*b*x^2*(35/9*e*x^4+14/9*d*x^2+
c)*a^2+8/5*b^2*(7/3*d*x^2+c)*x^4*a-16/5*b^3*c*x^6)*(b*x^2+a)^(1/2)/x^7/a^4
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.71

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^8\sqrt{a + bx^2}}\,dx$$
$$= \frac{((48\,b^3c - 56\,ab^2d + 70\,a^2be - 105\,a^3f)x^6 - (24\,ab^2c - 28\,a^2bd + 35\,a^3e)x^4 - 15\,a^3c + 3\,(6\,a^2bc - 7\,a^3d)x^2)}{105\,a^4x^7}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*((48*b^3*c - 56*a*b^2*d + 70*a^2*b*e - 105*a^3*f)*x^6 - (24*a*b^2*c -
 28*a^2*b*d + 35*a^3*e)*x^4 - 15*a^3*c + 3*(6*a^2*b*c - 7*a^3*d)*x^2)*sqrt(
b*x^2 + a)/(a^4*x^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 891 vs. 2(136) = 272.

Time = 2.12 (sec) , antiderivative size = 891, normalized size of antiderivative = 6.36

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx = -\frac{5a^6b^{\frac{19}{2}}c\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}} - \frac{9a^5b^{\frac{21}{2}}cx^2\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}} - \frac{5a^4b^{\frac{23}{2}}cx^4\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}} - \frac{3a^4b^{\frac{9}{2}}d\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} + \frac{5a^3b^{\frac{25}{2}}cx^6\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}} - \frac{2a^3b^{\frac{11}{2}}dx^2\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} + \frac{30a^2b^{\frac{27}{2}}cx^8\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}} - \frac{3a^2b^{\frac{13}{2}}dx^4\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} + \frac{40ab^{\frac{29}{2}}cx^{10}\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}} - \frac{12ab^{\frac{15}{2}}dx^6\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} + \frac{16b^{\frac{31}{2}}cx^{12}\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}} - \frac{8b^{\frac{17}{2}}dx^8\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{\sqrt{b}e\sqrt{\frac{a}{bx^2}+1}}{3ax^2} - \frac{\sqrt{b}f\sqrt{\frac{a}{bx^2}+1}}{a} + \frac{2b^{\frac{3}{2}}e\sqrt{\frac{a}{bx^2}+1}}{3a^2}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a)**(1/2),x)
```

```
[Out] -5*a**6*b**(19/2)*c*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**1
0*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 9*a**5*b**(21/2)*c*x
**2*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**
5*b**11*x**10 + 35*a**4*b**12*x**12) - 5*a**4*b**(23/2)*c*x**4*sqrt(a/(b*x*
*2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 +
35*a**4*b**12*x**12) - 3*a**4*b**(9/2)*d*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4
*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) + 5*a**3*b**(25/2)*c*x**6*sq
rt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**1
1*x**10 + 35*a**4*b**12*x**12) - 2*a**3*b**(11/2)*d*x**2*sqrt(a/(b*x**2) +
1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) + 30*a**2*b*
```

```
*(27/2)*c*x**8*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**
8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 3*a**2*b**(13/2)*d*x**4*s
qrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x
**8) + 40*a*b**(29/2)*c*x**10*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105
*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 12*a*b**(1
5/2)*d*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 1
5*a**3*b**6*x**8) + 16*b**(31/2)*c*x**12*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9
*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) -
 8*b**(17/2)*d*x**8*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*
x**6 + 15*a**3*b**6*x**8) - sqrt(b)*e*sqrt(a/(b*x**2) + 1)/(3*a*x**2) - sqr
t(b)*f*sqrt(a/(b*x**2) + 1)/a + 2*b**(3/2)*e*sqrt(a/(b*x**2) + 1)/(3*a**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.38

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx = \frac{16\sqrt{bx^2+a}b^3c}{35\,a^4x} - \frac{8\sqrt{bx^2+a}b^2d}{15\,a^3x} + \frac{2\sqrt{bx^2+a}be}{3\,a^2x} - \frac{\sqrt{bx^2+a}f}{ax} - \frac{8\sqrt{bx^2+a}b^2c}{35\,a^3x^3} + \frac{4\sqrt{bx^2+a}bd}{15\,a^2x^3} - \frac{\sqrt{bx^2+a}e}{3\,ax^3} + \frac{6\sqrt{bx^2+a}bc}{35\,a^2x^5} - \frac{\sqrt{bx^2+a}d}{5\,ax^5} - \frac{\sqrt{bx^2+a}c}{7\,ax^7}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 16/35*sqrt(b*x^2 + a)*b^3*c/(a^4*x) - 8/15*sqrt(b*x^2 + a)*b^2*d/(a^3*x) +
2/3*sqrt(b*x^2 + a)*b*e/(a^2*x) - sqrt(b*x^2 + a)*f/(a*x) - 8/35*sqrt(b*x^2
 + a)*b^2*c/(a^3*x^3) + 4/15*sqrt(b*x^2 + a)*b*d/(a^2*x^3) - 1/3*sqrt(b*x^2
 + a)*e/(a*x^3) + 6/35*sqrt(b*x^2 + a)*b*c/(a^2*x^5) - 1/5*sqrt(b*x^2 + a)*
d/(a*x^5) - 1/7*sqrt(b*x^2 + a)*c/(a*x^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 548 vs. 2(124) = 248.

Time = 0.32 (sec) , antiderivative size = 548, normalized size of antiderivative = 3.91

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx = \frac{2\left(105\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}\sqrt{b}f + 210\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}b^{\frac{3}{2}}e - 630\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a\sqrt{b}f + 560\left(\right.\right.}{}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a))^12*sqrt(b)*f + 210*(sqrt(b)*x - sq
rt(b*x^2 + a))^10*b^(3/2)*e - 630*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a*sqrt(b
)*f + 560*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(5/2)*d - 910*(sqrt(b)*x - sqrt
(b*x^2 + a))^8*a*b^(3/2)*e + 1575*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*sqrt(
b)*f + 1680*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(7/2)*c - 1400*(sqrt(b)*x - s
qrt(b*x^2 + a))^6*a*b^(5/2)*d + 1540*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^
(3/2)*e - 2100*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*sqrt(b)*f - 1008*(sqrt(b
)*x - sqrt(b*x^2 + a))^4*a*b^(7/2)*c + 1176*(sqrt(b)*x - sqrt(b*x^2 + a))^4
*a^2*b^(5/2)*d - 1260*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(3/2)*e + 1575*
(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*sqrt(b)*f + 336*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*a^2*b^(7/2)*c - 392*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(5/2)*d
+ 490*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*b^(3/2)*e - 630*(sqrt(b)*x - sqrt
(b*x^2 + a))^2*a^5*sqrt(b)*f - 48*a^3*b^(7/2)*c + 56*a^4*b^(5/2)*d - 70*a^5
*b^(3/2)*e + 105*a^6*sqrt(b)*f)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.89

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx = \frac{\sqrt{b\,x^2+a}\,\left(-105\,f\,a^3+70\,e\,a^2\,b-56\,d\,a\,b^2+48\,c\,b^3\right)}{105\,a^4\,x} - \frac{\sqrt{b\,x^2+a}\,\left(7\,a\,d-6\,b\,c\right)}{35\,a^2\,x^5} - \frac{\sqrt{b\,x^2+a}\,\left(35\,e\,a^2-28\,d\,a\,b+24\,c\,b^2\right)}{105\,a^3\,x^3} - \frac{c\,\sqrt{b\,x^2+a}}{7\,a\,x^7}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^(1/2)),x)
```

```
[Out] ((a + b*x^2)^(1/2)*(48*b^3*c - 105*a^3*f - 56*a*b^2*d + 70*a^2*b*e))/(105*a
^4*x) - ((a + b*x^2)^(1/2)*(7*a*d - 6*b*c))/(35*a^2*x^5) - ((a + b*x^2)^(1/
2)*(24*b^2*c + 35*a^2*e - 28*a*b*d))/(105*a^3*x^3) - (c*(a + b*x^2)^(1/2))/
(7*a*x^7)
```

3.158 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx$

Optimal result 962
Rubi [A] (verified) 962
Mathematica [A] (verified) 964
Maple [A] (verified) 965
Fricas [A] (verification not implemented) 965
Sympy [B] (verification not implemented) 966
Maxima [A] (verification not implemented) 967
Giac [B] (verification not implemented) 967
Mupad [B] (verification not implemented) 968

Optimal result

Integrand size = 32, antiderivative size = 189

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx = -\frac{c\sqrt{a+bx^2}}{9ax^9}+\frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7} - \frac{(16b^2c-18abd+21a^2e)\sqrt{a+bx^2}}{105a^3x^5} + \frac{(64b^3c-72ab^2d+84a^2be-105a^3f)\sqrt{a+bx^2}}{315a^4x^3} - \frac{2b(64b^3c-72ab^2d+84a^2be-105a^3f)\sqrt{a+bx^2}}{315a^5x}$$

```
[Out] -1/9*c*(b*x^2+a)^(1/2)/a/x^9+1/63*(-9*a*d+8*b*c)*(b*x^2+a)^(1/2)/a^2/x^7-1/
105*(21*a^2*e-18*a*b*d+16*b^2*c)*(b*x^2+a)^(1/2)/a^3/x^5+1/315*(-105*a^3*f+
84*a^2*b*e-72*a*b^2*d+64*b^3*c)*(b*x^2+a)^(1/2)/a^4/x^3-2/315*b*(-105*a^3*f
+84*a^2*b*e-72*a*b^2*d+64*b^3*c)*(b*x^2+a)^(1/2)/a^5/x
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{1817, 12, 277, 270\}$

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(8bc-9ad)}{63a^2x^7} - \frac{\sqrt{a+bx^2}(21a^2e-18abd+16b^2c)}{105a^3x^5} - \frac{2b\sqrt{a+bx^2}(-105a^3f+84a^2be-72ab^2d+64b^3c)}{315a^5x} + \frac{\sqrt{a+bx^2}(-105a^3f+84a^2be-72ab^2d+64b^3c)}{315a^4x^3} - \frac{c\sqrt{a+bx^2}}{9ax^9}$$

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/9*(c*Sqrt[a + b*x^2])/(a*x^9) + ((8*b*c - 9*a*d)*Sqrt[a + b*x^2])/(63*a^
2*x^7) - ((16*b^2*c - 18*a*b*d + 21*a^2*e)*Sqrt[a + b*x^2])/(105*a^3*x^5) +
 ((64*b^3*c - 72*a*b^2*d + 84*a^2*b*e - 105*a^3*f)*Sqrt[a + b*x^2])/(315*a^
4*x^3) - (2*b*(64*b^3*c - 72*a*b^2*d + 84*a^2*b*e - 105*a^3*f)*Sqrt[a + b*x
^2])/(315*a^5*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{c\sqrt{a+bx^2}}{9ax^9} - \frac{\int \frac{8bc-9a(d+ex^2+fx^4)}{x^8\sqrt{a+bx^2}}\,dx}{9a}$$

$$= -\frac{c\sqrt{a+bx^2}}{9ax^9} + \frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7} + \frac{\int \frac{6b(8bc-9ad)-7a(-9ae-9afx^2)}{x^6\sqrt{a+bx^2}}\,dx}{63a^2}$$

$$= -\frac{c\sqrt{a+bx^2}}{9ax^9} + \frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7} - \frac{(16b^2c-18abd+21a^2e)\sqrt{a+bx^2}}{105a^3x^5} - \frac{\int \frac{4b(48b^2c-54abd+63a^2e)-315a^3f}{x^4\sqrt{a+bx^2}}\,dx}{315a^3}$$

$$= -\frac{c\sqrt{a+bx^2}}{9ax^9} + \frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7} - \frac{(16b^2c-18abd+21a^2e)\sqrt{a+bx^2}}{105a^3x^5} - \frac{(64b^3c-72ab^2d+84a^2be-105a^3f)\int \frac{1}{x^4\sqrt{a+bx^2}}\,dx}{105a^3}$$

$$= -\frac{c\sqrt{a+bx^2}}{9ax^9} + \frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7} - \frac{(16b^2c-18abd+21a^2e)\sqrt{a+bx^2}}{105a^3x^5} + \frac{(64b^3c-72ab^2d+84a^2be-105a^3f)\sqrt{a+bx^2}}{315a^4x^3} + \frac{(2b(64b^3c-72ab^2d+84a^2be-105a^3f))\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx}{315a^4}$$

$$= -\frac{c\sqrt{a+bx^2}}{9ax^9} + \frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7} - \frac{(16b^2c-18abd+21a^2e)\sqrt{a+bx^2}}{105a^3x^5} + \frac{(64b^3c-72ab^2d+84a^2be-105a^3f)\sqrt{a+bx^2}}{315a^4x^3} - \frac{2b(64b^3c-72ab^2d+84a^2be-105a^3f)\sqrt{a+bx^2}}{315a^5x}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.71

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{a+bx^2}(128b^4cx^8-16ab^3x^6(4c+9dx^2)+24a^2b^2x^4(2c+3dx^2+7ex^4)-2a^3bx^2(20c+27dx^2+42ex^4}{315a^5x^9}$$

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/315*(Sqrt[a + b*x^2]*(128*b^4*c*x^8 - 16*a*b^3*x^6*(4*c + 9*d*x^2) + 24*
a^2*b^2*x^4*(2*c + 3*d*x^2 + 7*e*x^4) - 2*a^3*b*x^2*(20*c + 27*d*x^2 + 42*e
*x^4 + 105*f*x^6) + a^4*(35*c + 45*d*x^2 + 63*e*x^4 + 105*f*x^6)))/(a^5*x^9
)
```

Maple [A] (verified)

Time = 3.68 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$-\frac{\left(\left(3f\,x^6+\frac{9}{5}e\,x^4+\frac{9}{7}d\,x^2+c\right)a^4-\frac{8\left(\frac{21}{4}f\,x^6+\frac{21}{10}e\,x^4+\frac{27}{20}d\,x^2+c\right)b\,x^2a^3}{7}+\frac{48b^2x^4\left(\frac{7}{2}e\,x^4+\frac{3}{2}d\,x^2+c\right)a^2}{35}-\frac{64b^3x^6\left(\frac{9d\,x^2}{4}+c\right)a}{35}+128\ldots}{9x^9a^5}$
gosper	$-\frac{\sqrt{b\,x^2+a}\,(-210a^3bf\,x^8+168a^2b^2e\,x^8-144a\,b^3d\,x^8+128b^4c\,x^8+105a^4f\,x^6-84a^3be\,x^6+72a^2b^2d\,x^6-64a\,b^3c\,x^6+63a^4e\,x^4\ldots}{315x^9a^5}$
trager	$-\frac{\sqrt{b\,x^2+a}\,(-210a^3bf\,x^8+168a^2b^2e\,x^8-144a\,b^3d\,x^8+128b^4c\,x^8+105a^4f\,x^6-84a^3be\,x^6+72a^2b^2d\,x^6-64a\,b^3c\,x^6+63a^4e\,x^4\ldots}{315x^9a^5}$
risch	$-\frac{\sqrt{b\,x^2+a}\,(-210a^3bf\,x^8+168a^2b^2e\,x^8-144a\,b^3d\,x^8+128b^4c\,x^8+105a^4f\,x^6-84a^3be\,x^6+72a^2b^2d\,x^6-64a\,b^3c\,x^6+63a^4e\,x^4\ldots}{315x^9a^5}$
default	$d\left(-\frac{\sqrt{b\,x^2+a}}{7a\,x^7}-\frac{6b\left(-\frac{\sqrt{b\,x^2+a}}{5a\,x^5}-\frac{4b\left(-\frac{\sqrt{b\,x^2+a}}{3a\,x^3}+\frac{2b\sqrt{b\,x^2+a}}{3a^2x}\right)}{5a}\right)}{7a}\right)+f\left(-\frac{\sqrt{b\,x^2+a}}{3a\,x^3}+\frac{2b\sqrt{b\,x^2+a}}{3a^2x}\right)+e\left(-\frac{\sqrt{b\ldots}}{5\ldots}\right.$

[In] int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/9*((3*f*x^6+9/5*e*x^4+9/7*d*x^2+c)*a^4-8/7*(21/4*f*x^6+21/10*e*x^4+27/20 *d*x^2+c)*b*x^2*a^3+48/35*b^2*x^4*(7/2*e*x^4+3/2*d*x^2+c)*a^2-64/35*b^3*x^6 *(9/4*d*x^2+c)*a+128/35*b^4*c*x^8)*(b*x^2+a)^(1/2)/x^9/a^5

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.75

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx = -\frac{(2\,(64\,b^4c-72\,ab^3d+84\,a^2b^2e-105\,a^3bf)x^8-(64\,ab^3c-72\,a^2b^2d+84\,a^3be-105\,a^4f)x^6+35\,a^4c+\ldots}{315\,a^5x^9}$$

[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] -1/315*(2*(64*b^4*c - 72*a*b^3*d + 84*a^2*b^2*e - 105*a^3*b*f)*x^8 - (64*a* b^3*c - 72*a^2*b^2*d + 84*a^3*b*e - 105*a^4*f)*x^6 + 35*a^4*c + 3*(16*a^2*b ^2*c - 18*a^3*b*d + 21*a^4*e)*x^4 - 5*(8*a^3*b*c - 9*a^4*d)*x^2)*sqrt(b*x^2 + a)/(a^5*x^9)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1642 vs. $2(190) = 380$.

Time = 2.92 (sec) , antiderivative size = 1642, normalized size of antiderivative = 8.69

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^{10}\sqrt{a + bx^2}}\, dx = \text{Too large to display}$$

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a)**(1/2),x)
```

```
[Out] -35*a**8*b**(33/2)*c*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*
b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b**2
0*x**16) - 100*a**7*b**(35/2)*c*x**2*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x
**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14
 + 315*a**5*b**20*x**16) - 98*a**6*b**(37/2)*c*x**4*sqrt(a/(b*x**2) + 1)/(3
15*a**9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a
**6*b**19*x**14 + 315*a**5*b**20*x**16) - 5*a**6*b**(19/2)*d*sqrt(a/(b*x**2
) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35
*a**4*b**12*x**12) - 28*a**5*b**(39/2)*c*x**6*sqrt(a/(b*x**2) + 1)/(315*a**
9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b*
*19*x**14 + 315*a**5*b**20*x**16) - 9*a**5*b**(21/2)*d*x**2*sqrt(a/(b*x**2)
 + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*
a**4*b**12*x**12) - 35*a**4*b**(41/2)*c*x**8*sqrt(a/(b*x**2) + 1)/(315*a**9
*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**
19*x**14 + 315*a**5*b**20*x**16) - 5*a**4*b**(23/2)*d*x**4*sqrt(a/(b*x**2)
+ 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a
**4*b**12*x**12) - 3*a**4*b**(9/2)*e*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**
4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 280*a**3*b**(43/2)*c*x**10*sqr
t(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*
b**18*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b**20*x**16) + 5*a**3*b**(25
/2)*d*x**6*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 +
105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 2*a**3*b**(11/2)*e*x**2*sqrt(
a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8)
 - 560*a**2*b**(45/2)*c*x**12*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1
260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14 + 315*
a**5*b**20*x**16) + 30*a**2*b**(27/2)*d*x**8*sqrt(a/(b*x**2) + 1)/(35*a**7*
b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**1
2) - 3*a**2*b**(13/2)*e*x**4*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a
**4*b**5*x**6 + 15*a**3*b**6*x**8) - 448*a*b**(47/2)*c*x**14*sqrt(a/(b*x**2
) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12
 + 1260*a**6*b**19*x**14 + 315*a**5*b**20*x**16) + 40*a*b**(29/2)*d*x**10*s
qrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**
11*x**10 + 35*a**4*b**12*x**12) - 12*a*b**(15/2)*e*x**6*sqrt(a/(b*x**2) + 1
)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 128*b**(49/
2)*c*x**16*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**1
```

```
0 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b**20*x**16) +
 16*b**(31/2)*d*x**12*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b*
*10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 8*b**(17/2)*e*x**8
*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6
*x**8) - sqrt(b)*f*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*b**(3/2)*f*sqrt(a/(b
*x**2) + 1)/(3*a**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.46

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx = -\frac{128\sqrt{bx^2+a}b^4c}{315\,a^5x} + \frac{16\sqrt{bx^2+a}b^3d}{35\,a^4x} - \frac{8\sqrt{bx^2+a}b^2e}{15\,a^3x} + \frac{2\sqrt{bx^2+a}bf}{3\,a^2x} + \frac{64\sqrt{bx^2+a}b^3c}{315\,a^4x^3} - \frac{8\sqrt{bx^2+a}b^2d}{35\,a^3x^3} + \frac{4\sqrt{bx^2+a}be}{15\,a^2x^3} - \frac{\sqrt{bx^2+a}f}{3\,ax^3} - \frac{16\sqrt{bx^2+a}b^2c}{105\,a^3x^5} + \frac{6\sqrt{bx^2+a}bd}{35\,a^2x^5} - \frac{\sqrt{bx^2+a}e}{5\,ax^5} + \frac{8\sqrt{bx^2+a}bc}{63\,a^2x^7} - \frac{\sqrt{bx^2+a}d}{7\,ax^7} - \frac{\sqrt{bx^2+a}c}{9\,ax^9}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -128/315*sqrt(b*x^2 + a)*b^4*c/(a^5*x) + 16/35*sqrt(b*x^2 + a)*b^3*d/(a^4*x
) - 8/15*sqrt(b*x^2 + a)*b^2*e/(a^3*x) + 2/3*sqrt(b*x^2 + a)*b*f/(a^2*x) +
64/315*sqrt(b*x^2 + a)*b^3*c/(a^4*x^3) - 8/35*sqrt(b*x^2 + a)*b^2*d/(a^3*x^
3) + 4/15*sqrt(b*x^2 + a)*b*e/(a^2*x^3) - 1/3*sqrt(b*x^2 + a)*f/(a*x^3) - 1
6/105*sqrt(b*x^2 + a)*b^2*c/(a^3*x^5) + 6/35*sqrt(b*x^2 + a)*b*d/(a^2*x^5)
- 1/5*sqrt(b*x^2 + a)*e/(a*x^5) + 8/63*sqrt(b*x^2 + a)*b*c/(a^2*x^7) - 1/7*
sqrt(b*x^2 + a)*d/(a*x^7) - 1/9*sqrt(b*x^2 + a)*c/(a*x^9)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 660 vs. 2(169) = 338.

Time = 0.34 (sec) , antiderivative size = 660, normalized size of antiderivative = 3.49

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx = \frac{4\left(315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}b^{\frac{3}{2}}f + 840\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}b^{\frac{5}{2}}e - 1995\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}ab^{\frac{3}{2}}f + 252\right.}{}$$

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 4/315*(315*(sqrt(b)*x - sqrt(b*x^2 + a))^14*b^(3/2)*f + 840*(sqrt(b)*x - sq
rt(b*x^2 + a))^12*b^(5/2)*e - 1995*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a*b^(3/
2)*f + 2520*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(7/2)*d - 3780*(sqrt(b)*x -
sqrt(b*x^2 + a))^10*a*b^(5/2)*e + 5355*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2
*b^(3/2)*f + 8064*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(9/2)*c - 6552*(sqrt(b)
*x - sqrt(b*x^2 + a))^8*a*b^(7/2)*d + 6804*(sqrt(b)*x - sqrt(b*x^2 + a))^8*
a^2*b^(5/2)*e - 7875*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(3/2)*f - 5376*(
sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(9/2)*c + 6048*(sqrt(b)*x - sqrt(b*x^2 +
 a))^6*a^2*b^(7/2)*d - 6216*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*b^(5/2)*e +
 6825*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(3/2)*f + 2304*(sqrt(b)*x - sqr
t(b*x^2 + a))^4*a^2*b^(9/2)*c - 2592*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^
(7/2)*d + 3024*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^(5/2)*e - 3465*(sqrt(b
)*x - sqrt(b*x^2 + a))^4*a^5*b^(3/2)*f - 576*(sqrt(b)*x - sqrt(b*x^2 + a))^
2*a^3*b^(9/2)*c + 648*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*b^(7/2)*d - 756*(
sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(5/2)*e + 945*(sqrt(b)*x - sqrt(b*x^2
+ a))^2*a^6*b^(3/2)*f + 64*a^4*b^(9/2)*c - 72*a^5*b^(7/2)*d + 84*a^6*b^(5/2
)*e - 105*a^7*b^(3/2)*f)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^9
```

Mupad [B] (verification not implemented)

Time = 6.14 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx = \frac{\sqrt{b\,x^2+a}\,\left(-105\,f\,a^3+84\,e\,a^2\,b-72\,d\,a\,b^2+64\,c\,b^3\right)}{315\,a^4\,x^3} - \frac{\sqrt{b\,x^2+a}\,\left(9\,a\,d-8\,b\,c\right)}{63\,a^2\,x^7} - \frac{\sqrt{b\,x^2+a}\,\left(21\,e\,a^2-18\,d\,a\,b+16\,c\,b^2\right)}{105\,a^3\,x^5} - \frac{\sqrt{b\,x^2+a}\,\left(-210\,f\,a^3\,b+168\,e\,a^2\,b^2-144\,d\,a\,b^3+128\,c\,b^4\right)}{315\,a^5\,x} - \frac{c\,\sqrt{b\,x^2+a}}{9\,a\,x^9}$$

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^(1/2)),x)
```

```
[Out] ((a + b*x^2)^(1/2)*(64*b^3*c - 105*a^3*f - 72*a*b^2*d + 84*a^2*b*e))/(315*a
^4*x^3) - ((a + b*x^2)^(1/2)*(9*a*d - 8*b*c))/(63*a^2*x^7) - ((a + b*x^2)^(
1/2)*(16*b^2*c + 21*a^2*e - 18*a*b*d))/(105*a^3*x^5) - ((a + b*x^2)^(1/2)*(
128*b^4*c + 168*a^2*b^2*e - 144*a*b^3*d - 210*a^3*b*f))/(315*a^5*x) - (c*(a
 + b*x^2)^(1/2))/(9*a*x^9)
```

3.159 $\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

Optimal result 969
Rubi [A] (verified) 970
Mathematica [A] (verified) 974
Maple [A] (verified) 975
Fricas [A] (verification not implemented) 977
Sympy [F(-1)] 978
Maxima [B] (verification not implemented) 978
Giac [A] (verification not implemented) 979
Mupad [F(-1)] 979

Optimal result

Integrand size = 32, antiderivative size = 381

$$\begin{aligned}\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx =& \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^9}{7a\,(a+bx^2)^{7/2}} \\ &-\frac{(2Ab^3-a(9b^2B-16abC+23a^2D))\,x^9}{35a^2b^3\,(a+bx^2)^{5/2}} \\ &-\frac{(16Ab^3-3a(24b^2B-66abC+143a^2D))\,x^7}{210a^2b^4\,(a+bx^2)^{3/2}} \\ &+\frac{Dx^9}{6b^3\,(a+bx^2)^{3/2}}-\frac{(16Ab^3-3a(24b^2B-66abC+143a^2D))\,x^5}{30a^2b^5\sqrt{a+bx^2}} \\ &-\frac{(16Ab^3-3a(24b^2B-66abC+143a^2D))\,x\sqrt{a+bx^2}}{16ab^7} \\ &+\frac{(16Ab^3-3a(24b^2B-66abC+143a^2D))\,x^3\sqrt{a+bx^2}}{24a^2b^6} \\ &+\frac{(16Ab^3-72ab^2B+198a^2bC-429a^3D)\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{15/2}}\end{aligned}$$

```
[Out] 1/7*(A-a*(B*b^2-C*a*b+D*a^2)/b^3)*x^9/a/(b*x^2+a)^(7/2)-1/35*(2*A*b^3-a*(9*
B*b^2-16*C*a*b+23*D*a^2))*x^9/a^2/b^3/(b*x^2+a)^(5/2)-1/210*(16*A*b^3-3*a*(
24*B*b^2-66*C*a*b+143*D*a^2))*x^7/a^2/b^4/(b*x^2+a)^(3/2)+1/6*D*x^9/b^3/(b*
x^2+a)^(3/2)+1/16*(16*A*b^3-72*B*a*b^2+198*C*a^2*b-429*D*a^3)*arctanh(x*b^(
1/2)/(b*x^2+a)^(1/2))/b^(15/2)-1/30*(16*A*b^3-3*a*(24*B*b^2-66*C*a*b+143*D*
a^2))*x^5/a^2/b^5/(b*x^2+a)^(1/2)-1/16*(16*A*b^3-3*a*(24*B*b^2-66*C*a*b+143
*D*a^2))*x*(b*x^2+a)^(1/2)/a/b^7+1/24*(16*A*b^3-3*a*(24*B*b^2-66*C*a*b+143*
D*a^2))*x^3*(b*x^2+a)^(1/2)/a^2/b^6
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {1818, 1599, 1277, 1598, 470, 294, 327, 223, 212}

$$\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = -\frac{x^9(2Ab^3-a(23a^2D-16abC+9b^2B))}{35a^2b^3(a+bx^2)^{5/2}} + \frac{x^9\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a(a+bx^2)^{7/2}} - \frac{x\sqrt{a+bx^2}(16Ab^3-3a(143a^2D-66abC+24b^2B))}{16ab^7} + \frac{x^3\sqrt{a+bx^2}(16Ab^3-3a(143a^2D-66abC+24b^2B))}{24a^2b^6} - \frac{x^5(16Ab^3-3a(143a^2D-66abC+24b^2B))}{30a^2b^5\sqrt{a+bx^2}} - \frac{x^7(16Ab^3-3a(143a^2D-66abC+24b^2B))}{210a^2b^4(a+bx^2)^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-429a^3D+198a^2bC-72ab^2B+16Ab^3)}{16b^{15/2}} + \frac{Dx^9}{6b^3(a+bx^2)^{3/2}}$$

```
[In] Int[(x^8*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^9)/(7*a*(a + b*x^2)^(7/2)) - ((2*A
*b^3 - a*(9*b^2*B - 16*a*b*C + 23*a^2*D))*x^9)/(35*a^2*b^3*(a + b*x^2)^(5/2
)) - ((16*A*b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*x^7)/(210*a^2*b^4*
(a + b*x^2)^(3/2)) + (D*x^9)/(6*b^3*(a + b*x^2)^(3/2)) - ((16*A*b^3 - 3*a*(
24*b^2*B - 66*a*b*C + 143*a^2*D))*x^5)/(30*a^2*b^5*Sqrt[a + b*x^2]) - ((16*
A*b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*x*Sqrt[a + b*x^2])/(16*a*b^7
) + ((16*A*b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*x^3*Sqrt[a + b*x^2]
)/(24*a^2*b^6) + ((16*A*b^3 - 72*a*b^2*B + 198*a^2*b*C - 429*a^3*D)*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(15/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, Simp[(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d
*f*(q + 1))), x] + Dist[f/(2*d*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1] &
& GtQ[m, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^9}{7a\,(a+bx^2)^{7/2}}-\frac{\int\frac{x^7\left(\left(2Ab-\frac{9a\left(b^2B-abC+a^2D\right)}{b^2}\right)x-7a\left(C-\frac{aD}{b}\right)x^3-7aDx^5\right)}{(a+bx^2)^{7/2}}\,dx}{7ab}$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^9}{7a\,(a+bx^2)^{7/2}}-\frac{\int\frac{x^8\left(2Ab-\frac{9a\left(b^2B-abC+a^2D\right)}{b^2}-7a\left(C-\frac{aD}{b}\right)x^2-7aDx^4\right)}{(a+bx^2)^{7/2}}\,dx}{7ab}$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^9}{7a\,(a+bx^2)^{7/2}}-\frac{(2Ab^3-a(9b^2B-16abC+23a^2D))\,x^9}{35a^2b^3\,(a+bx^2)^{5/2}}+\frac{\int\frac{x^7\left(\left(8Ab-\frac{9a\left(4b^2B-11abC+18a^2D\right)}{b^2}\right)x+\frac{35a^2Dx^3}{b}\right)}{(a+bx^2)^{5/2}}\,dx}{35a^2b}$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^9}{7a\,(a+bx^2)^{7/2}}-\frac{(2Ab^3-a(9b^2B-16abC+23a^2D))\,x^9}{35a^2b^3\,(a+bx^2)^{5/2}}+\frac{\int\frac{x^8\left(8Ab-\frac{9a\left(4b^2B-11abC+18a^2D\right)}{b^2}+\frac{35a^2Dx^2}{b}\right)}{(a+bx^2)^{5/2}}\,dx}{35a^2b}$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^9}{7a\,(a+bx^2)^{7/2}}-\frac{(2Ab^3-a(9b^2B-16abC+23a^2D))\,x^9}{35a^2b^3\,(a+bx^2)^{5/2}}+\frac{Dx^9}{6b^3\,(a+bx^2)^{3/2}}-\frac{\left(\frac{315a^3D}{b}-6b\left(8Ab-\frac{9a(4b^2B-11abC+18a^2D)}{b^2}\right)\right)\int\frac{x^8}{(a+bx^2)^{5/2}}\,dx}{210a^2b^2}$$

$$= \frac{\left(A - \frac{a(b^2B - abC + a^2D)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3 - a(9b^2B - 16abC + 23a^2D)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} - \frac{\left(16Ab^3 - 72ab^2B + 198a^2bC - 429a^3D\right)x^7}{210a^2b^4\left(a+bx^2\right)^{3/2}} + \frac{Dx^9}{6b^3\left(a+bx^2\right)^{3/2}} - \frac{\left(\frac{315a^3D}{b} - 6b\left(8Ab - \frac{9a(4b^2B - 11abC + 18a^2D)}{b^2}\right)\right)\int\frac{x^6}{(a+bx^2)^{3/2}}\,dx}{90a^2b^3}$$

$$= \frac{\left(A - \frac{a(b^2B - abC + a^2D)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3 - a(9b^2B - 16abC + 23a^2D)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} - \frac{\left(16Ab^3 - 72ab^2B + 198a^2bC - 429a^3D\right)x^7}{210a^2b^4\left(a+bx^2\right)^{3/2}} + \frac{Dx^9}{6b^3\left(a+bx^2\right)^{3/2}} - \frac{\left(16Ab^3 - 72ab^2B + 198a^2bC - 429a^3D\right)x^5}{30a^2b^5\sqrt{a+bx^2}} - \frac{\left(\frac{315a^3D}{b} - 6b\left(8Ab - \frac{9a(4b^2B - 11abC + 18a^2D)}{b^2}\right)\right)\int\frac{x^4}{\sqrt{a+bx^2}}\,dx}{18a^2b^4}$$

$$= \frac{\left(A - \frac{a(b^2B - abC + a^2D)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3 - a(9b^2B - 16abC + 23a^2D)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} - \frac{\left(16Ab^3 - 72ab^2B + 198a^2bC - 429a^3D\right)x^7}{210a^2b^4\left(a+bx^2\right)^{3/2}} + \frac{Dx^9}{6b^3\left(a+bx^2\right)^{3/2}} - \frac{\left(16Ab^3 - 72ab^2B + 198a^2bC - 429a^3D\right)x^5}{30a^2b^5\sqrt{a+bx^2}} + \frac{\left(16Ab^3 - 72ab^2B + 198a^2bC - 429a^3D\right)x^3\sqrt{a+bx^2}}{24a^2b^6} + \frac{\left(\frac{315a^3D}{b} - 6b\left(8Ab - \frac{9a(4b^2B - 11abC + 18a^2D)}{b^2}\right)\right)\int\frac{x^2}{\sqrt{a+bx^2}}\,dx}{24ab^5}$$

$$= \frac{\left(A - \frac{a(b^2B - abC + a^2D)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3 - a(9b^2B - 16abC + 23a^2D)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} - \frac{\left(16Ab^3 - 72ab^2B + 198a^2bC - 429a^3D\right)x^7}{210a^2b^4\left(a+bx^2\right)^{3/2}} + \frac{Dx^9}{6b^3\left(a+bx^2\right)^{3/2}} - \frac{\left(16Ab^3 - 72ab^2B + 198a^2bC - 429a^3D\right)x^5}{30a^2b^5\sqrt{a+bx^2}} - \frac{\left(16Ab^3 - 72ab^2B + 198a^2bC - 429a^3D\right)x\sqrt{a+bx^2}}{16ab^7} + \frac{\left(16Ab^3 - 72ab^2B + 198a^2bC - 429a^3D\right)x^3\sqrt{a+bx^2}}{24a^2b^6} + \frac{\left(16Ab^3 - 72ab^2B + 198a^2bC - 429a^3D\right)\int\frac{1}{\sqrt{a+bx^2}}\,dx}{16b^7}$$

$$= \frac{\left(A - \frac{a(b^2B - abC + a^2D)}{b^3}\right)x^9}{7a(a+bx^2)^{7/2}} - \frac{(2Ab^3 - a(9b^2B - 16abC + 23a^2D))x^9}{35a^2b^3(a+bx^2)^{5/2}} - \frac{(16Ab^3 - 72ab^2B + 198a^2bC - 429a^3D)x^7}{210a^2b^4(a+bx^2)^{3/2}} + \frac{Dx^9}{6b^3(a+bx^2)^{3/2}} - \frac{(16Ab^3 - 72ab^2B + 198a^2bC - 429a^3D)x^5}{30a^2b^5\sqrt{a+bx^2}} - \frac{(16Ab^3 - 72ab^2B + 198a^2bC - 429a^3D)x\sqrt{a+bx^2}}{16ab^7} + \frac{(16Ab^3 - 72ab^2B + 198a^2bC - 429a^3D)x^3\sqrt{a+bx^2}}{24a^2b^6} + \frac{(16Ab^3 - 72ab^2B + 198a^2bC - 429a^3D)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{16b^7}$$

$$= \frac{\left(A - \frac{a(b^2B - abC + a^2D)}{b^3}\right)x^9}{7a(a+bx^2)^{7/2}} - \frac{(2Ab^3 - a(9b^2B - 16abC + 23a^2D))x^9}{35a^2b^3(a+bx^2)^{5/2}} - \frac{(16Ab^3 - 72ab^2B + 198a^2bC - 429a^3D)x^7}{210a^2b^4(a+bx^2)^{3/2}} + \frac{Dx^9}{6b^3(a+bx^2)^{3/2}} - \frac{(16Ab^3 - 72ab^2B + 198a^2bC - 429a^3D)x^5}{30a^2b^5\sqrt{a+bx^2}} - \frac{(16Ab^3 - 72ab^2B + 198a^2bC - 429a^3D)x\sqrt{a+bx^2}}{16ab^7} + \frac{(16Ab^3 - 72ab^2B + 198a^2bC - 429a^3D)x^3\sqrt{a+bx^2}}{24a^2b^6} + \frac{(16Ab^3 - 72ab^2B + 198a^2bC - 429a^3D)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{15/2}}$$

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.68

$$\int \frac{x^8(A + Bx^2 + Cx^4 + Dx^6)}{(a+bx^2)^{9/2}}\,dx = \frac{x(45045a^6D - 2310a^5b(9C - 65Dx^2) + 42a^4b^2(180B - 1650Cx^2 + 4147}{} + \frac{(16Ab^3 - 3a(24b^2B - 66abC + 143a^2D))\,\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{8b^{15/2}}$$

```
[In] Integrate[(x^8*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] (x*(45045*a^6*D - 2310*a^5*b*(9*C - 65*D*x^2) + 42*a^4*b^2*(180*B - 1650*C*
x^2 + 4147*D*x^4) - 12*a^3*b^3*(140*A - 2100*B*x^2 + 6699*C*x^4 - 6292*D*x^
6) - 2*a*b^5*x^4*(3248*A - 6336*B*x^2 + 1155*C*x^4 + 455*D*x^6) + a^2*b^4*x
```

```
^2*(-5600*A + 29232*B*x^2 - 34848*C*x^4 + 5005*D*x^6) + 4*b^6*x^6*(-704*A +
 35*(6*B*x^2 + 3*C*x^4 + 2*D*x^6))))/(1680*b^7*(a + b*x^2)^(7/2)) + ((16*A*
b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a]
+ Sqrt[a + b*x^2])])/(8*b^(15/2))
```

Maple [A] (verified)

Time = 3.75 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$\frac{-\frac{58a\,x^5\left(\frac{65}{464}Dx^6+\frac{165}{464}C\,x^4-\frac{396}{203}x^2B+A\right)b^{\frac{11}{2}}}{15}+\left(\frac{1}{6}Dx^{13}+\frac{1}{4}C\,x^{11}+\frac{1}{2}B\,x^9-\frac{176}{105}A\,x^7\right)b^{\frac{13}{2}}-\frac{99\left(-\frac{65Dx^2}{9}+C\right)a^5x\,b^{\frac{3}{2}}}{8}+\frac{9a^4x\left(\frac{4147}{180}\right.}{}}{}$
default	$A\left(-\frac{x^7}{7b(b\,x^2+a)^{\frac{7}{2}}}+\frac{-\frac{x^5}{5b\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{-\frac{x^3}{3b\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}}{b}}{b}}{b}\right)+D\left(\frac{x^{13}}{6b(b\,x^2+a)^{\frac{7}{2}}}-\frac{13a}{}\right.$

```
[In] int(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b*x^2+a)^(7/2)/b^(15/2)*(-58/15*a*x^5*(65/464*D*x^6+165/464*C*x^4-396/20
3*x^2*B+A)*b^(11/2)+(1/6*D*x^13+1/4*C*x^11+1/2*B*x^9-176/105*A*x^7)*b^(13/2
)-99/8*(-65/9*D*x^2+C)*a^5*x*b^(3/2)+9/2*a^4*x*(4147/180*D*x^4-55/6*C*x^2+B
)*b^(5/2)-a^3*(-1573/35*D*x^6+957/20*C*x^4-15*x^2*B+A)*x*b^(7/2)-10/3*a^2*x
^3*(-143/160*D*x^6+1089/175*C*x^4-261/50*x^2*B+A)*b^(9/2)+429/16*D*b^(1/2)*
a^6*x+(b^3*A-9/2*a*b^2*B+99/8*C*a^2*b-429/16*D*a^3)*(b*x^2+a)^(7/2)*arctanh
((b*x^2+a)^(1/2)/x/b^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 987, normalized size of antiderivative = 2.59

$$\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \left[\frac{105\left(\left(429\,Da^3b^4-198\,Ca^2b^5+72\,Bab^6-16\,Ab^7\right)x^8+429\,Da^7-198\right.}{}\right.$$

```
[In] integrate(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/3360*(105*((429*D*a^3*b^4 - 198*C*a^2*b^5 + 72*B*a*b^6 - 16*A*b^7)*x^8 +
 429*D*a^7 - 198*C*a^6*b + 72*B*a^5*b^2 - 16*A*a^4*b^3 + 4*(429*D*a^4*b^3 -
 198*C*a^3*b^4 + 72*B*a^2*b^5 - 16*A*a*b^6)*x^6 + 6*(429*D*a^5*b^2 - 198*C*
a^4*b^3 + 72*B*a^3*b^4 - 16*A*a^2*b^5)*x^4 + 4*(429*D*a^6*b - 198*C*a^5*b^2
 + 72*B*a^4*b^3 - 16*A*a^3*b^4)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 +
a)*sqrt(b)*x - a) + 2*(280*D*b^7*x^13 - 70*(13*D*a*b^6 - 6*C*b^7)*x^11 + 35
*(143*D*a^2*b^5 - 66*C*a*b^6 + 24*B*b^7)*x^9 + 176*(429*D*a^3*b^4 - 198*C*a
^2*b^5 + 72*B*a*b^6 - 16*A*b^7)*x^7 + 406*(429*D*a^4*b^3 - 198*C*a^3*b^4 +
72*B*a^2*b^5 - 16*A*a*b^6)*x^5 + 350*(429*D*a^5*b^2 - 198*C*a^4*b^3 + 72*B*
a^3*b^4 - 16*A*a^2*b^5)*x^3 + 105*(429*D*a^6*b - 198*C*a^5*b^2 + 72*B*a^4*b
^3 - 16*A*a^3*b^4)*x)*sqrt(b*x^2 + a))/(b^12*x^8 + 4*a*b^11*x^6 + 6*a^2*b^1
0*x^4 + 4*a^3*b^9*x^2 + a^4*b^8), 1/1680*(105*((429*D*a^3*b^4 - 198*C*a^2*b
^5 + 72*B*a*b^6 - 16*A*b^7)*x^8 + 429*D*a^7 - 198*C*a^6*b + 72*B*a^5*b^2 -
16*A*a^4*b^3 + 4*(429*D*a^4*b^3 - 198*C*a^3*b^4 + 72*B*a^2*b^5 - 16*A*a*b^6
)*x^6 + 6*(429*D*a^5*b^2 - 198*C*a^4*b^3 + 72*B*a^3*b^4 - 16*A*a^2*b^5)*x^4
 + 4*(429*D*a^6*b - 198*C*a^5*b^2 + 72*B*a^4*b^3 - 16*A*a^3*b^4)*x^2)*sqrt(
-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (280*D*b^7*x^13 - 70*(13*D*a*b^6 -
 6*C*b^7)*x^11 + 35*(143*D*a^2*b^5 - 66*C*a*b^6 + 24*B*b^7)*x^9 + 176*(429*
D*a^3*b^4 - 198*C*a^2*b^5 + 72*B*a*b^6 - 16*A*b^7)*x^7 + 406*(429*D*a^4*b^3
 - 198*C*a^3*b^4 + 72*B*a^2*b^5 - 16*A*a*b^6)*x^5 + 350*(429*D*a^5*b^2 - 19
8*C*a^4*b^3 + 72*B*a^3*b^4 - 16*A*a^2*b^5)*x^3 + 105*(429*D*a^6*b - 198*C*a
^5*b^2 + 72*B*a^4*b^3 - 16*A*a^3*b^4)*x)*sqrt(b*x^2 + a))/(b^12*x^8 + 4*a*b
^11*x^6 + 6*a^2*b^10*x^4 + 4*a^3*b^9*x^2 + a^4*b^8)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**8*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1221 vs. $2(345) = 690$.

Time = 0.22 (sec) , antiderivative size = 1221, normalized size of antiderivative = 3.20

$$\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
[Out] 1/6*D*x^13/((b*x^2 + a)^(7/2)*b) - 13/24*D*a*x^11/((b*x^2 + a)^(7/2)*b^2) +
 1/4*C*x^11/((b*x^2 + a)^(7/2)*b) + 143/48*D*a^2*x^9/((b*x^2 + a)^(7/2)*b^3
) - 11/8*C*a*x^9/((b*x^2 + a)^(7/2)*b^2) + 1/2*B*x^9/((b*x^2 + a)^(7/2)*b)
- 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 5
6*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*A*x + 4
29/560*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 5
6*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*D*a^3*x
/b^3 - 99/280*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b
^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*
C*a^2*x/b^2 + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7
/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b
^4))*B*a*x/b + 143/80*D*a^3*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*
x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^4 - 33/40*C*a^2*x*(1
5*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*
x^2 + a)^(5/2)*b^3))/b^3 + 3/10*B*a*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*
x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2 - 1/15*A*x
*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/(
(b*x^2 + a)^(5/2)*b^3))/b + 143/16*D*a^3*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2
*a/((b*x^2 + a)^(3/2)*b^2))/b^5 - 33/8*C*a^2*x*(3*x^2/((b*x^2 + a)^(3/2)*b)
 + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^4 + 3/2*B*a*x*(3*x^2/((b*x^2 + a)^(3/2)*b
) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3 - 1/3*A*x*(3*x^2/((b*x^2 + a)^(3/2)*b)
 + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 429/16*D*a^4*x^3/((b*x^2 + a)^(5/2)*b
^6) - 99/8*C*a^3*x^3/((b*x^2 + a)^(5/2)*b^5) + 9/2*B*a^2*x^3/((b*x^2 + a)^(
5/2)*b^4) - A*a*x^3/((b*x^2 + a)^(5/2)*b^3) - 19877/560*D*a^3*x/(sqrt(b*x^2
 + a)*b^7) - 2431/560*D*a^4*x/((b*x^2 + a)^(3/2)*b^7) + 12441/560*D*a^5*x/(
```

```
(b*x^2 + a)^(5/2)*b^7) + 4587/280*C*a^2*x/(sqrt(b*x^2 + a)*b^6) + 561/280*C
*a^3*x/((b*x^2 + a)^(3/2)*b^6) - 2871/280*C*a^4*x/((b*x^2 + a)^(5/2)*b^6) -
 417/70*B*a*x/(sqrt(b*x^2 + a)*b^5) - 51/70*B*a^2*x/((b*x^2 + a)^(3/2)*b^5)
 + 261/70*B*a^3*x/((b*x^2 + a)^(5/2)*b^5) + 139/105*A*x/(sqrt(b*x^2 + a)*b^
4) + 17/105*A*a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*A*a^2*x/((b*x^2 + a)^(5/2
)*b^4) - 429/16*D*a^3*arcsinh(b*x/sqrt(a*b))/b^(15/2) + 99/8*C*a^2*arcsinh(
b*x/sqrt(a*b))/b^(13/2) - 9/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(11/2) + A*arcsi
nh(b*x/sqrt(a*b))/b^(9/2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.90

$$\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(\left(\left(35\left(2\left(\frac{4Dx^2}{b}-\frac{13Da^4b^{11}-6Ca^3b^{12}}{a^3b^{13}}\right)x^2+\frac{143Da^5b^{10}-66Ca^4b^{11}+24Ba^3b^{12}}{a^3b^{13}}\right)\right.\right.\right.\right.}{}$$
$$+\frac{(429\,Da^3-198\,Ca^2b+72\,Bab^2-16\,Ab^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{15}{2}}}$$

```
[In] integrate(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/1680*((((35*(2*(4*D*x^2/b - (13*D*a^4*b^11 - 6*C*a^3*b^12)/(a^3*b^13))*x^
2 + (143*D*a^5*b^10 - 66*C*a^4*b^11 + 24*B*a^3*b^12)/(a^3*b^13))*x^2 + 176*
(429*D*a^6*b^9 - 198*C*a^5*b^10 + 72*B*a^4*b^11 - 16*A*a^3*b^12)/(a^3*b^13)
)*x^2 + 406*(429*D*a^7*b^8 - 198*C*a^6*b^9 + 72*B*a^5*b^10 - 16*A*a^4*b^11)
/(a^3*b^13))*x^2 + 350*(429*D*a^8*b^7 - 198*C*a^7*b^8 + 72*B*a^6*b^9 - 16*A
*a^5*b^10)/(a^3*b^13))*x^2 + 105*(429*D*a^9*b^6 - 198*C*a^8*b^7 + 72*B*a^7*
b^8 - 16*A*a^6*b^9)/(a^3*b^13))*x/(b*x^2 + a)^(7/2) + 1/16*(429*D*a^3 - 198
*C*a^2*b + 72*B*a*b^2 - 16*A*b^3)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^
(15/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \int \frac{x^8\,(A+B\,x^2+C\,x^4+x^6\,D)}{(b\,x^2+a)^{9/2}}\,dx$$

```
[In] int((x^8*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2),x)
```

```
[Out] int((x^8*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2), x)
```

3.160 $\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}} dx$

Optimal result 980
Rubi [A] (verified) 981
Mathematica [A] (verified) 984
Maple [A] (verified) 985
Fricas [A] (verification not implemented) 987
Sympy [F(-1)] 988
Maxima [B] (verification not implemented) 988
Giac [A] (verification not implemented) 989
Mupad [F(-1)] 989

Optimal result

Integrand size = 32, antiderivative size = 279

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx=\frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^7}{7a(a+bx^2)^{7/2}}+\frac{(b^2B-2abC+3a^2D)\,x^7}{5ab^3(a+bx^2)^{5/2}}+\frac{(8b^2B-36abC+99a^2D)\,x^5}{60ab^4(a+bx^2)^{3/2}}+\frac{Dx^7}{4b^3(a+bx^2)^{3/2}}+\frac{(8b^2B-36abC+99a^2D)\,x^3}{12ab^5\sqrt{a+bx^2}}-\frac{(8b^2B-36abC+99a^2D)\,x\sqrt{a+bx^2}}{8ab^6}+\frac{(8b^2B-36abC+99a^2D)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{13/2}}$$

```
[Out] 1/7*(A-a*(B*b^2-C*a*b+D*a^2)/b^3)*x^7/a/(b*x^2+a)^(7/2)+1/5*(B*b^2-2*C*a*b+
3*D*a^2)*x^7/a/b^3/(b*x^2+a)^(5/2)+1/60*(8*B*b^2-36*C*a*b+99*D*a^2)*x^5/a/b
^4/(b*x^2+a)^(3/2)+1/4*D*x^7/b^3/(b*x^2+a)^(3/2)+1/8*(8*B*b^2-36*C*a*b+99*D
*a^2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(13/2)+1/12*(8*B*b^2-36*C*a*b+99
*D*a^2)*x^3/a/b^5/(b*x^2+a)^(1/2)-1/8*(8*B*b^2-36*C*a*b+99*D*a^2)*x*(b*x^2+
a)^(1/2)/a/b^6
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {1818, 1599, 1277, 1598, 470, 294, 327, 223, 212}

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \frac{x^7\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\,(a+bx^2)^{7/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(99a^2D-36abC+8b^2B)}{8b^{13/2}} - \frac{x\sqrt{a+bx^2}(99a^2D-36abC+8b^2B)}{8ab^6} + \frac{x^3(99a^2D-36abC+8b^2B)}{12ab^5\sqrt{a+bx^2}} + \frac{x^5(99a^2D-36abC+8b^2B)}{60ab^4\,(a+bx^2)^{3/2}} + \frac{x^7(3a^2D-2abC+b^2B)}{5ab^3\,(a+bx^2)^{5/2}} + \frac{Dx^7}{4b^3\,(a+bx^2)^{3/2}}$$

```
[In] Int[(x^6*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^7)/(7*a*(a + b*x^2)^(7/2)) + ((b^2
*B - 2*a*b*C + 3*a^2*D)*x^7)/(5*a*b^3*(a + b*x^2)^(5/2)) + ((8*b^2*B - 36*a
*b*C + 99*a^2*D)*x^5)/(60*a*b^4*(a + b*x^2)^(3/2)) + (D*x^7)/(4*b^3*(a + b*
x^2)^(3/2)) + ((8*b^2*B - 36*a*b*C + 99*a^2*D)*x^3)/(12*a*b^5*Sqrt[a + b*x^
2]) - ((8*b^2*B - 36*a*b*C + 99*a^2*D)*x*Sqrt[a + b*x^2])/(8*a*b^6) + ((8*b
^2*B - 36*a*b*C + 99*a^2*D)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(13/
2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, Simp[(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d
*f*(q + 1))), x] + Dist[f/(2*d*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1] &
& GtQ[m, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
```

`b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]`

Rubi steps

$$\text{integral} = \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^7}{7a(a+bx^2)^{7/2}} - \frac{\int \frac{x^5\left(-7a\left(B-\frac{a(bC-aD)}{b^2}\right)x-7a\left(C-\frac{aD}{b}\right)x^3-7aDx^5\right)}{(a+bx^2)^{7/2}}\,dx}{7ab}$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^7}{7a(a+bx^2)^{7/2}} - \frac{\int \frac{x^6\left(-7a\left(B-\frac{a(bC-aD)}{b^2}\right)-7a\left(C-\frac{aD}{b}\right)x^2-7aDx^4\right)}{(a+bx^2)^{7/2}}\,dx}{7ab}$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^7}{7a(a+bx^2)^{7/2}} + \frac{(b^2B-2abC+3a^2D)\,x^7}{5ab^3(a+bx^2)^{5/2}} + \frac{\int \frac{x^5\left(-7a\left(2B-\frac{a(9bC-16aD)}{b^2}\right)x+\frac{35a^2Dx^3}{b}\right)}{(a+bx^2)^{5/2}}\,dx}{35a^2b}$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^7}{7a(a+bx^2)^{7/2}} + \frac{(b^2B-2abC+3a^2D)\,x^7}{5ab^3(a+bx^2)^{5/2}} + \frac{\int \frac{x^6\left(-7a\left(2B-\frac{a(9bC-16aD)}{b^2}\right)+\frac{35a^2Dx^2}{b}\right)}{(a+bx^2)^{5/2}}\,dx}{35a^2b}$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^7}{7a(a+bx^2)^{7/2}} + \frac{(b^2B-2abC+3a^2D)\,x^7}{5ab^3(a+bx^2)^{5/2}} + \frac{Dx^7}{4b^3(a+bx^2)^{3/2}} - \frac{(8b^2B-36abC+99a^2D)\int \frac{x^6}{(a+bx^2)^{5/2}}\,dx}{20ab^3}$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^7}{7a(a+bx^2)^{7/2}} + \frac{(b^2B-2abC+3a^2D)\,x^7}{5ab^3(a+bx^2)^{5/2}} + \frac{(8b^2B-36abC+99a^2D)\,x^5}{60ab^4(a+bx^2)^{3/2}} + \frac{Dx^7}{4b^3(a+bx^2)^{3/2}} - \frac{(8b^2B-36abC+99a^2D)\int \frac{x^4}{(a+bx^2)^{3/2}}\,dx}{12ab^4}$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^7}{7a(a+bx^2)^{7/2}} + \frac{(b^2B-2abC+3a^2D)\,x^7}{5ab^3(a+bx^2)^{5/2}} + \frac{(8b^2B-36abC+99a^2D)\,x^5}{60ab^4(a+bx^2)^{3/2}} + \frac{Dx^7}{4b^3(a+bx^2)^{3/2}} + \frac{(8b^2B-36abC+99a^2D)\,x^3}{12ab^5\sqrt{a+bx^2}} - \frac{(8b^2B-36abC+99a^2D)\int \frac{x^2}{\sqrt{a+bx^2}}\,dx}{4ab^5}$$

$$= \frac{\left(A - \frac{a(b^2B - abC + a^2D)}{b^3}\right)x^7}{7a(a+bx^2)^{7/2}} + \frac{(b^2B - 2abC + 3a^2D)x^7}{5ab^3(a+bx^2)^{5/2}} + \frac{(8b^2B - 36abC + 99a^2D)x^5}{60ab^4(a+bx^2)^{3/2}} + \frac{Dx^7}{4b^3(a+bx^2)^{3/2}} + \frac{(8b^2B - 36abC + 99a^2D)x^3}{12ab^5\sqrt{a+bx^2}} - \frac{(8b^2B - 36abC + 99a^2D)x\sqrt{a+bx^2}}{8ab^6} + \frac{(8b^2B - 36abC + 99a^2D)\int \frac{1}{\sqrt{a+bx^2}}dx}{8b^6}$$

$$= \frac{\left(A - \frac{a(b^2B - abC + a^2D)}{b^3}\right)x^7}{7a(a+bx^2)^{7/2}} + \frac{(b^2B - 2abC + 3a^2D)x^7}{5ab^3(a+bx^2)^{5/2}} + \frac{(8b^2B - 36abC + 99a^2D)x^5}{60ab^4(a+bx^2)^{3/2}} + \frac{Dx^7}{4b^3(a+bx^2)^{3/2}} + \frac{(8b^2B - 36abC + 99a^2D)x^3}{12ab^5\sqrt{a+bx^2}} - \frac{(8b^2B - 36abC + 99a^2D)x\sqrt{a+bx^2}}{8ab^6} + \frac{(8b^2B - 36abC + 99a^2D)\operatorname{Subst}\left(\int \frac{1}{1-bx^2}dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{8b^6}$$

$$= \frac{\left(A - \frac{a(b^2B - abC + a^2D)}{b^3}\right)x^7}{7a(a+bx^2)^{7/2}} + \frac{(b^2B - 2abC + 3a^2D)x^7}{5ab^3(a+bx^2)^{5/2}} + \frac{(8b^2B - 36abC + 99a^2D)x^5}{60ab^4(a+bx^2)^{3/2}} + \frac{Dx^7}{4b^3(a+bx^2)^{3/2}} + \frac{(8b^2B - 36abC + 99a^2D)x^3}{12ab^5\sqrt{a+bx^2}} - \frac{(8b^2B - 36abC + 99a^2D)x\sqrt{a+bx^2}}{8ab^6} + \frac{(8b^2B - 36abC + 99a^2D)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{13/2}}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.77

$$\int \frac{x^6(A + Bx^2 + Cx^4 + Dx^6)}{(a+bx^2)^{9/2}}dx = \frac{x(-10395a^6D + 120Ab^6x^6 + 630a^5b(6C - 55Dx^2) + a^2b^4x^4(-3248B +}{} + \frac{(8b^2B - 36abC + 99a^2D)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{4b^{13/2}}$$

```
[In] Integrate[(x^6*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] (x*(-10395*a^6*D + 120*A*b^6*x^6 + 630*a^5*b*(6*C - 55*D*x^2) + a^2*b^4*x^4
*(-3248*B + 6336*C*x^2 - 1155*D*x^4) - 42*a^4*b^2*(20*B - 300*C*x^2 + 957*D
*x^4) - 8*a^3*b^3*x^2*(350*B - 1827*C*x^2 + 2178*D*x^4) + 2*a*b^5*x^6*(-704
*B + 105*(2*C*x^2 + D*x^4))))/(840*a*b^6*(a + b*x^2)^(7/2)) + ((8*b^2*B - 3
```

```
6*a*b*C + 99*a^2*D)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(4*b
^(13/2))
```

Maple [A] (verified)

Time = 3.67 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$\frac{7a(b\,x^2+a)^{\frac{7}{2}}\left(B\,b^2-\frac{9}{2}Cab+\frac{99}{8}Da^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)+x\left(-\frac{176a\left(-\frac{105}{704}Dx^4-\frac{105}{352}C\,x^2+B\right)x^6b^{\frac{11}{2}}}{15}-7a^4\left(\frac{957}{20}Dx^4-15C\,x^2+\right.\right.}{7(b\,x^2+}$
default	$B\left(-\frac{x^7}{7b(b\,x^2+a)^{\frac{7}{2}}}+\frac{-\frac{x^5}{5b\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{-\frac{x^3}{3b\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}}{b}}{b}}{b}\right)+A\left(-\frac{x^5}{2b(b\,x^2+a)^{\frac{7}{2}}}+\right.$

```
[In] int(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7/(b*x^2+a)^(7/2)*(7*a*(b*x^2+a)^(7/2)*(B*b^2-9/2*C*a*b+99/8*D*a^2)*arcta
nh((b*x^2+a)^(1/2)/x/b^(1/2))+x*(-176/15*a*(-105/704*D*x^4-105/352*C*x^2+B)
*x^6*b^(11/2)-7*a^4*(957/20*D*x^4-15*C*x^2+B)*b^(5/2)-70/3*a^3*x^2*(1089/17
5*D*x^4-261/50*C*x^2+B)*b^(7/2)-406/15*a^2*x^4*(165/464*D*x^4-396/203*C*x^2
+B)*b^(9/2)+63/2*a^5*(-55/6*D*x^2+C)*b^(3/2)+A*b^(13/2)*x^6-693/8*D*b^(1/2)
*a^6))/b^(13/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 816, normalized size of antiderivative = 2.92

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \Bigg[\frac{105\,((99\,Da^3b^4-36\,Ca^2b^5+8\,Bab^6)x^8+99\,Da^7-36\,Ca^6b+8\,Ba^5b}{}$$

$$-\frac{105\,((99\,Da^3b^4-36\,Ca^2b^5+8\,Bab^6)x^8+99\,Da^7-36\,Ca^6b+8\,Ba^5b^2+4\,(99\,Da^4b^3-36\,Ca^3b^4+8\,Ba^2}{}$$

```
[In] integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/1680*(105*((99*D*a^3*b^4 - 36*C*a^2*b^5 + 8*B*a*b^6)*x^8 + 99*D*a^7 - 36
*C*a^6*b + 8*B*a^5*b^2 + 4*(99*D*a^4*b^3 - 36*C*a^3*b^4 + 8*B*a^2*b^5)*x^6
+ 6*(99*D*a^5*b^2 - 36*C*a^4*b^3 + 8*B*a^3*b^4)*x^4 + 4*(99*D*a^6*b - 36*C*
a^5*b^2 + 8*B*a^4*b^3)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b
)*x - a) + 2*(210*D*a*b^6*x^11 - 105*(11*D*a^2*b^5 - 4*C*a*b^6)*x^9 - 8*(21
78*D*a^3*b^4 - 792*C*a^2*b^5 + 176*B*a*b^6 - 15*A*b^7)*x^7 - 406*(99*D*a^4*
b^3 - 36*C*a^3*b^4 + 8*B*a^2*b^5)*x^5 - 350*(99*D*a^5*b^2 - 36*C*a^4*b^3 +
8*B*a^3*b^4)*x^3 - 105*(99*D*a^6*b - 36*C*a^5*b^2 + 8*B*a^4*b^3)*x)*sqrt(b*
x^2 + a))/(a*b^11*x^8 + 4*a^2*b^10*x^6 + 6*a^3*b^9*x^4 + 4*a^4*b^8*x^2 + a^
5*b^7), -1/840*(105*((99*D*a^3*b^4 - 36*C*a^2*b^5 + 8*B*a*b^6)*x^8 + 99*D*a
^7 - 36*C*a^6*b + 8*B*a^5*b^2 + 4*(99*D*a^4*b^3 - 36*C*a^3*b^4 + 8*B*a^2*b^
5)*x^6 + 6*(99*D*a^5*b^2 - 36*C*a^4*b^3 + 8*B*a^3*b^4)*x^4 + 4*(99*D*a^6*b
- 36*C*a^5*b^2 + 8*B*a^4*b^3)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
a)) - (210*D*a*b^6*x^11 - 105*(11*D*a^2*b^5 - 4*C*a*b^6)*x^9 - 8*(2178*D*a^
3*b^4 - 792*C*a^2*b^5 + 176*B*a*b^6 - 15*A*b^7)*x^7 - 406*(99*D*a^4*b^3 - 3
6*C*a^3*b^4 + 8*B*a^2*b^5)*x^5 - 350*(99*D*a^5*b^2 - 36*C*a^4*b^3 + 8*B*a^3
*b^4)*x^3 - 105*(99*D*a^6*b - 36*C*a^5*b^2 + 8*B*a^4*b^3)*x)*sqrt(b*x^2 + a
))/(a*b^11*x^8 + 4*a^2*b^10*x^6 + 6*a^3*b^9*x^4 + 4*a^4*b^8*x^2 + a^5*b^7)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**6*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 986 vs. $2(247) = 494$.

Time = 0.23 (sec) , antiderivative size = 986, normalized size of antiderivative = 3.53

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
[Out] 1/4*D*x^11/((b*x^2 + a)^(7/2)*b) - 11/8*D*a*x^9/((b*x^2 + a)^(7/2)*b^2) + 1
/2*C*x^9/((b*x^2 + a)^(7/2)*b) - 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*
x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/(
(b*x^2 + a)^(7/2)*b^4))*B*x - 99/280*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x
^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((
b*x^2 + a)^(7/2)*b^4))*D*a^2*x/b^2 + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*b) + 7
0*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a
^3/((b*x^2 + a)^(7/2)*b^4))*C*a*x/b - 33/40*D*a^2*x*(15*x^4/((b*x^2 + a)^(5
/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/
b^3 + 3/10*C*a*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2
)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2 - 1/15*B*x*(15*x^4/((b*x^2 + a)
^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3
))/b - 1/2*A*x^5/((b*x^2 + a)^(7/2)*b) - 33/8*D*a^2*x*(3*x^2/((b*x^2 + a)^(
3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^4 + 3/2*C*a*x*(3*x^2/((b*x^2 + a)^
(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3 - 1/3*B*x*(3*x^2/((b*x^2 + a)^(
3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 - 99/8*D*a^3*x^3/((b*x^2 + a)^(5
/2)*b^5) + 9/2*C*a^2*x^3/((b*x^2 + a)^(5/2)*b^4) - B*a*x^3/((b*x^2 + a)^(5/
2)*b^3) - 5/8*A*a*x^3/((b*x^2 + a)^(7/2)*b^2) + 4587/280*D*a^2*x/(sqrt(b*x^
2 + a)*b^6) + 561/280*D*a^3*x/((b*x^2 + a)^(3/2)*b^6) - 2871/280*D*a^4*x/((
b*x^2 + a)^(5/2)*b^6) - 417/70*C*a*x/(sqrt(b*x^2 + a)*b^5) - 51/70*C*a^2*x/
((b*x^2 + a)^(3/2)*b^5) + 261/70*C*a^3*x/((b*x^2 + a)^(5/2)*b^5) + 139/105*
B*x/(sqrt(b*x^2 + a)*b^4) + 17/105*B*a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*B*
a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14*A*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*A*x/
(sqrt(b*x^2 + a)*a*b^3) + 3/56*A*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*A*a^2*
x/((b*x^2 + a)^(7/2)*b^3) + 99/8*D*a^2*arcsinh(b*x/sqrt(a*b))/b^(13/2) - 9/
2*C*a*arcsinh(b*x/sqrt(a*b))/b^(11/2) + B*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.95

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(\left(\left(105\left(\frac{2\,Dx^2}{b}-\frac{11\,Da^4b^9-4\,Ca^3b^{10}}{a^3b^{11}}\right)x^2-\frac{8\,(2178\,Da^5b^8-792\,Ca^4b^9+176\,Ba^3b^{10}-}{a^3b^{11}}\right.\right.\right.\right.}{}$$

$$-\frac{(99\,Da^2-36\,Cab+8\,Bb^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{13}{2}}}$$

```
[In] integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/840*((((105*(2*D*x^2/b - (11*D*a^4*b^9 - 4*C*a^3*b^10)/(a^3*b^11))*x^2 -
8*(2178*D*a^5*b^8 - 792*C*a^4*b^9 + 176*B*a^3*b^10 - 15*A*a^2*b^11)/(a^3*b^
11))*x^2 - 406*(99*D*a^6*b^7 - 36*C*a^5*b^8 + 8*B*a^4*b^9)/(a^3*b^11))*x^2
- 350*(99*D*a^7*b^6 - 36*C*a^6*b^7 + 8*B*a^5*b^8)/(a^3*b^11))*x^2 - 105*(99
*D*a^8*b^5 - 36*C*a^7*b^6 + 8*B*a^6*b^7)/(a^3*b^11))*x/(b*x^2 + a)^(7/2) -
1/8*(99*D*a^2 - 36*C*a*b + 8*B*b^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/
b^(13/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \int \frac{x^6\,(A+B\,x^2+C\,x^4+x^6\,D)}{(b\,x^2+a)^{9/2}}\,dx$$

```
[In] int((x^6*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2),x)
```

```
[Out] int((x^6*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2), x)
```

3.161 $\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

Optimal result 990
Rubi [A] (verified) 990
Mathematica [A] (verified) 994
Maple [A] (verified) 994
Fricas [A] (verification not implemented) 996
Sympy [B] (verification not implemented) 996
Maxima [B] (verification not implemented) 1000
Giac [A] (verification not implemented) 1001
Mupad [F(-1)] 1002

Optimal result

Integrand size = 32, antiderivative size = 210

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^5}{7a(a+bx^2)^{7/2}} + \frac{(2Ab^3+a(5b^2B-12abC+19a^2D))\,x^5}{35a^2b^3(a+bx^2)^{5/2}} + \frac{a(bC-3aD)x}{3b^5(a+bx^2)^{3/2}} - \frac{(4bC-15aD)x}{3b^5\sqrt{a+bx^2}} + \frac{Dx\sqrt{a+bx^2}}{2b^5} + \frac{(2bC-9aD)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}$$

```
[Out] 1/7*(A-a*(B*b^2-C*a*b+D*a^2)/b^3)*x^5/a/(b*x^2+a)^(7/2)+1/35*(2*A*b^3+a*(5*
B*b^2-12*C*a*b+19*D*a^2))*x^5/a^2/b^3/(b*x^2+a)^(5/2)+1/3*a*(C*b-3*D*a)*x/b
^5/(b*x^2+a)^(3/2)+1/2*(2*C*b-9*D*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(
11/2)-1/3*(4*C*b-15*D*a)*x/b^5/(b*x^2+a)^(1/2)+1/2*D*x*(b*x^2+a)^(1/2)/b^5
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {1818, 1599, 1277, 1598, 466, 1171, 396, 223, 212}

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \frac{x^5(a(19a^2D-12abC+5b^2B)+2Ab^3)}{35a^2b^3(a+bx^2)^{5/2}} + \frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a(a+bx^2)^{7/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bC-9aD)}{2b^{11/2}} - \frac{x(4bC-15aD)}{3b^5\sqrt{a+bx^2}} + \frac{ax(bC-3aD)}{3b^5(a+bx^2)^{3/2}} + \frac{Dx\sqrt{a+bx^2}}{2b^5}$$

```
[In] Int[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^5)/(7*a*(a + b*x^2)^(7/2)) + ((2*A
*b^3 + a*(5*b^2*B - 12*a*b*C + 19*a^2*D))*x^5)/(35*a^2*b^3*(a + b*x^2)^(5/2
)) + (a*(b*C - 3*a*D)*x)/(3*b^5*(a + b*x^2)^(3/2)) - ((4*b*C - 15*a*D)*x)/(
3*b^5*Sqrt[a + b*x^2]) + (D*x*Sqrt[a + b*x^2])/(2*b^5) + ((2*b*C - 9*a*D)*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, Simp[(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d
*f*(q + 1))), x] + Dist[f/(2*d*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1] &
& GtQ[m, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(A - \frac{a(b^2B - abC + a^2D)}{b^3}\right)x^5}{7a\,(a+bx^2)^{7/2}} - \frac{\int \frac{x^3\left(-\left(\left(2Ab + \frac{5a(b^2B - abC + a^2D)}{b^2}\right)x\right) - 7a\left(C - \frac{aD}{b}\right)x^3 - 7aDx^5\right)}{(a+bx^2)^{7/2}}\,dx}{7ab}$$

$$= \frac{\left(A - \frac{a(b^2B - abC + a^2D)}{b^3}\right)x^5}{7a\,(a+bx^2)^{7/2}} - \frac{\int \frac{x^4\left(-2Ab - \frac{5a(b^2B - abC + a^2D)}{b^2} - 7a\left(C - \frac{aD}{b}\right)x^2 - 7aDx^4\right)}{(a+bx^2)^{7/2}}\,dx}{7ab}$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(2Ab^3+a(5b^2B-12abC+19a^2D)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x^3\left(\frac{35a^2(bC-2aD)x}{b^2}+\frac{35a^2Dx^3}{b}\right)}{(a+bx^2)^{5/2}}\,dx}{35a^2b}$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(2Ab^3+a(5b^2B-12abC+19a^2D)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x^4\left(\frac{35a^2(bC-2aD)}{b^2}+\frac{35a^2Dx^2}{b}\right)}{(a+bx^2)^{5/2}}\,dx}{35a^2b}$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(2Ab^3+a(5b^2B-12abC+19a^2D)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{a(bC-3aD)x}{3b^5\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{\frac{35a^3(bC-3aD)}{b}-105a^2(bC-3aD)x^2-105a^2bDx^4}{(a+bx^2)^{3/2}}\,dx}{105a^2b^4}$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(2Ab^3+a(5b^2B-12abC+19a^2D)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{a(bC-3aD)x}{3b^5\left(a+bx^2\right)^{3/2}} - \frac{(4bC-15aD)x}{3b^5\sqrt{a+bx^2}} + \frac{\int \frac{\frac{105a^3(bC-4aD)}{b}+105a^3Dx^2}{\sqrt{a+bx^2}}\,dx}{105a^3b^4}$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(2Ab^3+a(5b^2B-12abC+19a^2D)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{a(bC-3aD)x}{3b^5\left(a+bx^2\right)^{3/2}} - \frac{(4bC-15aD)x}{3b^5\sqrt{a+bx^2}} + \frac{Dx\sqrt{a+bx^2}}{2b^5} + \frac{(2bC-9aD)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b^5}$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(2Ab^3+a(5b^2B-12abC+19a^2D)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{a(bC-3aD)x}{3b^5\left(a+bx^2\right)^{3/2}} - \frac{(4bC-15aD)x}{3b^5\sqrt{a+bx^2}} + \frac{Dx\sqrt{a+bx^2}}{2b^5} + \frac{(2bC-9aD)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b^5}$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(2Ab^3+a(5b^2B-12abC+19a^2D)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{a(bC-3aD)x}{3b^5\left(a+bx^2\right)^{3/2}} - \frac{(4bC-15aD)x}{3b^5\sqrt{a+bx^2}} + \frac{Dx\sqrt{a+bx^2}}{2b^5} + \frac{(2bC-9aD)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.85

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \frac{x(945a^6D+12Ab^6x^6+6ab^5x^4(7A+5Bx^2)-210a^5b(C-15Dx^2)+a^2b}{210} + \frac{(2bC-9aD)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{11/2}}$$

```
[In] Integrate[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] (x*(945*a^6*D + 12*A*b^6*x^6 + 6*a*b^5*x^4*(7*A + 5*B*x^2) - 210*a^5*b*(C -
 15*D*x^2) + a^2*b^4*x^6*(-352*C + 105*D*x^2) + 14*a^4*b^2*x^2*(-50*C + 261
*D*x^2) + 4*a^3*b^3*x^4*(-203*C + 396*D*x^2)))/(210*a^2*b^5*(a + b*x^2)^(7/
2)) + ((2*b*C - 9*a*D)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/b
^(11/2)
```

Maple [A] (verified)

Time = 3.65 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{\frac{a\left(\frac{5x^2B}{7}+A\right)x^5b^{\frac{11}{2}}}{5}+\frac{2Ab^{\frac{13}{2}}x^7}{35}+a^2\left(-x\,a^3\left(-15Dx^2+C\right)b^{\frac{3}{2}}-\frac{10a^2\left(-\frac{261Dx^2}{50}+C\right)x^3b^{\frac{5}{2}}}{3}-\frac{58a\,x^5\left(-\frac{396Dx^2}{203}+C\right)b^{\frac{7}{2}}}{15}+\left(\frac{1}{2}Dx^9\right.\right.}{\left(b\,x^2+a\right)^{\frac{7}{2}}b^{\frac{11}{2}}a^2}$
default	$C\left(-\frac{x^7}{7b(b\,x^2+a)^{\frac{7}{2}}}+\frac{-\frac{x^5}{5b\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{-\frac{x^3}{3b\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}}{b}}{b}}{b}\right)+B\left(-\frac{x^5}{2b(b\,x^2+a)^{\frac{7}{2}}}+\right.$

```
[In] int(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b*x^2+a)^(7/2)*(1/5*a*(5/7*x^2*B+A)*x^5*b^(11/2)+2/35*A*b^(13/2)*x^7+a^2
*(-x*a^3*(-15*D*x^2+C)*b^(3/2)-10/3*a^2*(-261/50*D*x^2+C)*x^3*b^(5/2)-58/15
*a*x^5*(-396/203*D*x^2+C)*b^(7/2)+(1/2*D*x^9-176/105*C*x^7)*b^(9/2)+9/2*D*b
^(1/2)*a^4*x+arctanh((b*x^2+a)^(1/2)/x/b^(1/2))*(b*x^2+a)^(7/2)*(C*b-9/2*D*
a)))/b^(11/2)/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 653, normalized size of antiderivative = 3.11

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \left[\frac{105\left((9\,Da^3b^4-2\,Ca^2b^5)x^8+9\,Da^7-2\,Ca^6b+4\,(9\,Da^4b^3-2\,Ca^3b^4)x\right.}{}\right.$$

```
[In] integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/420*(105*((9*D*a^3*b^4 - 2*C*a^2*b^5)*x^8 + 9*D*a^7 - 2*C*a^6*b + 4*(9*D
*a^4*b^3 - 2*C*a^3*b^4)*x^6 + 6*(9*D*a^5*b^2 - 2*C*a^4*b^3)*x^4 + 4*(9*D*a^
6*b - 2*C*a^5*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x
- a) + 2*(105*D*a^2*b^5*x^9 + 2*(792*D*a^3*b^4 - 176*C*a^2*b^5 + 15*B*a*b^6
 + 6*A*b^7)*x^7 + 14*(261*D*a^4*b^3 - 58*C*a^3*b^4 + 3*A*a*b^6)*x^5 + 350*(
9*D*a^5*b^2 - 2*C*a^4*b^3)*x^3 + 105*(9*D*a^6*b - 2*C*a^5*b^2)*x)*sqrt(b*x^
2 + a))/(a^2*b^10*x^8 + 4*a^3*b^9*x^6 + 6*a^4*b^8*x^4 + 4*a^5*b^7*x^2 + a^6
*b^6), 1/210*(105*((9*D*a^3*b^4 - 2*C*a^2*b^5)*x^8 + 9*D*a^7 - 2*C*a^6*b +
4*(9*D*a^4*b^3 - 2*C*a^3*b^4)*x^6 + 6*(9*D*a^5*b^2 - 2*C*a^4*b^3)*x^4 + 4*(
9*D*a^6*b - 2*C*a^5*b^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) +
 (105*D*a^2*b^5*x^9 + 2*(792*D*a^3*b^4 - 176*C*a^2*b^5 + 15*B*a*b^6 + 6*A*b
^7)*x^7 + 14*(261*D*a^4*b^3 - 58*C*a^3*b^4 + 3*A*a*b^6)*x^5 + 350*(9*D*a^5*
b^2 - 2*C*a^4*b^3)*x^3 + 105*(9*D*a^6*b - 2*C*a^5*b^2)*x)*sqrt(b*x^2 + a))/
(a^2*b^10*x^8 + 4*a^3*b^9*x^6 + 6*a^4*b^8*x^4 + 4*a^5*b^7*x^2 + a^6*b^6)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6467 vs. 2(199) = 398.

Time = 94.78 (sec) , antiderivative size = 6467, normalized size of antiderivative = 30.80

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x**4*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 +
 b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x
**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a
**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/
a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a))) + B*x**7/(7*a**(9/2)*sqrt(1
 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 21*a**(5/2)*b**2*x**
4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + C*(105*a*
*(205/2)*b**45*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*
b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x*
*2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2
)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt
(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a
**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 630*a**(203/2)*b**46*x**2*
sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(
1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a*
*(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x*
*6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a)
+ 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(1
11/2)*x**12*sqrt(1 + b*x**2/a)) + 1575*a**(201/2)*b**47*x**4*sqrt(1 + b*x**
2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a)
+ 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(1
03/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*
x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/
2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sq
rt(1 + b*x**2/a)) + 2100*a**(199/2)*b**48*x**6*sqrt(1 + b*x**2/a)*asinh(sqr
t(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/
2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqr
t(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575
*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*
x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/
a)) + 1575*a**(197/2)*b**49*x**8*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a)
)/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*
x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a
) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b*
*(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 +
 b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 630*a**(
195/2)*b**50*x**10*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205
/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 +
b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(1
99/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*
sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 1
05*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 105*a**(193/2)*b**51*x
**12*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*
sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 15
75*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/
```

2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**
2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*
b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 105*a**102*b**(91/2)*x/(105*a**(205/
2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b
*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(19
9/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*s
qrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 10
5*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 665*a**101*b**(93/2)*x*
*3/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)
*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/
a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b
(107/2)*x8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1
+ b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 1771*a*
*100*b**(95/2)*x**5/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(
203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4
*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) +
1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109
/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x
2/a)) - 2549*a99*b**(97/2)*x**7/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x*
*2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)
*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(
1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a*
*(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x*
*12*sqrt(1 + b*x**2/a)) - 2096*a**98*b**(99/2)*x**9/(105*a**(205/2)*b**(99/
2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) +
1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(1
05/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*
x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/
2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 934*a**97*b**(101/2)*x**11/(105*a
(205/2)*b(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqr
t(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100
*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)
*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/
a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 176*a**96*b**(10
3/2)*x**13/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b*
*(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 +
b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(
197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10
*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)))
+ D*(-315*a**(311/2)*b**66*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*
a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*s
qrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 14
00*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/
2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**
2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 1890*a**(309/2)

```
*b**67*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(
143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/
a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b
**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1
+ b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(2
97/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 4725*a**(307/2)*b**68*x**4*sqr
t(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/2)*sqrt(1 +
 b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(3
05/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*
sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 4
20*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2
)*x**12*sqrt(1 + b*x**2/a)) - 6300*a**(305/2)*b**69*x**6*sqrt(1 + b*x**2/a)
*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 42
0*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2
)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2
/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b
**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1
+ b*x**2/a)) - 4725*a**(303/2)*b**70*x**8*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*
x/sqrt(a))/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b*
*(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 +
 b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(
301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10
*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) -
1890*a**(301/2)*b**71*x**10*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70
*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*
sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1
400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151
/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x*
*2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 315*a**(299/2)
*b**72*x**12*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**
(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2
/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*
b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1
 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(
297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 315*a**155*b**(133/2)*x/(70*a
**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sq
rt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 140
0*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2
)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2
/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 1995*a**154*b**(
135/2)*x**3/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b
**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1
+ b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**
(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**1
0*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) +
```

```
 5313*a**153*b**(137/2)*x**5/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) +
 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(14
7/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x
**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2
)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt
(1 + b*x**2/a)) + 7647*a**152*b**(139/2)*x**7/(70*a**(309/2)*b**(143/2)*sqr
t(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*
a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*
x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a
) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(
155/2)*x**12*sqrt(1 + b*x**2/a)) + 6323*a**151*b**(141/2)*x**9/(70*a**(309/
2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 +
b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(3
03/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*
sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 7
0*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 2907*a**150*b**(143/2)*
x**11/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145
/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x*
*2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2
)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt
(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 633*a
**149*b**(145/2)*x**13/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a
**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x
**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a)
 + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(
153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b
*x**2/a)) + 35*a**148*b**(147/2)*x**15/(70*a**(309/2)*b**(143/2)*sqrt(1 + b
*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305
/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sq
rt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420
*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*
x**12*sqrt(1 + b*x**2/a)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 753 vs. $2(183) = 366$.

Time = 0.23 (sec) , antiderivative size = 753, normalized size of antiderivative = 3.59

$$\int \frac{x^4(A + Bx^2 + Cx^4 + Dx^6)}{(a + bx^2)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/2*D*x^9/((b*x^2 + a)^(7/2)*b) - 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a
*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/
```

```
((b*x^2 + a)^(7/2)*b^4))*C*x + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^
4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b
*x^2 + a)^(7/2)*b^4))*D*a*x/b + 3/10*D*a*x*(15*x^4/((b*x^2 + a)^(5/2)*b) +
20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2 - 1/1
5*C*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*
a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*B*x^5/((b*x^2 + a)^(7/2)*b) + 3/2*D*a*
x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3 - 1/3*C*x
*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 9/2*D*a^
2*x^3/((b*x^2 + a)^(5/2)*b^4) - C*a*x^3/((b*x^2 + a)^(5/2)*b^3) - 5/8*B*a*x
^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*A*x^3/((b*x^2 + a)^(7/2)*b) - 417/70*D*a*x
/(sqrt(b*x^2 + a)*b^5) - 51/70*D*a^2*x/((b*x^2 + a)^(3/2)*b^5) + 261/70*D*a
^3*x/((b*x^2 + a)^(5/2)*b^5) + 139/105*C*x/(sqrt(b*x^2 + a)*b^4) + 17/105*C
*a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*C*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14
*B*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*B*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*B*a*x
/((b*x^2 + a)^(5/2)*b^3) - 15/56*B*a^2*x/((b*x^2 + a)^(7/2)*b^3) + 3/140*A*
x/((b*x^2 + a)^(5/2)*b^2) + 2/35*A*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*A*x/(
(b*x^2 + a)^(3/2)*a*b^2) - 3/28*A*a*x/((b*x^2 + a)^(7/2)*b^2) - 9/2*D*a*arc
sinh(b*x/sqrt(a*b))/b^(11/2) + C*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.97

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(\left(\left(\frac{105\,Dx^2}{b} + \frac{2\,(792\,Da^4b^7-176\,Ca^3b^8+15\,Ba^2b^9+6\,Aab^{10})}{a^3b^9}\right)x^2 + \frac{14\,(261\,Da^5b^6-58\,C\cdots}{a^3b^9}\cdots}{210\,(bx^2+\cdots} + \frac{(9\,Da-2\,Cb)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{11}{2}}}$$

```
[In] integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/210*((((105*D*x^2/b + 2*(792*D*a^4*b^7 - 176*C*a^3*b^8 + 15*B*a^2*b^9 + 6
*A*a*b^10)/(a^3*b^9))*x^2 + 14*(261*D*a^5*b^6 - 58*C*a^4*b^7 + 3*A*a^2*b^9)
/(a^3*b^9))*x^2 + 350*(9*D*a^6*b^5 - 2*C*a^5*b^6)/(a^3*b^9))*x^2 + 105*(9*D
*a^7*b^4 - 2*C*a^6*b^5)/(a^3*b^9))*x/(b*x^2 + a)^(7/2) + 1/2*(9*D*a - 2*C*b
)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx=\int \frac{x^4\,(A+B\,x^2+C\,x^4+x^6\,D)}{(b\,x^2+a)^{9/2}}\,dx$$

```
[In] int((x^4*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2),x)
[Out] int((x^4*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2), x)
```

3.162 $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

Optimal result 1003
Rubi [A] (verified) 1003
Mathematica [A] (verified) 1006
Maple [A] (verified) 1007
Fricas [A] (verification not implemented) 1009
Sympy [B] (verification not implemented) 1009
Maxima [B] (verification not implemented) 1012
Giac [A] (verification not implemented) 1013
Mupad [F(-1)] 1013

Optimal result

Integrand size = 32, antiderivative size = 179

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = -\frac{a^3Dx}{b^4(a+bx^2)^{7/2}} + \frac{(Ab^3-10a^3D)x^3}{3ab^3(a+bx^2)^{7/2}} + \frac{(4Ab^3+3ab^2B-58a^3D)x^5}{15a^2b^2(a+bx^2)^{7/2}} + \frac{(8Ab^3+6ab^2B+15a^2bC-176a^3D)x^7}{105a^3b(a+bx^2)^{7/2}} + \frac{D\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

```
[Out] -a^3*D*x/b^4/(b*x^2+a)^(7/2)+1/3*(A*b^3-10*D*a^3)*x^3/a/b^3/(b*x^2+a)^(7/2)
+1/15*(4*A*b^3+3*B*a*b^2-58*D*a^3)*x^5/a^2/b^2/(b*x^2+a)^(7/2)+1/105*(8*A*b
^3+6*B*a*b^2+15*C*a^2*b-176*D*a^3)*x^7/a^3/b/(b*x^2+a)^(7/2)+D*arctanh(x*b^
(1/2)/(b*x^2+a)^(1/2))/b^(9/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1818, 1599, 1277, 1598, 463, 294, 223, 212}

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \frac{x^3(a(17a^2D-10abC+3b^2B)+4Ab^3)}{35a^2b^3(a+bx^2)^{5/2}} + \frac{x^3\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a(a+bx^2)^{7/2}} + \frac{x^3(a(-71a^2D+15abC+6b^2B)+8Ab^3)}{105a^3b^3(a+bx^2)^{3/2}} + \frac{D\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}} - \frac{Dx}{b^4\sqrt{a+bx^2}}$$

```
[In] Int[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^3)/(7*a*(a + b*x^2)^(7/2)) + ((4*A
*b^3 + a*(3*b^2*B - 10*a*b*C + 17*a^2*D))*x^3)/(35*a^2*b^3*(a + b*x^2)^(5/2
)) + ((8*A*b^3 + a*(6*b^2*B + 15*a*b*C - 71*a^2*D))*x^3)/(105*a^3*b^3*(a +
b*x^2)^(3/2)) - (D*x)/(b^4*Sqrt[a + b*x^2]) + (D*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/b^(9/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, Simp[(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d
*f*(q + 1))), x] + Dist[f/(2*d*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1] &
& GtQ[m, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x\left(-\left(\left(4Ab+\frac{3a\left(b^2B-abC+a^2D\right)}{b^2}\right)x\right)-7a\left(C-\frac{aD}{b}\right)x^3-7aDx^5\right)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^2\left(-4Ab-\frac{3a\left(b^2B-abC+a^2D\right)}{b^2}-7a\left(C-\frac{aD}{b}\right)x^2-7aDx^4\right)}{\left(a+bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^3+a(3b^2B-10abC+17a^2D)\right)x^3}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x\left(\left(8Ab+\frac{3a\left(2b^2B+5abC-12a^2D\right)}{b^2}\right)x+\frac{35a^2Dx^3}{b}\right)}{\left(a+bx^2\right)^{5/2}}\,dx}{35a^2b}$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^3+a(3b^2B-10abC+17a^2D)\right)x^3}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{x^2\left(8Ab+\frac{3a\left(2b^2B+5abC-12a^2D\right)}{b^2}+\frac{35a^2Dx^2}{b}\right)}{\left(a+bx^2\right)^{5/2}}\,dx}{35a^2b}$$

$$= \frac{\left(A - \frac{a(b^2B - abC + a^2D)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^3 + a(3b^2B - 10abC + 17a^2D)\right)x^3}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8Ab^3 + a(6b^2B + 15abC - 71a^2D)\right)x^3}{105a^3b^3\left(a+bx^2\right)^{3/2}} + \frac{D\int \frac{x^2}{(a+bx^2)^{3/2}}\,dx}{b^3}$$

$$= \frac{\left(A - \frac{a(b^2B - abC + a^2D)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^3 + a(3b^2B - 10abC + 17a^2D)\right)x^3}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8Ab^3 + a(6b^2B + 15abC - 71a^2D)\right)x^3}{105a^3b^3\left(a+bx^2\right)^{3/2}} - \frac{Dx}{b^4\sqrt{a+bx^2}} + \frac{D\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b^4}$$

$$= \frac{\left(A - \frac{a(b^2B - abC + a^2D)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^3 + a(3b^2B - 10abC + 17a^2D)\right)x^3}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8Ab^3 + a(6b^2B + 15abC - 71a^2D)\right)x^3}{105a^3b^3\left(a+bx^2\right)^{3/2}} - \frac{Dx}{b^4\sqrt{a+bx^2}} + \frac{D\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{b^4}$$

$$= \frac{\left(A - \frac{a(b^2B - abC + a^2D)}{b^3}\right)x^3}{7a\left(a+bx^2\right)^{7/2}} + \frac{\left(4Ab^3 + a(3b^2B - 10abC + 17a^2D)\right)x^3}{35a^2b^3\left(a+bx^2\right)^{5/2}} + \frac{\left(8Ab^3 + a(6b^2B + 15abC - 71a^2D)\right)x^3}{105a^3b^3\left(a+bx^2\right)^{3/2}} - \frac{Dx}{b^4\sqrt{a+bx^2}} + \frac{D\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.82

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \frac{-105a^6Dx - 350a^5bDx^3 - 406a^4b^2Dx^5 + 8Ab^6x^7 - 176a^3b^3Dx^7 + 2ab^5}{105a^3b^4\left(a+bx^2\right)^{7/2}} - \frac{D\log\left(-\sqrt{b}x + \sqrt{a+bx^2}\right)}{b^{9/2}}$$

```
[In] Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-105*a^6*D*x - 350*a^5*b*D*x^3 - 406*a^4*b^2*D*x^5 + 8*A*b^6*x^7 - 176*a^3
*b^3*D*x^7 + 2*a*b^5*x^5*(14*A + 3*B*x^2) + a^2*b^4*x^3*(35*A + 21*B*x^2 +
15*C*x^4))/(105*a^3*b^4*(a + b*x^2)^(7/2)) - (D*Log[-(Sqrt[b]*x) + Sqrt[a +
 b*x^2]])/b^(9/2)
```

Maple [A] (verified)

Time = 3.58 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{\frac{a^2x^3\left(\frac{3}{7}Cx^4+\frac{3}{5}x^2B+A\right)b^{\frac{9}{2}}}{3}+\frac{4a\left(\frac{3x^2B}{14}+A\right)x^5b^{\frac{11}{2}}}{15}+\frac{8Ab^{\frac{13}{2}}x^7}{105}+a^3\left((bx^2+a)^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx^2+a}}{x\sqrt{b}}\right)-\frac{176x^7b^{\frac{7}{2}}}{105}-\frac{58b^{\frac{5}{2}}ax^5}{15}-\frac{10b^{\frac{3}{2}}}{}\right.}{(bx^2+a)^{\frac{7}{2}}b^{\frac{9}{2}}a^3}$
default	$D\left(-\frac{x^7}{7b(bx^2+a)^{\frac{7}{2}}}+\frac{-\frac{x^5}{5b\left(bx^2+a\right)^{\frac{5}{2}}}+\frac{-\frac{x^3}{3b\left(bx^2+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{bx^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{b^{\frac{3}{2}}}}{b}}{b}}{b}\right)+C\left(-\frac{x^5}{2b(bx^2+a)^{\frac{7}{2}}}+\right.$ 5

```
[In] int(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b*x^2+a)^(7/2)*(1/3*a^2*x^3*(3/7*C*x^4+3/5*x^2*B+A)*b^(9/2)+4/15*a*(3/14
*x^2*B+A)*x^5*b^(11/2)+8/105*A*b^(13/2)*x^7+a^3*((b*x^2+a)^(7/2)*arctanh((b
*x^2+a)^(1/2)/x/b^(1/2))-176/105*x^7*b^(7/2)-58/15*b^(5/2)*a*x^5-10/3*b^(3/
2)*a^2*x^3-b^(1/2)*a^3*x)*D)/b^(9/2)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 491, normalized size of antiderivative = 2.74

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \left[\frac{105\left(Da^3b^4x^8+4\,Da^4b^3x^6+6\,Da^5b^2x^4+4\,Da^6bx^2+Da^7\right)\sqrt{b}\log\left(-\right.}{}\right.$$

$$-\frac{105\left(Da^3b^4x^8+4\,Da^4b^3x^6+6\,Da^5b^2x^4+4\,Da^6bx^2+Da^7\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(105\,Da^6bx+(176\,D}{105\,(a^3b^9x^8+4\,a^4b^8x^6}$$

```
[In] integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(D*a^3*b^4*x^8 + 4*D*a^4*b^3*x^6 + 6*D*a^5*b^2*x^4 + 4*D*a^6*b*
x^2 + D*a^7)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(1
05*D*a^6*b*x + (176*D*a^3*b^4 - 15*C*a^2*b^5 - 6*B*a*b^6 - 8*A*b^7)*x^7 + 7
*(58*D*a^4*b^3 - 3*B*a^2*b^5 - 4*A*a*b^6)*x^5 + 35*(10*D*a^5*b^2 - A*a^2*b^
5)*x^3)*sqrt(b*x^2 + a))/(a^3*b^9*x^8 + 4*a^4*b^8*x^6 + 6*a^5*b^7*x^4 + 4*a
^6*b^6*x^2 + a^7*b^5), -1/105*(105*(D*a^3*b^4*x^8 + 4*D*a^4*b^3*x^6 + 6*D*a
^5*b^2*x^4 + 4*D*a^6*b*x^2 + D*a^7)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
 a)) + (105*D*a^6*b*x + (176*D*a^3*b^4 - 15*C*a^2*b^5 - 6*B*a*b^6 - 8*A*b^7
)*x^7 + 7*(58*D*a^4*b^3 - 3*B*a^2*b^5 - 4*A*a*b^6)*x^5 + 35*(10*D*a^5*b^2 -
 A*a^2*b^5)*x^3)*sqrt(b*x^2 + a))/(a^3*b^9*x^8 + 4*a^4*b^8*x^6 + 6*a^5*b^7*
x^4 + 4*a^6*b^6*x^2 + a^7*b^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3803 vs. 2(178) = 356.

Time = 66.25 (sec) , antiderivative size = 3803, normalized size of antiderivative = 21.25

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x**2*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sq
rt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/
```

```
2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a
)) + 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**
2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**
(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x*
*2/a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2
)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) +
420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1
 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a*
*(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2
/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*
sqrt(1 + b*x**2/a))) + B*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a
**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/
a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqr
t(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**
2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a))) + C*
x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a)
 + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt(1 +
 b*x**2/a)) + D*(105*a**(205/2)*b**45*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sq
rt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(10
1/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x
**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/
2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqr
t(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 630
*a**(203/2)*b**46*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**
(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(
1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a
**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x
**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a)
 + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 1575*a**(201/2)*b*
*47*x**4*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99
/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a)
+ 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(
105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b
*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193
/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 2100*a**(199/2)*b**48*x**6*sqrt(
1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b
*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201
/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sq
rt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630
*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)
*x**12*sqrt(1 + b*x**2/a)) + 1575*a**(197/2)*b**49*x**8*sqrt(1 + b*x**2/a)*
asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630
*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)
*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/
a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b*
```

```
*(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1
+ b*x**2/a)) + 630*a**(195/2)*b**50*x**10*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*
x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b*
*(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 +
 b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(
197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10
*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) +
 105*a**(193/2)*b**51*x**12*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(10
5*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*
sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2
100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107
/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x*
*2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 105*a**102*b*
*(91/2)*x/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**
(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 +
b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(1
97/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*
sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) -
665*a**101*b**(93/2)*x**3/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 63
0*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2
)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2
/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b
**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1
 + b*x**2/a)) - 1771*a**100*b**(95/2)*x**5/(105*a**(205/2)*b**(99/2)*sqrt(1
 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**
(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**
6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) +
 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(11
1/2)*x**12*sqrt(1 + b*x**2/a)) - 2549*a**99*b**(97/2)*x**7/(105*a**(205/2)*
b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x*
*2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2
)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt
(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a
**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 2096*a**98*b**(99/2)*x**9/
(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x*
*2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a)
+ 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(
107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b
*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 934*a**97*
b**(101/2)*x**11/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203
/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sq
rt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 157
5*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)
*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2
/a)) - 176*a**96*b**(103/2)*x**13/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2
```

```
/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b
**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1
+ b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(
195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**1
2*sqrt(1 + b*x**2/a)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(160) = 320$.

Time = 0.25 (sec) , antiderivative size = 533, normalized size of antiderivative = 2.98

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = -\frac{1}{35}\left(\frac{35\,x^6}{(bx^2+a)^{\frac{7}{2}}b}+\frac{70\,ax^4}{(bx^2+a)^{\frac{7}{2}}b^2}+\frac{56\,a^2x^2}{(bx^2+a)^{\frac{7}{2}}b^3}+\frac{16\,a^3}{(bx^2+a)^{\frac{7}{2}}b^4}\right)Dx - \frac{Dx\left(\frac{15\,x^4}{(bx^2+a)^{\frac{5}{2}}b}+\frac{20\,ax^2}{(bx^2+a)^{\frac{5}{2}}b^2}+\frac{8\,a^2}{(bx^2+a)^{\frac{5}{2}}b^3}\right)}{15\,b} - \frac{Cx^5}{2\,(bx^2+a)^{\frac{7}{2}}b} - \frac{Dx\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{3\,b^2} - \frac{Dax^3}{(bx^2+a)^{\frac{5}{2}}b^3} - \frac{5\,Cax^3}{8\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Bx^3}{4\,(bx^2+a)^{\frac{7}{2}}b} + \frac{139\,Dx}{105\,\sqrt{bx^2+a}b^4} + \frac{17\,Dax}{105\,(bx^2+a)^{\frac{3}{2}}b^4} - \frac{29\,Da^2x}{35\,(bx^2+a)^{\frac{5}{2}}b^4} + \frac{Cx}{14\,(bx^2+a)^{\frac{3}{2}}b^3} + \frac{Cx}{7\,\sqrt{bx^2+a}ab^3} + \frac{3\,Cax}{56\,(bx^2+a)^{\frac{5}{2}}b^3} - \frac{15\,Ca^2x}{56\,(bx^2+a)^{\frac{7}{2}}b^3} + \frac{3\,Bx}{140\,(bx^2+a)^{\frac{5}{2}}b^2} + \frac{2\,Bx}{35\,\sqrt{bx^2+a}a^2b^2} + \frac{Bx}{35\,(bx^2+a)^{\frac{3}{2}}ab^2} - \frac{3\,Bax}{28\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Ax}{7\,(bx^2+a)^{\frac{7}{2}}b} + \frac{8\,Ax}{105\,\sqrt{bx^2+a}a^3b} + \frac{4\,Ax}{105\,(bx^2+a)^{\frac{3}{2}}a^2b} + \frac{Ax}{35\,(bx^2+a)^{\frac{5}{2}}ab} + \frac{D\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{9}{2}}}$$

```
[In] integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56
*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*D*x - 1/
15*D*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8
*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*C*x^5/((b*x^2 + a)^(7/2)*b) - 1/3*D*x
*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 - D*a*x^3/
((b*x^2 + a)^(5/2)*b^3) - 5/8*C*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*B*x^3/(
(b*x^2 + a)^(7/2)*b) + 139/105*D*x/(sqrt(b*x^2 + a)*b^4) + 17/105*D*a*x/((b
*x^2 + a)^(3/2)*b^4) - 29/35*D*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14*C*x/((b
```

```
*x^2 + a)^(3/2)*b^3) + 1/7*C*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*C*a*x/((b*x^2
 + a)^(5/2)*b^3) - 15/56*C*a^2*x/((b*x^2 + a)^(7/2)*b^3) + 3/140*B*x/((b*x^
2 + a)^(5/2)*b^2) + 2/35*B*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*B*x/((b*x^2 +
 a)^(3/2)*a*b^2) - 3/28*B*a*x/((b*x^2 + a)^(7/2)*b^2) - 1/7*A*x/((b*x^2 + a
)^(7/2)*b) + 8/105*A*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*A*x/((b*x^2 + a)^(3/
2)*a^2*b) + 1/35*A*x/((b*x^2 + a)^(5/2)*a*b) + D*arcsinh(b*x/sqrt(a*b))/b^(
9/2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.89

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = -\frac{\left(\left(x^2\left(\frac{(176\,Da^3b^6-15\,Ca^2b^7-6\,Bab^8-8\,Ab^9)x^2}{a^3b^7}+\frac{7\,(58\,Da^4b^5-3\,Ba^2b^7-4\,Aab^8)}{a^3b^7}\right)+\frac{35\,(10\,Da^5b^4-Aa^2b^7)}{a^3b^7}\right)x^2+\frac{105\,Da^3}{b^4}\right)x}{105\,(bx^2+a)^{\frac{7}{2}}} - \frac{D\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}$$

```
[In] integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*((x^2*((176*D*a^3*b^6 - 15*C*a^2*b^7 - 6*B*a*b^8 - 8*A*b^9)*x^2/(a^3
*b^7) + 7*(58*D*a^4*b^5 - 3*B*a^2*b^7 - 4*A*a*b^8)/(a^3*b^7)) + 35*(10*D*a^
5*b^4 - A*a^2*b^7)/(a^3*b^7))*x^2 + 105*D*a^3/b^4)*x/(b*x^2 + a)^(7/2) - D*
log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \int \frac{x^2\left(A+B\,x^2+C\,x^4+x^6\,D\right)}{{\left(b\,x^2+a\right)}^{9/2}}\,dx$$

```
[In] int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2),x)
```

```
[Out] int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2), x)
```

3.163 $\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx$

Optimal result 1014
Rubi [A] (verified) 1014
Mathematica [A] (verified) 1016
Maple [A] (verified) 1016
Fricas [A] (verification not implemented) 1018
Sympy [B] (verification not implemented) 1018
Maxima [B] (verification not implemented) 1020
Giac [A] (verification not implemented) 1020
Mupad [F(-1)] 1021

Optimal result

Integrand size = 29, antiderivative size = 134

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx = \frac{Ax}{a(a+bx^2)^{7/2}} + \frac{(6Ab+aB)x^3}{3a^2(a+bx^2)^{7/2}} + \frac{(24Ab^2+a(4bB+3aC))\,x^5}{15a^3(a+bx^2)^{7/2}} + \frac{(48Ab^3+a(8b^2B+6abC+15a^2D))\,x^7}{105a^4(a+bx^2)^{7/2}}$$

```
[Out] A*x/a/(b*x^2+a)^(7/2)+1/3*(6*A*b+B*a)*x^3/a^2/(b*x^2+a)^(7/2)+1/15*(24*A*b^
2+a*(4*B*b+3*C*a))*x^5/a^3/(b*x^2+a)^(7/2)+1/105*(48*A*b^3+a*(8*B*b^2+6*C*a
*b+15*D*a^2))*x^7/a^4/(b*x^2+a)^(7/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1827, 1817, 12, 270}

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx = \frac{x^5(a(3aC+4bB)+24Ab^2)}{15a^3(a+bx^2)^{7/2}} + \frac{x^3(aB+6Ab)}{3a^2(a+bx^2)^{7/2}} + \frac{x^7(a(15a^2D+6abC+8b^2B)+48Ab^3)}{105a^4(a+bx^2)^{7/2}} + \frac{Ax}{a(a+bx^2)^{7/2}}$$

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(9/2),x]
```

```
[Out] (A*x)/(a*(a + b*x^2)^(7/2)) + ((6*A*b + a*B)*x^3)/(3*a^2*(a + b*x^2)^(7/2))
 + ((24*A*b^2 + a*(4*b*B + 3*a*C))*x^5)/(15*a^3*(a + b*x^2)^(7/2)) + ((48*A
*b^3 + a*(8*b^2*B + 6*a*b*C + 15*a^2*D))*x^7)/(105*a^4*(a + b*x^2)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 1827

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*x*((a + b*
x^2)^(p + 1)/a), x] + Dist[1/a, Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3))
, x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ[
Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{Ax}{a\left(a+bx^2\right)^{7/2}}+\frac{\int \frac{x^2\left(6Ab+a\left(B+Cx^2+Dx^4\right)\right)}{\left(a+bx^2\right)^{9/2}}\,dx}{a} \\
&= \frac{Ax}{a\left(a+bx^2\right)^{7/2}}+\frac{(6Ab+aB)x^3}{3a^2\left(a+bx^2\right)^{7/2}}+\frac{\int \frac{x^4\left(4b(6Ab+aB)+3a\left(aC+aDx^2\right)\right)}{\left(a+bx^2\right)^{9/2}}\,dx}{3a^2} \\
&= \frac{Ax}{a\left(a+bx^2\right)^{7/2}}+\frac{(6Ab+aB)x^3}{3a^2\left(a+bx^2\right)^{7/2}}+\frac{\left(24Ab^2+a(4bB+3aC)\right)x^5}{15a^3\left(a+bx^2\right)^{7/2}} \\
&\quad+\frac{\int \frac{\left(2b\left(24Ab^2+4abB+3a^2C\right)+15a^3D\right)x^6}{\left(a+bx^2\right)^{9/2}}\,dx}{15a^3} \\
&= \frac{Ax}{a\left(a+bx^2\right)^{7/2}}+\frac{(6Ab+aB)x^3}{3a^2\left(a+bx^2\right)^{7/2}}+\frac{\left(24Ab^2+a(4bB+3aC)\right)x^5}{15a^3\left(a+bx^2\right)^{7/2}} \\
&\quad+\frac{\left(48Ab^3+a\left(8b^2B+6abC+15a^2D\right)\right)\int \frac{x^6}{\left(a+bx^2\right)^{9/2}}\,dx}{15a^3}
\end{aligned}
$$

$$= \frac{Ax}{a\left(a+bx^2\right)^{7/2}} + \frac{(6Ab+aB)x^3}{3a^2\left(a+bx^2\right)^{7/2}} + \frac{\left(24Ab^2+a(4bB+3aC)\right)x^5}{15a^3\left(a+bx^2\right)^{7/2}} + \frac{\left(48Ab^3+a(8b^2B+6abC+15a^2D)\right)x^7}{105a^4\left(a+bx^2\right)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{\left(a+bx^2\right)^{9/2}}\,dx = \frac{48Ab^3x^7+8ab^2x^5(21A+Bx^2)+2a^2bx^3(105A+14Bx^2+3Cx^4)+a^3(105A}{105a^4\left(a+bx^2\right)^{7/2}}$$

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(9/2),x]
```

```
[Out] (48*A*b^3*x^7 + 8*a*b^2*x^5*(21*A + B*x^2) + 2*a^2*b*x^3*(105*A + 14*B*x^2
+ 3*C*x^4) + a^3*(105*A*x + 35*B*x^3 + 21*C*x^5 + 15*D*x^7))/(105*a^4*(a +
b*x^2)^(7/2))
```

Maple [A] (verified)

Time = 3.58 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{\left(\left(\frac{1}{7}Dx^{6}+\frac{1}{5}C\,x^{4}+\frac{1}{3}x^{2}B+A\right)a^{3}+2\left(\frac{1}{35}C\,x^{4}+\frac{2}{15}x^{2}B+A\right)b\,x^{2}a^{2}+\frac{8\left(\frac{x^{2}B}{21}+A\right)b^{2}x^{4}a}{5}+\frac{16x^{6}b^{3}A}{35}\right)x}{\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{4}}$
gosper	$\frac{x\left(48x^{6}b^{3}A+8Ba\,b^{2}x^{6}+6a^{2}bC\,x^{6}+15Da^{3}x^{6}+168aA\,b^{2}x^{4}+28B\,a^{2}b\,x^{4}+21a^{3}C\,x^{4}+210a^{2}Ab\,x^{2}+35B\,a^{3}x^{2}+105a^{3}A\right)}{105\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{4}}$
trager	$\frac{x\left(48x^{6}b^{3}A+8Ba\,b^{2}x^{6}+6a^{2}bC\,x^{6}+15Da^{3}x^{6}+168aA\,b^{2}x^{4}+28B\,a^{2}b\,x^{4}+21a^{3}C\,x^{4}+210a^{2}Ab\,x^{2}+35B\,a^{3}x^{2}+105a^{3}A\right)}{105\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{4}}$
default	$A\left(\frac{x}{7a\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{b\,x^{2}+a}}\right)}{7a}}{a}\right)+D\left(-\frac{x^{5}}{2b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{5a\left(-\frac{x^{3}}{4b\left(b\,x^{2}+a\right)\cdots}\right.}{\cdots}\right.$

```
[In] int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((1/7*D*x^6+1/5*C*x^4+1/3*x^2*B+A)*a^3+2*(1/35*C*x^4+2/15*x^2*B+A)*b*x^2*a^
2+8/5*(1/21*x^2*B+A)*b^2*x^4*a+16/35*x^6*b^3*A)*x/(b*x^2+a)^(7/2)/a^4
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx = \frac{((15\,Da^3+6\,Ca^2b+8\,Bab^2+48\,Ab^3)x^7+7\,(3\,Ca^3+4\,Ba^2b+24\,Aab^2)x^5+}{105\,(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7b}$$

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*((15*D*a^3 + 6*C*a^2*b + 8*B*a*b^2 + 48*A*b^3)*x^7 + 7*(3*C*a^3 + 4*B
*a^2*b + 24*A*a*b^2)*x^5 + 105*A*a^3*x + 35*(B*a^3 + 6*A*a^2*b)*x^3)*sqrt(b
*x^2 + a)/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2088 vs. 2(129) = 258.

Time = 47.67 (sec) , antiderivative size = 2088, normalized size of antiderivative = 15.58

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(
1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*
b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) +
 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt
(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a
**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**
2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8
*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(
25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sq
rt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*
b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) +
 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt
(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b*
*3*x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*
x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x
```

```
**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a
**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b
*x**2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(
35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a
) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sq
rt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/
2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 104*a**9*b**5*x**11/(35*a**(37/2)*sqrt(
1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**
2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 52
5*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1
+ b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x*
*13/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2
/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*
sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(2
7/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**
2/a))) + B*(35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*
b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 42
0*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 +
 b*x**2/a)) + 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17
/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a)
+ 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt
(1 + b*x**2/a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420
*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x
**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x*
*8*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a)
 + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1
 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b
**4*x**8*sqrt(1 + b*x**2/a))) + C*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a
) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1
+ b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(
11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**
(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/
a))) + D*x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 +
b*x**2/a) + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6
*sqrt(1 + b*x**2/a))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(120) = 240.

Time = 0.20 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.50

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx = -\frac{Dx^5}{2\,(bx^2+a)^{\frac{7}{2}}b} - \frac{5\,Dax^3}{8\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Cx^3}{4\,(bx^2+a)^{\frac{7}{2}}b} + \frac{16\,Ax}{35\,\sqrt{bx^2+a}a^4} + \frac{8\,Ax}{35\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{6\,Ax}{35\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{Ax}{7\,(bx^2+a)^{\frac{7}{2}}a} + \frac{Dx}{14\,(bx^2+a)^{\frac{3}{2}}b^3} + \frac{Dx}{7\,\sqrt{bx^2+a}ab^3} + \frac{3\,Dax}{56\,(bx^2+a)^{\frac{5}{2}}b^3} - \frac{15\,Da^2x}{56\,(bx^2+a)^{\frac{7}{2}}b^3} + \frac{3\,Cx}{140\,(bx^2+a)^{\frac{5}{2}}b^2} + \frac{2\,Cx}{35\,\sqrt{bx^2+a}a^2b^2} + \frac{Cx}{35\,(bx^2+a)^{\frac{3}{2}}ab^2} - \frac{3\,Cax}{28\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Bx}{7\,(bx^2+a)^{\frac{7}{2}}b} + \frac{8\,Bx}{105\,\sqrt{bx^2+a}a^3b} + \frac{4\,Bx}{105\,(bx^2+a)^{\frac{3}{2}}a^2b} + \frac{Bx}{35\,(bx^2+a)^{\frac{5}{2}}ab}$$

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/2*D*x^5/((b*x^2 + a)^(7/2)*b) - 5/8*D*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/
4*C*x^3/((b*x^2 + a)^(7/2)*b) + 16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*A*x/
((b*x^2 + a)^(3/2)*a^3) + 6/35*A*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((b*x^
2 + a)^(7/2)*a) + 1/14*D*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*D*x/(sqrt(b*x^2 +
a)*a*b^3) + 3/56*D*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*D*a^2*x/((b*x^2 + a)
^(7/2)*b^3) + 3/140*C*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*C*x/(sqrt(b*x^2 + a)
*a^2*b^2) + 1/35*C*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*C*a*x/((b*x^2 + a)^(7
/2)*b^2) - 1/7*B*x/((b*x^2 + a)^(7/2)*b) + 8/105*B*x/(sqrt(b*x^2 + a)*a^3*b
) + 4/105*B*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*B*x/((b*x^2 + a)^(5/2)*a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{(15\,Da^3b^3+6\,Ca^2b^4+8\,Bab^5+48\,Ab^6)x^2}{a^4b^3} + \frac{7\,(3\,Ca^3b^3+4\,Ba^2b^4+24\,Aab^5)}{a^4b^3}\right) + \frac{35\,(Ba^3b^3+6}{a^4b^3}\right.\right.}{105\,(bx^2+a)^{\frac{7}{2}}}$$

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((15*D*a^3*b^3 + 6*C*a^2*b^4 + 8*B*a*b^5 + 48*A*b^6)*x^2/(a^4*b
^3) + 7*(3*C*a^3*b^3 + 4*B*a^2*b^4 + 24*A*a*b^5)/(a^4*b^3)) + 35*(B*a^3*b^3
 + 6*A*a^2*b^4)/(a^4*b^3))*x^2 + 105*A/a)*x/(b*x^2 + a)^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{(a + bx^2)^{9/2}} \, dx = \int \frac{A + B\,x^2 + C\,x^4 + x^6\,D}{(b\,x^2 + a)^{9/2}} \, dx$$

```
[In] int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(9/2),x)
[Out] int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(9/2), x)
```

3.164 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx$

Optimal result 1022
Rubi [A] (verified) 1022
Mathematica [A] (verified) 1024
Maple [A] (verified) 1025
Fricas [A] (verification not implemented) 1026
Sympy [B] (verification not implemented) 1026
Maxima [A] (verification not implemented) 1028
Giac [A] (verification not implemented) 1028
Mupad [F(-1)] 1029

Optimal result

Integrand size = 32, antiderivative size = 185

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx=-\frac{A}{ax\left(a+bx^2\right)^{7/2}}-\frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}}-\frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{\left(4b(48Ab^2-a(6bB+aC))-3a^3D\right)x^5}{15a^4\left(a+bx^2\right)^{7/2}}-\frac{2b\left(4b(48Ab^2-a(6bB+aC))-3a^3D\right)x^7}{105a^5\left(a+bx^2\right)^{7/2}}$$

```
[Out] -A/a/x/(b*x^2+a)^(7/2)-(8*A*b-B*a)*x/a^2/(b*x^2+a)^(7/2)-1/3*(48*A*b^2-a*(6
*B*b+C*a))*x^3/a^3/(b*x^2+a)^(7/2)-1/15*(4*b*(48*A*b^2-a*(6*B*b+C*a))-3*D*a
^3)*x^5/a^4/(b*x^2+a)^(7/2)-2/105*b*(4*b*(48*A*b^2-a*(6*B*b+C*a))-3*D*a^3)*
x^7/a^5/(b*x^2+a)^(7/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1817, 1827, 12, 277, 270}

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx=-\frac{x^3\left(48Ab^2-a(aC+6bB)\right)}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{x(8Ab-aB)}{a^2\left(a+bx^2\right)^{7/2}}-\frac{2bx^7\left(-3a^3D-4ab(aC+6bB)+192Ab^3\right)}{105a^5\left(a+bx^2\right)^{7/2}}-\frac{x^5\left(-3a^3D-4ab(aC+6bB)+192Ab^3\right)}{15a^4\left(a+bx^2\right)^{7/2}}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] -(A/(a*x*(a + b*x^2)^(7/2))) - ((8*A*b - a*B)*x)/(a^2*(a + b*x^2)^(7/2)) -
((48*A*b^2 - a*(6*b*B + a*C))*x^3)/(3*a^3*(a + b*x^2)^(7/2)) - ((192*A*b^3
- 4*a*b*(6*b*B + a*C) - 3*a^3*D)*x^5)/(15*a^4*(a + b*x^2)^(7/2)) - (2*b*(19
2*A*b^3 - 4*a*b*(6*b*B + a*C) - 3*a^3*D)*x^7)/(105*a^5*(a + b*x^2)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 1827

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*x*((a + b*
x^2)^(p + 1)/a), x] + Dist[1/a, Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3))
, x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ[
Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{A}{ax\,(a+bx^2)^{7/2}} - \frac{\int \frac{8Ab-a(B+Cx^2+Dx^4)}{(a+bx^2)^{9/2}}\,dx}{a}$$

$$= -\frac{A}{ax(a+bx^2)^{7/2}} - \frac{(8Ab-aB)x}{a^2(a+bx^2)^{7/2}} - \frac{\int \frac{x^2(6b(8Ab-aB)+a(-aC-aDx^2))}{(a+bx^2)^{9/2}}dx}{a^2}$$

$$= -\frac{A}{ax(a+bx^2)^{7/2}} - \frac{(8Ab-aB)x}{a^2(a+bx^2)^{7/2}} - \frac{(48Ab^2-a(6bB+aC))x^3}{3a^3(a+bx^2)^{7/2}} - \frac{\int \frac{(4b(48Ab^2-6abB-a^2C)-3a^3D)x^4}{(a+bx^2)^{9/2}}dx}{3a^3}$$

$$= -\frac{A}{ax(a+bx^2)^{7/2}} - \frac{(8Ab-aB)x}{a^2(a+bx^2)^{7/2}} - \frac{(48Ab^2-a(6bB+aC))x^3}{3a^3(a+bx^2)^{7/2}} - \frac{(192Ab^3-4ab(6bB+aC)-3a^3D)\int \frac{x^4}{(a+bx^2)^{9/2}}dx}{3a^3}$$

$$= -\frac{A}{ax(a+bx^2)^{7/2}} - \frac{(8Ab-aB)x}{a^2(a+bx^2)^{7/2}} - \frac{(48Ab^2-a(6bB+aC))x^3}{3a^3(a+bx^2)^{7/2}} - \frac{(192Ab^3-4ab(6bB+aC)-3a^3D)x^5}{15a^4(a+bx^2)^{7/2}} - \frac{(2b(192Ab^3-4ab(6bB+aC)-3a^3D))\int \frac{x^6}{(a+bx^2)^{9/2}}dx}{15a^4}$$

$$= -\frac{A}{ax(a+bx^2)^{7/2}} - \frac{(8Ab-aB)x}{a^2(a+bx^2)^{7/2}} - \frac{(48Ab^2-a(6bB+aC))x^3}{3a^3(a+bx^2)^{7/2}} - \frac{(192Ab^3-4ab(6bB+aC)-3a^3D)x^5}{15a^4(a+bx^2)^{7/2}} - \frac{2b(192Ab^3-4ab(6bB+aC)-3a^3D)x^7}{105a^5(a+bx^2)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(a+bx^2)^{9/2}}dx = \frac{-384Ab^4x^8+48ab^3x^6(-28A+Bx^2)+8a^2b^2x^4(-210A+21Bx^2+Cx^4)-\cdots}{105a^5x\cdots}$$

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-384*A*b^4*x^8 + 48*a*b^3*x^6*(-28*A + B*x^2) + 8*a^2*b^2*x^4*(-210*A + 21
*B*x^2 + C*x^4) - 7*a^4*(15*A - 15*B*x^2 - 5*C*x^4 - 3*D*x^6) + 2*a^3*b*x^2
*(-420*A + 105*B*x^2 + 14*C*x^4 + 3*D*x^6))/(105*a^5*x*(a + b*x^2)^(7/2))
```

Maple [A] (verified)

Time = 3.55 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$-\frac{\left(-\frac{1}{5} D x^{6}-\frac{1}{3} C \,x^{4}-x^{2} B +A \right) a^{4}+8 b \left(-\frac{1}{140} D x^{6}-\frac{1}{30} C \,x^{4}-\frac{1}{4} x^{2} B +A \right) x^{2} a^{3}+16 \left(-\frac{1}{210} C \,x^{4}-\frac{1}{10} x^{2} B +A \right) b^{2} x^{4} a^{2}+\frac{64 b^{3} x^{6} \left(-\right.}{5}}{\left(b \,x^{2}+a \right)^{\frac{7}{2}} x \,a^{5}}$
gosper	$-\frac{384 A \,b^{4} x^{8}-48 B a \,b^{3} x^{8}-8 C \,a^{2} b^{2} x^{8}-6 D \,a^{3} b \,x^{8}+1344 A a \,b^{3} x^{6}-168 B \,a^{2} b^{2} x^{6}-28 C \,a^{3} b \,x^{6}-21 D \,a^{4} x^{6}+1680 A \,a^{2} b^{2} x^{4}-210}{105 x \left(b \,x^{2}+a \right)^{\frac{7}{2}} a^{5}}$
trager	$-\frac{384 A \,b^{4} x^{8}-48 B a \,b^{3} x^{8}-8 C \,a^{2} b^{2} x^{8}-6 D \,a^{3} b \,x^{8}+1344 A a \,b^{3} x^{6}-168 B \,a^{2} b^{2} x^{6}-28 C \,a^{3} b \,x^{6}-21 D \,a^{4} x^{6}+1680 A \,a^{2} b^{2} x^{4}-210}{105 x \left(b \,x^{2}+a \right)^{\frac{7}{2}} a^{5}}$
default	$B \left(\frac{x}{7 a \left(b \,x^{2}+a \right)^{\frac{7}{2}}}+\frac{\frac{6 x}{35 a \left(b \,x^{2}+a \right)^{\frac{5}{2}}}+\frac{6 \left(\frac{4 x}{15 a \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{8 x}{15 a^{2} \sqrt{b \,x^{2}+a}}\right)}{7 a}}{a}\right)+D \left(-\frac{x^{3}}{4 b \left(b \,x^{2}+a \right)^{\frac{7}{2}}}+\frac{3 a \left(-\frac{x}{6 b \left(b \,x^{2}+a \right)}\right.}{}\right.$

[In] int((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)

[Out] -((-1/5*D*x^6-1/3*C*x^4-x^2*B+A)*a^4+8*b*(-1/140*D*x^6-1/30*C*x^4-1/4*x^2*B+A)*x^2*a^3+16*(-1/210*C*x^4-1/10*x^2*B+A)*b^2*x^4*a^2+64/5*b^3*x^6*(-1/28*x^2*B+A)*a+128/35*A*b^4*x^8)/(b*x^2+a)^(7/2)/x/a^5

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \frac{(2\,(3\,Da^3b+4\,Ca^2b^2+24\,Bab^3-192\,Ab^4)x^8+7\,(3\,Da^4+4\,Ca^3b+24\,Ba^2b}{105\,(a^5b^4x^9+4}$$

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(2*(3*D*a^3*b + 4*C*a^2*b^2 + 24*B*a*b^3 - 192*A*b^4)*x^8 + 7*(3*D*a^
4 + 4*C*a^3*b + 24*B*a^2*b^2 - 192*A*a*b^3)*x^6 - 105*A*a^4 + 35*(C*a^4 + 6
*B*a^3*b - 48*A*a^2*b^2)*x^4 + 105*(B*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*x^2 + a)
/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2392 vs. $2(170) = 340$.

Time = 77.57 (sec) , antiderivative size = 2392, normalized size of antiderivative = 12.93

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**2/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(-35*a**4*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*
x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 28
0*a**3*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*
x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 56
0*a**2*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*
x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 44
8*a*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**
2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 128*b
**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 2
10*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)) + B*(35*a**
14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**
2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6
*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(
27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x*
*2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*
b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 70
0*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 +
 b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**
6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sqrt(1 + b*
```

```
x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4
*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(
29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x*
*2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b**3*x**7/(
35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) +
 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(
1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*
b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a))
 + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x
**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a
**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*
x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x
**12*sqrt(1 + b*x**2/a)) + 104*a**9*b**5*x**11/(35*a**(37/2)*sqrt(1 + b*x**
2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sq
rt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/
2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/
a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x**13/(35*a
**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525
*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 +
b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5
*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a))) +
C*(35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sq
rt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/
2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a
)) + 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**
2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**
(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x*
*2/a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2
)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) +
420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1
 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a*
*(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2
/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*
sqrt(1 + b*x**2/a))) + D*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a
**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/
a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqr
t(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**
2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.69

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx = -\frac{Dx^3}{4\left(bx^2+a\right)^{\frac{7}{2}}b} + \frac{16\,Bx}{35\sqrt{bx^2+a}a^4} + \frac{8\,Bx}{35\left(bx^2+a\right)^{\frac{3}{2}}a^3} + \frac{6\,Bx}{35\left(bx^2+a\right)^{\frac{5}{2}}a^2} + \frac{Bx}{7\left(bx^2+a\right)^{\frac{7}{2}}a} + \frac{3\,Dx}{140\left(bx^2+a\right)^{\frac{5}{2}}b^2} + \frac{2\,Dx}{35\sqrt{bx^2+a}a^2b^2} + \frac{Dx}{35\left(bx^2+a\right)^{\frac{3}{2}}ab^2} - \frac{3\,Dax}{28\left(bx^2+a\right)^{\frac{7}{2}}b^2} - \frac{Cx}{7\left(bx^2+a\right)^{\frac{7}{2}}b} + \frac{8\,Cx}{105\sqrt{bx^2+a}a^3b} + \frac{4\,Cx}{105\left(bx^2+a\right)^{\frac{3}{2}}a^2b} + \frac{Cx}{35\left(bx^2+a\right)^{\frac{5}{2}}ab} - \frac{128\,Abx}{35\sqrt{bx^2+a}a^5} - \frac{64\,Abx}{35\left(bx^2+a\right)^{\frac{3}{2}}a^4} - \frac{48\,Abx}{35\left(bx^2+a\right)^{\frac{5}{2}}a^3} - \frac{8\,Abx}{7\left(bx^2+a\right)^{\frac{7}{2}}a^2} - \frac{A}{\left(bx^2+a\right)^{\frac{7}{2}}ax}$$

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*D*x^3/((b*x^2 + a)^(7/2)*b) + 16/35*B*x/(sqrt(b*x^2 + a)*a^4) + 8/35*B
*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*B*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*B*x/((b
*x^2 + a)^(7/2)*a) + 3/140*D*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*D*x/(sqrt(b*x
^2 + a)*a^2*b^2) + 1/35*D*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*D*a*x/((b*x^2
+ a)^(7/2)*b^2) - 1/7*C*x/((b*x^2 + a)^(7/2)*b) + 8/105*C*x/(sqrt(b*x^2 + a
)*a^3*b) + 4/105*C*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*C*x/((b*x^2 + a)^(5/2
)*a*b) - 128/35*A*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*A*b*x/((b*x^2 + a)^(3/2
)*a^4) - 48/35*A*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*A*b*x/((b*x^2 + a)^(7/2)
*a^2) - A/((b*x^2 + a)^(7/2)*a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{\left(6\,Da^{12}b^4+8\,Ca^{11}b^5+48\,Ba^{10}b^6-279\,Aa^9b^7\right)x^2}{a^{14}b^3} + \frac{7\left(3\,Da^{13}b^3+4\,Ca^{12}b^4+24\,Ba^{11}b^5-132\,Aa\right.}{a^{14}b^3}\right.\right.\right.}{105\left(bx^2+a\right)^{\frac{7}{2}}} + \frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^4}$$

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((6*D*a^12*b^4 + 8*C*a^11*b^5 + 48*B*a^10*b^6 - 279*A*a^9*b^7)*
x^2/(a^14*b^3) + 7*(3*D*a^13*b^3 + 4*C*a^12*b^4 + 24*B*a^11*b^5 - 132*A*a^1
0*b^6)/(a^14*b^3)) + 35*(C*a^13*b^3 + 6*B*a^12*b^4 - 30*A*a^11*b^5)/(a^14*b
^3))*x^2 + 105*(B*a^13*b^3 - 4*A*a^12*b^4)/(a^14*b^3))*x/(b*x^2 + a)^(7/2)
+ 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^2\,{\left(b\,x^2+a\right)}^{9/2}}\,dx$$

```
[In] int((A + B*x^2 + C*x^4 + x^6*D)/(x^2*(a + b*x^2)^(9/2)),x)
[Out] int((A + B*x^2 + C*x^4 + x^6*D)/(x^2*(a + b*x^2)^(9/2)), x)
```

3.165 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^{9/2}}\,dx$

Optimal result 1030
Rubi [A] (verified) 1030
Mathematica [A] (verified) 1033
Maple [A] (verified) 1033
Fricas [A] (verification not implemented) 1035
Sympy [B] (verification not implemented) 1035
Maxima [A] (verification not implemented) 1037
Giac [A] (verification not implemented) 1038
Mupad [F(-1)] 1038

Optimal result

Integrand size = 32, antiderivative size = 242

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^{9/2}}\,dx &= -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}+\frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}}\\&\quad+\frac{(80Ab^2-3a(8bB-aC))\,x}{3a^3\left(a+bx^2\right)^{7/2}}+\frac{(160Ab^3-a(48b^2B-6abC-a^2D))\,x^3}{3a^4\left(a+bx^2\right)^{7/2}}\\&\quad+\frac{4b(160Ab^3-a(48b^2B-6abC-a^2D))\,x^5}{15a^5\left(a+bx^2\right)^{7/2}}\\&\quad+\frac{8b^2(160Ab^3-a(48b^2B-6abC-a^2D))\,x^7}{105a^6\left(a+bx^2\right)^{7/2}}\end{aligned}$$

```
[Out] -1/3*A/a/x^3/(b*x^2+a)^(7/2)+1/3*(10*A*b-3*B*a)/a^2/x/(b*x^2+a)^(7/2)+1/3*(
80*A*b^2-3*a*(8*B*b-C*a))*x/a^3/(b*x^2+a)^(7/2)+1/3*(160*A*b^3-a*(48*B*b^2-
6*C*a*b-D*a^2))*x^3/a^4/(b*x^2+a)^(7/2)+4/15*b*(160*A*b^3-a*(48*B*b^2-6*C*a
*b-D*a^2))*x^5/a^5/(b*x^2+a)^(7/2)+8/105*b^2*(160*A*b^3-a*(48*B*b^2-6*C*a*b
-D*a^2))*x^7/a^6/(b*x^2+a)^(7/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used

$= \{1817, 1827, 12, 277, 270\}$

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^{9/2}}\,dx = \frac{x(80Ab^2-3a(8bB-aC))}{3a^3\left(a+bx^2\right)^{7/2}} + \frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{8b^2x^7(160Ab^3-a(a^2(-D)-6abC+48b^2B))}{105a^6\left(a+bx^2\right)^{7/2}} + \frac{4bx^5(160Ab^3-a(a^2(-D)-6abC+48b^2B))}{15a^5\left(a+bx^2\right)^{7/2}} + \frac{x^3(160Ab^3-a(a^2(-D)-6abC+48b^2B))}{3a^4\left(a+bx^2\right)^{7/2}} - \frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(a + b*x^2)^(9/2)),x]
```

```
[Out] -1/3*A/(a*x^3*(a + b*x^2)^(7/2)) + (10*A*b - 3*a*B)/(3*a^2*x*(a + b*x^2)^(7
/2)) + ((80*A*b^2 - 3*a*(8*b*B - a*C))*x)/(3*a^3*(a + b*x^2)^(7/2)) + ((160
*A*b^3 - a*(48*b^2*B - 6*a*b*C - a^2*D))*x^3)/(3*a^4*(a + b*x^2)^(7/2)) + (
4*b*(160*A*b^3 - a*(48*b^2*B - 6*a*b*C - a^2*D))*x^5)/(15*a^5*(a + b*x^2)^(
7/2)) + (8*b^2*(160*A*b^3 - a*(48*b^2*B - 6*a*b*C - a^2*D))*x^7)/(105*a^6*(
a + b*x^2)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
```

```
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 1827

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*x*((a + b*
x^2)^(p + 1)/a), x] + Dist[1/a, Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3))
, x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ[
Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{A}{3ax^3(a+bx^2)^{7/2}} - \frac{\int \frac{10Ab-3a(B+Cx^2+Dx^4)}{x^2(a+bx^2)^{9/2}}\,dx}{3a} \\
&= -\frac{A}{3ax^3(a+bx^2)^{7/2}} + \frac{10Ab-3aB}{3a^2x(a+bx^2)^{7/2}} + \frac{\int \frac{8b(10Ab-3aB)-a(-3aC-3aDx^2)}{(a+bx^2)^{9/2}}\,dx}{3a^2} \\
&= -\frac{A}{3ax^3(a+bx^2)^{7/2}} + \frac{10Ab-3aB}{3a^2x(a+bx^2)^{7/2}} \\
&\quad + \frac{(80Ab^2-3a(8bB-aC))x}{3a^3(a+bx^2)^{7/2}} + \frac{\int \frac{(6b(80Ab^2-24abB+3a^2C)+3a^3D)x^2}{(a+bx^2)^{9/2}}\,dx}{3a^3} \\
&= -\frac{A}{3ax^3(a+bx^2)^{7/2}} + \frac{10Ab-3aB}{3a^2x(a+bx^2)^{7/2}} + \frac{(80Ab^2-3a(8bB-aC))x}{3a^3(a+bx^2)^{7/2}} \\
&\quad + \frac{(160Ab^3-a(48b^2B-6abC-a^2D))\int \frac{x^2}{(a+bx^2)^{9/2}}\,dx}{a^3} \\
&= -\frac{A}{3ax^3(a+bx^2)^{7/2}} + \frac{10Ab-3aB}{3a^2x(a+bx^2)^{7/2}} + \frac{(80Ab^2-3a(8bB-aC))x}{3a^3(a+bx^2)^{7/2}} \\
&\quad + \frac{(160Ab^3-a(48b^2B-6abC-a^2D))x^3}{3a^4(a+bx^2)^{7/2}} \\
&\quad + \frac{(4b(160Ab^3-a(48b^2B-6abC-a^2D)))\int \frac{x^4}{(a+bx^2)^{9/2}}\,dx}{3a^4} \\
&= -\frac{A}{3ax^3(a+bx^2)^{7/2}} + \frac{10Ab-3aB}{3a^2x(a+bx^2)^{7/2}} + \frac{(80Ab^2-3a(8bB-aC))x}{3a^3(a+bx^2)^{7/2}} \\
&\quad + \frac{(160Ab^3-a(48b^2B-6abC-a^2D))x^3}{3a^4(a+bx^2)^{7/2}} \\
&\quad + \frac{4b(160Ab^3-a(48b^2B-6abC-a^2D))x^5}{15a^5(a+bx^2)^{7/2}} \\
&\quad + \frac{(8b^2(160Ab^3-a(48b^2B-6abC-a^2D)))\int \frac{x^6}{(a+bx^2)^{9/2}}\,dx}{15a^5}
\end{aligned}
$$

$$= -\frac{A}{3ax^3(a+bx^2)^{7/2}} + \frac{10Ab-3aB}{3a^2x(a+bx^2)^{7/2}} + \frac{(80Ab^2-3a(8bB-aC))\,x}{3a^3(a+bx^2)^{7/2}} + \frac{(160Ab^3-a(48b^2B-6abC-a^2D))\,x^3}{3a^4(a+bx^2)^{7/2}} + \frac{4b(160Ab^3-a(48b^2B-6abC-a^2D))\,x^5}{15a^5(a+bx^2)^{7/2}} + \frac{8b^2(160Ab^3-a(48b^2B-6abC-a^2D))\,x^7}{105a^6(a+bx^2)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(a+bx^2)^{9/2}}\,dx = \frac{1280Ab^5x^{10}+128ab^4x^8(35A-3Bx^2)+16a^2b^3x^6(350A-84Bx^2+3Cx^4)}{}$$

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(a + b*x^2)^(9/2)),x]
```

```
[Out] (1280*A*b^5*x^10 + 128*a*b^4*x^8*(35*A - 3*B*x^2) + 16*a^2*b^3*x^6*(350*A -
 84*B*x^2 + 3*C*x^4) - 35*a^5*(A + 3*B*x^2 - 3*C*x^4 - D*x^6) + 8*a^3*b^2*x
^4*(350*A - 210*B*x^2 + 21*C*x^4 + D*x^6) + 14*a^4*b*x^2*(25*A - 60*B*x^2 +
 15*C*x^4 + 2*D*x^6))/(105*a^6*x^3*(a + b*x^2)^(7/2))
```

Maple [A] (verified)

Time = 3.55 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{7\left(D x^{6}+3 C \,x^{4}-3 x^{2} B -A \right) a^{5}+70 b \,x^{2} \left(\frac{2}{25} D x^{6}+\frac{3}{5} C \,x^{4}-\frac{12}{5} x^{2} B +A \right) a^{4}+560 b^{2} x^{4} \left(\frac{1}{350} D x^{6}+\frac{3}{50} C \,x^{4}-\frac{3}{5} x^{2} B +A \right) a^{3}+1120 \left(\frac{3}{350} C\right.}{21 \left(b \,x^{2}+a \right)^{\frac{7}{2}} x^{3} a^{6}}$
gosper	$-\frac{-1280 A \,b^{5} x^{10}+384 B a \,b^{4} x^{10}-48 C \,a^{2} b^{3} x^{10}-8 D \,a^{3} b^{2} x^{10}-4480 a A \,b^{4} x^{8}+1344 B \,a^{2} b^{3} x^{8}-168 C \,a^{3} b^{2} x^{8}-28 D \,a^{4} b \,x^{8}-5600 a^{2}}{105 x^{3} (b}$
trager	$-\frac{-1280 A \,b^{5} x^{10}+384 B a \,b^{4} x^{10}-48 C \,a^{2} b^{3} x^{10}-8 D \,a^{3} b^{2} x^{10}-4480 a A \,b^{4} x^{8}+1344 B \,a^{2} b^{3} x^{8}-168 C \,a^{3} b^{2} x^{8}-28 D \,a^{4} b \,x^{8}-5600 a^{2}}{105 x^{3} (b}$
default	$C \left(\frac{x}{7 a \left(b \,x^{2}+a \right)^{\frac{7}{2}}}+\frac{\frac{6 x}{35 a \left(b \,x^{2}+a \right)^{\frac{5}{2}}}+\frac{6 \left(\frac{4 x}{15 a \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{8 x}{15 a^{2} \sqrt{b \,x^{2}+a}}\right)}{7 a}}{a}\right)+D \left(-\frac{x}{6 b \left(b \,x^{2}+a \right)^{\frac{7}{2}}}+\frac{a \left(\frac{x}{7 a \left(b \,x^{2}+a \right)^{\frac{7}{2}}}+\right.}{}\right.$

[In] int((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)

[Out] 1/21*(7*(D*x^6+3*C*x^4-3*B*x^2-A)*a^5+70*b*x^2*(2/25*D*x^6+3/5*C*x^4-12/5*x^2*B+A)*a^4+560*b^2*x^4*(1/350*D*x^6+3/50*C*x^4-3/5*x^2*B+A)*a^3+1120*(3/350*C*x^4-6/25*x^2*B+A)*b^3*x^6*a^2+896*(-3/35*x^2*B+A)*b^4*x^8*a+256*A*b^5*x^10)/(b*x^2+a)^(7/2)/x^3/a^6

Fricas [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(8\left(Da^3b^2+6\,Ca^2b^3-48\,Bab^4+160\,Ab^5\right)x^{10}+28\left(Da^4b+6\,Ca^3b^2-48\,B\right.}{}$$

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(D*a^3*b^2 + 6*C*a^2*b^3 - 48*B*a*b^4 + 160*A*b^5)*x^10 + 28*(D*a^
4*b + 6*C*a^3*b^2 - 48*B*a^2*b^3 + 160*A*a*b^4)*x^8 + 35*(D*a^5 + 6*C*a^4*b
 - 48*B*a^3*b^2 + 160*A*a^2*b^3)*x^6 - 35*A*a^5 + 35*(3*C*a^5 - 24*B*a^4*b
+ 80*A*a^3*b^2)*x^4 - 35*(3*B*a^5 - 10*A*a^4*b)*x^2)*sqrt(b*x^2 + a)/(a^6*b
^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2861 vs. 2(224) = 448.

Time = 120.75 (sec) , antiderivative size = 2861, normalized size of antiderivative = 11.82

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**4/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(-7*a**6*b**(51/2)*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*
b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**
10 + 21*a**6*b**30*x**12) + 63*a**5*b**(53/2)*x**2*sqrt(a/(b*x**2) + 1)/(21
*a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b
**28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12) + 630*a**4*b**(55/2
)*x**4*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 2
10*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b
**30*x**12) + 1680*a**3*b**(57/2)*x**6*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25
*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 +
105*a**7*b**29*x**10 + 21*a**6*b**30*x**12) + 2016*a**2*b**(59/2)*x**8*sqrt
(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**
27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12)
 + 1152*a*b**(61/2)*x**10*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a
**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**2
9*x**10 + 21*a**6*b**30*x**12) + 256*b**(63/2)*x**12*sqrt(a/(b*x**2) + 1)/(
21*a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8
*b**28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12)) + B*(-35*a**4*b*
*(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 210*a**
```

```
7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 280*a**3*b**(35/
2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 210*a**
7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 560*a**2*b**(37/
2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 210*a**
7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 448*a*b**(39/2)*
x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 210*a**7*b
**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 128*b**(41/2)*x**8*
sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 210*a**7*b**18*
x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)) + C*(35*a**14*x/(35*a**(3
7/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**
(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x*
*2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**
10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 175*a
**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1
+ b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b*
*3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 2
10*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1
 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*
a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x*
*2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**
8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**
(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b**3*x**7/(35*a**(37/2)*s
qrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)
*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a)
+ 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqr
t(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 286*a**10*b
**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b
*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*
x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*
a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 +
b*x**2/a)) + 104*a**9*b**5*x**11/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**
(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/
a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*s
qrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25
/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x**13/(35*a**(37/2)*sqrt(
1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**
2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 52
5*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1
+ b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a))) + D*(35*a**5*x**
3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/
a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*s
qrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 63*a**4*b
*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x
**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x*
*6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 36*a*
```

```
*3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(
1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*
b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a))
+ 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2
*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(
13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**
2/a)))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.39

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\,(a+bx^2)^{9/2}}\,dx = \frac{16\,Cx}{35\,\sqrt{bx^2+a}a^4}+\frac{8\,Cx}{35\,(bx^2+a)^{\frac{3}{2}}a^3}+\frac{6\,Cx}{35\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{Cx}{7\,(bx^2+a)^{\frac{7}{2}}a}-\frac{Dx}{7\,(bx^2+a)^{\frac{7}{2}}b}+\frac{8\,Dx}{105\,\sqrt{bx^2+a}a^3b}+\frac{4\,Dx}{105\,(bx^2+a)^{\frac{3}{2}}a^2b} + \frac{Dx}{35\,(bx^2+a)^{\frac{5}{2}}ab}-\frac{128\,Bbx}{35\,\sqrt{bx^2+a}a^5}-\frac{64\,Bbx}{35\,(bx^2+a)^{\frac{3}{2}}a^4}-\frac{48\,Bbx}{35\,(bx^2+a)^{\frac{5}{2}}a^3} - \frac{8\,Bbx}{7\,(bx^2+a)^{\frac{7}{2}}a^2}+\frac{256\,Ab^2x}{21\,\sqrt{bx^2+a}a^6}+\frac{128\,Ab^2x}{21\,(bx^2+a)^{\frac{3}{2}}a^5}+\frac{32\,Ab^2x}{7\,(bx^2+a)^{\frac{5}{2}}a^4} + \frac{80\,Ab^2x}{21\,(bx^2+a)^{\frac{7}{2}}a^3}-\frac{B}{(bx^2+a)^{\frac{7}{2}}ax}+\frac{10\,Ab}{3\,(bx^2+a)^{\frac{7}{2}}a^2x}-\frac{A}{3\,(bx^2+a)^{\frac{7}{2}}ax^3}$$

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*C*x/(sqrt(b*x^2 + a)*a^4) + 8/35*C*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*C
*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*C*x/((b*x^2 + a)^(7/2)*a) - 1/7*D*x/((b*x^
2 + a)^(7/2)*b) + 8/105*D*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*D*x/((b*x^2 + a
)^(3/2)*a^2*b) + 1/35*D*x/((b*x^2 + a)^(5/2)*a*b) - 128/35*B*b*x/(sqrt(b*x^
2 + a)*a^5) - 64/35*B*b*x/((b*x^2 + a)^(3/2)*a^4) - 48/35*B*b*x/((b*x^2 + a
)^(5/2)*a^3) - 8/7*B*b*x/((b*x^2 + a)^(7/2)*a^2) + 256/21*A*b^2*x/(sqrt(b*x
^2 + a)*a^6) + 128/21*A*b^2*x/((b*x^2 + a)^(3/2)*a^5) + 32/7*A*b^2*x/((b*x^
2 + a)^(5/2)*a^4) + 80/21*A*b^2*x/((b*x^2 + a)^(7/2)*a^3) - B/((b*x^2 + a)^
(7/2)*a*x) + 10/3*A*b/((b*x^2 + a)^(7/2)*a^2*x) - 1/3*A/((b*x^2 + a)^(7/2)*
a*x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.44

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{(8\,Da^{15}b^5+48\,Ca^{14}b^6-279\,Ba^{13}b^7+790\,Aa^{12}b^8)x^2}{a^{18}b^3}+\frac{7\,(4\,Da^{16}b^4+24\,Ca^{15}b^5-132\,Ba^{14}b^6+3}{a^{18}b^3}\right.\right.\right.}{105}$$

$$+\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba\sqrt{b}-12\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ab^{\frac{3}{2}}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^2\sqrt{b}+30\left(\sqrt{b}x-\right.\right.}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3a^5}$$

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((8*D*a^15*b^5 + 48*C*a^14*b^6 - 279*B*a^13*b^7 + 790*A*a^12*b^
8)*x^2/(a^18*b^3) + 7*(4*D*a^16*b^4 + 24*C*a^15*b^5 - 132*B*a^14*b^6 + 365*
A*a^13*b^7)/(a^18*b^3)) + 35*(D*a^17*b^3 + 6*C*a^16*b^4 - 30*B*a^15*b^5 + 8
0*A*a^14*b^6)/(a^18*b^3))*x^2 + 105*(C*a^17*b^3 - 4*B*a^16*b^4 + 10*A*a^15*
b^5)/(a^18*b^3))*x/(b*x^2 + a)^(7/2) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))
^4*B*a*sqrt(b) - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/2) - 6*(sqrt(b)*
x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b) + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A
*a*b^(3/2) + 3*B*a^3*sqrt(b) - 14*A*a^2*b^(3/2))/(((sqrt(b)*x - sqrt(b*x^2
+ a))^2 - a)^3*a^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^{9/2}}\,dx = \int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^4\,{\left(b\,x^2+a\right)}^{9/2}} \,dx$$

```
[In] int((A + B*x^2 + C*x^4 + x^6*D)/(x^4*(a + b*x^2)^(9/2)),x)
```

```
[Out] int((A + B*x^2 + C*x^4 + x^6*D)/(x^4*(a + b*x^2)^(9/2)), x)
```

3.166 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{9/2}}\,dx$

Optimal result . . . 1039
Rubi [A] (verified) . . . 1040
Mathematica [A] (verified) . . . 1042
Maple [A] (verified) . . . 1043
Fricas [A] (verification not implemented) . . . 1045
Sympy [F(-1)] . . . 1045
Maxima [A] (verification not implemented) . . . 1045
Giac [B] (verification not implemented) . . . 1046
Mupad [B] (verification not implemented) . . . 1047

Optimal result

Integrand size = 32, antiderivative size = 281

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{9/2}}\,dx &= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}+\frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}}\\ &-\frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}}-\frac{\left(192Ab^3-a\left(80b^2B-24abC+3a^2D\right)\right)x}{21a^4\left(a+bx^2\right)^{7/2}}\\ &-\frac{2\left(192Ab^3-a\left(80b^2B-24abC+3a^2D\right)\right)x}{35a^5\left(a+bx^2\right)^{5/2}}\\ &-\frac{8\left(192Ab^3-a\left(80b^2B-24abC+3a^2D\right)\right)x}{105a^6\left(a+bx^2\right)^{3/2}}\\ &-\frac{16\left(192Ab^3-a\left(80b^2B-24abC+3a^2D\right)\right)x}{105a^7\sqrt{a+bx^2}}\end{aligned}$$

```
[Out] -1/5*A/a/x^5/(b*x^2+a)^(7/2)+1/15*(12*A*b-5*B*a)/a^2/x^3/(b*x^2+a)^(7/2)+1/
3*(-24*A*b^2+a*(10*B*b-3*C*a))/a^3/x/(b*x^2+a)^(7/2)-1/21*(192*A*b^3-a*(80*
B*b^2-24*C*a*b+3*D*a^2))*x/a^4/(b*x^2+a)^(7/2)-2/35*(192*A*b^3-a*(80*B*b^2-
24*C*a*b+3*D*a^2))*x/a^5/(b*x^2+a)^(5/2)-8/105*(192*A*b^3-a*(80*B*b^2-24*C*
a*b+3*D*a^2))*x/a^6/(b*x^2+a)^(3/2)-16/105*(192*A*b^3-a*(80*B*b^2-24*C*a*b+
3*D*a^2))*x/a^7/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1817, 12, 198, 197}

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{9/2}}dx=-\frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}}+\frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}}-\frac{16x(-3a^3D-8ab(10bB-3aC)+192Ab^3)}{105a^7\sqrt{a+bx^2}}-\frac{8x(192Ab^3-a(3a^2D-24abC+80b^2B))}{105a^6\left(a+bx^2\right)^{3/2}}-\frac{2x(-3a^3D-8ab(10bB-3aC)+192Ab^3)}{35a^5\left(a+bx^2\right)^{5/2}}-\frac{x(-3a^3D-8ab(10bB-3aC)+192Ab^3)}{21a^4\left(a+bx^2\right)^{7/2}}-\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(a + b*x^2)^(9/2)),x]
```

```
[Out] -1/5*A/(a*x^5*(a + b*x^2)^(7/2)) + (12*A*b - 5*a*B)/(15*a^2*x^3*(a + b*x^2)
^(7/2)) - (24*A*b^2 - a*(10*b*B - 3*a*C))/(3*a^3*x*(a + b*x^2)^(7/2)) - ((1
92*A*b^3 - 8*a*b*(10*b*B - 3*a*C) - 3*a^3*D)*x)/(21*a^4*(a + b*x^2)^(7/2))
- (2*(192*A*b^3 - 8*a*b*(10*b*B - 3*a*C) - 3*a^3*D)*x)/(35*a^5*(a + b*x^2)^
(5/2)) - (8*(192*A*b^3 - a*(80*b^2*B - 24*a*b*C + 3*a^2*D))*x)/(105*a^6*(a
+ b*x^2)^(3/2)) - (16*(192*A*b^3 - 8*a*b*(10*b*B - 3*a*C) - 3*a^3*D)*x)/(10
5*a^7*Sqrt[a + b*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{A}{5ax^5(a+bx^2)^{7/2}} - \frac{\int \frac{12Ab-5a(B+Cx^2+Dx^4)}{x^4(a+bx^2)^{9/2}}\,dx}{5a}$$

$$= -\frac{A}{5ax^5(a+bx^2)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3(a+bx^2)^{7/2}} + \frac{\int \frac{10b(12Ab-5aB)-3a(-5aC-5aDx^2)}{x^2(a+bx^2)^{9/2}}\,dx}{15a^2}$$

$$= -\frac{A}{5ax^5(a+bx^2)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3(a+bx^2)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x(a+bx^2)^{7/2}} - \frac{\int \frac{8b(120Ab^2-50abB+15a^2C)-15a^3D}{(a+bx^2)^{9/2}}\,dx}{15a^3}$$

$$= -\frac{A}{5ax^5(a+bx^2)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3(a+bx^2)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x(a+bx^2)^{7/2}} - \frac{(192Ab^3-8ab(10bB-3aC)-3a^3D)\int \frac{1}{(a+bx^2)^{9/2}}\,dx}{3a^3}$$

$$= -\frac{A}{5ax^5(a+bx^2)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3(a+bx^2)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x(a+bx^2)^{7/2}} - \frac{(192Ab^3-8ab(10bB-3aC)-3a^3D)\,x}{21a^4(a+bx^2)^{7/2}} - \frac{(2(192Ab^3-8ab(10bB-3aC)-3a^3D))\int \frac{1}{(a+bx^2)^{7/2}}\,dx}{7a^4}$$

$$= -\frac{A}{5ax^5(a+bx^2)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3(a+bx^2)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x(a+bx^2)^{7/2}} - \frac{(192Ab^3-8ab(10bB-3aC)-3a^3D)\,x}{21a^4(a+bx^2)^{7/2}} - \frac{2(192Ab^3-8ab(10bB-3aC)-3a^3D)\,x}{35a^5(a+bx^2)^{5/2}} - \frac{(8(192Ab^3-8ab(10bB-3aC)-3a^3D))\int \frac{1}{(a+bx^2)^{5/2}}\,dx}{35a^5}$$

$$= -\frac{A}{5ax^5(a+bx^2)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3(a+bx^2)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x(a+bx^2)^{7/2}} - \frac{(192Ab^3-8ab(10bB-3aC)-3a^3D)\,x}{21a^4(a+bx^2)^{7/2}} - \frac{2(192Ab^3-8ab(10bB-3aC)-3a^3D)\,x}{35a^5(a+bx^2)^{5/2}} - \frac{8(192Ab^3-8ab(10bB-3aC)-3a^3D)\,x}{105a^6(a+bx^2)^{3/2}} - \frac{(16(192Ab^3-8ab(10bB-3aC)-3a^3D))\int\frac{1}{(a+bx^2)^{3/2}}\,dx}{105a^6}$$

$$= -\frac{A}{5ax^5(a+bx^2)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3(a+bx^2)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x(a+bx^2)^{7/2}} - \frac{(192Ab^3-8ab(10bB-3aC)-3a^3D)\,x}{21a^4(a+bx^2)^{7/2}} - \frac{2(192Ab^3-8ab(10bB-3aC)-3a^3D)\,x}{35a^5(a+bx^2)^{5/2}} - \frac{8(192Ab^3-8ab(10bB-3aC)-3a^3D)\,x}{105a^6(a+bx^2)^{3/2}} - \frac{16(192Ab^3-8ab(10bB-3aC)-3a^3D)\,x}{105a^7\sqrt{a+bx^2}}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6(a+bx^2)^{9/2}}\,dx = \frac{-3072Ab^6x^{12}+256ab^5x^{10}(-42A+5Bx^2)-128a^2b^4x^8(105A-35Bx^2+3C}{}$$

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-3072*A*b^6*x^12 + 256*a*b^5*x^10*(-42*A + 5*B*x^2) - 128*a^2*b^4*x^8*(105
*A - 35*B*x^2 + 3*C*x^4) + 16*a^3*b^3*x^6*(-420*A + 350*B*x^2 - 84*C*x^4 +
3*D*x^6) + 56*a^4*b^2*x^4*(-15*A + 50*B*x^2 - 30*C*x^4 + 3*D*x^6) + 14*a^5*
b*x^2*(6*A + 25*B*x^2 - 60*C*x^4 + 15*D*x^6) - 7*a^6*(3*A + 5*x^2*(B + 3*C*
x^2 - 3*D*x^4)))/(105*a^7*x^5*(a + b*x^2)^(7/2))
```

Maple [A] (verified)

Time = 3.64 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$-\frac{\left(-5D x^{6}+5C x^{4}+\frac{5}{3}x^{2}B+A\right)a^{6}-4\left(\frac{5}{2}D x^{6}-10C x^{4}+\frac{25}{6}x^{2}B+A\right)b x^{2}a^{5}+40\left(-\frac{1}{5}D x^{6}+2C x^{4}-\frac{10}{3}x^{2}B+A\right)b^{2}x^{4}a^{4}+320b^{3}x^{6}(}{5\left(b x^{2}+a\right)^{\frac{7}{2}}x^{5}a^{7}}$
gosper	$-\frac{3072A b^{6}x^{12}-1280B a b^{5}x^{12}+384C a^{2}b^{4}x^{12}-48D a^{3}b^{3}x^{12}+10752A a b^{5}x^{10}-4480B a^{2}b^{4}x^{10}+1344C a^{3}b^{3}x^{10}-168D a^{4}b^{2}x^{1}}{}$
trager	$-\frac{3072A b^{6}x^{12}-1280B a b^{5}x^{12}+384C a^{2}b^{4}x^{12}-48D a^{3}b^{3}x^{12}+10752A a b^{5}x^{10}-4480B a^{2}b^{4}x^{10}+1344C a^{3}b^{3}x^{10}-168D a^{4}b^{2}x^{1}}{}$
default	$D\left(\frac{x}{7a\left(b x^{2}+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b x^{2}+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{b x^{2}+a}}\right)}{7a}}{a}\right)+B\left(-\frac{1}{3a x^{3}\left(b x^{2}+a\right)^{\frac{7}{2}}}-\frac{10b\left(-\frac{1}{a x\left(b x^{2}+\right.}\right.}{}\right.$

```
[In] int((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*((-5*D*x^6+5*C*x^4+5/3*x^2*B+A)*a^6-4*(5/2*D*x^6-10*C*x^4+25/6*x^2*B+A
)*b*x^2*a^5+40*(-1/5*D*x^6+2*C*x^4-10/3*x^2*B+A)*b^2*x^4*a^4+320*b^3*x^6*(-
1/140*D*x^6+1/5*C*x^4-5/6*x^2*B+A)*a^3+640*(1/35*C*x^4-1/3*x^2*B+A)*b^4*x^8
*a^2+512*(-5/42*x^2*B+A)*b^5*x^10*a+1024/7*A*b^6*x^12)/(b*x^2+a)^(7/2)/x^5/
a^7
```

Fricas [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{9/2}}\,dx = \frac{(16\,(3\,Da^3b^3-24\,Ca^2b^4+80\,Bab^5-192\,Ab^6)x^{12}+56\,(3\,Da^4b^2-24\,Ca^3b^3}{}$$

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(16*(3*D*a^3*b^3 - 24*C*a^2*b^4 + 80*B*a*b^5 - 192*A*b^6)*x^12 + 56*(
3*D*a^4*b^2 - 24*C*a^3*b^3 + 80*B*a^2*b^4 - 192*A*a*b^5)*x^10 + 70*(3*D*a^5
*b - 24*C*a^4*b^2 + 80*B*a^3*b^3 - 192*A*a^2*b^4)*x^8 - 21*A*a^6 + 35*(3*D*
a^6 - 24*C*a^5*b + 80*B*a^4*b^2 - 192*A*a^3*b^3)*x^6 - 35*(3*C*a^6 - 10*B*a
^5*b + 24*A*a^4*b^2)*x^4 - 7*(5*B*a^6 - 12*A*a^5*b)*x^2)*sqrt(b*x^2 + a)/(a
^7*b^4*x^13 + 4*a^8*b^3*x^11 + 6*a^9*b^2*x^9 + 4*a^10*b*x^7 + a^11*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**6/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.42

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{9/2}}\,dx = \frac{16\,Dx}{35\,\sqrt{bx^2+a}a^4} + \frac{8\,Dx}{35\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{6\,Dx}{35\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{Dx}{7\,(bx^2+a)^{\frac{7}{2}}a} - \frac{128\,Cbx}{35\,\sqrt{bx^2+a}a^5} - \frac{64\,Cbx}{35\,(bx^2+a)^{\frac{3}{2}}a^4} - \frac{48\,Cbx}{35\,(bx^2+a)^{\frac{5}{2}}a^3} - \frac{8\,Cbx}{7\,(bx^2+a)^{\frac{7}{2}}a^2} + \frac{256\,Bb^2x}{21\,\sqrt{bx^2+a}a^6} + \frac{128\,Bb^2x}{21\,(bx^2+a)^{\frac{3}{2}}a^5} + \frac{32\,Bb^2x}{7\,(bx^2+a)^{\frac{5}{2}}a^4} + \frac{80\,Bb^2x}{21\,(bx^2+a)^{\frac{7}{2}}a^3} - \frac{1024\,Ab^3x}{35\,\sqrt{bx^2+a}a^7} - \frac{512\,Ab^3x}{35\,(bx^2+a)^{\frac{3}{2}}a^6} - \frac{384\,Ab^3x}{35\,(bx^2+a)^{\frac{5}{2}}a^5} - \frac{64\,Ab^3x}{7\,(bx^2+a)^{\frac{7}{2}}a^4} - \frac{C}{(bx^2+a)^{\frac{7}{2}}ax} + \frac{10\,Bb}{3\,(bx^2+a)^{\frac{7}{2}}a^2x} - \frac{8\,Ab^2}{(bx^2+a)^{\frac{7}{2}}a^3x} - \frac{B}{3\,(bx^2+a)^{\frac{7}{2}}ax^3} + \frac{4\,Ab}{5\,(bx^2+a)^{\frac{7}{2}}a^2x^3} - \frac{A}{5\,(bx^2+a)^{\frac{7}{2}}ax^5}$$

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*D*x/(sqrt(b*x^2 + a)*a^4) + 8/35*D*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*D
*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*D*x/((b*x^2 + a)^(7/2)*a) - 128/35*C*b*x/(
sqrt(b*x^2 + a)*a^5) - 64/35*C*b*x/((b*x^2 + a)^(3/2)*a^4) - 48/35*C*b*x/((
b*x^2 + a)^(5/2)*a^3) - 8/7*C*b*x/((b*x^2 + a)^(7/2)*a^2) + 256/21*B*b^2*x/
(sqrt(b*x^2 + a)*a^6) + 128/21*B*b^2*x/((b*x^2 + a)^(3/2)*a^5) + 32/7*B*b^2
*x/((b*x^2 + a)^(5/2)*a^4) + 80/21*B*b^2*x/((b*x^2 + a)^(7/2)*a^3) - 1024/3
5*A*b^3*x/(sqrt(b*x^2 + a)*a^7) - 512/35*A*b^3*x/((b*x^2 + a)^(3/2)*a^6) -
384/35*A*b^3*x/((b*x^2 + a)^(5/2)*a^5) - 64/7*A*b^3*x/((b*x^2 + a)^(7/2)*a^
4) - C/((b*x^2 + a)^(7/2)*a*x) + 10/3*B*b/((b*x^2 + a)^(7/2)*a^2*x) - 8*A*b
^2/((b*x^2 + a)^(7/2)*a^3*x) - 1/3*B/((b*x^2 + a)^(7/2)*a*x^3) + 4/5*A*b/((
b*x^2 + a)^(7/2)*a^2*x^3) - 1/5*A/((b*x^2 + a)^(7/2)*a*x^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. 2(252) = 504.

Time = 0.30 (sec) , antiderivative size = 592, normalized size of antiderivative = 2.11

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{(48\,Da^{18}b^6-279\,Ca^{17}b^7+790\,Ba^{16}b^8-1686\,Aa^{15}b^9)x^2}{a^{22}b^3} + \frac{7\,(24\,Da^{19}b^5-132\,Ca^{18}b^6+365\,Ba^{1}}{a^{22}b^3}\right.\right.\right.}{} + \frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Ca^2\sqrt{b} - 60\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Bab^{\frac{3}{2}} + 150\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Ab^{\frac{5}{2}} - 60\left(\sqrt{b}\right.\right.}{}$$

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((48*D*a^18*b^6 - 279*C*a^17*b^7 + 790*B*a^16*b^8 - 1686*A*a^15
*b^9)*x^2/(a^22*b^3) + 7*(24*D*a^19*b^5 - 132*C*a^18*b^6 + 365*B*a^17*b^7 -
 768*A*a^16*b^8)/(a^22*b^3)) + 35*(6*D*a^20*b^4 - 30*C*a^19*b^5 + 80*B*a^18
*b^6 - 165*A*a^17*b^7)/(a^22*b^3))*x^2 + 105*(D*a^21*b^3 - 4*C*a^20*b^4 + 1
0*B*a^19*b^5 - 20*A*a^18*b^6)/(a^22*b^3))*x/(b*x^2 + a)^(7/2) + 2/15*(15*(s
qrt(b)*x - sqrt(b*x^2 + a))^8*C*a^2*sqrt(b) - 60*(sqrt(b)*x - sqrt(b*x^2 +
a))^8*B*a*b^(3/2) + 150*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5/2) - 60*(sqr
t(b)*x - sqrt(b*x^2 + a))^6*C*a^3*sqrt(b) + 270*(sqrt(b)*x - sqrt(b*x^2 + a
))^6*B*a^2*b^(3/2) - 720*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a*b^(5/2) + 90*(
sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^4*sqrt(b) - 430*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*B*a^3*b^(3/2) + 1260*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(5/2)
- 60*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^5*sqrt(b) + 290*(sqrt(b)*x - sqrt(
b*x^2 + a))^2*B*a^4*b^(3/2) - 840*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b^(
5/2) + 15*C*a^6*sqrt(b) - 70*B*a^5*b^(3/2) + 198*A*a^4*b^(5/2))/(((sqrt(b)*
x - sqrt(b*x^2 + a))^2 - a)^5*a^6)
```

Mupad [B] (verification not implemented)

Time = 7.41 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.44

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6 (a+bx^2)^{9/2}}\,dx = \frac{\frac{61\,A\,b}{35\,a^3}+\frac{78\,A\,b^2\,x^2}{35\,a^4}}{x^3\,{\left(b\,x^2+a\right)}^{5/2}}+\frac{\frac{128\,B\,b}{21\,a^5}+\frac{256\,B\,b^2\,x^2}{21\,a^6}}{x\,\sqrt{b\,x^2+a}}+\frac{x\,D}{{\left(b\,x^2+a\right)}^{9/2}} - \frac{\frac{B}{3\,a^2}+\frac{19\,B\,b\,x^2}{21\,a^3}}{x^3\,{\left(b\,x^2+a\right)}^{5/2}}-\frac{\frac{C}{a^4}+\frac{128\,C\,b\,x^2}{35\,a^5}}{x\,\sqrt{b\,x^2+a}}-\frac{\frac{512\,A\,b^2}{35\,a^6}+\frac{1024\,A\,b^3\,x^2}{35\,a^7}}{x\,\sqrt{b\,x^2+a}}-\frac{A\,\sqrt{b\,x^2+a}}{5\,a^5\,x^5} + \frac{18\,b^2\,x^5\,D}{5\,a^2\,{\left(b\,x^2+a\right)}^{9/2}}+\frac{72\,b^3\,x^7\,D}{35\,a^3\,{\left(b\,x^2+a\right)}^{9/2}}+\frac{16\,b^4\,x^9\,D}{35\,a^4\,{\left(b\,x^2+a\right)}^{9/2}}-\frac{A\,b}{7\,a^2\,x^3\,{\left(b\,x^2+a\right)}^{7/2}} - \frac{32\,B\,b}{21\,a^4\,x\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{B\,b^2\,x}{7\,a^3\,{\left(b\,x^2+a\right)}^{7/2}}+\frac{27\,A\,b^2}{7\,a^5\,x\,{\left(b\,x^2+a\right)}^{3/2}} + \frac{3\,b\,x^3\,D}{a\,{\left(b\,x^2+a\right)}^{9/2}}-\frac{29\,C\,b\,x}{35\,a^4\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{13\,C\,b\,x}{35\,a^3\,{\left(b\,x^2+a\right)}^{5/2}}-\frac{C\,b\,x}{7\,a^2\,{\left(b\,x^2+a\right)}^{7/2}}$$

```
[In] int((A + B*x^2 + C*x^4 + x^6*D)/(x^6*(a + b*x^2)^(9/2)),x)
```

```
[Out] ((61*A*b)/(35*a^3) + (78*A*b^2*x^2)/(35*a^4))/(x^3*(a + b*x^2)^(5/2)) + ((1
28*B*b)/(21*a^5) + (256*B*b^2*x^2)/(21*a^6))/(x*(a + b*x^2)^(1/2)) + (x*D)/
(a + b*x^2)^(9/2) - (B/(3*a^2) + (19*B*b*x^2)/(21*a^3))/(x^3*(a + b*x^2)^(5
/2)) - (C/a^4 + (128*C*b*x^2)/(35*a^5))/(x*(a + b*x^2)^(1/2)) - ((512*A*b^2
)/(35*a^6) + (1024*A*b^3*x^2)/(35*a^7))/(x*(a + b*x^2)^(1/2)) - (A*(a + b*x
^2)^(1/2))/(5*a^5*x^5) + (18*b^2*x^5*D)/(5*a^2*(a + b*x^2)^(9/2)) + (72*b^3
*x^7*D)/(35*a^3*(a + b*x^2)^(9/2)) + (16*b^4*x^9*D)/(35*a^4*(a + b*x^2)^(9/
2)) - (A*b)/(7*a^2*x^3*(a + b*x^2)^(7/2)) - (32*B*b)/(21*a^4*x*(a + b*x^2)^
```

```
(3/2)) + (B*b^2*x)/(7*a^3*(a + b*x^2)^(7/2)) + (27*A*b^2)/(7*a^5*x*(a + b*x
^2)^(3/2)) + (3*b*x^3*D)/(a*(a + b*x^2)^(9/2)) - (29*C*b*x)/(35*a^4*(a + b*
x^2)^(3/2)) - (13*C*b*x)/(35*a^3*(a + b*x^2)^(5/2)) - (C*b*x)/(7*a^2*(a + b
*x^2)^(7/2))
```

3.167 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(a+bx^2)^{9/2}}\,dx$

Optimal result 1049
Rubi [A] (verified) 1050
Mathematica [A] (verified) 1053
Maple [A] (verified) 1054
Fricas [A] (verification not implemented) 1056
Sympy [F(-1)] 1056
Maxima [A] (verification not implemented) 1057
Giac [B] (verification not implemented) 1058
Mupad [B] (verification not implemented) 1059

Optimal result

Integrand size = 32, antiderivative size = 334

$$
\begin{aligned}
\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)^{9/2}}\,dx = & -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}}+\frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} \\
& -\frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}}+\frac{48Ab^3-a(24b^2B-10abC+3a^2D)}{3a^4x\left(a+bx^2\right)^{7/2}} \\
& +\frac{8b(48Ab^3-a(24b^2B-10abC+3a^2D))\,x}{21a^5\left(a+bx^2\right)^{7/2}} \\
& +\frac{16b(48Ab^3-a(24b^2B-10abC+3a^2D))\,x}{35a^6\left(a+bx^2\right)^{5/2}} \\
& +\frac{64b(48Ab^3-a(24b^2B-10abC+3a^2D))\,x}{105a^7\left(a+bx^2\right)^{3/2}} \\
& +\frac{128b(48Ab^3-a(24b^2B-10abC+3a^2D))\,x}{105a^8\sqrt{a+bx^2}}
\end{aligned}
$$

```
[Out] -1/7*A/a/x^7/(b*x^2+a)^(7/2)+1/5*(2*A*b-B*a)/a^2/x^5/(b*x^2+a)^(7/2)+1/15*(
-24*A*b^2+a*(12*B*b-5*C*a))/a^3/x^3/(b*x^2+a)^(7/2)+1/3*(48*A*b^3-a*(24*B*b
^2-10*C*a*b+3*D*a^2))/a^4/x/(b*x^2+a)^(7/2)+8/21*b*(48*A*b^3-a*(24*B*b^2-10
*C*a*b+3*D*a^2))*x/a^5/(b*x^2+a)^(7/2)+16/35*b*(48*A*b^3-a*(24*B*b^2-10*C*a
*b+3*D*a^2))*x/a^6/(b*x^2+a)^(5/2)+64/105*b*(48*A*b^3-a*(24*B*b^2-10*C*a*b+
3*D*a^2))*x/a^7/(b*x^2+a)^(3/2)+128/105*b*(48*A*b^3-a*(24*B*b^2-10*C*a*b+3*
D*a^2))*x/a^8/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1817, 12, 277, 198, 197}

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)^{9/2}}\,dx &= -\frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}} \\ &+\frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}}+\frac{128bx(48Ab^3-a(3a^2D-10abC+24b^2B))}{105a^8\sqrt{a+bx^2}} \\ &+\frac{64bx(48Ab^3-a(3a^2D-10abC+24b^2B))}{105a^7\left(a+bx^2\right)^{3/2}} \\ &+\frac{16bx(48Ab^3-a(3a^2D-10abC+24b^2B))}{35a^6\left(a+bx^2\right)^{5/2}} \\ &+\frac{8bx(48Ab^3-a(3a^2D-10abC+24b^2B))}{21a^5\left(a+bx^2\right)^{7/2}} \\ &+\frac{48Ab^3-a(3a^2D-10abC+24b^2B)}{3a^4x\left(a+bx^2\right)^{7/2}}-\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}}\end{aligned}$$

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(a + b*x^2)^(9/2)),x]
```

```
[Out] -1/7*A/(a*x^7*(a + b*x^2)^(7/2)) + (2*A*b - a*B)/(5*a^2*x^5*(a + b*x^2)^(7/
2)) - (24*A*b^2 - a*(12*b*B - 5*a*C))/(15*a^3*x^3*(a + b*x^2)^(7/2)) + (48*
A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))/(3*a^4*x*(a + b*x^2)^(7/2)) + (8
*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))*x)/(21*a^5*(a + b*x^2)^(7
/2)) + (16*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))*x)/(35*a^6*(a +
 b*x^2)^(5/2)) + (64*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))*x)/(1
05*a^7*(a + b*x^2)^(3/2)) + (128*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a
^2*D))*x)/(105*a^8*Sqrt[a + b*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
```

```
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{14Ab-7a\left(B+Cx^2+Dx^4\right)}{x^6\left(a+bx^2\right)^{9/2}}\,dx}{7a}\\
&= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}}+\frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}}+\frac{\int \frac{12b(14Ab-7aB)-5a\left(-7aC-7aDx^2\right)}{x^4\left(a+bx^2\right)^{9/2}}\,dx}{35a^2}\\
&= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}}+\frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}}\\
&\quad -\frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{10b\left(168Ab^2-84abB+35a^2C\right)-105a^3D}{x^2\left(a+bx^2\right)^{9/2}}\,dx}{105a^3}\\
&= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}}+\frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}}-\frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}}\\
&\quad -\frac{\left(48Ab^3-a\left(24b^2B-10abC+3a^2D\right)\right)\int \frac{1}{x^2\left(a+bx^2\right)^{9/2}}\,dx}{3a^3}\\
&= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}}+\frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}}\\
&\quad -\frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}}+\frac{48Ab^3-a\left(24b^2B-10abC+3a^2D\right)}{3a^4x\left(a+bx^2\right)^{7/2}}\\
&\quad +\frac{\left(8b\left(48Ab^3-a\left(24b^2B-10abC+3a^2D\right)\right)\right)\int \frac{1}{\left(a+bx^2\right)^{9/2}}\,dx}{3a^4}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} \\
&\quad - \frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}} + \frac{48Ab^3-a(24b^2B-10abC+3a^2D)}{3a^4x\left(a+bx^2\right)^{7/2}} \\
&\quad + \frac{8b(48Ab^3-a(24b^2B-10abC+3a^2D))\,x}{21a^5\left(a+bx^2\right)^{7/2}} \\
&\quad + \frac{(16b(48Ab^3-a(24b^2B-10abC+3a^2D)))\int \frac{1}{\left(a+bx^2\right)^{7/2}}\,dx}{7a^5} \\
&= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}} \\
&\quad + \frac{48Ab^3-a(24b^2B-10abC+3a^2D)}{3a^4x\left(a+bx^2\right)^{7/2}} \\
&\quad + \frac{8b(48Ab^3-a(24b^2B-10abC+3a^2D))\,x}{21a^5\left(a+bx^2\right)^{7/2}} \\
&\quad + \frac{16b(48Ab^3-a(24b^2B-10abC+3a^2D))\,x}{35a^6\left(a+bx^2\right)^{5/2}} \\
&\quad + \frac{(64b(48Ab^3-a(24b^2B-10abC+3a^2D)))\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx}{35a^6} \\
&= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}} \\
&\quad + \frac{48Ab^3-a(24b^2B-10abC+3a^2D)}{3a^4x\left(a+bx^2\right)^{7/2}} \\
&\quad + \frac{8b(48Ab^3-a(24b^2B-10abC+3a^2D))\,x}{21a^5\left(a+bx^2\right)^{7/2}} \\
&\quad + \frac{16b(48Ab^3-a(24b^2B-10abC+3a^2D))\,x}{35a^6\left(a+bx^2\right)^{5/2}} \\
&\quad + \frac{64b(48Ab^3-a(24b^2B-10abC+3a^2D))\,x}{105a^7\left(a+bx^2\right)^{3/2}} \\
&\quad + \frac{(128b(48Ab^3-a(24b^2B-10abC+3a^2D)))\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{105a^7}
\end{aligned}
$$

$$= -\frac{A}{7ax^7(a+bx^2)^{7/2}} + \frac{2Ab-aB}{5a^2x^5(a+bx^2)^{7/2}} - \frac{24Ab^2-a(12bB-5aC)}{15a^3x^3(a+bx^2)^{7/2}}$$
$$+ \frac{48Ab^3-a(24b^2B-10abC+3a^2D)}{3a^4x(a+bx^2)^{7/2}}$$
$$+ \frac{8b(48Ab^3-a(24b^2B-10abC+3a^2D))\,x}{21a^5(a+bx^2)^{7/2}}$$
$$+ \frac{16b(48Ab^3-a(24b^2B-10abC+3a^2D))\,x}{35a^6(a+bx^2)^{5/2}}$$
$$+ \frac{64b(48Ab^3-a(24b^2B-10abC+3a^2D))\,x}{105a^7(a+bx^2)^{3/2}}$$
$$+ \frac{128b(48Ab^3-a(24b^2B-10abC+3a^2D))\,x}{105a^8\sqrt{a+bx^2}}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(a+bx^2)^{9/2}}\,dx = \frac{6144Ab^7x^{14} - 3072ab^6x^{12}(-7A+Bx^2) + 256a^2b^5x^{10}(105A-42Bx^2+5C}{}$$

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(a + b*x^2)^(9/2)),x]
```

```
[Out] (6144*A*b^7*x^14 - 3072*a*b^6*x^12*(-7*A + B*x^2) + 256*a^2*b^5*x^10*(105*A
 - 42*B*x^2 + 5*C*x^4) + 14*a^6*b*x^2*(3*A + 6*B*x^2 + 25*C*x^4 - 60*D*x^6)
 + 112*a^4*b^3*x^6*(15*A - 60*B*x^2 + 50*C*x^4 - 12*D*x^6) + 128*a^3*b^4*x^
8*(105*A - 105*B*x^2 + 35*C*x^4 - 3*D*x^6) - 56*a^5*b^2*x^4*(3*A + 15*B*x^2
 - 50*C*x^4 + 30*D*x^6) - a^7*(15*A + 21*B*x^2 + 35*x^4*(C + 3*D*x^2)))/(10
5*a^8*x^7*(a + b*x^2)^(7/2))
```

Maple [A] (verified)

Time = 3.58 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$\frac{(-105Dx^6-35Cx^4-21x^2B-15A)a^7+42bx^2\left(-20Dx^6+\frac{25}{3}Cx^4+2x^2B+A\right)a^6-168b^2x^4\left(10Dx^6-\frac{50}{3}Cx^4+5x^2B+A\right)a^5+}{}$
gosper	$-\frac{-6144Ab^7x^{14}+3072Bab^6x^{14}-1280Ca^2b^5x^{14}+384Da^3b^4x^{14}-21504Aab^6x^{12}+10752Ba^2b^5x^{12}-4480Ca^3b^4x^{12}+1344}{}$
trager	$-\frac{-6144Ab^7x^{14}+3072Bab^6x^{14}-1280Ca^2b^5x^{14}+384Da^3b^4x^{14}-21504Aab^6x^{12}+10752Ba^2b^5x^{12}-4480Ca^3b^4x^{12}+1344}{}$
	$\left(2b\left(-\frac{1}{5a\,x^5\left(b\,x^2+a\right)^{\frac{7}{2}}}-\frac{12b\left(-\frac{1}{3a\,x^3\left(b\,x^2+a\right)^{\frac{7}{2}}}-\frac{10b\left(-\frac{1}{ax\left(b\,x^2+a\right)^{\frac{7}{2}}}-\frac{8b\left(\frac{x}{7a\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{6}{35a\left(b\,x\right.}\right.}{}\right)}{3}\right)}{5a}\right.\right.$

```
[In] int((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*((-105*D*x^6-35*C*x^4-21*B*x^2-15*A)*a^7+42*b*x^2*(-20*D*x^6+25/3*C*x
^4+2*x^2*B+A)*a^6-168*b^2*x^4*(10*D*x^6-50/3*C*x^4+5*x^2*B+A)*a^5+1680*(-4/
5*D*x^6+10/3*C*x^4-4*x^2*B+A)*b^3*x^6*a^4+13440*(-1/35*D*x^6+1/3*C*x^4-x^2*
B+A)*b^4*x^8*a^3+26880*(1/21*C*x^4-2/5*x^2*B+A)*b^5*x^10*a^2+21504*b^6*x^12
*(-1/7*x^2*B+A)*a+6144*A*b^7*x^14)/(b*x^2+a)^(7/2)/x^7/a^8
```

Fricas [A] (verification not implemented)

none

Time = 0.70 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)^{9/2}}\,dx = -\frac{(128\,(3\,Da^3b^4-10\,Ca^2b^5+24\,Bab^6-48\,Ab^7)x^{14}+448\,(3\,Da^4b^3-10\,Ca^3b^4+24\,Ba^2b^5-48\,Aab^6)x^{12}+5}{}$$

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/105*(128*(3*D*a^3*b^4 - 10*C*a^2*b^5 + 24*B*a*b^6 - 48*A*b^7)*x^14 + 448
*(3*D*a^4*b^3 - 10*C*a^3*b^4 + 24*B*a^2*b^5 - 48*A*a*b^6)*x^12 + 560*(3*D*a
^5*b^2 - 10*C*a^4*b^3 + 24*B*a^3*b^4 - 48*A*a^2*b^5)*x^10 + 280*(3*D*a^6*b
- 10*C*a^5*b^2 + 24*B*a^4*b^3 - 48*A*a^3*b^4)*x^8 + 15*A*a^7 + 35*(3*D*a^7
- 10*C*a^6*b + 24*B*a^5*b^2 - 48*A*a^4*b^3)*x^6 + 7*(5*C*a^7 - 12*B*a^6*b +
 24*A*a^5*b^2)*x^4 + 21*(B*a^7 - 2*A*a^6*b)*x^2)*sqrt(b*x^2 + a)/(a^8*b^4*x
^15 + 4*a^9*b^3*x^13 + 6*a^10*b^2*x^11 + 4*a^11*b*x^9 + a^12*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**8/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.46

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8 (a+bx^2)^{9/2}}\,dx = -\frac{128\,Dbx}{35\,\sqrt{bx^2+a}a^5} - \frac{64\,Dbx}{35\,(bx^2+a)^{\frac{3}{2}}a^4} - \frac{48\,Dbx}{35\,(bx^2+a)^{\frac{5}{2}}a^3} - \frac{8\,Dbx}{7\,(bx^2+a)^{\frac{7}{2}}a^2} + \frac{256\,Cb^2x}{21\,\sqrt{bx^2+a}a^6} + \frac{128\,Cb^2x}{21\,(bx^2+a)^{\frac{3}{2}}a^5} + \frac{32\,Cb^2x}{7\,(bx^2+a)^{\frac{5}{2}}a^4} + \frac{80\,Cb^2x}{21\,(bx^2+a)^{\frac{7}{2}}a^3} - \frac{1024\,Bb^3x}{35\,\sqrt{bx^2+a}a^7} - \frac{512\,Bb^3x}{35\,(bx^2+a)^{\frac{3}{2}}a^6} - \frac{384\,Bb^3x}{35\,(bx^2+a)^{\frac{5}{2}}a^5} - \frac{64\,Bb^3x}{7\,(bx^2+a)^{\frac{7}{2}}a^4} + \frac{2048\,Ab^4x}{35\,\sqrt{bx^2+a}a^8} + \frac{1024\,Ab^4x}{35\,(bx^2+a)^{\frac{3}{2}}a^7} + \frac{768\,Ab^4x}{35\,(bx^2+a)^{\frac{5}{2}}a^6} + \frac{128\,Ab^4x}{7\,(bx^2+a)^{\frac{7}{2}}a^5} - \frac{D}{(bx^2+a)^{\frac{7}{2}}ax} + \frac{10\,Cb}{3\,(bx^2+a)^{\frac{7}{2}}a^2x} - \frac{8\,Bb^2}{(bx^2+a)^{\frac{7}{2}}a^3x} + \frac{16\,Ab^3}{(bx^2+a)^{\frac{7}{2}}a^4x} - \frac{C}{3\,(bx^2+a)^{\frac{7}{2}}ax^3} + \frac{4\,Bb}{5\,(bx^2+a)^{\frac{7}{2}}a^2x^3} - \frac{8\,Ab^2}{5\,(bx^2+a)^{\frac{7}{2}}a^3x^3} - \frac{B}{5\,(bx^2+a)^{\frac{7}{2}}ax^5} + \frac{2\,Ab}{5\,(bx^2+a)^{\frac{7}{2}}a^2x^5} - \frac{A}{7\,(bx^2+a)^{\frac{7}{2}}ax^7}$$

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -128/35*D*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*D*b*x/((b*x^2 + a)^(3/2)*a^4) -
 48/35*D*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*D*b*x/((b*x^2 + a)^(7/2)*a^2) +
256/21*C*b^2*x/(sqrt(b*x^2 + a)*a^6) + 128/21*C*b^2*x/((b*x^2 + a)^(3/2)*a^
5) + 32/7*C*b^2*x/((b*x^2 + a)^(5/2)*a^4) + 80/21*C*b^2*x/((b*x^2 + a)^(7/2
)*a^3) - 1024/35*B*b^3*x/(sqrt(b*x^2 + a)*a^7) - 512/35*B*b^3*x/((b*x^2 + a
)^(3/2)*a^6) - 384/35*B*b^3*x/((b*x^2 + a)^(5/2)*a^5) - 64/7*B*b^3*x/((b*x^
2 + a)^(7/2)*a^4) + 2048/35*A*b^4*x/(sqrt(b*x^2 + a)*a^8) + 1024/35*A*b^4*x
/((b*x^2 + a)^(3/2)*a^7) + 768/35*A*b^4*x/((b*x^2 + a)^(5/2)*a^6) + 128/7*A
*b^4*x/((b*x^2 + a)^(7/2)*a^5) - D/((b*x^2 + a)^(7/2)*a*x) + 10/3*C*b/((b*x
^2 + a)^(7/2)*a^2*x) - 8*B*b^2/((b*x^2 + a)^(7/2)*a^3*x) + 16*A*b^3/((b*x^2
 + a)^(7/2)*a^4*x) - 1/3*C/((b*x^2 + a)^(7/2)*a*x^3) + 4/5*B*b/((b*x^2 + a)
^(7/2)*a^2*x^3) - 8/5*A*b^2/((b*x^2 + a)^(7/2)*a^3*x^3) - 1/5*B/((b*x^2 + a
)^(7/2)*a*x^5) + 2/5*A*b/((b*x^2 + a)^(7/2)*a^2*x^5) - 1/7*A/((b*x^2 + a)^(
7/2)*a*x^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 938 vs. $2(300) = 600$.

Time = 0.32 (sec) , antiderivative size = 938, normalized size of antiderivative = 2.81

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^8 (a + bx^2)^{9/2}} \, dx =$$

$$-\frac{\left(\left(x^2\left(\frac{(279\,Da^{21}b^7 - 790\,Ca^{20}b^8 + 1686\,Ba^{19}b^9 - 3072\,Aa^{18}b^{10})x^2}{a^{26}b^3} + \frac{7\,(132\,Da^{22}b^6 - 365\,Ca^{21}b^7 + 768\,Ba^{20}b^8 - 1386\,Aa^{19}b^9)}{a^{26}b^3}\right) + \frac{35\,(30\,Da^{23}\cdots}{\cdots}\right.\right.\cdots}{105\,(bx^2 + a)^{\frac{7}{2}}}$$

$$+\frac{2\left(105\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^{12} Da^3\sqrt{b} - 420\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^{12} Ca^2b^{\frac{3}{2}} + 1050\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^{12} Bab^{\frac{5}{2}} - \cdots\right.}{\cdots}$$

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*((x^2*((279*D*a^21*b^7 - 790*C*a^20*b^8 + 1686*B*a^19*b^9 - 3072*A*a
^18*b^10)*x^2/(a^26*b^3) + 7*(132*D*a^22*b^6 - 365*C*a^21*b^7 + 768*B*a^20*
b^8 - 1386*A*a^19*b^9)/(a^26*b^3)) + 35*(30*D*a^23*b^5 - 80*C*a^22*b^6 + 16
5*B*a^21*b^7 - 294*A*a^20*b^8)/(a^26*b^3))*x^2 + 105*(4*D*a^24*b^4 - 10*C*a
^23*b^5 + 20*B*a^22*b^6 - 35*A*a^21*b^7)/(a^26*b^3))*x/(b*x^2 + a)^(7/2) +
2/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a))^12*D*a^3*sqrt(b) - 420*(sqrt(b)*x
- sqrt(b*x^2 + a))^12*C*a^2*b^(3/2) + 1050*(sqrt(b)*x - sqrt(b*x^2 + a))^12
*B*a*b^(5/2) - 2100*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*b^(7/2) - 630*(sqrt(
b)*x - sqrt(b*x^2 + a))^10*D*a^4*sqrt(b) + 2730*(sqrt(b)*x - sqrt(b*x^2 + a
))^10*C*a^3*b^(3/2) - 7140*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2*b^(5/2) +
 14700*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a*b^(7/2) + 1575*(sqrt(b)*x - sqr
t(b*x^2 + a))^8*D*a^5*sqrt(b) - 7210*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^4*
b^(3/2) + 19950*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^3*b^(5/2) - 42840*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*A*a^2*b^(7/2) - 2100*(sqrt(b)*x - sqrt(b*x^2 + a
))^6*D*a^6*sqrt(b) + 9940*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^5*b^(3/2) - 2
8560*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^4*b^(5/2) + 64680*(sqrt(b)*x - sqr
t(b*x^2 + a))^6*A*a^3*b^(7/2) + 1575*(sqrt(b)*x - sqrt(b*x^2 + a))^4*D*a^7*
sqrt(b) - 7560*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^6*b^(3/2) + 21966*(sqrt(
b)*x - sqrt(b*x^2 + a))^4*B*a^5*b^(5/2) - 49812*(sqrt(b)*x - sqrt(b*x^2 + a
))^4*A*a^4*b^(7/2) - 630*(sqrt(b)*x - sqrt(b*x^2 + a))^2*D*a^8*sqrt(b) + 30
10*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^7*b^(3/2) - 8652*(sqrt(b)*x - sqrt(b
*x^2 + a))^2*B*a^6*b^(5/2) + 19404*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^5*b^
(7/2) + 105*D*a^9*sqrt(b) - 490*C*a^8*b^(3/2) + 1386*B*a^7*b^(5/2) - 3072*A
*a^6*b^(7/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7*a^7)
```

Mupad [B] (verification not implemented)

Time = 7.74 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)^{9/2}}\,dx = \frac{\frac{61\,B\,b}{35\,a^3}+\frac{78\,B\,b^2\,x^2}{35\,a^4}}{x^3\,{\left(b\,x^2+a\right)}^{5/2}}+\frac{\frac{128\,C\,b}{21\,a^5}+\frac{256\,C\,b^2\,x^2}{21\,a^6}}{x\,\sqrt{b\,x^2+a}} - \frac{\frac{C}{3\,a^2}+\frac{19\,C\,b\,x^2}{21\,a^3}}{x^3\,{\left(b\,x^2+a\right)}^{5/2}}-\frac{\frac{167\,A\,b^2}{35\,a^4}+\frac{191\,A\,b^3\,x^2}{35\,a^5}}{x^3\,{\left(b\,x^2+a\right)}^{5/2}}+\frac{\frac{1024\,A\,b^3}{35\,a^7}+\frac{2048\,A\,b^4\,x^2}{35\,a^8}}{x\,\sqrt{b\,x^2+a}}-\frac{\frac{512\,B\,b^2}{35\,a^6}+\frac{1024\,B\,b^3\,x^2}{35\,a^7}}{x\,\sqrt{b\,x^2+a}} - \frac{A\,\sqrt{b\,x^2+a}}{7\,a^5\,x^7}-\frac{B\,\sqrt{b\,x^2+a}}{5\,a^5\,x^5}-\frac{{\left(\frac{a}{b\,x^2}+1\right)}^{9/2}\,D\,{}_2\mathrm{F}_1\left(\frac{9}{2},5;\ 6;\ -\frac{a}{b\,x^2}\right)}{10\,x\,{\left(b\,x^2+a\right)}^{9/2}} + \frac{34\,A\,b\,\sqrt{b\,x^2+a}}{35\,a^6\,x^5}-\frac{B\,b}{7\,a^2\,x^3\,{\left(b\,x^2+a\right)}^{7/2}}-\frac{32\,C\,b}{21\,a^4\,x\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{C\,b^2\,x}{7\,a^3\,{\left(b\,x^2+a\right)}^{7/2}} - \frac{58\,A\,b^3}{7\,a^6\,x\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{A\,b^2}{7\,a^3\,x^3\,{\left(b\,x^2+a\right)}^{7/2}}+\frac{27\,B\,b^2}{7\,a^5\,x\,{\left(b\,x^2+a\right)}^{3/2}}$$

```
[In] int((A + B*x^2 + C*x^4 + x^6*D)/(x^8*(a + b*x^2)^(9/2)),x)
```

```
[Out] ((61*B*b)/(35*a^3) + (78*B*b^2*x^2)/(35*a^4))/(x^3*(a + b*x^2)^(5/2)) + ((1
28*C*b)/(21*a^5) + (256*C*b^2*x^2)/(21*a^6))/(x*(a + b*x^2)^(1/2)) - (C/(3*
a^2) + (19*C*b*x^2)/(21*a^3))/(x^3*(a + b*x^2)^(5/2)) - ((167*A*b^2)/(35*a^
4) + (191*A*b^3*x^2)/(35*a^5))/(x^3*(a + b*x^2)^(5/2)) + ((1024*A*b^3)/(35*
a^7) + (2048*A*b^4*x^2)/(35*a^8))/(x*(a + b*x^2)^(1/2)) - ((512*B*b^2)/(35*
a^6) + (1024*B*b^3*x^2)/(35*a^7))/(x*(a + b*x^2)^(1/2)) - (A*(a + b*x^2)^(1
/2))/(7*a^5*x^7) - (B*(a + b*x^2)^(1/2))/(5*a^5*x^5) - ((a/(b*x^2) + 1)^(9/
2)*D*hypergeom([9/2, 5], 6, -a/(b*x^2)))/(10*x*(a + b*x^2)^(9/2)) + (34*A*b
*(a + b*x^2)^(1/2))/(35*a^6*x^5) - (B*b)/(7*a^2*x^3*(a + b*x^2)^(7/2)) - (3
2*C*b)/(21*a^4*x*(a + b*x^2)^(3/2)) + (C*b^2*x)/(7*a^3*(a + b*x^2)^(7/2)) -
 (58*A*b^3)/(7*a^6*x*(a + b*x^2)^(3/2)) + (A*b^2)/(7*a^3*x^3*(a + b*x^2)^(7
/2)) + (27*B*b^2)/(7*a^5*x*(a + b*x^2)^(3/2))
```

3.168 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}(a+bx^2)^{9/2}}\,dx$

Optimal result 1060
Rubi [A] (verified) 1061
Mathematica [A] (verified) 1064
Maple [A] (verified) 1065
Fricas [A] (verification not implemented) 1067
Sympy [F(-1)] 1067
Maxima [A] (verification not implemented) 1068
Giac [B] (verification not implemented) 1069
Mupad [F(-1)] 1070

Optimal result

Integrand size = 32, antiderivative size = 392

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{9/2}}\,dx &= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}+\frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}}\\ &-\frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}}+\frac{128Ab^3-3a(24b^2B-12abC+5a^2D)}{45a^4x^3\left(a+bx^2\right)^{7/2}}\\ &-\frac{2b(128Ab^3-3a(24b^2B-12abC+5a^2D))}{9a^5x\left(a+bx^2\right)^{7/2}}\\ &-\frac{16b^2(128Ab^3-3a(24b^2B-12abC+5a^2D))\,x}{63a^6\left(a+bx^2\right)^{7/2}}\\ &-\frac{32b^2(128Ab^3-3a(24b^2B-12abC+5a^2D))\,x}{105a^7\left(a+bx^2\right)^{5/2}}\\ &-\frac{128b^2(128Ab^3-3a(24b^2B-12abC+5a^2D))\,x}{315a^8\left(a+bx^2\right)^{3/2}}\\ &-\frac{256b^2(128Ab^3-3a(24b^2B-12abC+5a^2D))\,x}{315a^9\sqrt{a+bx^2}}\end{aligned}$$

```
[Out] -1/9*A/a/x^9/(b*x^2+a)^(7/2)+1/63*(16*A*b-9*B*a)/a^2/x^7/(b*x^2+a)^(7/2)+1/
45*(-32*A*b^2+9*a*(2*B*b-C*a))/a^3/x^5/(b*x^2+a)^(7/2)+1/45*(128*A*b^3-3*a*
(24*B*b^2-12*C*a*b+5*D*a^2))/a^4/x^3/(b*x^2+a)^(7/2)-2/9*b*(128*A*b^3-3*a*(
24*B*b^2-12*C*a*b+5*D*a^2))/a^5/x/(b*x^2+a)^(7/2)-16/63*b^2*(128*A*b^3-3*a*
(24*B*b^2-12*C*a*b+5*D*a^2))*x/a^6/(b*x^2+a)^(7/2)-32/105*b^2*(128*A*b^3-3*
a*(24*B*b^2-12*C*a*b+5*D*a^2))*x/a^7/(b*x^2+a)^(5/2)-128/315*b^2*(128*A*b^3
-3*a*(24*B*b^2-12*C*a*b+5*D*a^2))*x/a^8/(b*x^2+a)^(3/2)-256/315*b^2*(128*A*
b^3-3*a*(24*B*b^2-12*C*a*b+5*D*a^2))*x/a^9/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1817, 12, 277, 198, 197}

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{9/2}}\,dx = -\frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{256b^2x(-15a^3D-36ab(2bB-aC)+128Ab^3)}{315a^9\sqrt{a+bx^2}} - \frac{128b^2x(-15a^3D-36ab(2bB-aC)+128Ab^3)}{315a^8\left(a+bx^2\right)^{3/2}} - \frac{32b^2x(-15a^3D-36ab(2bB-aC)+128Ab^3)}{105a^7\left(a+bx^2\right)^{5/2}} - \frac{16b^2x(-15a^3D-36ab(2bB-aC)+128Ab^3)}{63a^6\left(a+bx^2\right)^{7/2}} - \frac{2b(-15a^3D-36ab(2bB-aC)+128Ab^3)}{9a^5x\left(a+bx^2\right)^{7/2}} + \frac{-15a^3D-36ab(2bB-aC)+128Ab^3}{45a^4x^3\left(a+bx^2\right)^{7/2}} - \frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*(a + b*x^2)^(9/2)),x]
```

```
[Out] -1/9*A/(a*x^9*(a + b*x^2)^(7/2)) + (16*A*b - 9*a*B)/(63*a^2*x^7*(a + b*x^2)
^(7/2)) - (32*A*b^2 - 9*a*(2*b*B - a*C))/(45*a^3*x^5*(a + b*x^2)^(7/2)) + (
128*A*b^3 - 36*a*b*(2*b*B - a*C) - 15*a^3*D)/(45*a^4*x^3*(a + b*x^2)^(7/2))
 - (2*b*(128*A*b^3 - 36*a*b*(2*b*B - a*C) - 15*a^3*D))/(9*a^5*x*(a + b*x^2)
^(7/2)) - (16*b^2*(128*A*b^3 - 36*a*b*(2*b*B - a*C) - 15*a^3*D)*x)/(63*a^6*
(a + b*x^2)^(7/2)) - (32*b^2*(128*A*b^3 - 36*a*b*(2*b*B - a*C) - 15*a^3*D)*
x)/(105*a^7*(a + b*x^2)^(5/2)) - (128*b^2*(128*A*b^3 - 36*a*b*(2*b*B - a*C)
 - 15*a^3*D)*x)/(315*a^8*(a + b*x^2)^(3/2)) - (256*b^2*(128*A*b^3 - 36*a*b*
(2*b*B - a*C) - 15*a^3*D)*x)/(315*a^9*Sqrt[a + b*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{16Ab-9a\left(B+Cx^2+Dx^4\right)}{x^8\left(a+bx^2\right)^{9/2}}\,dx}{9a}\\
&= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}+\frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}}+\frac{\int \frac{14b(16Ab-9aB)-7a\left(-9aC-9aDx^2\right)}{x^6\left(a+bx^2\right)^{9/2}}\,dx}{63a^2}\\
&= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}+\frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}}\\
&\quad-\frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{12b\left(224Ab^2-126abB+63a^2C\right)-315a^3D}{x^4\left(a+bx^2\right)^{9/2}}\,dx}{315a^3}\\
&= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}+\frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}}-\frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}}\\
&\quad-\frac{\left(128Ab^3-36ab(2bB-aC)-15a^3D\right)\int \frac{1}{x^4\left(a+bx^2\right)^{9/2}}\,dx}{15a^3}
\end{aligned}
$$

$$= -\frac{A}{9ax^9(a+bx^2)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7(a+bx^2)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5(a+bx^2)^{7/2}} + \frac{128Ab^3-36ab(2bB-aC)-15a^3D}{45a^4x^3(a+bx^2)^{7/2}} + \frac{(2b(128Ab^3-36ab(2bB-aC)-15a^3D))\int \frac{1}{x^2(a+bx^2)^{9/2}}\,dx}{9a^4}$$

$$= -\frac{A}{9ax^9(a+bx^2)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7(a+bx^2)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5(a+bx^2)^{7/2}} + \frac{128Ab^3-36ab(2bB-aC)-15a^3D}{45a^4x^3(a+bx^2)^{7/2}} - \frac{2b(128Ab^3-36ab(2bB-aC)-15a^3D)}{9a^5x(a+bx^2)^{7/2}} - \frac{(16b^2(128Ab^3-36ab(2bB-aC)-15a^3D))\int \frac{1}{(a+bx^2)^{9/2}}\,dx}{9a^5}$$

$$= -\frac{A}{9ax^9(a+bx^2)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7(a+bx^2)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5(a+bx^2)^{7/2}} + \frac{128Ab^3-36ab(2bB-aC)-15a^3D}{45a^4x^3(a+bx^2)^{7/2}} - \frac{2b(128Ab^3-36ab(2bB-aC)-15a^3D)}{9a^5x(a+bx^2)^{7/2}} - \frac{16b^2(128Ab^3-36ab(2bB-aC)-15a^3D)\,x}{63a^6(a+bx^2)^{7/2}} - \frac{(32b^2(128Ab^3-36ab(2bB-aC)-15a^3D))\int \frac{1}{(a+bx^2)^{7/2}}\,dx}{21a^6}$$

$$= -\frac{A}{9ax^9(a+bx^2)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7(a+bx^2)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5(a+bx^2)^{7/2}} + \frac{128Ab^3-36ab(2bB-aC)-15a^3D}{45a^4x^3(a+bx^2)^{7/2}} - \frac{2b(128Ab^3-36ab(2bB-aC)-15a^3D)}{9a^5x(a+bx^2)^{7/2}} - \frac{16b^2(128Ab^3-36ab(2bB-aC)-15a^3D)\,x}{63a^6(a+bx^2)^{7/2}} - \frac{32b^2(128Ab^3-36ab(2bB-aC)-15a^3D)\,x}{105a^7(a+bx^2)^{5/2}} - \frac{(128b^2(128Ab^3-36ab(2bB-aC)-15a^3D))\int \frac{1}{(a+bx^2)^{5/2}}\,dx}{105a^7}$$

$$
\begin{aligned}
&= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} \\
&\quad + \frac{128Ab^3-36ab(2bB-aC)-15a^3D}{45a^4x^3\left(a+bx^2\right)^{7/2}} - \frac{2b(128Ab^3-36ab(2bB-aC)-15a^3D)}{9a^5x\left(a+bx^2\right)^{7/2}} \\
&\quad - \frac{16b^2(128Ab^3-36ab(2bB-aC)-15a^3D)\,x}{63a^6\left(a+bx^2\right)^{7/2}} \\
&\quad - \frac{32b^2(128Ab^3-36ab(2bB-aC)-15a^3D)\,x}{105a^7\left(a+bx^2\right)^{5/2}} \\
&\quad - \frac{128b^2(128Ab^3-36ab(2bB-aC)-15a^3D)\,x}{315a^8\left(a+bx^2\right)^{3/2}} \\
&\quad - \frac{(256b^2(128Ab^3-36ab(2bB-aC)-15a^3D))\int \frac{1}{(a+bx^2)^{3/2}}\,dx}{315a^8} \\
&= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}} + \frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}} - \frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}} \\
&\quad + \frac{128Ab^3-36ab(2bB-aC)-15a^3D}{45a^4x^3\left(a+bx^2\right)^{7/2}} - \frac{2b(128Ab^3-36ab(2bB-aC)-15a^3D)}{9a^5x\left(a+bx^2\right)^{7/2}} \\
&\quad - \frac{16b^2(128Ab^3-36ab(2bB-aC)-15a^3D)\,x}{63a^6\left(a+bx^2\right)^{7/2}} \\
&\quad - \frac{32b^2(128Ab^3-36ab(2bB-aC)-15a^3D)\,x}{105a^7\left(a+bx^2\right)^{5/2}} \\
&\quad - \frac{128b^2(128Ab^3-36ab(2bB-aC)-15a^3D)\,x}{315a^8\left(a+bx^2\right)^{3/2}} \\
&\quad - \frac{256b^2(128Ab^3-36ab(2bB-aC)-15a^3D)\,x}{315a^9\sqrt{a+bx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.69

$$
\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{9/2}}\,dx = \frac{-32768Ab^8x^{16}+2048ab^7x^{14}(-56A+9Bx^2)-1024a^2b^6x^{12}(140A-63Bx^2
$$

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-32768*A*b^8*x^16 + 2048*a*b^7*x^14*(-56*A + 9*B*x^2) - 1024*a^2*b^6*x^12*
(140*A - 63*B*x^2 + 9*C*x^4) - 56*a^6*b^2*x^4*(4*A + 9*B*x^2 + 45*C*x^4 - 1
50*D*x^6) + 4480*a^4*b^4*x^8*(-2*A + 9*B*x^2 - 9*C*x^4 + 3*D*x^6) + 256*a^3
*b^5*x^10*(-280*A + 315*B*x^2 - 126*C*x^4 + 15*D*x^6) - a^8*(35*A + 45*B*x^
2 + 63*C*x^4 + 105*D*x^6) + 112*a^5*b^3*x^6*(8*A + 45*B*x^2 - 180*C*x^4 + 1
50*D*x^6) + 2*a^7*b*x^2*(40*A + 21*(3*B*x^2 + 6*C*x^4 + 25*D*x^6)))/(315*a^
9*x^9*(a + b*x^2)^(7/2))
```

Maple [A] (verified)

Time = 3.65 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{\left(-105 D x^{6}-63 C \,x^{4}-45 x^{2} B -35 A \right) a^{8}+80 b \,x^{2} \left(\frac{105}{8} D x^{6}+\frac{63}{20} C \,x^{4}+\frac{63}{40} x^{2} B +A \right) a^{7}-224 \left(-\frac{75}{2} D x^{6}+\frac{45}{4} C \,x^{4}+\frac{9}{4} x^{2} B +A \right) b^{2} x^{4} a^{6}}{}$
gosper	$-\frac{32768 A \,b^{8} x^{16}-18432 B a \,b^{7} x^{16}+9216 C \,a^{2} b^{6} x^{16}-3840 D a^{3} b^{5} x^{16}+114688 A a \,b^{7} x^{14}-64512 B \,a^{2} b^{6} x^{14}+32256 C \,a^{3} b^{5} x^{14}-134}{}$
trager	$-\frac{32768 A \,b^{8} x^{16}-18432 B a \,b^{7} x^{16}+9216 C \,a^{2} b^{6} x^{16}-3840 D a^{3} b^{5} x^{16}+114688 A a \,b^{7} x^{14}-64512 B \,a^{2} b^{6} x^{14}+32256 C \,a^{3} b^{5} x^{14}-134}{}$

```
[In] int((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/315*((-105*D*x^6-63*C*x^4-45*B*x^2-35*A)*a^8+80*b*x^2*(105/8*D*x^6+63/20*
C*x^4+63/40*x^2*B+A)*a^7-224*(-75/2*D*x^6+45/4*C*x^4+9/4*x^2*B+A)*b^2*x^4*a
^6+896*(75/4*D*x^6-45/2*C*x^4+45/8*x^2*B+A)*b^3*x^6*a^5-8960*b^4*x^8*(-3/2*
D*x^6+9/2*C*x^4-9/2*x^2*B+A)*a^4-71680*(-3/56*D*x^6+9/20*C*x^4-9/8*x^2*B+A)
*b^5*x^10*a^3-143360*(9/140*C*x^4-9/20*x^2*B+A)*b^6*x^12*a^2-114688*(-9/56*
x^2*B+A)*b^7*x^14*a-32768*A*b^8*x^16)/(b*x^2+a)^(7/2)/x^9/a^9
```

Fricas [A] (verification not implemented)

none

Time = 0.97 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{9/2}}\,dx = \frac{(256\,(15\,Da^3b^5-36\,Ca^2b^6+72\,Bab^7-128\,Ab^8)x^{16}+896\,(15\,Da^4b^4-36\,C$$

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/315*(256*(15*D*a^3*b^5 - 36*C*a^2*b^6 + 72*B*a*b^7 - 128*A*b^8)*x^16 + 89
6*(15*D*a^4*b^4 - 36*C*a^3*b^5 + 72*B*a^2*b^6 - 128*A*a*b^7)*x^14 + 1120*(1
5*D*a^5*b^3 - 36*C*a^4*b^4 + 72*B*a^3*b^5 - 128*A*a^2*b^6)*x^12 + 560*(15*D
*a^6*b^2 - 36*C*a^5*b^3 + 72*B*a^4*b^4 - 128*A*a^3*b^5)*x^10 - 35*A*a^8 + 7
0*(15*D*a^7*b - 36*C*a^6*b^2 + 72*B*a^5*b^3 - 128*A*a^4*b^4)*x^8 - 7*(15*D*
a^8 - 36*C*a^7*b + 72*B*a^6*b^2 - 128*A*a^5*b^3)*x^6 - 7*(9*C*a^8 - 18*B*a^
7*b + 32*A*a^6*b^2)*x^4 - 5*(9*B*a^8 - 16*A*a^7*b)*x^2)*sqrt(b*x^2 + a)/(a^
9*b^4*x^17 + 4*a^10*b^3*x^15 + 6*a^11*b^2*x^13 + 4*a^12*b*x^11 + a^13*x^9)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**10/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.48

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{9/2}}\,dx = \frac{256\,Db^2x}{21\,\sqrt{bx^2+a}a^6} + \frac{128\,Db^2x}{21\,(bx^2+a)^{\frac{3}{2}}a^5} + \frac{32\,Db^2x}{7\,(bx^2+a)^{\frac{5}{2}}a^4} + \frac{80\,Db^2x}{21\,(bx^2+a)^{\frac{7}{2}}a^3} - \frac{1024\,Cb^3x}{35\,\sqrt{bx^2+a}a^7} - \frac{512\,Cb^3x}{35\,(bx^2+a)^{\frac{3}{2}}a^6} - \frac{384\,Cb^3x}{35\,(bx^2+a)^{\frac{5}{2}}a^5} - \frac{64\,Cb^3x}{7\,(bx^2+a)^{\frac{7}{2}}a^4} + \frac{2048\,Bb^4x}{35\,\sqrt{bx^2+a}a^8} + \frac{1024\,Bb^4x}{35\,(bx^2+a)^{\frac{3}{2}}a^7} + \frac{768\,Bb^4x}{35\,(bx^2+a)^{\frac{5}{2}}a^6} + \frac{128\,Bb^4x}{7\,(bx^2+a)^{\frac{7}{2}}a^5} - \frac{32768\,Ab^5x}{315\,\sqrt{bx^2+a}a^9} - \frac{16384\,Ab^5x}{315\,(bx^2+a)^{\frac{3}{2}}a^8} - \frac{4096\,Ab^5x}{105\,(bx^2+a)^{\frac{5}{2}}a^7} - \frac{2048\,Ab^5x}{63\,(bx^2+a)^{\frac{7}{2}}a^6} + \frac{10\,Db}{3\,(bx^2+a)^{\frac{7}{2}}a^2x} - \frac{8\,Cb^2}{(bx^2+a)^{\frac{7}{2}}a^3x} + \frac{16\,Bb^3}{(bx^2+a)^{\frac{7}{2}}a^4x} - \frac{256\,Ab^4}{9\,(bx^2+a)^{\frac{7}{2}}a^5x} - \frac{D}{3\,(bx^2+a)^{\frac{7}{2}}ax^3} + \frac{4\,Cb}{5\,(bx^2+a)^{\frac{7}{2}}a^2x^3} - \frac{8\,Bb^2}{5\,(bx^2+a)^{\frac{7}{2}}a^3x^3} + \frac{128\,Ab^3}{45\,(bx^2+a)^{\frac{7}{2}}a^4x^3} - \frac{C}{5\,(bx^2+a)^{\frac{7}{2}}ax^5} + \frac{2\,Bb}{5\,(bx^2+a)^{\frac{7}{2}}a^2x^5} - \frac{32\,Ab^2}{45\,(bx^2+a)^{\frac{7}{2}}a^3x^5} - \frac{B}{7\,(bx^2+a)^{\frac{7}{2}}ax^7} + \frac{16\,Ab}{63\,(bx^2+a)^{\frac{7}{2}}a^2x^7} - \frac{A}{9\,(bx^2+a)^{\frac{7}{2}}ax^9}$$

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 256/21*D*b^2*x/(sqrt(b*x^2 + a)*a^6) + 128/21*D*b^2*x/((b*x^2 + a)^(3/2)*a^
5) + 32/7*D*b^2*x/((b*x^2 + a)^(5/2)*a^4) + 80/21*D*b^2*x/((b*x^2 + a)^(7/2
)*a^3) - 1024/35*C*b^3*x/(sqrt(b*x^2 + a)*a^7) - 512/35*C*b^3*x/((b*x^2 + a
)^(3/2)*a^6) - 384/35*C*b^3*x/((b*x^2 + a)^(5/2)*a^5) - 64/7*C*b^3*x/((b*x^
2 + a)^(7/2)*a^4) + 2048/35*B*b^4*x/(sqrt(b*x^2 + a)*a^8) + 1024/35*B*b^4*x
/((b*x^2 + a)^(3/2)*a^7) + 768/35*B*b^4*x/((b*x^2 + a)^(5/2)*a^6) + 128/7*B
*b^4*x/((b*x^2 + a)^(7/2)*a^5) - 32768/315*A*b^5*x/(sqrt(b*x^2 + a)*a^9) -
16384/315*A*b^5*x/((b*x^2 + a)^(3/2)*a^8) - 4096/105*A*b^5*x/((b*x^2 + a)^(
5/2)*a^7) - 2048/63*A*b^5*x/((b*x^2 + a)^(7/2)*a^6) + 10/3*D*b/((b*x^2 + a)
^(7/2)*a^2*x) - 8*C*b^2/((b*x^2 + a)^(7/2)*a^3*x) + 16*B*b^3/((b*x^2 + a)^(
7/2)*a^4*x) - 256/9*A*b^4/((b*x^2 + a)^(7/2)*a^5*x) - 1/3*D/((b*x^2 + a)^(7
/2)*a*x^3) + 4/5*C*b/((b*x^2 + a)^(7/2)*a^2*x^3) - 8/5*B*b^2/((b*x^2 + a)^(
7/2)*a^3*x^3) + 128/45*A*b^3/((b*x^2 + a)^(7/2)*a^4*x^3) - 1/5*C/((b*x^2 +
a)^(7/2)*a*x^5) + 2/5*B*b/((b*x^2 + a)^(7/2)*a^2*x^5) - 32/45*A*b^2/((b*x^2
 + a)^(7/2)*a^3*x^5) - 1/7*B/((b*x^2 + a)^(7/2)*a*x^7) + 16/63*A*b/((b*x^2
+ a)^(7/2)*a^2*x^7) - 1/9*A/((b*x^2 + a)^(7/2)*a*x^9)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1162 vs. $2(355) = 710$.

Time = 0.31 (sec) , antiderivative size = 1162, normalized size of antiderivative = 2.96

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^{10} (a + bx^2)^{9/2}} dx = \text{Too large to display}$$

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((790*D*a^24*b^8 - 1686*C*a^23*b^9 + 3072*B*a^22*b^10 - 5053*A*
a^21*b^11)*x^2/(a^30*b^3) + 7*(365*D*a^25*b^7 - 768*C*a^24*b^8 + 1386*B*a^2
3*b^9 - 2264*A*a^22*b^10)/(a^30*b^3)) + 35*(80*D*a^26*b^6 - 165*C*a^25*b^7
+ 294*B*a^24*b^8 - 476*A*a^23*b^9)/(a^30*b^3))*x^2 + 105*(10*D*a^27*b^5 - 2
0*C*a^26*b^6 + 35*B*a^25*b^7 - 56*A*a^24*b^8)/(a^30*b^3))*x/(b*x^2 + a)^(7/
2) - 2/315*(1260*(sqrt(b)*x - sqrt(b*x^2 + a))^16*D*a^3*b^(3/2) - 3150*(sqr
t(b)*x - sqrt(b*x^2 + a))^16*C*a^2*b^(5/2) + 6300*(sqrt(b)*x - sqrt(b*x^2 +
 a))^16*B*a*b^(7/2) - 11025*(sqrt(b)*x - sqrt(b*x^2 + a))^16*A*b^(9/2) - 10
710*(sqrt(b)*x - sqrt(b*x^2 + a))^14*D*a^4*b^(3/2) + 27720*(sqrt(b)*x - sqr
t(b*x^2 + a))^14*C*a^3*b^(5/2) - 56700*(sqrt(b)*x - sqrt(b*x^2 + a))^14*B*a
^2*b^(7/2) + 100800*(sqrt(b)*x - sqrt(b*x^2 + a))^14*A*a*b^(9/2) + 39270*(s
qrt(b)*x - sqrt(b*x^2 + a))^12*D*a^5*b^(3/2) - 105840*(sqrt(b)*x - sqrt(b*x
^2 + a))^12*C*a^4*b^(5/2) + 223020*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a^3*b
^(7/2) - 405300*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*a^2*b^(9/2) - 81270*(sqr
t(b)*x - sqrt(b*x^2 + a))^10*D*a^6*b^(3/2) + 226800*(sqrt(b)*x - sqrt(b*x^2
 + a))^10*C*a^5*b^(5/2) - 495180*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^4*b^(
7/2) + 927360*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a^3*b^(9/2) + 103950*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*D*a^7*b^(3/2) - 297108*(sqrt(b)*x - sqrt(b*x^2 +
 a))^8*C*a^6*b^(5/2) + 666036*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^5*b^(7/2)
 - 1291374*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^4*b^(9/2) - 84210*(sqrt(b)*x
 - sqrt(b*x^2 + a))^6*D*a^8*b^(3/2) + 243432*(sqrt(b)*x - sqrt(b*x^2 + a))^
6*C*a^7*b^(5/2) - 551124*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^6*b^(7/2) + 10
73856*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^5*b^(9/2) + 42210*(sqrt(b)*x - sq
rt(b*x^2 + a))^4*D*a^9*b^(3/2) - 121968*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a
^8*b^(5/2) + 275076*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^7*b^(7/2) - 533124*
(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^6*b^(9/2) - 11970*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*D*a^10*b^(3/2) + 34272*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^9*b^(
5/2) - 76644*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^8*b^(7/2) + 147456*(sqrt(b
)*x - sqrt(b*x^2 + a))^2*A*a^7*b^(9/2) + 1470*D*a^11*b^(3/2) - 4158*C*a^10*
b^(5/2) + 9216*B*a^9*b^(7/2) - 17609*A*a^8*b^(9/2))/(((sqrt(b)*x - sqrt(b*x
^2 + a))^2 - a)^9*a^8)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{9/2}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^{10}\,{\left(b\,x^2+a\right)}^{9/2}} \,d x$$

```
[In] int((A + B*x^2 + C*x^4 + x^6*D)/(x^10*(a + b*x^2)^(9/2)),x)
[Out] int((A + B*x^2 + C*x^4 + x^6*D)/(x^10*(a + b*x^2)^(9/2)), x)
```

3.169 $\int \frac{cx^5+dx^7+ex^9+fx^{11}}{\sqrt{a+bx^2}}\,dx$

Optimal result 1071
Rubi [A] (verified) 1071
Mathematica [A] (verified) 1073
Maple [A] (verified) 1073
Fricas [A] (verification not implemented) 1074
Sympy [B] (verification not implemented) 1075
Maxima [A] (verification not implemented) 1076
Giac [A] (verification not implemented) 1076
Mupad [B] (verification not implemented) 1077

Optimal result

Integrand size = 33, antiderivative size = 214

$$\int \frac{cx^5+dx^7+ex^9+fx^{11}}{\sqrt{a+bx^2}}\,dx = \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^6} - \frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)(a+bx^2)^{3/2}}{3b^6} + \frac{(b^3c-3ab^2d+6a^2be-10a^3f)(a+bx^2)^{5/2}}{5b^6} + \frac{(b^2d-4abe+10a^2f)(a+bx^2)^{7/2}}{7b^6} + \frac{(be-5af)(a+bx^2)^{9/2}}{9b^6} + \frac{f(a+bx^2)^{11/2}}{11b^6}$$

```
[Out] -1/3*a*(-5*a^3*f+4*a^2*b*e-3*a*b^2*d+2*b^3*c)*(b*x^2+a)^(3/2)/b^6+1/5*(-10*
a^3*f+6*a^2*b*e-3*a*b^2*d+b^3*c)*(b*x^2+a)^(5/2)/b^6+1/7*(10*a^2*f-4*a*b*e+
b^2*d)*(b*x^2+a)^(7/2)/b^6+1/9*(-5*a*f+b*e)*(b*x^2+a)^(9/2)/b^6+1/11*f*(b*x
^2+a)^(11/2)/b^6+a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x^2+a)^(1/2)/b^6
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used

$= \{1825, 1813, 1864\}$

$$\int \frac{cx^5 + dx^7 + ex^9 + fx^{11}}{\sqrt{a+bx^2}}\,dx = \frac{(a+bx^2)^{7/2}(10a^2f - 4abe + b^2d)}{7b^6} + \frac{(a+bx^2)^{5/2}(-10a^3f + 6a^2be - 3ab^2d + b^3c)}{5b^6} - \frac{a(a+bx^2)^{3/2}(-5a^3f + 4a^2be - 3ab^2d + 2b^3c)}{3b^6} + \frac{a^2\sqrt{a+bx^2}(a^3(-f) + a^2be - ab^2d + b^3c)}{b^6} + \frac{(a+bx^2)^{9/2}(be - 5af)}{9b^6} + \frac{f(a+bx^2)^{11/2}}{11b^6}$$

```
[In] Int[(c*x^5 + d*x^7 + e*x^9 + f*x^11)/Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^6 - (a*(2*b^3*c
 - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*(a + b*x^2)^(3/2))/(3*b^6) + ((b^3*c -
3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*(a + b*x^2)^(5/2))/(5*b^6) + ((b^2*d - 4*
a*b*e + 10*a^2*f)*(a + b*x^2)^(7/2))/(7*b^6) + ((b*e - 5*a*f)*(a + b*x^2)^(
9/2))/(9*b^6) + (f*(a + b*x^2)^(11/2))/(11*b^6)
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1825

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\text{integral} = \int \frac{x(cx^4 + dx^6 + ex^8 + fx^{10})}{\sqrt{a+bx^2}}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{cx^2 + dx^3 + ex^4 + fx^5}{\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^2(-b^3c+ab^2d-a^2be+a^3f)}{b^5\sqrt{a+bx}} + \frac{a(-2b^3c+3ab^2d-4a^2be+5a^3f)\sqrt{a+bx}}{b^5} + \frac{(b^3c-3ab^2d+6a^2be-10a^3f)(a+bx)^{3/2}}{b^5} + \frac{(b^2d-4abe+10a^2f)(a+bx)^{5/2}}{b^5} + \frac{(be-5af)(a+bx)^{7/2}}{b^5} + \frac{f(a+bx)^{9/2}}{b^5}\right) dx, x, x^2\right)$$

$$= \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^6} - \frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)(a+bx^2)^{3/2}}{3b^6} + \frac{(b^3c-3ab^2d+6a^2be-10a^3f)(a+bx^2)^{5/2}}{5b^6} + \frac{(b^2d-4abe+10a^2f)(a+bx^2)^{7/2}}{7b^6} + \frac{(be-5af)(a+bx^2)^{9/2}}{9b^6} + \frac{f(a+bx^2)^{11/2}}{11b^6}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.74

$$\int \frac{cx^5+dx^7+ex^9+fx^{11}}{\sqrt{a+bx^2}}\,dx$$

$$= \frac{\sqrt{a+bx^2}(-1280a^5f+128a^4b(11e+5fx^2)-16a^3b^2(99d+44ex^2+30fx^4)+8a^2b^3(231c+99dx^2+66e}{3465}$$

```
[In] Integrate[(c*x^5 + d*x^7 + e*x^9 + f*x^11)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-1280*a^5*f + 128*a^4*b*(11*e + 5*f*x^2) - 16*a^3*b^2*(99
*d + 44*e*x^2 + 30*f*x^4) + 8*a^2*b^3*(231*c + 99*d*x^2 + 66*e*x^4 + 50*f*x
^6) - 2*a*b^4*x^2*(462*c + 297*d*x^2 + 220*e*x^4 + 175*f*x^6) + b^5*x^4*(69
3*c + 5*(99*d*x^2 + 77*e*x^4 + 63*f*x^6))))/(3465*b^6)
```

Maple [A] (verified)

Time = 3.68 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$-\frac{256\left(-\frac{693\left(\frac{5}{11} f x^6+\frac{5}{9} e x^4+\frac{5}{7} d x^2+c\right) x^4 b^5}{1280}+\frac{231\left(\frac{25}{66} f x^6+\frac{10}{21} e x^4+\frac{9}{14} d x^2+c\right) x^2 a b^4}{320}-\frac{231\left(\frac{50}{231} f x^6+\frac{2}{7} e x^4+\frac{3}{7} d x^2+c\right) a^2 b^3}{160}+\frac{99\left(\frac{10}{33}\cdots\right.}{}\cdots\right)}{693 b^6}$
gosper	$-\frac{\sqrt{b x^2+a}\,(-315 f x^{10} b^5+350 a b^4 f x^8-385 b^5 e x^8-400 a^2 b^3 f x^6+440 a b^4 e x^6-495 b^5 d x^6+480 a^3 b^2 f x^4-528 a^2 b^3 e x^4+594 a b\cdots}{3465 b\cdots}$
trager	$-\frac{\sqrt{b x^2+a}\,(-315 f x^{10} b^5+350 a b^4 f x^8-385 b^5 e x^8-400 a^2 b^3 f x^6+440 a b^4 e x^6-495 b^5 d x^6+480 a^3 b^2 f x^4-528 a^2 b^3 e x^4+594 a b\cdots}{3465 b\cdots}$
risch	$-\frac{\sqrt{b x^2+a}\,(-315 f x^{10} b^5+350 a b^4 f x^8-385 b^5 e x^8-400 a^2 b^3 f x^6+440 a b^4 e x^6-495 b^5 d x^6+480 a^3 b^2 f x^4-528 a^2 b^3 e x^4+594 a b\cdots}{3465 b\cdots}$
default	$e\left(\frac{x^8\sqrt{b x^2+a}}{9b}-\frac{8a\left(\frac{x^6\sqrt{b x^2+a}}{7b}-\frac{6a\left(\frac{x^4\sqrt{b x^2+a}}{5b}-\frac{4a\left(\frac{x^2\sqrt{b x^2+a}}{3b}-\frac{2a\sqrt{b x^2+a}}{3b^2}\right)}{5b}\right)}{7b}\right)}{9b}\right)+d\left(\frac{x^6\sqrt{b x^2+a}}{7b}-\frac{6a\left(\frac{x^4\sqrt{\cdots}}{}\cdots\right.}{}\cdots\right.$

```
[In] int((f*x^11+e*x^9+d*x^7+c*x^5)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -256/693*(-693/1280*(5/11*f*x^6+5/9*e*x^4+5/7*d*x^2+c)*x^4*b^5+231/320*(25/
66*f*x^6+10/21*e*x^4+9/14*d*x^2+c)*x^2*a*b^4-231/160*(50/231*f*x^6+2/7*e*x^
4+3/7*d*x^2+c)*a^2*b^3+99/80*(10/33*f*x^4+4/9*e*x^2+d)*a^3*b^2-11/10*(5/11*
f*x^2+e)*a^4*b+f*a^5)*(b*x^2+a)^(1/2)/b^6
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.83

$$\int \frac{cx^5+dx^7+ex^9+fx^{11}}{\sqrt{a+bx^2}}\,dx$$
$$=\frac{(315\,b^5fx^{10}+35\,(11\,b^5e-10\,ab^4f)x^8+5\,(99\,b^5d-88\,ab^4e+80\,a^2b^3f)x^6+1848\,a^2b^3c-1584\,a^3b^2d+140\cdots}{\cdots}$$

```
[In] integrate((f*x^11+e*x^9+d*x^7+c*x^5)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*b^5*f*x^10 + 35*(11*b^5*e - 10*a*b^4*f)*x^8 + 5*(99*b^5*d - 88*
a*b^4*e + 80*a^2*b^3*f)*x^6 + 1848*a^2*b^3*c - 1584*a^3*b^2*d + 1408*a^4*b*
e - 1280*a^5*f + 3*(231*b^5*c - 198*a*b^4*d + 176*a^2*b^3*e - 160*a^3*b^2*f
)*x^4 - 4*(231*a*b^4*c - 198*a^2*b^3*d + 176*a^3*b^2*e - 160*a^4*b*f)*x^2)*
sqrt(b*x^2 + a)/b^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(214) = 428$.

Time = 0.46 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.07

$$\int \frac{cx^5 + dx^7 + ex^9 + fx^{11}}{\sqrt{a + bx^2}}\,dx = \begin{cases} -\frac{256a^5f\sqrt{a+bx^2}}{693b^6} + \frac{128a^4e\sqrt{a+bx^2}}{315b^5} + \frac{128a^4fx^2\sqrt{a+bx^2}}{693b^5} - \frac{16a^3d\sqrt{a+bx^2}}{35b^4} - \frac{64a^3ex^2\sqrt{a+bx^2}}{315b^4} - \frac{32a^3fx^4\sqrt{a+bx^2}}{231b^4} + \frac{8a^2c\sqrt{a+bx^2}}{15b^3} \\ \frac{\frac{cx^6}{6} + \frac{dx^8}{8} + \frac{ex^{10}}{10} + \frac{fx^{12}}{12}}{\sqrt{a}} \end{cases}$$

```
[In] integrate((f*x**11+e*x**9+d*x**7+c*x**5)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-256*a**5*f*sqrt(a + b*x**2)/(693*b**6) + 128*a**4*e*sqrt(a + b*
x**2)/(315*b**5) + 128*a**4*f*x**2*sqrt(a + b*x**2)/(693*b**5) - 16*a**3*d*
sqrt(a + b*x**2)/(35*b**4) - 64*a**3*e*x**2*sqrt(a + b*x**2)/(315*b**4) - 3
2*a**3*f*x**4*sqrt(a + b*x**2)/(231*b**4) + 8*a**2*c*sqrt(a + b*x**2)/(15*b
**3) + 8*a**2*d*x**2*sqrt(a + b*x**2)/(35*b**3) + 16*a**2*e*x**4*sqrt(a + b
*x**2)/(105*b**3) + 80*a**2*f*x**6*sqrt(a + b*x**2)/(693*b**3) - 4*a*c*x**2
*sqrt(a + b*x**2)/(15*b**2) - 6*a*d*x**4*sqrt(a + b*x**2)/(35*b**2) - 8*a*e
*x**6*sqrt(a + b*x**2)/(63*b**2) - 10*a*f*x**8*sqrt(a + b*x**2)/(99*b**2) +
 c*x**4*sqrt(a + b*x**2)/(5*b) + d*x**6*sqrt(a + b*x**2)/(7*b) + e*x**8*sqr
t(a + b*x**2)/(9*b) + f*x**10*sqrt(a + b*x**2)/(11*b), Ne(b, 0)), ((c*x**6/
6 + d*x**8/8 + e*x**10/10 + f*x**12/12)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.62

$$\int \frac{cx^5 + dx^7 + ex^9 + fx^{11}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}fx^{10}}{11\,b} + \frac{\sqrt{bx^2+a}ex^8}{9\,b} - \frac{10\,\sqrt{bx^2+a}afx^8}{99\,b^2} + \frac{\sqrt{bx^2+a}dx^6}{7\,b} - \frac{8\,\sqrt{bx^2+a}aex^6}{63\,b^2} + \frac{80\,\sqrt{bx^2+a}a^2fx^6}{693\,b^3} + \frac{\sqrt{bx^2+a}cx^4}{5\,b} - \frac{6\,\sqrt{bx^2+a}adx^4}{35\,b^2} + \frac{16\,\sqrt{bx^2+a}a^2ex^4}{105\,b^3} - \frac{32\,\sqrt{bx^2+a}a^3fx^4}{231\,b^4} - \frac{4\,\sqrt{bx^2+a}acx^2}{15\,b^2} + \frac{8\,\sqrt{bx^2+a}a^2dx^2}{35\,b^3} - \frac{64\,\sqrt{bx^2+a}a^3ex^2}{315\,b^4} + \frac{128\,\sqrt{bx^2+a}a^4fx^2}{693\,b^5} + \frac{8\,\sqrt{bx^2+a}a^2c}{15\,b^3} - \frac{16\,\sqrt{bx^2+a}a^3d}{35\,b^4} + \frac{128\,\sqrt{bx^2+a}a^4e}{315\,b^5} - \frac{256\,\sqrt{bx^2+a}a^5f}{693\,b^6}$$

```
[In] integrate((f*x^11+e*x^9+d*x^7+c*x^5)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/11*sqrt(b*x^2 + a)*f*x^10/b + 1/9*sqrt(b*x^2 + a)*e*x^8/b - 10/99*sqrt(b*
x^2 + a)*a*f*x^8/b^2 + 1/7*sqrt(b*x^2 + a)*d*x^6/b - 8/63*sqrt(b*x^2 + a)*a
*e*x^6/b^2 + 80/693*sqrt(b*x^2 + a)*a^2*f*x^6/b^3 + 1/5*sqrt(b*x^2 + a)*c*x
^4/b - 6/35*sqrt(b*x^2 + a)*a*d*x^4/b^2 + 16/105*sqrt(b*x^2 + a)*a^2*e*x^4/
b^3 - 32/231*sqrt(b*x^2 + a)*a^3*f*x^4/b^4 - 4/15*sqrt(b*x^2 + a)*a*c*x^2/b
^2 + 8/35*sqrt(b*x^2 + a)*a^2*d*x^2/b^3 - 64/315*sqrt(b*x^2 + a)*a^3*e*x^2/
b^4 + 128/693*sqrt(b*x^2 + a)*a^4*f*x^2/b^5 + 8/15*sqrt(b*x^2 + a)*a^2*c/b^
3 - 16/35*sqrt(b*x^2 + a)*a^3*d/b^4 + 128/315*sqrt(b*x^2 + a)*a^4*e/b^5 - 2
56/693*sqrt(b*x^2 + a)*a^5*f/b^6
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.21

$$\int \frac{cx^5 + dx^7 + ex^9 + fx^{11}}{\sqrt{a+bx^2}}\,dx = \frac{(a^2b^3c - a^3b^2d + a^4be - a^5f)\sqrt{bx^2+a}}{b^6} + \frac{693\,(bx^2+a)^{\frac{5}{2}}b^3c - 2310\,(bx^2+a)^{\frac{3}{2}}ab^3c + 495\,(bx^2+a)^{\frac{7}{2}}b^2d - 2079\,(bx^2+a)^{\frac{5}{2}}ab^2d + 3465\,(bx^2+a)^{\frac{3}{2}}a}{}$$

```
[In] integrate((f*x^11+e*x^9+d*x^7+c*x^5)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] (a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*sqrt(b*x^2 + a)/b^6 + 1/3465*(693
*(b*x^2 + a)^(5/2)*b^3*c - 2310*(b*x^2 + a)^(3/2)*a*b^3*c + 495*(b*x^2 + a)
```

```
^(7/2)*b^2*d - 2079*(b*x^2 + a)^(5/2)*a*b^2*d + 3465*(b*x^2 + a)^(3/2)*a^2*
b^2*d + 385*(b*x^2 + a)^(9/2)*b*e - 1980*(b*x^2 + a)^(7/2)*a*b*e + 4158*(b*
x^2 + a)^(5/2)*a^2*b*e - 4620*(b*x^2 + a)^(3/2)*a^3*b*e + 315*(b*x^2 + a)^(
11/2)*f - 1925*(b*x^2 + a)^(9/2)*a*f + 4950*(b*x^2 + a)^(7/2)*a^2*f - 6930*
(b*x^2 + a)^(5/2)*a^3*f + 5775*(b*x^2 + a)^(3/2)*a^4*f)/b^6
```

Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.87

$$\int \frac{cx^5+dx^7+ex^9+fx^{11}}{\sqrt{a+bx^2}}\,dx=\sqrt{b\,x^2+a}\left(\frac{x^6\,\left(400\,f\,a^2\,b^3-440\,e\,a\,b^4+495\,d\,b^5\right)}{3465\,b^6}-\frac{1280\,f\,a^5-1408\,e\,a^4\,b+1584\,d\,a^3\,b^2-1848\,c\,a^2\,b^3}{3465\,b^6}+\frac{x^4\,\left(-480\,f\,a^3\,b^2+528\,e\,a^2\,b^3-594\,d\,a\,b^4+693\,c\,b^5\right)}{3465\,b^6}+\frac{f\,x^{10}}{11\,b}+\frac{x^8\,\left(385\,b^5\,e-350\,a\,b^4\,f\right)}{3465\,b^6}-\frac{4\,a\,x^2\,\left(-160\,f\,a^3+176\,e\,a^2\,b-198\,d\,a\,b^2+231\,c\,b^3\right)}{3465\,b^5}\right)$$

```
[In] int((c*x^5 + d*x^7 + e*x^9 + f*x^11)/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((x^6*(495*b^5*d + 400*a^2*b^3*f - 440*a*b^4*e))/(3465*b^
6) - (1280*a^5*f - 1848*a^2*b^3*c + 1584*a^3*b^2*d - 1408*a^4*b*e)/(3465*b^
6) + (x^4*(693*b^5*c + 528*a^2*b^3*e - 480*a^3*b^2*f - 594*a*b^4*d))/(3465*
b^6) + (f*x^10)/(11*b) + (x^8*(385*b^5*e - 350*a*b^4*f))/(3465*b^6) - (4*a*
x^2*(231*b^3*c - 160*a^3*f - 198*a*b^2*d + 176*a^2*b*e))/(3465*b^5))
```

3.170 $\int \frac{cx^3+dx^5+ex^7+fx^9}{\sqrt{a+bx^2}}\,dx$

Optimal result 1078
Rubi [A] (verified) 1078
Mathematica [A] (verified) 1080
Maple [A] (verified) 1080
Fricas [A] (verification not implemented) 1081
Sympy [B] (verification not implemented) 1081
Maxima [A] (verification not implemented) 1082
Giac [A] (verification not implemented) 1082
Mupad [B] (verification not implemented) 1083

Optimal result

Integrand size = 33, antiderivative size = 167

$$\int \frac{cx^3+dx^5+ex^7+fx^9}{\sqrt{a+bx^2}}\,dx = -\frac{a(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^5} + \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\left(a+bx^2\right)^{3/2}}{3b^5} + \frac{(b^2d-3abe+6a^2f)\left(a+bx^2\right)^{5/2}}{5b^5} + \frac{(be-4af)\left(a+bx^2\right)^{7/2}}{7b^5} + \frac{f\left(a+bx^2\right)^{9/2}}{9b^5}$$

```
[Out] 1/3*(-4*a^3*f+3*a^2*b*e-2*a*b^2*d+b^3*c)*(b*x^2+a)^(3/2)/b^5+1/5*(6*a^2*f-3
*a*b*e+b^2*d)*(b*x^2+a)^(5/2)/b^5+1/7*(-4*a*f+b*e)*(b*x^2+a)^(7/2)/b^5+1/9*
f*(b*x^2+a)^(9/2)/b^5-a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x^2+a)^(1/2)/b^5
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1825, 1813, 1864}

$$\int \frac{cx^3+dx^5+ex^7+fx^9}{\sqrt{a+bx^2}}\,dx = \frac{\left(a+bx^2\right)^{5/2}(6a^2f-3abe+b^2d)}{5b^5} + \frac{\left(a+bx^2\right)^{3/2}(-4a^3f+3a^2be-2ab^2d+b^3c)}{3b^5} - \frac{a\sqrt{a+bx^2}(a^3(-f)+a^2be-ab^2d+b^3c)}{b^5} + \frac{\left(a+bx^2\right)^{7/2}(be-4af)}{7b^5} + \frac{f\left(a+bx^2\right)^{9/2}}{9b^5}$$

```
[In] Int[(c*x^3 + d*x^5 + e*x^7 + f*x^9)/Sqrt[a + b*x^2],x]
```

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^5) + ((b^3*c -
2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*(a + b*x^2)^(3/2))/(3*b^5) + ((b^2*d - 3*a
*b*e + 6*a^2*f)*(a + b*x^2)^(5/2))/(5*b^5) + ((b*e - 4*a*f)*(a + b*x^2)^(7/
2))/(7*b^5) + (f*(a + b*x^2)^(9/2))/(9*b^5)
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1825

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x(cx^2+dx^4+ex^6+fx^8)}{\sqrt{a+bx^2}}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{cx+dx^2+ex^3+fx^4}{\sqrt{a+bx}}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a(-b^3c+ab^2d-a^2be+a^3f)}{b^4\sqrt{a+bx}}+\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\sqrt{a+bx}}{b^4}\right.\right. \\
&\qquad \left.\left. +\frac{(b^2d-3abe+6a^2f)(a+bx)^{3/2}}{b^4}+\frac{(be-4af)(a+bx)^{5/2}}{b^4}+\frac{f(a+bx)^{7/2}}{b^4}\right)dx, x, x^2\right) \\
&= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^5}+\frac{(b^3c-2ab^2d+3a^2be-4a^3f)(a+bx^2)^{3/2}}{3b^5} \\
&\quad +\frac{(b^2d-3abe+6a^2f)(a+bx^2)^{5/2}}{5b^5}+\frac{(be-4af)(a+bx^2)^{7/2}}{7b^5}+\frac{f(a+bx^2)^{9/2}}{9b^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

$$\int \frac{cx^3 + dx^5 + ex^7 + fx^9}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a+bx^2}(128a^4f - 16a^3b(9e+4fx^2) + 24a^2b^2(7d+3ex^2+2fx^4) - 2ab^3(105c+42dx^2+27ex^4+20fx^6)}{315b^5}$$

[In] Integrate[(c*x^3 + d*x^5 + e*x^7 + f*x^9)/Sqrt[a + b*x^2],x]

[Out] (Sqrt[a + b*x^2]*(128*a^4*f - 16*a^3*b*(9*e + 4*f*x^2) + 24*a^2*b^2*(7*d + 3*e*x^2 + 2*f*x^4) - 2*a*b^3*(105*c + 42*d*x^2 + 27*e*x^4 + 20*f*x^6) + b^4 *x^2*(105*c + 63*d*x^2 + 45*e*x^4 + 35*f*x^6)))/(315*b^5)

Maple [A] (verified)

Time = 3.51 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{128\left(\frac{105\left(\frac{1}{3}f\,x^6+\frac{3}{7}e\,x^4+\frac{3}{5}d\,x^2+c\right)x^2b^4}{128}-\frac{105\left(\frac{4}{21}f\,x^6+\frac{9}{35}e\,x^4+\frac{2}{5}d\,x^2+c\right)a\,b^3}{64}+\frac{21\left(\frac{2}{7}f\,x^4+\frac{3}{7}e\,x^2+d\right)a^2b^2}{16}-\frac{9\left(\frac{4f\,x^2}{9}+e\right)a^3b}{8}+a^4f\right)}{315b^5}$
gosper	$\frac{\sqrt{b\,x^2+a}\,(35f\,x^8b^4-40a\,b^3f\,x^6+45b^4e\,x^6+48a^2b^2f\,x^4-54a\,b^3e\,x^4+63b^4d\,x^4-64a^3bf\,x^2+72a^2b^2e\,x^2-84a\,b^3d\,x^2+105b^4c\,x^2}{315b^5}$
trager	$\frac{\sqrt{b\,x^2+a}\,(35f\,x^8b^4-40a\,b^3f\,x^6+45b^4e\,x^6+48a^2b^2f\,x^4-54a\,b^3e\,x^4+63b^4d\,x^4-64a^3bf\,x^2+72a^2b^2e\,x^2-84a\,b^3d\,x^2+105b^4c\,x^2}{315b^5}$
risch	$\frac{\sqrt{b\,x^2+a}\,(35f\,x^8b^4-40a\,b^3f\,x^6+45b^4e\,x^6+48a^2b^2f\,x^4-54a\,b^3e\,x^4+63b^4d\,x^4-64a^3bf\,x^2+72a^2b^2e\,x^2-84a\,b^3d\,x^2+105b^4c\,x^2}{315b^5}$
default	$f\left(\frac{x^8\sqrt{b\,x^2+a}}{9b}-\frac{8a\left(\frac{x^6\sqrt{b\,x^2+a}}{7b}-\frac{6a\left(\frac{x^4\sqrt{b\,x^2+a}}{5b}-\frac{4a\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)}{5b}\right)}{7b}\right)}{9b}\right)+e\left(\frac{x^6\sqrt{b\,x^2+a}}{7b}-\frac{6a\left(\frac{x^4\sqrt{}}{}\right.}{}\right.$

[In] int((f*x^9+e*x^7+d*x^5+c*x^3)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 128/315*(105/128*(1/3*f*x^6+3/7*e*x^4+3/5*d*x^2+c)*x^2*b^4-105/64*(4/21*f*x ^6+9/35*e*x^4+2/5*d*x^2+c)*a*b^3+21/16*(2/7*f*x^4+3/7*e*x^2+d)*a^2*b^2-9/8* (4/9*f*x^2+e)*a^3*b+a^4*f)*(b*x^2+a)^(1/2)/b^5

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.80

$$\int \frac{cx^3 + dx^5 + ex^7 + fx^9}{\sqrt{a + bx^2}}\,dx = \frac{(35\,b^4fx^8 + 5\,(9\,b^4e - 8\,ab^3f)x^6 - 210\,ab^3c + 168\,a^2b^2d - 144\,a^3be + 128\,a^4f + 3\,(21\,b^4d - 18\,ab^3e + 16\,a}{315\,b^5}$$

```
[In] integrate((f*x^9+e*x^7+d*x^5+c*x^3)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^4*f*x^8 + 5*(9*b^4*e - 8*a*b^3*f)*x^6 - 210*a*b^3*c + 168*a^2*b
^2*d - 144*a^3*b*e + 128*a^4*f + 3*(21*b^4*d - 18*a*b^3*e + 16*a^2*b^2*f)*x
^4 + (105*b^4*c - 84*a*b^3*d + 72*a^2*b^2*e - 64*a^3*b*f)*x^2)*sqrt(b*x^2 +
 a)/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. 2(163) = 326.

Time = 0.37 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.04

$$\int \frac{cx^3 + dx^5 + ex^7 + fx^9}{\sqrt{a + bx^2}}\,dx = \begin{cases} \frac{128a^4f\sqrt{a+bx^2}}{315b^5} - \frac{16a^3e\sqrt{a+bx^2}}{35b^4} - \frac{64a^3fx^2\sqrt{a+bx^2}}{315b^4} + \frac{8a^2d\sqrt{a+bx^2}}{15b^3} + \frac{8a^2ex^2\sqrt{a+bx^2}}{35b^3} + \frac{16a^2fx^4\sqrt{a+bx^2}}{105b^3} - \frac{2ac\sqrt{a+bx^2}}{3b^2} - \frac{4ad}{} \\ \frac{\frac{cx^4}{4} + \frac{dx^6}{6} + \frac{ex^8}{8} + \frac{fx^{10}}{10}}{\sqrt{a}} \end{cases}$$

```
[In] integrate((f*x**9+e*x**7+d*x**5+c*x**3)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((128*a**4*f*sqrt(a + b*x**2)/(315*b**5) - 16*a**3*e*sqrt(a + b*x*
*2)/(35*b**4) - 64*a**3*f*x**2*sqrt(a + b*x**2)/(315*b**4) + 8*a**2*d*sqrt(
a + b*x**2)/(15*b**3) + 8*a**2*e*x**2*sqrt(a + b*x**2)/(35*b**3) + 16*a**2*
f*x**4*sqrt(a + b*x**2)/(105*b**3) - 2*a*c*sqrt(a + b*x**2)/(3*b**2) - 4*a*
d*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*e*x**4*sqrt(a + b*x**2)/(35*b**2) -
 8*a*f*x**6*sqrt(a + b*x**2)/(63*b**2) + c*x**2*sqrt(a + b*x**2)/(3*b) + d*
x**4*sqrt(a + b*x**2)/(5*b) + e*x**6*sqrt(a + b*x**2)/(7*b) + f*x**8*sqrt(a
 + b*x**2)/(9*b), Ne(b, 0)), ((c*x**4/4 + d*x**6/6 + e*x**8/8 + f*x**10/10)
/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.57

$$\int \frac{cx^3+dx^5+ex^7+fx^9}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}fx^8}{9\,b} + \frac{\sqrt{bx^2+a}ex^6}{7\,b} - \frac{8\,\sqrt{bx^2+a}afx^6}{63\,b^2} + \frac{\sqrt{bx^2+a}dx^4}{5\,b} - \frac{6\,\sqrt{bx^2+a}aex^4}{35\,b^2} + \frac{16\,\sqrt{bx^2+a}a^2fx^4}{105\,b^3} + \frac{\sqrt{bx^2+a}cx^2}{3\,b} - \frac{4\,\sqrt{bx^2+a}adx^2}{15\,b^2} + \frac{8\,\sqrt{bx^2+a}a^2ex^2}{35\,b^3} - \frac{64\,\sqrt{bx^2+a}a^3fx^2}{315\,b^4} - \frac{2\,\sqrt{bx^2+a}ac}{3\,b^2} + \frac{8\,\sqrt{bx^2+a}a^2d}{15\,b^3} - \frac{16\,\sqrt{bx^2+a}a^3e}{35\,b^4} + \frac{128\,\sqrt{bx^2+a}a^4f}{315\,b^5}$$

```
[In] integrate((f*x^9+e*x^7+d*x^5+c*x^3)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(b*x^2 + a)*f*x^8/b + 1/7*sqrt(b*x^2 + a)*e*x^6/b - 8/63*sqrt(b*x^2
 + a)*a*f*x^6/b^2 + 1/5*sqrt(b*x^2 + a)*d*x^4/b - 6/35*sqrt(b*x^2 + a)*a*e*
x^4/b^2 + 16/105*sqrt(b*x^2 + a)*a^2*f*x^4/b^3 + 1/3*sqrt(b*x^2 + a)*c*x^2/
b - 4/15*sqrt(b*x^2 + a)*a*d*x^2/b^2 + 8/35*sqrt(b*x^2 + a)*a^2*e*x^2/b^3 -
 64/315*sqrt(b*x^2 + a)*a^3*f*x^2/b^4 - 2/3*sqrt(b*x^2 + a)*a*c/b^2 + 8/15*
sqrt(b*x^2 + a)*a^2*d/b^3 - 16/35*sqrt(b*x^2 + a)*a^3*e/b^4 + 128/315*sqrt(
b*x^2 + a)*a^4*f/b^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.16

$$\int \frac{cx^3+dx^5+ex^7+fx^9}{\sqrt{a+bx^2}}\,dx = -\frac{(ab^3c-a^2b^2d+a^3be-a^4f)\sqrt{bx^2+a}}{b^5} + \frac{105\,(bx^2+a)^{\frac{3}{2}}b^3c+63\,(bx^2+a)^{\frac{5}{2}}b^2d-210\,(bx^2+a)^{\frac{3}{2}}ab^2d+45\,(bx^2+a)^{\frac{7}{2}}be-189\,(bx^2+a)^{\frac{5}{2}}abe+315}{315\,b^5}$$

```
[In] integrate((f*x^9+e*x^7+d*x^5+c*x^3)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*sqrt(b*x^2 + a)/b^5 + 1/315*(105*(
b*x^2 + a)^(3/2)*b^3*c + 63*(b*x^2 + a)^(5/2)*b^2*d - 210*(b*x^2 + a)^(3/2)
*a*b^2*d + 45*(b*x^2 + a)^(7/2)*b*e - 189*(b*x^2 + a)^(5/2)*a*b*e + 315*(b*
x^2 + a)^(3/2)*a^2*b*e + 35*(b*x^2 + a)^(9/2)*f - 180*(b*x^2 + a)^(7/2)*a*f
 + 378*(b*x^2 + a)^(5/2)*a^2*f - 420*(b*x^2 + a)^(3/2)*a^3*f)/b^5
```

Mupad [B] (verification not implemented)

Time = 5.89 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.87

$$\int \frac{cx^3+dx^5+ex^7+fx^9}{\sqrt{a+bx^2}}\,dx=\sqrt{b\,x^2+a}\,\left(\frac{128\,f\,a^4-144\,e\,a^3\,b+168\,d\,a^2\,b^2-210\,c\,a\,b^3}{315\,b^5}+\frac{x^4\,\left(48\,f\,a^2\,b^2-54\,e\,a\,b^3+63\,d\,b^4\right)}{315\,b^5}+\frac{f\,x^8}{9\,b}+\frac{x^6\,\left(45\,b^4\,e-40\,a\,b^3\,f\right)}{315\,b^5}+\frac{x^2\,\left(-64\,f\,a^3\,b+72\,e\,a^2\,b^2-84\,d\,a\,b^3+105\,c\,b^4\right)}{315\,b^5}\right)$$

```
[In] int((c*x^3 + d*x^5 + e*x^7 + f*x^9)/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((128*a^4*f + 168*a^2*b^2*d - 210*a*b^3*c - 144*a^3*b*e)/
(315*b^5) + (x^4*(63*b^4*d + 48*a^2*b^2*f - 54*a*b^3*e))/(315*b^5) + (f*x^8
)/(9*b) + (x^6*(45*b^4*e - 40*a*b^3*f))/(315*b^5) + (x^2*(105*b^4*c + 72*a^
2*b^2*e - 84*a*b^3*d - 64*a^3*b*f))/(315*b^5))
```

3.171 $\int \frac{cx+dx^3+ex^5+fx^7}{\sqrt{a+bx^2}} dx$

Optimal result 1084
Rubi [A] (verified) 1084
Mathematica [A] (verified) 1086
Maple [A] (verified) 1086
Fricas [A] (verification not implemented) 1087
Sympy [B] (verification not implemented) 1087
Maxima [A] (verification not implemented) 1088
Giac [A] (verification not implemented) 1088
Mupad [B] (verification not implemented) 1089

Optimal result

Integrand size = 31, antiderivative size = 121

$$\int \frac{cx+dx^3+ex^5+fx^7}{\sqrt{a+bx^2}} dx = \frac{(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^4} + \frac{(b^2d-2abe+3a^2f)(a+bx^2)^{3/2}}{3b^4} + \frac{(be-3af)(a+bx^2)^{5/2}}{5b^4} + \frac{f(a+bx^2)^{7/2}}{7b^4}$$

```
[Out] 1/3*(3*a^2*f-2*a*b*e+b^2*d)*(b*x^2+a)^(3/2)/b^4+1/5*(-3*a*f+b*e)*(b*x^2+a)^
(5/2)/b^4+1/7*f*(b*x^2+a)^(7/2)/b^4+(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x^2+a
)^(1/2)/b^4
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1825, 1813, 1864}

$$\int \frac{cx+dx^3+ex^5+fx^7}{\sqrt{a+bx^2}} dx = \frac{(a+bx^2)^{3/2}(3a^2f-2abe+b^2d)}{3b^4} + \frac{\sqrt{a+bx^2}(a^3(-f)+a^2be-ab^2d+b^3c)}{b^4} + \frac{(a+bx^2)^{5/2}(be-3af)}{5b^4} + \frac{f(a+bx^2)^{7/2}}{7b^4}$$

```
[In] Int[(c*x + d*x^3 + e*x^5 + f*x^7)/Sqrt[a + b*x^2],x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^4 + ((b^2*d - 2*a*b
*e + 3*a^2*f)*(a + b*x^2)^(3/2))/(3*b^4) + ((b*e - 3*a*f)*(a + b*x^2)^(5/2)
)/(5*b^4) + (f*(a + b*x^2)^(7/2))/(7*b^4)
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1825

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b^3c-ab^2d+a^2be-a^3f}{b^3\sqrt{a+bx}}+\frac{(b^2d-2abe+3a^2f)\sqrt{a+bx}}{b^3}\right.\right. \\
&\qquad \left.\left.+\frac{(be-3af)(a+bx)^{3/2}}{b^3}+\frac{f(a+bx)^{5/2}}{b^3}\right)dx, x, x^2\right) \\
&= \frac{(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^4}+\frac{(b^2d-2abe+3a^2f)\,(a+bx^2)^{3/2}}{3b^4} \\
&\quad +\frac{(be-3af)\,(a+bx^2)^{5/2}}{5b^4}+\frac{f(a+bx^2)^{7/2}}{7b^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{cx+dx^3+ex^5+fx^7}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(-48a^3f+8a^2b(7e+3fx^2)-2ab^2(35d+14ex^2+9fx^4)+b^3(105c+35dx^2+21ex^4+15fx^6))}{105b^4}$$

```
[In] Integrate[(c*x + d*x^3 + e*x^5 + f*x^7)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-48*a^3*f + 8*a^2*b*(7*e + 3*f*x^2) - 2*a*b^2*(35*d + 14*
e*x^2 + 9*f*x^4) + b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6)))/(105*b^4)
```

Maple [A] (verified)

Time = 3.55 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$-\frac{16\left(\frac{\left(-5f\,x^{6}-7e\,x^{4}-\frac{35}{3}d\,x^{2}-35c\right)b^{3}}{16}+\frac{35\left(\frac{9}{35}f\,x^{4}+\frac{2}{5}e\,x^{2}+d\right)a\,b^{2}}{24}-\frac{7\left(\frac{3f\,x^{2}}{7}+e\right)a^{2}b}{6}+f\,a^{3}\right)\sqrt{b\,x^{2}+a}}{35b^{4}}$
gosper	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-15f\,x^{6}b^{3}+18a\,b^{2}f\,x^{4}-21b^{3}e\,x^{4}-24a^{2}bf\,x^{2}+28a\,b^{2}e\,x^{2}-35b^{3}d\,x^{2}+48f\,a^{3}-56a^{2}be+70a\,b^{2}d-105b^{3}c\right)}{105b^{4}}$
trager	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-15f\,x^{6}b^{3}+18a\,b^{2}f\,x^{4}-21b^{3}e\,x^{4}-24a^{2}bf\,x^{2}+28a\,b^{2}e\,x^{2}-35b^{3}d\,x^{2}+48f\,a^{3}-56a^{2}be+70a\,b^{2}d-105b^{3}c\right)}{105b^{4}}$
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-15f\,x^{6}b^{3}+18a\,b^{2}f\,x^{4}-21b^{3}e\,x^{4}-24a^{2}bf\,x^{2}+28a\,b^{2}e\,x^{2}-35b^{3}d\,x^{2}+48f\,a^{3}-56a^{2}be+70a\,b^{2}d-105b^{3}c\right)}{105b^{4}}$
default	$f\left(\frac{x^{6}\sqrt{b\,x^{2}+a}}{7b}-\frac{6a\left(\frac{x^{4}\sqrt{b\,x^{2}+a}}{5b}-\frac{4a\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+a}}{3b^{2}}\right)}{5b}\right)}{7b}\right)+e\left(\frac{x^{4}\sqrt{b\,x^{2}+a}}{5b}-\frac{4a\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+a}}{3b^{2}}\right.}{5b}\right.$

```
[In] int((f*x^7+e*x^5+d*x^3+c*x)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -16/35*(1/16*(-5*f*x^6-7*e*x^4-35/3*d*x^2-35*c)*b^3+35/24*(9/35*f*x^4+2/5*e
*x^2+d)*a*b^2-7/6*(3/7*f*x^2+e)*a^2*b+f*a^3)*(b*x^2+a)^(1/2)/b^4
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.78

$$\int \frac{cx+dx^3+ex^5+fx^7}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{(15\,b^3fx^6+3\,(7\,b^3e-6\,ab^2f)x^4+105\,b^3c-70\,ab^2d+56\,a^2be-48\,a^3f+(35\,b^3d-28\,ab^2e+24\,a^2bf)x^2)}{105\,b^4}$$

```
[In] integrate((f*x^7+e*x^5+d*x^3+c*x)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^3*f*x^6 + 3*(7*b^3*e - 6*a*b^2*f)*x^4 + 105*b^3*c - 70*a*b^2*d
+ 56*a^2*b*e - 48*a^3*f + (35*b^3*d - 28*a*b^2*e + 24*a^2*b*f)*x^2)*sqrt(b*
x^2 + a)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(112) = 224.

Time = 0.29 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.97

$$\int \frac{cx+dx^3+ex^5+fx^7}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} -\frac{16a^3f\sqrt{a+bx^2}}{35b^4}+\frac{8a^2e\sqrt{a+bx^2}}{15b^3}+\frac{8a^2fx^2\sqrt{a+bx^2}}{35b^3}-\frac{2ad\sqrt{a+bx^2}}{3b^2}-\frac{4aex^2\sqrt{a+bx^2}}{15b^2}-\frac{6afx^4\sqrt{a+bx^2}}{35b^2}+\frac{c\sqrt{a+bx^2}}{b}+\frac{dx^2\sqrt{a+bx^2}}{3b} \\ \frac{\frac{cx^2}{2}+\frac{dx^4}{4}+\frac{ex^6}{6}+\frac{fx^8}{8}}{\sqrt{a}} \end{cases}$$

```
[In] integrate((f*x**7+e*x**5+d*x**3+c*x)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-16*a**3*f*sqrt(a + b*x**2)/(35*b**4) + 8*a**2*e*sqrt(a + b*x**2
)/(15*b**3) + 8*a**2*f*x**2*sqrt(a + b*x**2)/(35*b**3) - 2*a*d*sqrt(a + b*x
**2)/(3*b**2) - 4*a*e*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*f*x**4*sqrt(a +
 b*x**2)/(35*b**2) + c*sqrt(a + b*x**2)/b + d*x**2*sqrt(a + b*x**2)/(3*b) +
 e*x**4*sqrt(a + b*x**2)/(5*b) + f*x**6*sqrt(a + b*x**2)/(7*b), Ne(b, 0)),
((c*x**2/2 + d*x**4/4 + e*x**6/6 + f*x**8/8)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.49

$$\int \frac{cx + dx^3 + ex^5 + fx^7}{\sqrt{a + bx^2}} \, dx = \frac{\sqrt{bx^2 + a}fx^6}{7\,b} + \frac{\sqrt{bx^2 + a}ex^4}{5\,b} - \frac{6\,\sqrt{bx^2 + a}afx^4}{35\,b^2} + \frac{\sqrt{bx^2 + a}dx^2}{3\,b} - \frac{4\,\sqrt{bx^2 + a}aex^2}{15\,b^2} + \frac{8\,\sqrt{bx^2 + a}a^2fx^2}{35\,b^3} + \frac{\sqrt{bx^2 + a}c}{b} - \frac{2\,\sqrt{bx^2 + a}ad}{3\,b^2} + \frac{8\,\sqrt{bx^2 + a}a^2e}{15\,b^3} - \frac{16\,\sqrt{bx^2 + a}a^3f}{35\,b^4}$$

```
[In] integrate((f*x^7+e*x^5+d*x^3+c*x)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*sqrt(b*x^2 + a)*f*x^6/b + 1/5*sqrt(b*x^2 + a)*e*x^4/b - 6/35*sqrt(b*x^2
 + a)*a*f*x^4/b^2 + 1/3*sqrt(b*x^2 + a)*d*x^2/b - 4/15*sqrt(b*x^2 + a)*a*e*
x^2/b^2 + 8/35*sqrt(b*x^2 + a)*a^2*f*x^2/b^3 + sqrt(b*x^2 + a)*c/b - 2/3*sq
rt(b*x^2 + a)*a*d/b^2 + 8/15*sqrt(b*x^2 + a)*a^2*e/b^3 - 16/35*sqrt(b*x^2 +
 a)*a^3*f/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.05

$$\int \frac{cx + dx^3 + ex^5 + fx^7}{\sqrt{a + bx^2}} \, dx = \frac{(b^3c - ab^2d + a^2be - a^3f)\sqrt{bx^2 + a}}{b^4} + \frac{35\,(bx^2 + a)^{\frac{3}{2}}b^2d + 21\,(bx^2 + a)^{\frac{5}{2}}be - 70\,(bx^2 + a)^{\frac{3}{2}}abe + 15\,(bx^2 + a)^{\frac{7}{2}}f - 63\,(bx^2 + a)^{\frac{5}{2}}af + 105\,(bx^2 +}{105\,b^4}$$

```
[In] integrate((f*x^7+e*x^5+d*x^3+c*x)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] (b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(b*x^2 + a)/b^4 + 1/105*(35*(b*x^2
+ a)^(3/2)*b^2*d + 21*(b*x^2 + a)^(5/2)*b*e - 70*(b*x^2 + a)^(3/2)*a*b*e +
15*(b*x^2 + a)^(7/2)*f - 63*(b*x^2 + a)^(5/2)*a*f + 105*(b*x^2 + a)^(3/2)*a
^2*f)/b^4
```

Mupad [B] (verification not implemented)

Time = 5.99 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.85

$$\int \frac{cx+dx^3+ex^5+fx^7}{\sqrt{a+bx^2}}\,dx=\sqrt{b\,x^2+a}\,\left(\frac{-48\,f\,a^3+56\,e\,a^2\,b-70\,d\,a\,b^2+105\,c\,b^3}{105\,b^4}+\frac{f\,x^6}{7\,b}+\frac{x^2\,\left(24\,f\,a^2\,b-28\,e\,a\,b^2+35\,d\,b^3\right)}{105\,b^4}+\frac{x^4\,\left(21\,b^3\,e-18\,a\,b^2\,f\right)}{105\,b^4}\right)$$

```
[In] int((c*x + d*x^3 + e*x^5 + f*x^7)/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((105*b^3*c - 48*a^3*f - 70*a*b^2*d + 56*a^2*b*e)/(105*b^
4) + (f*x^6)/(7*b) + (x^2*(35*b^3*d - 28*a*b^2*e + 24*a^2*b*f))/(105*b^4) +
 (x^4*(21*b^3*e - 18*a*b^2*f))/(105*b^4))
```

3.172 $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx$

Optimal result 1090
Rubi [A] (verified) 1090
Mathematica [A] (verified) 1094
Maple [A] (verified) 1095
Fricas [A] (verification not implemented) 1097
Sympy [B] (verification not implemented) 1098
Maxima [B] (verification not implemented) 1102
Giac [A] (verification not implemented) 1103
Mupad [F(-1)] 1103

Optimal result

Integrand size = 37, antiderivative size = 261

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx = \frac{(Ab^4-a(b^3B-ab^2C+a^2bD-a^3F))\,x^3}{7ab^4\,(a+bx^2)^{7/2}} + \frac{(4Ab^4+a(3b^3B-10ab^2C+17a^2bD-24a^3F))\,x^3}{35a^2b^4\,(a+bx^2)^{5/2}} + \frac{(8Ab^4+a(6b^3B+15ab^2C-71a^2bD+162a^3F))\,x^3}{105a^3b^4\,(a+bx^2)^{3/2}} - \frac{(bD-4aF)x}{b^5\sqrt{a+bx^2}} + \frac{Fx\sqrt{a+bx^2}}{2b^5} + \frac{(2bD-9aF)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}$$

```
[Out] 1/7*(A*b^4-a*(B*b^3-C*a*b^2+D*a^2*b-F*a^3))*x^3/a/b^4/(b*x^2+a)^(7/2)+1/35*
(4*A*b^4+a*(3*B*b^3-10*C*a*b^2+17*D*a^2*b-24*F*a^3))*x^3/a^2/b^4/(b*x^2+a)^
(5/2)+1/105*(8*A*b^4+a*(6*B*b^3+15*C*a*b^2-71*D*a^2*b+162*F*a^3))*x^3/a^3/b
^4/(b*x^2+a)^(3/2)+1/2*(2*D*b-9*F*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(
11/2)-(D*b-4*F*a)*x/b^5/(b*x^2+a)^(1/2)+1/2*F*x*(b*x^2+a)^(1/2)/b^5
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used

$= \{1818, 1814, 1599, 1277, 1598, 466, 396, 223, 212\}$

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx = \frac{x^3(a(162a^3F-71a^2bD+15ab^2C+6b^3B)+8Ab^4)}{105a^3b^4\,(a+bx^2)^{3/2}} + \frac{x^3(a(-24a^3F+17a^2bD-10ab^2C+3b^3B)+4Ab^4)}{35a^2b^4\,(a+bx^2)^{5/2}} + \frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\,(a+bx^2)^{7/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bD-9aF)}{2b^{11/2}} - \frac{x(bD-4aF)}{b^5\sqrt{a+bx^2}} + \frac{Fx\sqrt{a+bx^2}}{2b^5}$$

```
[In] Int[(x^2*(A + B*x^2 + C*x^4 + D*x^6 + F*x^8))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A/a - (b^3*B - a*b^2*C + a^2*b*D - a^3*F)/b^4)*x^3)/(7*(a + b*x^2)^(7/2))
 + ((4*A*b^4 + a*(3*b^3*B - 10*a*b^2*C + 17*a^2*b*D - 24*a^3*F))*x^3)/(35*a
^2*b^4*(a + b*x^2)^(5/2)) + ((8*A*b^4 + a*(6*b^3*B + 15*a*b^2*C - 71*a^2*b*
D + 162*a^3*F))*x^3)/(105*a^3*b^4*(a + b*x^2)^(3/2)) - ((b*D - 4*a*F)*x)/(b
^5*Sqrt[a + b*x^2]) + (F*x*Sqrt[a + b*x^2])/(2*b^5) + ((2*b*D - 9*a*F)*ArcT
anh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, Simp[(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d
*f*(q + 1))), x] + Dist[f/(2*d*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1] &
& GtQ[m, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1814

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist
[1/c, Int[(c*x)^(m + 1)*PolynomialQuotient[Pq, x, x]*(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && EqQ[Coeff[Pq, x, 0], 0]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(\frac{A}{a} - \frac{b^3B - ab^2C + a^2bD - a^3F}{b^4}\right)x^3}{7\left(a + bx^2\right)^{7/2}} - \frac{\int \frac{x\left(-\left(\left(\left(4Ab + \frac{3a\left(b^3B - ab^2C + a^2bD - a^3F\right)}{b^3}\right)x\right) - \frac{7a\left(b^2C - abD + a^2F\right)x^3}{b^2} - 7a\left(D - \frac{aF}{b}\right)x^5 - 7aFx^7\right)}{\left(a + bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= \frac{\left(\frac{A}{a} - \frac{b^3B - ab^2C + a^2bD - a^3F}{b^4}\right)x^3}{7\left(a + bx^2\right)^{7/2}} - \frac{\int \frac{x^2\left(-4Ab - \frac{3a\left(b^3B - ab^2C + a^2bD - a^3F\right)}{b^3} - \frac{7a\left(b^2C - abD + a^2F\right)x^2}{b^2} - 7a\left(D - \frac{aF}{b}\right)x^4 - 7aFx^6\right)}{\left(a + bx^2\right)^{7/2}}\,dx}{7ab}$$

$$= \frac{\left(\frac{A}{a} - \frac{b^3B - ab^2C + a^2bD - a^3F}{b^4}\right)x^3}{7\left(a + bx^2\right)^{7/2}} + \frac{\left(4Ab^4 + a\left(3b^3B - 10ab^2C + 17a^2bD - 24a^3F\right)\right)x^3}{35a^2b^4\left(a + bx^2\right)^{5/2}} + \frac{\int \frac{x\left(\left(8Ab^2 + 3a\left(2bB + 5aC - \frac{12a^2D}{b} + \frac{19a^3F}{b^2}\right)\right)x + 35a^2\left(D - \frac{2aF}{b}\right)x^3 + 35a^2Fx^5\right)}{\left(a + bx^2\right)^{5/2}}\,dx}{35a^2b^2}$$

$$= \frac{\left(\frac{A}{a} - \frac{b^3B - ab^2C + a^2bD - a^3F}{b^4}\right)x^3}{7\left(a + bx^2\right)^{7/2}} + \frac{\left(4Ab^4 + a\left(3b^3B - 10ab^2C + 17a^2bD - 24a^3F\right)\right)x^3}{35a^2b^4\left(a + bx^2\right)^{5/2}} + \frac{\int \frac{x^2\left(8Ab^2 + 3a\left(2bB + 5aC - \frac{12a^2D}{b} + \frac{19a^3F}{b^2}\right) + 35a^2\left(D - \frac{2aF}{b}\right)x^2 + 35a^2Fx^4\right)}{\left(a + bx^2\right)^{5/2}}\,dx}{35a^2b^2}$$

$$= \frac{\left(\frac{A}{a} - \frac{b^3B - ab^2C + a^2bD - a^3F}{b^4}\right)x^3}{7\left(a + bx^2\right)^{7/2}} + \frac{\left(4Ab^4 + a\left(3b^3B - 10ab^2C + 17a^2bD - 24a^3F\right)\right)x^3}{35a^2b^4\left(a + bx^2\right)^{5/2}} + \frac{\left(8Ab^4 + a\left(6b^3B + 15ab^2C - 71a^2bD + 162a^3F\right)\right)x^3}{105a^3b^4\left(a + bx^2\right)^{3/2}} - \frac{\int \frac{x\left(-\frac{105a^3(bD - 3aF)x}{b^2} - \frac{105a^3Fx^3}{b}\right)}{\left(a + bx^2\right)^{3/2}}\,dx}{105a^3b^2}$$

$$= \frac{\left(\frac{A}{a} - \frac{b^3B - ab^2C + a^2bD - a^3F}{b^4}\right)x^3}{7\left(a + bx^2\right)^{7/2}} + \frac{\left(4Ab^4 + a\left(3b^3B - 10ab^2C + 17a^2bD - 24a^3F\right)\right)x^3}{35a^2b^4\left(a + bx^2\right)^{5/2}} + \frac{\left(8Ab^4 + a\left(6b^3B + 15ab^2C - 71a^2bD + 162a^3F\right)\right)x^3}{105a^3b^4\left(a + bx^2\right)^{3/2}} - \frac{\int \frac{x^2\left(-\frac{105a^3(bD - 3aF)}{b^2} - \frac{105a^3Fx^2}{b}\right)}{\left(a + bx^2\right)^{3/2}}\,dx}{105a^3b^2}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}}+\frac{\left(4Ab^4+a(3b^3B-10ab^2C+17a^2bD-24a^3F)\right)x^3}{35a^2b^4\left(a+bx^2\right)^{5/2}}$$
$$+\frac{\left(8Ab^4+a(6b^3B+15ab^2C-71a^2bD+162a^3F)\right)x^3}{105a^3b^4\left(a+bx^2\right)^{3/2}}$$
$$-\frac{(bD-4aF)x}{b^5\sqrt{a+bx^2}}+\frac{\int\frac{\frac{105a^3(bD-4aF)}{b}+105a^3Fx^2}{\sqrt{a+bx^2}}dx}{105a^3b^4}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}}+\frac{\left(4Ab^4+a(3b^3B-10ab^2C+17a^2bD-24a^3F)\right)x^3}{35a^2b^4\left(a+bx^2\right)^{5/2}}$$
$$+\frac{\left(8Ab^4+a(6b^3B+15ab^2C-71a^2bD+162a^3F)\right)x^3}{105a^3b^4\left(a+bx^2\right)^{3/2}}$$
$$-\frac{(bD-4aF)x}{b^5\sqrt{a+bx^2}}+\frac{Fx\sqrt{a+bx^2}}{2b^5}+\frac{(2bD-9aF)\int\frac{1}{\sqrt{a+bx^2}}dx}{2b^5}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}}+\frac{\left(4Ab^4+a(3b^3B-10ab^2C+17a^2bD-24a^3F)\right)x^3}{35a^2b^4\left(a+bx^2\right)^{5/2}}$$
$$+\frac{\left(8Ab^4+a(6b^3B+15ab^2C-71a^2bD+162a^3F)\right)x^3}{105a^3b^4\left(a+bx^2\right)^{3/2}}-\frac{(bD-4aF)x}{b^5\sqrt{a+bx^2}}$$
$$+\frac{Fx\sqrt{a+bx^2}}{2b^5}+\frac{(2bD-9aF)\text{Subst}\left(\int\frac{1}{1-bx^2}dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2b^5}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\left(a+bx^2\right)^{7/2}}+\frac{\left(4Ab^4+a(3b^3B-10ab^2C+17a^2bD-24a^3F)\right)x^3}{35a^2b^4\left(a+bx^2\right)^{5/2}}$$
$$+\frac{\left(8Ab^4+a(6b^3B+15ab^2C-71a^2bD+162a^3F)\right)x^3}{105a^3b^4\left(a+bx^2\right)^{3/2}}$$
$$-\frac{(bD-4aF)x}{b^5\sqrt{a+bx^2}}+\frac{Fx\sqrt{a+bx^2}}{2b^5}+\frac{(2bD-9aF)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}$$

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.79

$$\int\frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{\left(a+bx^2\right)^{9/2}}dx=\frac{x(945a^7F+16Ab^7x^6+4ab^6x^4(14A+3Bx^2)-210a^6b(D-15Fx^2)\cdots}{}$$
$$+\frac{(2bD-9aF)\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{11/2}}$$

```
[In] Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6 + F*x^8))/(a + b*x^2)^(9/2),x]
```

```
[Out] (x*(945*a^7*F + 16*A*b^7*x^6 + 4*a*b^6*x^4*(14*A + 3*B*x^2) - 210*a^6*b*(D
- 15*F*x^2) + a^3*b^4*x^6*(-352*D + 105*F*x^2) + 14*a^5*b^2*x^2*(-50*D + 26
1*F*x^2) + 4*a^4*b^3*x^4*(-203*D + 396*F*x^2) + 2*a^2*b^5*x^2*(35*A + 21*B*
x^2 + 15*C*x^4)))/(210*a^3*b^5*(a + b*x^2)^(7/2)) + ((2*b*D - 9*a*F)*ArcTan
h[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/b^(11/2)
```

Maple [A] (verified)

Time = 3.64 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$\frac{3\left(b\,x^{2}+a\right)^{\frac{7}{2}} a^{3} \left(D b-\frac{9 F a}{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x \sqrt{b}}\right)+\left(x^{2} a^{2} \left(\frac{3}{7} C\,x^{4}+\frac{3}{5} x^{2} B+A\right) b^{\frac{11}{2}}+\frac{4 a \left(\frac{3 x^{2} B}{14}+A\right) x^{4} b^{\frac{13}{2}}}{5}-3 a^{6} \left(-15 F\,x^{2}+D\right) b^{\frac{3}{2}}\right.}{3 b^{\frac{11}{2}} \left(b\,x^{2}+a\right)^{\frac{7}{2}} a}$
default	$D \left(-\frac{x^{7}}{7 b \left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{-\frac{x^{5}}{5 b \left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{-\frac{x^{3}}{3 b \left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b \sqrt{b\,x^{2}+a}}+\frac{\ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}}{b}}{b}\right)+C \left(-\frac{x^{5}}{2 b \left(b\,x^{2}+a\right)^{\frac{7}{2}}}+ \right.$ 5

```
[In] int(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/3/b^(11/2)*(3*(b*x^2+a)^(7/2)*a^3*(D*b-9/2*F*a)*arctanh((b*x^2+a)^(1/2)/x
/b^(1/2))+(x^2*a^2*(3/7*C*x^4+3/5*x^2*B+A)*b^(11/2)+4/5*a*(3/14*x^2*B+A)*x^
4*b^(13/2)-3*a^6*(-15*F*x^2+D)*b^(3/2)-10*a^5*(-261/50*F*x^2+D)*x^2*b^(5/2)
-58/5*a^4*(-396/203*F*x^2+D)*x^4*b^(7/2)-176/35*(-105/352*F*x^2+D)*a^3*x^6*
b^(9/2)+8/35*A*b^(15/2)*x^6+27/2*F*b^(1/2)*a^7)*x)/(b*x^2+a)^(7/2)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 705, normalized size of antiderivative = 2.70

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx = \left[-\frac{105\,(9\,Fa^8 - 2\,Da^7b + (9\,Fa^4b^4 - 2\,Da^3b^5)x^8 + 4\,(9\,Fa^5b^3 -\right.$$

```
[In] integrate(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fri
cas")
```

```
[Out] [-1/420*(105*(9*F*a^8 - 2*D*a^7*b + (9*F*a^4*b^4 - 2*D*a^3*b^5)*x^8 + 4*(9*
F*a^5*b^3 - 2*D*a^4*b^4)*x^6 + 6*(9*F*a^6*b^2 - 2*D*a^5*b^3)*x^4 + 4*(9*F*a
^7*b - 2*D*a^6*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x
 - a) - 2*(105*F*a^3*b^5*x^9 + 2*(792*F*a^4*b^4 - 176*D*a^3*b^5 + 15*C*a^2*
b^6 + 6*B*a*b^7 + 8*A*b^8)*x^7 + 14*(261*F*a^5*b^3 - 58*D*a^4*b^4 + 3*B*a^2
*b^6 + 4*A*a*b^7)*x^5 + 70*(45*F*a^6*b^2 - 10*D*a^5*b^3 + A*a^2*b^6)*x^3 +
105*(9*F*a^7*b - 2*D*a^6*b^2)*x)*sqrt(b*x^2 + a))/(a^3*b^10*x^8 + 4*a^4*b^9
*x^6 + 6*a^5*b^8*x^4 + 4*a^6*b^7*x^2 + a^7*b^6), 1/210*(105*(9*F*a^8 - 2*D*
a^7*b + (9*F*a^4*b^4 - 2*D*a^3*b^5)*x^8 + 4*(9*F*a^5*b^3 - 2*D*a^4*b^4)*x^6
 + 6*(9*F*a^6*b^2 - 2*D*a^5*b^3)*x^4 + 4*(9*F*a^7*b - 2*D*a^6*b^2)*x^2)*sqr
t(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (105*F*a^3*b^5*x^9 + 2*(792*F*a^
4*b^4 - 176*D*a^3*b^5 + 15*C*a^2*b^6 + 6*B*a*b^7 + 8*A*b^8)*x^7 + 14*(261*F
*a^5*b^3 - 58*D*a^4*b^4 + 3*B*a^2*b^6 + 4*A*a*b^7)*x^5 + 70*(45*F*a^6*b^2 -
 10*D*a^5*b^3 + A*a^2*b^6)*x^3 + 105*(9*F*a^7*b - 2*D*a^6*b^2)*x)*sqrt(b*x^
2 + a))/(a^3*b^10*x^8 + 4*a^4*b^9*x^6 + 6*a^5*b^8*x^4 + 4*a^6*b^7*x^2 + a^7
*b^6)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6987 vs. $2(253) = 506$.

Time = 108.77 (sec) , antiderivative size = 6987, normalized size of antiderivative = 26.77

$$\int \frac{x^2(A + Bx^2 + Cx^4 + Dx^6 + Fx^8)}{(a + bx^2)^{9/2}}\, dx = \text{Too large to display}$$

```
[In] integrate(x**2*(F*x**8+D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sq
rt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/
2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a
)) + 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**
2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**
(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x*
*2/a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2
)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) +
420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1
 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a*
*(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2
/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*
sqrt(1 + b*x**2/a))) + B*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a
**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/
a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqr
t(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**
2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a))) + C*
x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a)
 + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt(1 +
 b*x**2/a)) + D*(105*a**(205/2)*b**45*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sq
rt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(10
1/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x
**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/
2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqr
t(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 630
*a**(203/2)*b**46*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**
(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(
1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a
**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x
**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a)
 + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 1575*a**(201/2)*b*
*47*x**4*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99
/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a)
+ 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(
105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b
```

```
*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193
/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 2100*a**(199/2)*b**48*x**6*sqrt(
1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b
*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201
/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sq
rt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630
*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)
*x**12*sqrt(1 + b*x**2/a)) + 1575*a**(197/2)*b**49*x**8*sqrt(1 + b*x**2/a)*
asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630
*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)
*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/
a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b*
*(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1
+ b*x**2/a)) + 630*a**(195/2)*b**50*x**10*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*
x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b*
*(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 +
 b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(
197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10
*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) +
 105*a**(193/2)*b**51*x**12*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(10
5*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*
sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2
100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107
/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x*
*2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 105*a**102*b*
*(91/2)*x/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**
(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 +
b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(1
97/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*
sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) -
665*a**101*b**(93/2)*x**3/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 63
0*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2
)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2
/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b
**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1
 + b*x**2/a)) - 1771*a**100*b**(95/2)*x**5/(105*a**(205/2)*b**(99/2)*sqrt(1
 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**
(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**
6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) +
 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(11
1/2)*x**12*sqrt(1 + b*x**2/a)) - 2549*a**99*b**(97/2)*x**7/(105*a**(205/2)*
b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x*
*2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2
)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt
(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a
```

```
**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 2096*a**98*b**(99/2)*x**9/
(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x*
*2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a)
+ 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(
107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b
*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 934*a**97*
b**(101/2)*x**11/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203
/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sq
rt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 157
5*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)
*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2
/a)) - 176*a**96*b**(103/2)*x**13/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2
/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b
**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1
+ b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(
195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**1
2*sqrt(1 + b*x**2/a))) + F*(-315*a**(311/2)*b**66*sqrt(1 + b*x**2/a)*asinh(
sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(3
07/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*
sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1
050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/
2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**
2/a)) - 1890*a**(309/2)*b**67*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(
a))/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2
)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2
/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*
b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1
 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 4725*a*
*(307/2)*b**68*x**4*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309
/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 +
 b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(
303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8
*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) +
70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 6300*a**(305/2)*b**69*
x**6*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/2)*
sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 10
50*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/
2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**
2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b
**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 4725*a**(303/2)*b**70*x**8*sqrt(1 + b
*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2
/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b
**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1
+ b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(
299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12
```

```
*sqrt(1 + b*x**2/a)) - 1890*a**(301/2)*b**71*x**10*sqrt(1 + b*x**2/a)*asinh
(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(
307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4
*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) +
1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153
/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x*
*2/a)) - 315*a**(299/2)*b**72*x**12*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt
(a))/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/
2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**
2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)
*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(
1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 315*a*
*155*b**(133/2)*x/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(30
7/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*s
qrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 10
50*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2
)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2
/a)) + 1995*a**154*b**(135/2)*x**3/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**
2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*
b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1
 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**
(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**1
2*sqrt(1 + b*x**2/a)) + 5313*a**153*b**(137/2)*x**5/(70*a**(309/2)*b**(143/
2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) +
 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(1
49/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*
x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2
)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 7647*a**152*b**(139/2)*x**7/(70*a*
*(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqr
t(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400
*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)
*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/
a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) + 6323*a**151*b**(1
41/2)*x**9/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b*
*(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 +
 b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(
301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10
*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) +
2907*a**150*b**(143/2)*x**11/(70*a**(309/2)*b**(143/2)*sqrt(1 + b*x**2/a) +
 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a**(305/2)*b**(14
7/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x
**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2
)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt
(1 + b*x**2/a)) + 633*a**149*b**(145/2)*x**13/(70*a**(309/2)*b**(143/2)*sqr
t(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*
```

```
a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(149/2)*
x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*x**2/a
) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/2)*b**(
155/2)*x**12*sqrt(1 + b*x**2/a)) + 35*a**148*b**(147/2)*x**15/(70*a**(309/2
)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b
*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(30
3/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*s
qrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70
*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 826 vs. $2(235) = 470$.

Time = 0.23 (sec) , antiderivative size = 826, normalized size of antiderivative = 3.16

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="max
ima")
```

```
[Out] 1/2*F*x^9/((b*x^2 + a)^(7/2)*b) - 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a
*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/
((b*x^2 + a)^(7/2)*b^4))*D*x + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^
4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b
*x^2 + a)^(7/2)*b^4))*F*a*x/b + 3/10*F*a*x*(15*x^4/((b*x^2 + a)^(5/2)*b) +
20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2 - 1/1
5*D*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*
a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*C*x^5/((b*x^2 + a)^(7/2)*b) + 3/2*F*a*
x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3 - 1/3*D*x
*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 9/2*F*a^
2*x^3/((b*x^2 + a)^(5/2)*b^4) - D*a*x^3/((b*x^2 + a)^(5/2)*b^3) - 5/8*C*a*x
^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*B*x^3/((b*x^2 + a)^(7/2)*b) - 417/70*F*a*x
/(sqrt(b*x^2 + a)*b^5) - 51/70*F*a^2*x/((b*x^2 + a)^(3/2)*b^5) + 261/70*F*a
^3*x/((b*x^2 + a)^(5/2)*b^5) + 139/105*D*x/(sqrt(b*x^2 + a)*b^4) + 17/105*D
*a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*D*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14
*C*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*C*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*C*a*x
/((b*x^2 + a)^(5/2)*b^3) - 15/56*C*a^2*x/((b*x^2 + a)^(7/2)*b^3) + 3/140*B*
x/((b*x^2 + a)^(5/2)*b^2) + 2/35*B*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*B*x/(
(b*x^2 + a)^(3/2)*a*b^2) - 3/28*B*a*x/((b*x^2 + a)^(7/2)*b^2) - 1/7*A*x/((b
*x^2 + a)^(7/2)*b) + 8/105*A*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*A*x/((b*x^2
+ a)^(3/2)*a^2*b) + 1/35*A*x/((b*x^2 + a)^(5/2)*a*b) - 9/2*F*a*arcsinh(b*x/
sqrt(a*b))/b^(11/2) + D*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.86

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(\left(\left(\frac{105\,Fx^2}{b}+\frac{2\,(792\,Fa^4b^7-176\,Da^3b^8+15\,Ca^2b^9+6\,Bab^{10}+8\,Ab^{11})}{a^3b^9}\right)x^2+14\cdots\right.\right.\right.}{} + \frac{(9\,Fa-2\,Db)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{11}{2}}}$$

```
[In] integrate(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="gia
c")
```

```
[Out] 1/210*((((105*F*x^2/b + 2*(792*F*a^4*b^7 - 176*D*a^3*b^8 + 15*C*a^2*b^9 + 6
*B*a*b^10 + 8*A*b^11)/(a^3*b^9))*x^2 + 14*(261*F*a^5*b^6 - 58*D*a^4*b^7 + 3
*B*a^2*b^9 + 4*A*a*b^10)/(a^3*b^9))*x^2 + 70*(45*F*a^6*b^5 - 10*D*a^5*b^6 +
 A*a^2*b^9)/(a^3*b^9))*x^2 + 105*(9*F*a^7*b^4 - 2*D*a^6*b^5)/(a^3*b^9))*x/(
b*x^2 + a)^(7/2) + 1/2*(9*F*a - 2*D*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)
))/b^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx = \int \frac{x^2\,(A+B\,x^2+C\,x^4+F\,x^8+x^6\,D)}{(b\,x^2+a)^{9/2}}\,dx$$

```
[In] int((x^2*(A + B*x^2 + C*x^4 + F*x^8 + x^6*D))/(a + b*x^2)^(9/2),x)
```

```
[Out] int((x^2*(A + B*x^2 + C*x^4 + F*x^8 + x^6*D))/(a + b*x^2)^(9/2), x)
```

3.173 $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}}\,dx$

Optimal result 1104
Rubi [A] (verified) 1104
Mathematica [A] (verified) 1107
Maple [A] (verified) 1107
Fricas [A] (verification not implemented) 1109
Sympy [B] (verification not implemented) 1110
Maxima [B] (verification not implemented) 1113
Giac [A] (verification not implemented) 1114
Mupad [F(-1)] 1114

Optimal result

Integrand size = 34, antiderivative size = 214

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}}\,dx &= \frac{(Ab^4-a^4F)\,x}{ab^4\,(a+bx^2)^{7/2}} \\ &+ \frac{(6Ab^4+ab^3B-10a^4F)\,x^3}{3a^2b^3\,(a+bx^2)^{7/2}} + \frac{(24Ab^4+a(4b^3B+3ab^2C-58a^3F))\,x^5}{15a^3b^2\,(a+bx^2)^{7/2}} \\ &+ \frac{(48Ab^4+a(8b^3B+6ab^2C+15a^2bD-176a^3F))\,x^7}{105a^4b\,(a+bx^2)^{7/2}} + \frac{F\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}\end{aligned}$$

```
[Out] (A*b^4-F*a^4)*x/a/b^4/(b*x^2+a)^(7/2)+1/3*(6*A*b^4+B*a*b^3-10*F*a^4)*x^3/a^
2/b^3/(b*x^2+a)^(7/2)+1/15*(24*A*b^4+a*(4*B*b^3+3*C*a*b^2-58*F*a^3))*x^5/a^
3/b^2/(b*x^2+a)^(7/2)+1/105*(48*A*b^4+a*(8*B*b^3+6*C*a*b^2+15*D*a^2*b-176*F
*a^3))*x^7/a^4/b/(b*x^2+a)^(7/2)+F*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(9/
2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used

= {1828, 1171, 393, 223, 212}

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}}\,dx = \frac{x\left(\frac{-176a^3F+15a^2bD+6ab^2C+8b^3B}{b^4}+\frac{48A}{a}\right)}{105a^3\sqrt{a+bx^2}} + \frac{x(a(122a^3F-45a^2bD+3ab^2C+4b^3B)+24Ab^4)}{105a^3b^4(a+bx^2)^{3/2}} + \frac{x\left(\frac{-22a^3F+15a^2bD-8ab^2C+b^3B}{b^4}+\frac{6A}{a}\right)}{35a(a+bx^2)^{5/2}} + \frac{x\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7(a+bx^2)^{7/2}} + \frac{F\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(a + b*x^2)^(9/2),x]
[Out] ((A/a - (b^3*B - a*b^2*C + a^2*b*D - a^3*F)/b^4)*x)/(7*(a + b*x^2)^(7/2)) +
 (((6*A)/a + (b^3*B - 8*a*b^2*C + 15*a^2*b*D - 22*a^3*F)/b^4)*x)/(35*a*(a +
 b*x^2)^(5/2)) + ((24*A*b^4 + a*(4*b^3*B + 3*a*b^2*C - 45*a^2*b*D + 122*a^3
*F))*x)/(105*a^3*b^4*(a + b*x^2)^(3/2)) + (((48*A)/a + (8*b^3*B + 6*a*b^2*C
 + 15*a^2*b*D - 176*a^3*F)/b^4)*x)/(105*a^3*Sqrt[a + b*x^2]) + (F*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(9/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
```

```
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\left(a+bx^2\right)^{7/2}} \\
&\quad-\frac{\int\frac{-6A-\frac{a\left(b^3B-ab^2C+a^2bD-a^3F\right)}{b^4}-\frac{7a\left(b^2C-abD+a^2F\right)x^2}{b^3}-\frac{7a(bD-aF)x^4}{b^2}-\frac{7aFx^6}{b}}{\left(a+bx^2\right)^{7/2}}\,dx}{7a} \\
&= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\left(a+bx^2\right)^{7/2}}+\frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a\left(a+bx^2\right)^{5/2}} \\
&\quad+\frac{\int\frac{\frac{24Ab^4+4ab^3B+3a^2b^2C-10a^3bD+17a^4F}{b^4}+\frac{35a^2(bD-2aF)x^2}{b^3}+\frac{35a^2Fx^4}{b^2}}{\left(a+bx^2\right)^{5/2}}\,dx}{35a^2} \\
&= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\left(a+bx^2\right)^{7/2}}+\frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a\left(a+bx^2\right)^{5/2}} \\
&\quad+\frac{\left(24Ab^4+a\left(4b^3B+3ab^2C-45a^2bD+122a^3F\right)\right)x}{105a^3b^4\left(a+bx^2\right)^{3/2}} \\
&\quad-\frac{\int\frac{-\frac{48Ab^4+8ab^3B+6a^2b^2C+15a^3bD-71a^4F}{b^4}-\frac{105a^3Fx^2}{b^3}}{\left(a+bx^2\right)^{3/2}}\,dx}{105a^3} \\
&= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\left(a+bx^2\right)^{7/2}}+\frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a\left(a+bx^2\right)^{5/2}} \\
&\quad+\frac{\left(24Ab^4+a\left(4b^3B+3ab^2C-45a^2bD+122a^3F\right)\right)x}{105a^3b^4\left(a+bx^2\right)^{3/2}} \\
&\quad+\frac{\left(\frac{48A}{a}+\frac{8b^3B+6ab^2C+15a^2bD-176a^3F}{b^4}\right)x}{105a^3\sqrt{a+bx^2}}+\frac{F\int\frac{1}{\sqrt{a+bx^2}}\,dx}{b^4}
\end{aligned}
$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\left(a+bx^2\right)^{7/2}}+\frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a\left(a+bx^2\right)^{5/2}}$$
$$+\frac{\left(24Ab^4+a(4b^3B+3ab^2C-45a^2bD+122a^3F)\right)x}{105a^3b^4\left(a+bx^2\right)^{3/2}}$$
$$+\frac{\left(\frac{48A}{a}+\frac{8b^3B+6ab^2C+15a^2bD-176a^3F}{b^4}\right)x}{105a^3\sqrt{a+bx^2}}+\frac{F\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{b^4}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\left(a+bx^2\right)^{7/2}}+\frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a\left(a+bx^2\right)^{5/2}}$$
$$+\frac{\left(24Ab^4+a(4b^3B+3ab^2C-45a^2bD+122a^3F)\right)x}{105a^3b^4\left(a+bx^2\right)^{3/2}}$$
$$+\frac{\left(\frac{48A}{a}+\frac{8b^3B+6ab^2C+15a^2bD-176a^3F}{b^4}\right)x}{105a^3\sqrt{a+bx^2}}+\frac{F\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.82

$$\int\frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{\left(a+bx^2\right)^{9/2}}\,dx=\frac{x(-105a^7F-350a^6bFx^2-406a^5b^2Fx^4+48Ab^7x^6-176a^4b^3Fx^6+$$
$$-\frac{F\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{9/2}}$$

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(a + b*x^2)^(9/2),x]
```

```
[Out] (x*(-105*a^7*F - 350*a^6*b*F*x^2 - 406*a^5*b^2*F*x^4 + 48*A*b^7*x^6 - 176*a
^4*b^3*F*x^6 + 8*a*b^6*x^4*(21*A + B*x^2) + 2*a^2*b^5*x^2*(105*A + 14*B*x^2
 + 3*C*x^4) + a^3*b^4*(105*A + 35*B*x^2 + 21*C*x^4 + 15*D*x^6)))/(105*a^4*b
^4*(a + b*x^2)^(7/2)) - (F*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(9/2)
```

Maple [A] (verified)

Time = 3.59 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{x\,a^{3}\left(\frac{1}{7}Dx^{6}+\frac{1}{5}C\,x^{4}+\frac{1}{3}x^{2}B+A\right)b^{\frac{9}{2}}+2\left(\frac{1}{35}C\,x^{4}+\frac{2}{15}x^{2}B+A\right)a^{2}x^{3}b^{\frac{11}{2}}+\frac{8\left(\frac{x^{2}B}{21}+A\right)a\,x^{5}b^{\frac{13}{2}}}{5}+\frac{16A\,b^{\frac{15}{2}}x^{7}}{35}+F\,a^{4}\left(\left(b\,x^{2}+a\right)^{\frac{7}{2}}\text{arc}\right.}{b^{\frac{9}{2}}\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{4}}$
default	$A\left(\frac{x}{7a(b\,x^{2}+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{b\,x^{2}+a}}\right)}{7a}}{a}\right)+F\left(-\frac{x^{7}}{7b(b\,x^{2}+a)^{\frac{7}{2}}}+\frac{-\frac{x^{5}}{5b\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\cdots}{\cdots}\right.$

```
[In] int((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^(9/2)*(x*a^3*(1/7*D*x^6+1/5*C*x^4+1/3*x^2*B+A)*b^(9/2)+2*(1/35*C*x^4+2/
15*x^2*B+A)*a^2*x^3*b^(11/2)+8/5*(1/21*x^2*B+A)*a*x^5*b^(13/2)+16/35*A*b^(1
5/2)*x^7+F*a^4*((b*x^2+a)^(7/2)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-176/105*
x^7*b^(7/2)-58/15*b^(5/2)*a*x^5-10/3*b^(3/2)*a^2*x^3-b^(1/2)*a^3*x))/(b*x^2
+a)^(7/2)/a^4
```

Fricas [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 567, normalized size of antiderivative = 2.65

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}}\,dx = \left[\frac{105\left(Fa^4b^4x^8+4\,Fa^5b^3x^6+6\,Fa^6b^2x^4+4\,Fa^7bx^2+Fa^8\right)\sqrt{b}\log\left(\right.}{}\right.$$

$$-\frac{105\left(Fa^4b^4x^8+4\,Fa^5b^3x^6+6\,Fa^6b^2x^4+4\,Fa^7bx^2+Fa^8\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\left(\left(176\,Fa^4b^4-15\,Da^3b\right.\right.}{1}$$

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas"
)
```

```
[Out] [1/210*(105*(F*a^4*b^4*x^8 + 4*F*a^5*b^3*x^6 + 6*F*a^6*b^2*x^4 + 4*F*a^7*b*
x^2 + F*a^8)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*((
176*F*a^4*b^4 - 15*D*a^3*b^5 - 6*C*a^2*b^6 - 8*B*a*b^7 - 48*A*b^8)*x^7 + 7*
(58*F*a^5*b^3 - 3*C*a^3*b^5 - 4*B*a^2*b^6 - 24*A*a*b^7)*x^5 + 35*(10*F*a^6*
b^2 - B*a^3*b^5 - 6*A*a^2*b^6)*x^3 + 105*(F*a^7*b - A*a^3*b^5)*x)*sqrt(b*x^
2 + a))/(a^4*b^9*x^8 + 4*a^5*b^8*x^6 + 6*a^6*b^7*x^4 + 4*a^7*b^6*x^2 + a^8*
b^5), -1/105*(105*(F*a^4*b^4*x^8 + 4*F*a^5*b^3*x^6 + 6*F*a^6*b^2*x^4 + 4*F*
a^7*b*x^2 + F*a^8)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + ((176*F*a^
4*b^4 - 15*D*a^3*b^5 - 6*C*a^2*b^6 - 8*B*a*b^7 - 48*A*b^8)*x^7 + 7*(58*F*a^
5*b^3 - 3*C*a^3*b^5 - 4*B*a^2*b^6 - 24*A*a*b^7)*x^5 + 35*(10*F*a^6*b^2 - B*
a^3*b^5 - 6*A*a^2*b^6)*x^3 + 105*(F*a^7*b - A*a^3*b^5)*x)*sqrt(b*x^2 + a))/
(a^4*b^9*x^8 + 4*a^5*b^8*x^6 + 6*a^6*b^7*x^4 + 4*a^7*b^6*x^2 + a^8*b^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5071 vs. $2(211) = 422$.

Time = 74.12 (sec) , antiderivative size = 5071, normalized size of antiderivative = 23.70

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6 + Fx^8}{(a + bx^2)^{9/2}}\, dx = \text{Too large to display}$$

```
[In] integrate((F*x**8+D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(
1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*
b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) +
 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt
(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a
**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**
2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8
*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(
25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sq
rt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*
b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) +
 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt
(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b*
*3*x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*
x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x
**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a
**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b
*x**2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(
35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a
) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sq
rt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/
2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 104*a**9*b**5*x**11/(35*a**(37/2)*sqrt(
1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**
2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 52
5*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1
+ b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x*
*13/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2
/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*
sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(2
7/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**
2/a))) + B*(35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*
b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 42
0*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 +
 b*x**2/a)) + 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17
/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a)
```

```
+ 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt
(1 + b*x**2/a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420
*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x
**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x*
*8*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a)
 + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1
 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b
**4*x**8*sqrt(1 + b*x**2/a))) + C*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a
) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1
+ b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(
11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**
(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/
a))) + D*x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 +
b*x**2/a) + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6
*sqrt(1 + b*x**2/a)) + F*(105*a**(205/2)*b**45*sqrt(1 + b*x**2/a)*asinh(sqr
t(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/
2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqr
t(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575
*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*
x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/
a)) + 630*a**(203/2)*b**46*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))
/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x
**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a)
 + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**
(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 +
b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 1575*a**(
201/2)*b**47*x**4*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/
2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b
*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(19
9/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*s
qrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 10
5*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 2100*a**(199/2)*b**48*x
**6*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*s
qrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 157
5*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2
)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2
/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b
**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 1575*a**(197/2)*b**49*x**8*sqrt(1 + b
*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2
/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b
**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1
+ b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(
195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**1
2*sqrt(1 + b*x**2/a)) + 630*a**(195/2)*b**50*x**10*sqrt(1 + b*x**2/a)*asinh
(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(
```

203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4
*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) +
1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109
/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x
2/a)) + 105*a(193/2)*b**51*x**12*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqr
t(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101
/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x*
*2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2
)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt
(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 105*
a**102*b**(91/2)*x/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(2
03/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*
sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1
575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/
2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x*
*2/a)) - 665*a**101*b**(93/2)*x**3/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**
2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*
b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1
 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**
(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**
12*sqrt(1 + b*x**2/a)) - 1771*a**100*b**(95/2)*x**5/(105*a**(205/2)*b**(99/
2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) +
 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(1
05/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*
x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/
2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 2549*a**99*b**(97/2)*x**7/(105*a*
*(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt
(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*
a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*
x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a
) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 2096*a**98*b**(99
/2)*x**9/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(
101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b
*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(19
7/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*s
qrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 9
34*a**97*b**(101/2)*x**11/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 63
0*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2
)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2
/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b
(109/2)*x10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1
 + b*x**2/a)) - 176*a**96*b**(103/2)*x**13/(105*a**(205/2)*b**(99/2)*sqrt(1
 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**
(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**
6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) +

```
630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(11
1/2)*x**12*sqrt(1 + b*x**2/a)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(198) = 396$.

Time = 0.24 (sec) , antiderivative size = 597, normalized size of antiderivative = 2.79

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6 + Fx^8}{(a + bx^2)^{9/2}}\,dx = -\frac{1}{35}\left(\frac{35\,x^6}{(bx^2+a)^{\frac{7}{2}}b} + \frac{70\,ax^4}{(bx^2+a)^{\frac{7}{2}}b^2} + \frac{56\,a^2x^2}{(bx^2+a)^{\frac{7}{2}}b^3} + \frac{16\,a^3}{(bx^2+a)^{\frac{7}{2}}b^4}\right)Fx - \frac{Fx\left(\frac{15\,x^4}{(bx^2+a)^{\frac{5}{2}}b} + \frac{20\,ax^2}{(bx^2+a)^{\frac{5}{2}}b^2} + \frac{8\,a^2}{(bx^2+a)^{\frac{5}{2}}b^3}\right)}{15\,b} - \frac{Dx^5}{2\,(bx^2+a)^{\frac{7}{2}}b} - \frac{Fx\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{3\,b^2} - \frac{Fax^3}{(bx^2+a)^{\frac{5}{2}}b^3} - \frac{5\,Dax^3}{8\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Cx^3}{4\,(bx^2+a)^{\frac{7}{2}}b} + \frac{16\,Ax}{35\,\sqrt{bx^2+a}a^4} + \frac{8\,Ax}{35\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{6\,Ax}{35\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{Ax}{7\,(bx^2+a)^{\frac{7}{2}}a} + \frac{139\,Fx}{105\,\sqrt{bx^2+a}b^4} + \frac{17\,Fax}{105\,(bx^2+a)^{\frac{3}{2}}b^4} - \frac{29\,Fa^2x}{35\,(bx^2+a)^{\frac{5}{2}}b^4} + \frac{Dx}{14\,(bx^2+a)^{\frac{3}{2}}b^3} + \frac{Dx}{7\,\sqrt{bx^2+a}ab^3} + \frac{3\,Dax}{56\,(bx^2+a)^{\frac{5}{2}}b^3} - \frac{15\,Da^2x}{56\,(bx^2+a)^{\frac{7}{2}}b^3} + \frac{3\,Cx}{140\,(bx^2+a)^{\frac{5}{2}}b^2} + \frac{2\,Cx}{35\,\sqrt{bx^2+a}a^2b^2} + \frac{Cx}{35\,(bx^2+a)^{\frac{3}{2}}ab^2} - \frac{3\,Cax}{28\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Bx}{7\,(bx^2+a)^{\frac{7}{2}}b} + \frac{8\,Bx}{105\,\sqrt{bx^2+a}a^3b} + \frac{4\,Bx}{105\,(bx^2+a)^{\frac{3}{2}}a^2b} + \frac{Bx}{35\,(bx^2+a)^{\frac{5}{2}}ab} + \frac{F\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{9}{2}}}$$

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima"
)
```

```
[Out] -1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56
*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*F*x - 1/
15*F*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8
*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*D*x^5/((b*x^2 + a)^(7/2)*b) - 1/3*F*x
*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 - F*a*x^3/
((b*x^2 + a)^(5/2)*b^3) - 5/8*D*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*C*x^3/(
(b*x^2 + a)^(7/2)*b) + 16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*A*x/((b*x^2 +
 a)^(3/2)*a^3) + 6/35*A*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((b*x^2 + a)^(7
```

```
/2)*a) + 139/105*F*x/(sqrt(b*x^2 + a)*b^4) + 17/105*F*a*x/((b*x^2 + a)^(3/2
)*b^4) - 29/35*F*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14*D*x/((b*x^2 + a)^(3/2
)*b^3) + 1/7*D*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*D*a*x/((b*x^2 + a)^(5/2)*b^
3) - 15/56*D*a^2*x/((b*x^2 + a)^(7/2)*b^3) + 3/140*C*x/((b*x^2 + a)^(5/2)*b
^2) + 2/35*C*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*C*x/((b*x^2 + a)^(3/2)*a*b^
2) - 3/28*C*a*x/((b*x^2 + a)^(7/2)*b^2) - 1/7*B*x/((b*x^2 + a)^(7/2)*b) + 8
/105*B*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*B*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/
35*B*x/((b*x^2 + a)^(5/2)*a*b) + F*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}}\,dx = -\frac{\left(\left(x^2\left(\frac{(176\,Fa^4b^6-15\,Da^3b^7-6\,Ca^2b^8-8\,Bab^9-48\,Ab^{10})x^2}{a^4b^7}+\frac{7\,(58\,Fa^5b^5-3\,Ca^3b^7-4\,Ba^2b^8-24\,Aab^9)}{a^4b^7}\right)+\frac{35\,(10\,Fa^6b^4-Ba^3b^7-6\,Aa^2b}{a^4b^7}\right.\right.}{105\,(bx^2+a)^{\frac{7}{2}}} - \frac{F\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}$$

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*((x^2*((176*F*a^4*b^6 - 15*D*a^3*b^7 - 6*C*a^2*b^8 - 8*B*a*b^9 - 48*
A*b^10)*x^2/(a^4*b^7) + 7*(58*F*a^5*b^5 - 3*C*a^3*b^7 - 4*B*a^2*b^8 - 24*A*
a*b^9)/(a^4*b^7)) + 35*(10*F*a^6*b^4 - B*a^3*b^7 - 6*A*a^2*b^8)/(a^4*b^7))*
x^2 + 105*(F*a^7*b^3 - A*a^3*b^7)/(a^4*b^7))*x/(b*x^2 + a)^(7/2) - F*log(ab
s(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}}\,dx = \int \frac{A+B\,x^2+C\,x^4+F\,x^8+x^6\,D}{(b\,x^2+a)^{9/2}}\,dx$$

```
[In] int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(a + b*x^2)^(9/2),x)
```

```
[Out] int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(a + b*x^2)^(9/2), x)
```

3.174 $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2(a+bx^2)^{9/2}}\,dx$

Optimal result 1115
Rubi [A] (verified) 1115
Mathematica [A] (verified) 1117
Maple [A] (verified) 1118
Fricas [A] (verification not implemented) 1120
Sympy [B] (verification not implemented) 1120
Maxima [B] (verification not implemented) 1122
Giac [A] (verification not implemented) 1123
Mupad [F(-1)] 1123

Optimal result

Integrand size = 37, antiderivative size = 193

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2(a+bx^2)^{9/2}}\,dx = -\frac{A}{ax(a+bx^2)^{7/2}}-\frac{(8Ab-aB)x}{a^2(a+bx^2)^{7/2}} -\frac{(48Ab^2-a(6bB+aC))x^3}{3a^3(a+bx^2)^{7/2}}-\frac{(192Ab^3-a(24b^2B+4abC+3a^2D))x^5}{15a^4(a+bx^2)^{7/2}} -\frac{(384Ab^4-a(48b^3B+8ab^2C+6a^2bD+15a^3F))x^7}{105a^5(a+bx^2)^{7/2}}$$

```
[Out] -A/a/x/(b*x^2+a)^(7/2)-(8*A*b-B*a)*x/a^2/(b*x^2+a)^(7/2)-1/3*(48*A*b^2-a*(6
*B*b+C*a))*x^3/a^3/(b*x^2+a)^(7/2)-1/15*(192*A*b^3-a*(24*B*b^2+4*C*a*b+3*D*
a^2))*x^5/a^4/(b*x^2+a)^(7/2)-1/105*(384*A*b^4-a*(48*B*b^3+8*C*a*b^2+6*D*a^
2*b+15*F*a^3))*x^7/a^5/(b*x^2+a)^(7/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {1817, 1827, 12, 270}

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2(a+bx^2)^{9/2}}\,dx = -\frac{x^3(48Ab^2-a(aC+6bB))}{3a^3(a+bx^2)^{7/2}} -\frac{x(8Ab-aB)}{a^2(a+bx^2)^{7/2}}-\frac{x^5(192Ab^3-a(3a^2D+4abC+24b^2B))}{15a^4(a+bx^2)^{7/2}} -\frac{x^7(384Ab^4-a(15a^3F+6a^2bD+8ab^2C+48b^3B))}{105a^5(a+bx^2)^{7/2}}-\frac{A}{ax(a+bx^2)^{7/2}}$$

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] -(A/(a*x*(a + b*x^2)^(7/2))) - ((8*A*b - a*B)*x)/(a^2*(a + b*x^2)^(7/2)) -
((48*A*b^2 - a*(6*b*B + a*C))*x^3)/(3*a^3*(a + b*x^2)^(7/2)) - ((192*A*b^3
- a*(24*b^2*B + 4*a*b*C + 3*a^2*D))*x^5)/(15*a^4*(a + b*x^2)^(7/2)) - ((384
*A*b^4 - a*(48*b^3*B + 8*a*b^2*C + 6*a^2*b*D + 15*a^3*F))*x^7)/(105*a^5*(a
+ b*x^2)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 1827

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*x*((a + b*
x^2)^(p + 1)/a), x] + Dist[1/a, Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3))
, x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ[
Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{A}{ax(a+bx^2)^{7/2}} - \frac{\int \frac{8Ab-a(B+Cx^2+Dx^4+Fx^6)}{(a+bx^2)^{9/2}}\,dx}{a}$$

$$= -\frac{A}{ax(a+bx^2)^{7/2}} - \frac{(8Ab-aB)x}{a^2(a+bx^2)^{7/2}} - \frac{\int \frac{x^2(6b(8Ab-aB)+a(-aC-aDx^2-aFx^4))}{(a+bx^2)^{9/2}}\,dx}{a^2}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^4\left(4b\left(48Ab^2-6abB-a^2C\right)+3a\left(-a^2D-a^2Fx^2\right)\right)}{\left(a+bx^2\right)^{9/2}}\,dx}{3a^3}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3-a\left(24b^2B+4abC+3a^2D\right)\right)x^5}{15a^4\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{\left(2b\left(192Ab^3-24ab^2B-4a^2bC-3a^3D\right)-15a^4F\right)x^6}{\left(a+bx^2\right)^{9/2}}\,dx}{15a^4}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3-a\left(24b^2B+4abC+3a^2D\right)\right)x^5}{15a^4\left(a+bx^2\right)^{7/2}} - \frac{\left(384Ab^4-48ab^3B-8a^2b^2C-6a^3bD-15a^4F\right)\int \frac{x^6}{\left(a+bx^2\right)^{9/2}}\,dx}{15a^4}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} - \frac{\left(192Ab^3-a\left(24b^2B+4abC+3a^2D\right)\right)x^5}{15a^4\left(a+bx^2\right)^{7/2}} - \frac{\left(384Ab^4-a\left(48b^3B+8ab^2C+6a^2bD+15a^3F\right)\right)x^7}{105a^5\left(a+bx^2\right)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \frac{-384Ab^4x^8+48ab^3x^6(-28A+Bx^2)+8a^2b^2x^4(-210A+21Bx^2+$$

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-384*A*b^4*x^8 + 48*a*b^3*x^6*(-28*A + B*x^2) + 8*a^2*b^2*x^4*(-210*A + 21
*B*x^2 + C*x^4) + 2*a^3*b*x^2*(-420*A + 105*B*x^2 + 14*C*x^4 + 3*D*x^6) + a
^4*(-105*A + 105*B*x^2 + 35*C*x^4 + 21*D*x^6 + 15*F*x^8))/(105*a^5*x*(a + b
*x^2)^(7/2))
```

Maple [A] (verified)

Time = 3.61 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$\frac{(15F\,x^8+21D x^6+35C\,x^4+105x^2B-105A)a^4-840b(-\frac{1}{140}D x^6-\frac{1}{30}C\,x^4-\frac{1}{4}x^2B+A)x^2a^3-1680(-\frac{1}{210}C\,x^4-\frac{1}{10}x^2B+A)b^2}{105(b\,x^2+a)^{\frac{7}{2}}x\,a^5}$
gosper	$-\frac{384A\,b^4x^8-48Ba\,b^3x^8-8C\,a^2b^2x^8-6Da^3b\,x^8-15F\,a^4x^8+1344Aa\,b^3x^6-168B\,a^2b^2x^6-28C\,a^3b\,x^6-21Da^4x^6+1680A\,a}{105x(b\,x^2+a)^{\frac{7}{2}}a^5}$
trager	$-\frac{384A\,b^4x^8-48Ba\,b^3x^8-8C\,a^2b^2x^8-6Da^3b\,x^8-15F\,a^4x^8+1344Aa\,b^3x^6-168B\,a^2b^2x^6-28C\,a^3b\,x^6-21Da^4x^6+1680A\,a}{105x(b\,x^2+a)^{\frac{7}{2}}a^5}$
default	$B\left(\frac{x}{7a(b\,x^2+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)+F\left(-\frac{x^5}{2b(b\,x^2+a)^{\frac{7}{2}}}+\frac{5a\left(-\frac{x^3}{4b\left(b\,x^2+a\right)}\right.}{}\right.$

```
[In] int((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/105*((15*F*x^8+21*D*x^6+35*C*x^4+105*B*x^2-105*A)*a^4-840*b*(-1/140*D*x^6
-1/30*C*x^4-1/4*x^2*B+A)*x^2*a^3-1680*(-1/210*C*x^4-1/10*x^2*B+A)*b^2*x^4*a
^2-1344*b^3*x^6*(-1/28*x^2*B+A)*a-384*A*b^4*x^8)/(b*x^2+a)^(7/2)/x/a^5
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \frac{((15\,Fa^4+6\,Da^3b+8\,Ca^2b^2+48\,Bab^3-384\,Ab^4)x^8+7\,(3\,Da^4+4\,C}{105}$$

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="fri
cas")
```

```
[Out] 1/105*((15*F*a^4 + 6*D*a^3*b + 8*C*a^2*b^2 + 48*B*a*b^3 - 384*A*b^4)*x^8 +
7*(3*D*a^4 + 4*C*a^3*b + 24*B*a^2*b^2 - 192*A*a*b^3)*x^6 - 105*A*a^4 + 35*(
C*a^4 + 6*B*a^3*b - 48*A*a^2*b^2)*x^4 + 105*(B*a^4 - 8*A*a^3*b)*x^2)*sqrt(b
*x^2 + a)/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*
x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2490 vs. 2(184) = 368.

Time = 98.39 (sec) , antiderivative size = 2490, normalized size of antiderivative = 12.90

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((F*x**8+D*x**6+C*x**4+B*x**2+A)/x**2/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(-35*a**4*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*
x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 28
0*a**3*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*
x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 56
0*a**2*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*
x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 44
8*a*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**
2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 128*b
**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 2
10*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)) + B*(35*a**
```

```
14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**
2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6
*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(
27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x*
*2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*
b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 70
0*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 +
 b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**
6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sqrt(1 + b*
x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4
*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(
29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x*
*2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b**3*x**7/(
35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) +
 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(
1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*
b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a))
 + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x
**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a
**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*
x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x
**12*sqrt(1 + b*x**2/a)) + 104*a**9*b**5*x**11/(35*a**(37/2)*sqrt(1 + b*x**
2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sq
rt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/
2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/
a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x**13/(35*a
**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525
*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 +
b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5
*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a))) +
C*(35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sq
rt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/
2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a
)) + 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**
2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**
(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x*
*2/a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2
)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) +
420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1
 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a*
*(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2
/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*
sqrt(1 + b*x**2/a))) + D*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a
**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/
a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqr
```

```
t(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**
2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a))) + F*
x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a)
 + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt(1 +
 b*x**2/a))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(175) = 350$.

Time = 0.21 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.18

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6 + Fx^8}{x^2 (a + bx^2)^{9/2}} dx = -\frac{Fx^5}{2 (bx^2 + a)^{\frac{7}{2}} b} - \frac{5 Fax^3}{8 (bx^2 + a)^{\frac{7}{2}} b^2} - \frac{Dx^3}{4 (bx^2 + a)^{\frac{7}{2}} b} + \frac{16 Bx}{35 \sqrt{bx^2 + a}a^4} + \frac{8 Bx}{35 (bx^2 + a)^{\frac{3}{2}} a^3} + \frac{6 Bx}{35 (bx^2 + a)^{\frac{5}{2}} a^2} + \frac{Bx}{7 (bx^2 + a)^{\frac{7}{2}} a} + \frac{Fx}{14 (bx^2 + a)^{\frac{3}{2}} b^3} + \frac{Fx}{7 \sqrt{bx^2 + a}ab^3} + \frac{3 Fax}{56 (bx^2 + a)^{\frac{5}{2}} b^3} - \frac{15 Fa^2x}{56 (bx^2 + a)^{\frac{7}{2}} b^3} + \frac{3 Dx}{140 (bx^2 + a)^{\frac{5}{2}} b^2} + \frac{2 Dx}{35 \sqrt{bx^2 + a}a^2b^2} + \frac{Dx}{35 (bx^2 + a)^{\frac{3}{2}} ab^2} - \frac{3 Dax}{28 (bx^2 + a)^{\frac{7}{2}} b^2} - \frac{Cx}{7 (bx^2 + a)^{\frac{7}{2}} b} + \frac{8 Cx}{105 \sqrt{bx^2 + a}a^3b} + \frac{4 Cx}{105 (bx^2 + a)^{\frac{3}{2}} a^2b} + \frac{Cx}{35 (bx^2 + a)^{\frac{5}{2}} ab} - \frac{128 Abx}{35 \sqrt{bx^2 + a}a^5} - \frac{64 Abx}{35 (bx^2 + a)^{\frac{3}{2}} a^4} - \frac{48 Abx}{35 (bx^2 + a)^{\frac{5}{2}} a^3} - \frac{8 Abx}{7 (bx^2 + a)^{\frac{7}{2}} a^2} - \frac{A}{(bx^2 + a)^{\frac{7}{2}} ax}$$

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="max
ima")
```

```
[Out] -1/2*F*x^5/((b*x^2 + a)^(7/2)*b) - 5/8*F*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/
4*D*x^3/((b*x^2 + a)^(7/2)*b) + 16/35*B*x/(sqrt(b*x^2 + a)*a^4) + 8/35*B*x/
((b*x^2 + a)^(3/2)*a^3) + 6/35*B*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*B*x/((b*x^
2 + a)^(7/2)*a) + 1/14*F*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*F*x/(sqrt(b*x^2 +
a)*a*b^3) + 3/56*F*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*F*a^2*x/((b*x^2 + a)
^(7/2)*b^3) + 3/140*D*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*D*x/(sqrt(b*x^2 + a)
*a^2*b^2) + 1/35*D*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*D*a*x/((b*x^2 + a)^(7
/2)*b^2) - 1/7*C*x/((b*x^2 + a)^(7/2)*b) + 8/105*C*x/(sqrt(b*x^2 + a)*a^3*b
) + 4/105*C*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*C*x/((b*x^2 + a)^(5/2)*a*b)
- 128/35*A*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*A*b*x/((b*x^2 + a)^(3/2)*a^4)
- 48/35*A*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*A*b*x/((b*x^2 + a)^(7/2)*a^2) -
 A/((b*x^2 + a)^(7/2)*a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{(15\,Fa^{13}b^3+6\,Da^{12}b^4+8\,Ca^{11}b^5+48\,Ba^{10}b^6-279\,Aa^9b^7)x^2}{a^{14}b^3}+\frac{7\,(3\,Da^{13}b^3+4\,Ca^{1}}{}\right.\right.\right.}{10}$$

$$+\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^4}$$

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="gia
c")
```

```
[Out] 1/105*((x^2*((15*F*a^13*b^3 + 6*D*a^12*b^4 + 8*C*a^11*b^5 + 48*B*a^10*b^6 -
 279*A*a^9*b^7)*x^2/(a^14*b^3) + 7*(3*D*a^13*b^3 + 4*C*a^12*b^4 + 24*B*a^11
*b^5 - 132*A*a^10*b^6)/(a^14*b^3)) + 35*(C*a^13*b^3 + 6*B*a^12*b^4 - 30*A*a
^11*b^5)/(a^14*b^3))*x^2 + 105*(B*a^13*b^3 - 4*A*a^12*b^4)/(a^14*b^3))*x/(b
*x^2 + a)^(7/2) + 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \int \frac{A+B\,x^2+C\,x^4+F\,x^8+x^6\,D}{x^2\,\left(b\,x^2+a\right)^{9/2}}\,dx$$

```
[In] int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(x^2*(a + b*x^2)^(9/2)),x)
```

```
[Out] int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(x^2*(a + b*x^2)^(9/2)), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions 1125

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
   m1 = max(map(expnType, list(expn.args)))
   return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
   return 9

#main function
def grade_antiderivative(result,optimal):

   #print ("Enter grade_antiderivative for sagemath")
   #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

   leaf_count_result  = leaf_count(result)
   leaf_count_optimal = leaf_count(optimal)

   #print("leaf_count_result=",leaf_count_result)
   #print("leaf_count_optimal=",leaf_count_optimal)

   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)

   if str(result).find("Integral") != -1:
      grade = "F"
      grade_annotation =""
   else:
      if expnType_result <= expnType_optimal:
         if result.has(I):
            if optimal.has(I): #both result and optimal complex
               if leaf_count_result <= 2*leaf_count_optimal:
                  grade = "A"
                  grade_annotation =""
               else:
                  grade = "B"
                  grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
            else: #result contains complex but optimal is not
               grade = "C"
               grade_annotation ="Result contains complex when optimal does not."
         else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
               grade = "A"
               grade_annotation =""
            else:
               grade = "B"
               grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
      else:
         grade = "C"
         grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
      if m:
         print ("func ", func , " is elementary_function")
      else:
         print ("func ", func , " is NOT elementary_function")


   return m

def is_special_function(func):
   #debug=False
   if debug:
      print ("type(func)=", type(func))

   m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

   if debug:
      print ("m=",m)
      if m:
         print ("func ", func ," is special_function")
      else:
         print ("func ", func ," is NOT special_function")


   return m


def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
  elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

  leaf_count_result  = tree_size(result) #leaf_count(result)
  leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

  #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


  expnType_result  = expnType(result)
  expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```